SOFT GLUON EVOLUTION AS GUIDING PRINCIPLE FOR COLOUR RECONNECTION*

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We present an idea of how to use a continuous extrapolation of the perturbative results of colour evolution at the amplitude level to the non-perturbative regime. Then we apply it as a guiding principle for a colour reconnection built on top of cluster hadronization model.

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1. Introduction and motivation

Monte Carlo event generators (GPMC) are central to High Energy Physics (HEP) and are an indispensable part of HEP experiments. Almost all measurements and discoveries in the modern era, including the discovery of the Higgs boson, have relied on GPMC. An overview of the physics embodied in the thee main GPMC: Herwig [1, 2], PYTHIA [3, 4] and Sherpa [5] was given in the review [6] and more recently, for example, in [7]. Here, we only very briefly summarize it, to set the scene for our studies. Event generators start from a perturbative description of a hard

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partonic interaction within a hadronic collision; they simulate the emission of additional partons as an evolution downwards in a kinematical variable using a parton shower algorithm. At small scales, perturbation theory is not valid and non-perturbative hadronization models are used to transform the system of partons to hadrons; here, additional, softer, partonic interactions (often called Multiple Partonic Interactions) may also take place between the original hadrons. Finally, many of the hadrons that are produced are unstable and decay models are used. It was also realised that to describe the experimental data, it was necessary to include a colour reconnection (CR) model which aims to improve the colour structure between various additional scatters in the Multiple Partonic Interactions (MPI) models. The starting point of CR is the idea of colour preconfinement [8]. While in a single hard interaction the colour structure is given by (the leading part of) the colour matrices that appear in the Feynman diagrams and also by the parton shower evolution, there is no such firm prescription for the assignment of colour lines or colour connections *between* individual additional scatters. Colour preconfinement leads us to the *assumption* that hard jets emerging from separate hard scatters should end up colour-connected when they are produced nearby in momentum space. As there is no such correlation in the non-perturbative modelling of the multiple hard interactions, we have to impose a model on it. The first time the CR model was introduced in [9] to explain the rising trend of $\langle p_{\rm t}\rangle$ *vs.* $N_{\rm ch}$ observed by UA1 [10]. However, realising the importance of the CR recently, there was significant research activity on the subject [11–21]. In this manuscript, we will briefly describe a new approach to CR [22] built on top of the cluster hadronization model [23]. The cluster model is based on mentioned above colour preconfinement *i.e.* it starts with the dominant colour structure of QCD diagrams provided by the parton showers evolution[1]. This colour structure with partons in the final-state features open colour lines which can be connected in such a way that pairs of partons form colour-singlet clusters with universal[2] mass distribution. The approach presented here is built on the idea of blurring the boundary between perturbative and non-perturbative QCD. In other words, on a continuous extrapolation of the perturbative results to non-perturbative regime. Similar ideas have been already successfully applied to the parton-showers [24] to explain the energy dependence of the 'intrinsic' transverse momentum of partons in the proton. This time, we considered soft-gluon evolution at the amplitude level to expose the structure of colour reconnection of the clusters. The document is organised as follows: in the next section, we describe the formalism of perturbative colour evolution, next, in

[1] In the model, after the parton shower evolution is terminated, all gluons follow non-perturbative isotropic decay to quark–antiquark pairs.

[2] To a good approximation, it does not depend on the hard process.

Section 3, we use the formalism to study analytical evolution of a two-cluster system. Following this, in Section 4, we present numerical results for bigger system up to five clusters and, at the end, we present summary and outlook.

2. Formalism: perturbative colour evolution

QCD amplitudes $|\mathcal{M}\rangle$ can be expressed in terms of contributing colour/spin[3] structures $|\sigma\rangle$

$$|\mathcal{M}\rangle = \sum_{\sigma} \mathcal{M}_\sigma |\sigma\rangle\,, \tag{1}$$

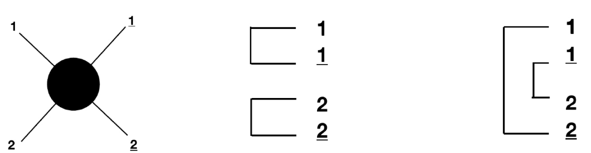
we use the colour flow basis in which $|\sigma\rangle$ can be labelled by permutations

$$|\sigma\rangle = \left| \begin{matrix} 1 & \cdots & n \\ \sigma(1) & \cdots & \sigma(n) \end{matrix} \right\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}}\,, \tag{2}$$

which describe how colour charge is flowing from one leg to another. An example of two possible colour flows in a simple situation of two clusters (four partons) is showed in Fig. 1, and more information and examples can be found in [25–27]. The bare amplitude can be related to the renormalized amplitude as

$$\left|\mathcal{M}\left(\{p\},\mu^2\right)\right\rangle = \boldsymbol{Z}^{-1}\left(\{p\},\mu^2,\epsilon\right)\left|\tilde{\mathcal{M}}\left(\{p\},\epsilon\right)\right\rangle\,, \tag{3}$$

where $\{p\}$ is the set of outgoing momenta, $\epsilon = (d-4)/2$ is the dimensional regularization parameter in d dimensions, and μ^2 is the scale at which the (infrared) renormalization has been performed. The renormalization constant $\boldsymbol{Z}$ is an operator in the space of colour structures and sums the infrared divergences to all orders, resulting in a finite renormalized amplitude. By taking a logarithmic derivative of Eq. (3) with respect to μ^2, we obtain



$$\left|\begin{matrix}\bar{1} & \bar{2}\\ 1 & 2\end{matrix}\right\rangle = |12\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad |21\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

Fig. 1. An example of two possible colour flows in a simple situation of two clusters (four partons).

[3] Since we are interested in the colour structure, we neglect spin space.

an evolution equation

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \left|\mathcal{M}\left(\{p\},\mu^2\right)\right\rangle = \boldsymbol{\Gamma}\left(\{p\},\mu^2\right) \left|\mathcal{M}\left(\{p\},\mu^2\right)\right\rangle \,, \tag{4}$$

where the pre-factor $\boldsymbol{\Gamma}$ is the soft anomalous dimension matrix which, in our case, is enough to consider at one loop [28]

$$\boldsymbol{\Gamma}\left(\{p\},\mu^2\right) = \sum_{i\neq j} \left(-\boldsymbol{T}_i \boldsymbol{T}_j\right) \Gamma_{\text{cusp}}\ \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i \,, \tag{5}$$

where $s_{ij} = 2p_i\, p_j$ for two outgoing or two incoming massless partons i and j and $s_{ij} = -2p_i\, p_j$ for incoming/outgoing or outgoing/incoming pairs, and $\Gamma_{\text{cusp}} = \alpha_{\text{s}}/4\pi$ in the lowest order. The solutions to the evolution equation take the form of

$$\left|\mathcal{M}\left(\{p\},\mu^2\right)\right\rangle = \boldsymbol{U}\left(\{p\},\mu^2,\left\{M_{ij}^2\right\}\right) \left|\mathcal{H}\left(\{p\},Q^2,\left\{M_{ij}^2\right\}\right)\right\rangle \,, \tag{6}$$

where $\mathcal{H}(\{p\},Q^2,\{M_{ij}^2\})$ represents the hard scattering amplitude before the evolution and $\boldsymbol{U}$ is the evolution operator. In our approach, the starting point for colour reconnection is an evolution of a cluster configuration given by pre-confiment and represented through a colour structure $|\tau\rangle$. Therefore, the initial step of colour reconnection is an evolution in colour space to scales of the order of μ^2 below the initial cluster masses (M_{ij}) and the parton shower infrared cutoff using the following evolution operator:

$$\boldsymbol{U}\left(\{p\},\mu^2,\left\{M_{ij}^2\right\}\right) = \exp\left(\sum_{i\neq j} (\boldsymbol{T}_i \boldsymbol{T}_j) \frac{\alpha_{\text{s}}}{2\pi} \left(\frac{1}{2}\ln^2\frac{M_{ij}^2}{\mu^2} - i\pi \ln\frac{M_{ij}^2}{\mu^2}\right)\right) . \tag{7}$$

In terms of the colour flow basis introduced earlier, the action of the above evolution operator can be summarised in iterating so-called colour reconnectors [25] which, once per action, will swap two indices of the permutation labelling the specific colour flow, and introduce longer sequences of transpositions when exponentiated. *Reconnection amplitude* is calculated as the overlap between the evolved amplitude and a new colour structure $|\sigma\rangle$

$$\mathcal{A}_{\tau\to\sigma} = \langle\sigma|\boldsymbol{U}\left(\{p\},\mu^2,\left\{M_{ij}^2\right\}\right)|\tau\rangle \,, \tag{8}$$

then the *reconnection probability* is given by

$$P_{\tau\to\sigma} = \frac{|\mathcal{A}_{\tau\to\sigma}|^2}{\sum_\rho |\mathcal{A}_{\tau\to\rho}|^2} \,, \tag{9}$$

where ρ runs over all possible colour flows.

3. Analytical results

In this section, we will study analytically the simplest possible situation of the evolution of a two-cluster system. As we saw in Fig. 1, in that case, there are just two possible colour flows and the evolution of the single state can be expressed in the following way:

$$|\sigma\rangle = \boldsymbol{U} \left| \begin{array}{cc} i & j \\ \bar{i} & \bar{j} \end{array} \right\rangle = \mathrm{e}^{\boldsymbol{\Omega}} \left| \begin{array}{cc} i & j \\ \bar{i} & \bar{j} \end{array} \right\rangle \equiv \sigma_{ij}\,|ij\rangle + \sigma_{ji}\,|ji\rangle \,. \tag{10}$$

In that case, we can provide an explicit expression for the exponent of the evolution operator

$$\boldsymbol{\Omega} = \begin{pmatrix} \frac{-3}{2}(\Omega_{23}+\Omega_{14}) & \frac{1}{2}(\Omega_{12}-\Omega_{23}-\Omega_{14}+\Omega_{34}) \\ \frac{1}{2}(\Omega_{12}-\Omega_{13}-\Omega_{24}+\Omega_{34}) & \frac{-3}{2}(\Omega_{13}+\Omega_{24}) \end{pmatrix}, \tag{11}$$

where $\Omega_{ij} = \frac{\alpha_{\mathrm{s}}}{2\pi}\left(\frac{1}{2}\ln^2\frac{M_{ij}^2}{\mu^2} - i\pi\ln\frac{M_{ij}^2}{\mu^2}\right)$. However, since our numerical investigation showed that the Coulomb term in Ω_{ij} has negligible effect on the massless cluster evolution, we can use the compact form for Ω_{ij} expressed in terms of the cluster masses or the partons four-momenta

$$\Omega_{ij} = \frac{\alpha_{\mathrm{s}}}{2\pi}\left[\ln^2\frac{M_{ij}^2}{\mu^2}\right] \sim \ln^2 M_{ij}^2 \sim \ln^2 2p_i\,p_j \,. \tag{12}$$

Taking this into account, the evolution matrix can be expressed in the following form:

$$\begin{aligned} \boldsymbol{U} &= \mathrm{e}^{\boldsymbol{\Omega}} = \frac{\mathrm{e}^{-\frac{3}{2}(a+b)}}{\sqrt{\Delta}}\sinh\left(\frac{\sqrt{\Delta}}{2}\right) \\ &\times \begin{pmatrix} \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{2}\right)+3(b-a) & 2(c-a) \\ 2(c-b) & \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{2}\right)+3(a-b) \end{pmatrix}, \end{aligned} \tag{13}$$

where we introduced new variables: $a = \ln^2\Omega_{23}^2 + \ln^2\Omega_{14}^2$, $b = \ln^2\Omega_{13}^2 + \ln^2\Omega_{24}^2$, $c = \ln^2\Omega_{12}^2 + \ln^2\Omega_{34}^2$ and $\Delta = 9a^2 - 4c(a+b) - 14ab + 9b^2 + 4c^2$. The reconnection probability when the initial colour flow is $|ij\rangle$ is given by

$$p_{\mathrm{rec}} = \frac{|\langle ji|\sigma\rangle|^2}{|\langle ji|\sigma\rangle|^2 + |\langle ij|\sigma\rangle|^2}\,. \tag{14}$$

We studied various kinematic limits of the above equation. We showed, for example, that when the initial partons in clusters have similar transverse momenta and are close in the rapidity, the colour reconnection is minimal. This result is in line with what one expects from the preconfinement. As we will see in the next section, where we present numerical results, a similar effect can be observed in the case of a bigger number of clusters.

4. Numerical results

In this section, we present the numerical results of the CR model for the five clusters case. We consider two different initial cluster configurations: one generated by the RAMBO method [29] (flat phase-space population), and the second by the Jadach algorithm [30] (used for the UA5 model [31]). In the case of UA5 kinematics, in order to enhance CR effects, we enforce random colour connections between the quarks and anti-quarks. In Fig. 2, we show the invariant mass distribution for five clusters before and after colour reconnection where the phase space was sampled with the two-phase space algorithms mentioned above. We see that from CR based on the soft

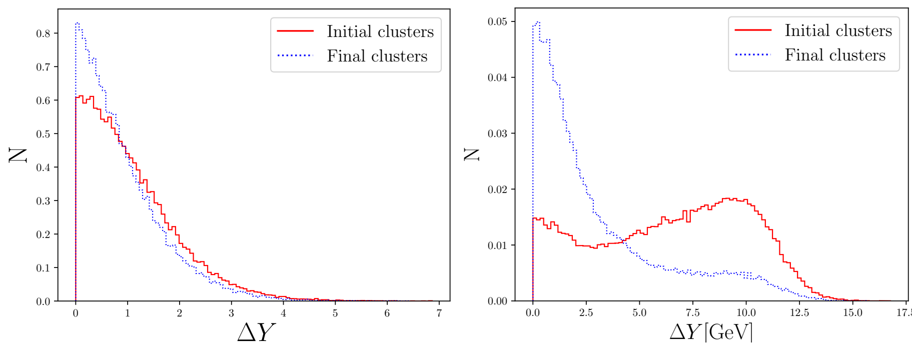


Fig. 2. Histogram of the ΔY values for the quark–anti-quark pairs of the original clusters that were reconnected and the ΔY value of the quark–anti-quark pairs of the reconnected clusters. Left: RAMBO phase space; right: UA5 phase space with random colour connections.

gluon evolution, we naturally obtained decreasing masses of the clusters after CR, feature which in other models like [11] has to be assumed as a guiding principle. In Fig. 3, we present rapidity difference ΔY between the quark–anti-quark pairs which were participating in the reconnection process. In both figures, we see that colour flows resulting in clusters consisting of quark–anti-quark pairs which are closer in rapidity are preferred which is expected from the preconfinement. The problem is that the full evolution in colour space is not feasible in a realistic situation at the LHC which needs to cope not just with five but with several tens to hundreds of clusters. However, we have found evidence that in the evolution of larger systems, the bulk of the reconnection effects is isolated in small subsystems of two to three clusters. In Fig. 4, we show the number of transpositions for four and five cluster evolution. For both cases, we note the peak at one transposition, *i.e.* a reconnection within a two cluster system between the initial and final state. This indicates the existence of independently evolving subsystems, where the contributions from the remainder of the event are suppressed. This observation might allow us to build a phenomenological model suitable for the LHC.

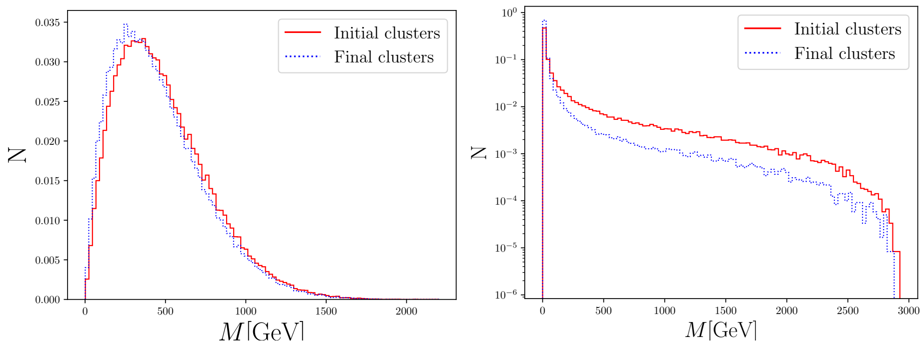


Fig. 3. Invariant mass distribution of 5 clusters before and after mesonic colour reconnection for RAMBO (left) and UA5 (right) kinematics.

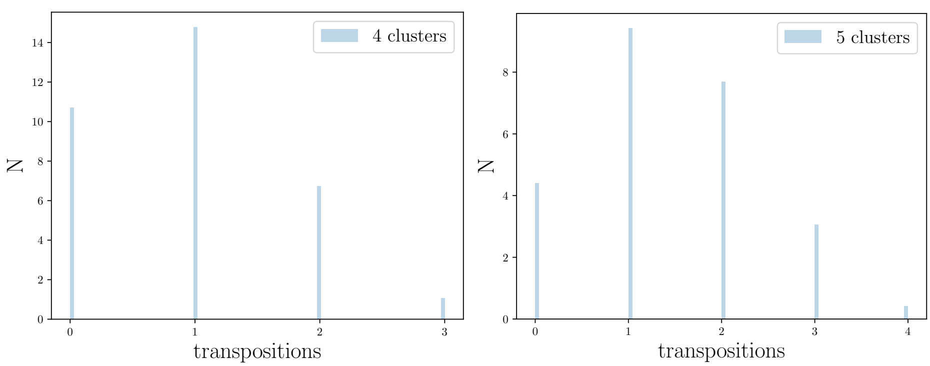


Fig. 4. Number of transpositions between the initial state and the reconnected final state for four and five cluster evolution.

5. Summary and outlook

We used an idea of continuous extrapolation of the perturbative results of colour evolution to non-perturbative regime of colour reconnection. We obtained input on how to improve or constrain existing colour reconnection models and insight on how to create new models built on top of the cluster hadronization approach. In particular, we have analytically solved the evolution of a two-cluster system and studied numerically the evolution of larger systems of up to five clusters. We have found that there is indeed a highly dynamic and non-trivial re-arrangement of colour structures already from a simple Ansatz using a one-loop soft anomalous dimension, which confirms earlier work on geometrically-inspired reconnection models [20]. The main complication is that the full evolution in colour space is not feasible for the case of typical LHC events, where we are dealing with much more than five clusters. Fortunately, we have found a piece of evidence that in the evolution of larger systems, the bulk of the reconnection effects is isolated in small subsystems of two to three clusters. Therefore, in the next step based on this observation, we plan to construct a full model which will be suitable for the LHC usage.

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