SOFT-DROPPED OBSERVABLES WITH CoLoRFuLNNLO*

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In this contribution, we present a detailed study of the effects that decrease the numerical precision in computing QCD radiative corrections to shape distributions at the next-to-next-to-leading order (NNLO) accuracy. For a specific example, we study the contributions to the distribution of soft-dropped heavy-jet mass. We focus on the edge of the phase space where the shape value becomes small and the cross section is dominated by large logarithmic contributions. We use the CoLoRFuLNNLO subtraction method that defines local subtractions in all single and double unresolved regions of the phase space.

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1. Introduction

Although high-energy lepton colliders do not operate at present, the interest for observables defined for these machines seems to endure. On the one hand, the reason is the large amount of high quality data collected at previous experiments on the Large Electron Positron collider, while on the other, lepton collisions offer a possible path for the construction of machines of the future, like the FCC-ee [1].

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The early discovery of the Higgs boson [2, 3] in the LHC program raised expectations of discovering several other new elementary particles as well, but as of now this hope did not fulfill. This puts a lot of pressure on both experimenters and theorists, as signatures of new physics have to be also searched differently. In absence of a clear peak signaling for a new particle created in particle collisions, distributions taken at experiments and predicted in model calculations have to be meticulously compared, too. In such searches, emphasis is put on precision, as differences between observation and model prediction can only be noticed if both have sufficiently small uncertainties. On the experimental side, uncertainties are decreased through acquiring more data and refining the detector apparatus. On the theory side, a straightforward, though tedious way to improve the precision of the predictions is to increase the perturbative order of the computation.

The never-ending quest for precision is important not only in new physics searches but in the measurement of fundamental parameters of the Standard Model, such as the strong coupling. If the aim is a measurement for strong coupling, the predictions used for that purpose have to fulfill some criteria [4], namely the fixed-order prediction should contain at least the NNLO QCD corrections, and for further refinement, it should be matched to analytic results of all-order resummations of large logarithmic contributions.

A significant source of uncertainty in the theoretical predictions is due to the modeling of the hadronization corrections. Besides a dispersive model [5] developed for lepton collisions, we have only phenomenological models for describing these effects. Without hadronization models derived from first principles, the only way to minimize uncertainties associated with it is to decrease the size of these contributions [6]. This can be achieved by defining new observables or modify existing ones such that they would become less sensitive to effects happening at scales of $\mathcal{O}(1\,\mathrm{GeV})$ where hadronization occurs. One possible way to modify existing observables is to apply the so-called soft-dropping technique [7] to some well-established observables. This technique alters the particle content of the event such that it removes particles particularly sensitive to hadronization. For some of these observables, it was possible to obtain all-order resummed predictions [8–13].

When all-order results are calculated, one important step in the computation is the validation of results. One possible way is offered by checking the resulting logarithmic structure with available fixed-order calculations in regions of phase space where these logarithms are enhanced. These checks stretch the fixed-order calculations to their limits because the computation has to be performed near the edges of the available phase space where logarithmic contributions dominate. In this report, we examine the soft-dropped version of heavy-jet mass ρ [8]. We examine the numerical precision that can be achieved for this observable in a computation at NNLO accuracy for very small values of ρ using the CoLoRFuLNNLO subtraction scheme [14, 15].

2. The soft-dropped heavy-jet mass

In our definition of heavy-jet mass, we first cluster all the partons in the final state according to the $e^+ e^-$ variant[1] of the $k_{\rm T}$ algorithm [16]. After the initial clustering, we end up with two, back-to-back jets. The directions of these two jets divide the event into two hemispheres, denoted arbitrarily by R and L. In each of these hemispheres, a Cambridge/Aachen jet algorithm [17] is used in such a way that records of all pseudojet mergings are kept. Next, we consider each hemisphere separately, and start with the last pseudojet. Undoing the last merging leading to this pseudojet, we test if the following inequality is satisfied:

$$\frac{\min\left[E_i, E_j\right]}{E_i + E_j} > z_{\rm cut}\,, \tag{1}$$

where E_i and E_j are the energies of the two pseudojets before the last merging, $z_{\rm cut} \in [0,1)$, and it governs the degree of soft drop applied on the pair. Note that this is a special version of soft drop defined in Ref. [8] with $\beta = 0$. If inequality (1) is fulfilled, we keep both pseudojets and continue with unmerging them and applying the same test for both unmergings separately. If the test fails, we discard the tracks building up the softer pseudojet and continue with unmerging only the harder pseudojet.

When the soft-drop procedure is iteratively applied to all pseudojets of both hemispheres, we obtain the hemisphere masses using only those tracks that survive after the previous steps of tests. The soft-dropped heavy-jet mass is obtained from

$$\rho = \frac{\max\left[m_{\rm R}^2, m_{\rm L}^2\right]}{E_J^2}\,, \tag{2}$$

where $m_{\rm R}$ and $m_{\rm L}$ correspond to the two hemisphere masses computed from the remaining tracks and E_J is the energy of the hemisphere having larger hemisphere mass.

In this contribution, we focus on computing the fixed-order predictions in NNLO QCD for $\rho \ll 1$.

3. Numerical stability issues related to subtraction schemes

Beyond leading order in QCD perturbation theory, kinematic singularities arise due to unresolved emissions of partons. After integrating over the phase space, these singularities cancel with virtual corrections order-by-order for all infrared-safe observables. In practical applications, due to complexity, it is not possible to perform the related phase-space integrals analytically. To overcome this problem when only one unresolved parton may be radiated, subtraction schemes [18–21] were invented where all kinematic singularities

[1] To obtain the $e^+ e^-$ variant, we turned off the possibility to recombine into a beam jet.

associated to unresolved emissions are subtracted from the real radiation cross section, and in order to leave the cross section unchanged, added back after summing and integrating over the unresolved degrees of freedom. In this way, all real emissions are regularized and the integrated subtraction terms ensure that the virtual corrections become finite in $d = 4$ dimensions.

To facilitate the required numerical integration, the computation has to be carried out with a numerical computer program. Due to internal organization of computers, real numbers can only be stored with a finite number of digits. This results in a loss of precision when sufficiently large numbers (regardless of sign) are stored. In practical terms, this means roughly fourteen significant digits in end results. This numerical precision is more than sufficient for the end results but when higher-order computations are carried out, this limitation in precision can be troublesome at intermediate steps. The closer the phase-space points are generated to the kinematic singularities of unresolved emissions, the larger the cross section becomes. Hence, the analytically correct subtraction terms may lead to large numerical mismatch due to the finite number of stored digits. Thus, we end up with ubiquitous incorrect large numbers spoiling the convergence of our Monte Carlo integration.

This behavior was observed early on when the first NLO QCD computations were performed and as a solution, a small (technical) cut was imposed on the physical phase space such that

$$y_{\min} \leq \min_{i,j} \frac{(p_i + p_j)^2}{Q^2}\,, \tag{3}$$

where Q is the CM energy of the collision and $y_{\min}$ is chosen not to have any observable effect on the computed cross section. This is achieved by computing the cross section for various values of $y_{\min}$ and the value is selected for which the cross section starts to saturate.

Strictly speaking, this operation should also be performed for all the distributions computed, and select a value for $y_{\min}$ for which *all* the distributions saturate. In NLO calculations, it was found that if a suitable value is found which corresponds to a saturated cross section, this value also works for distributions.

When distributions are required for small values of an observable, extra care has to be taken because it is very easy to significantly cut into important regions of phase space altering the shape of the distribution without having a major effect on the cross section. Hence when beyond leading order calculations are carried out for small values of an observable, *i.e.* close to the edge of the phase space, it is inevitable to investigate the behavior of shape of the distribution as a function of the $y_{\min}$ parameter. This is even more important in NNLO computations where the singularity structure is much more complex than at NLO.

4. NNLO QCD contribution to the soft-dropped heavy-jet mass for small values

The cross section for the soft-dropped heavy-jet mass, treating the strong coupling as the perturbative parameter, can be written as

$$\sigma[\rho] = \sigma^{\mathrm{LO}}[\rho] + \sigma^{\mathrm{NLO}}[\rho] + \sigma^{\mathrm{NNLO}}[\rho] + \dots\,, \tag{4}$$

where the beyond LO contributions can be decomposed as

$$\begin{aligned} \sigma^{\mathrm{NLO}}[\rho] &= \sigma^{\mathrm{R}}_{\mathrm{NLO}}[\rho] + \sigma^{\mathrm{V}}_{\mathrm{NLO}}[\rho]\,, \\ \sigma^{\mathrm{NNLO}}[\rho] &= \sigma^{\mathrm{RR}}_{\mathrm{NNLO}}[\rho] + \sigma^{\mathrm{RV}}_{\mathrm{NNLO}}[\rho] + \sigma^{\mathrm{VV}}_{\mathrm{NNLO}}[\rho]\,. \end{aligned} \tag{5}$$

In (5), the R and V superscripts indicate the presence and number of real and virtual parton emissions. All contributions to the distribution of the heavy-jet mass have to be analyzed separately to prove the independence of the predictions of the technical cut parameter.

We show the double-virtual, real-virtual and double-real contributions to the distribution of ρ in Figs. 1–3 obtained at least at four different values of y_{min}. For illustrative purposes, we chose the range of $\rho \in [\mathrm{e}^{-14} \simeq 10^{-6}, 1]$, and used $z_{\mathrm{cut}} = 10^{-4}$ to parametrize the soft-drop procedure. We see that over this full range, the distribution of $\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}[\rho]$ is independent of the technical cut for $y_{\mathrm{min}} \leq 10^{-7}$. The distribution of $\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}[\rho]$ is independent of the technical cut if $y_{\mathrm{min}} \leq 10^{-8}$ except maybe for the first bin. From the point of view of numerical stability, the $\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}[\rho]$ term is the most challenging. The distributions with $y_{\mathrm{min}} \leq 10^{-8}$ appear independent of the technical cut

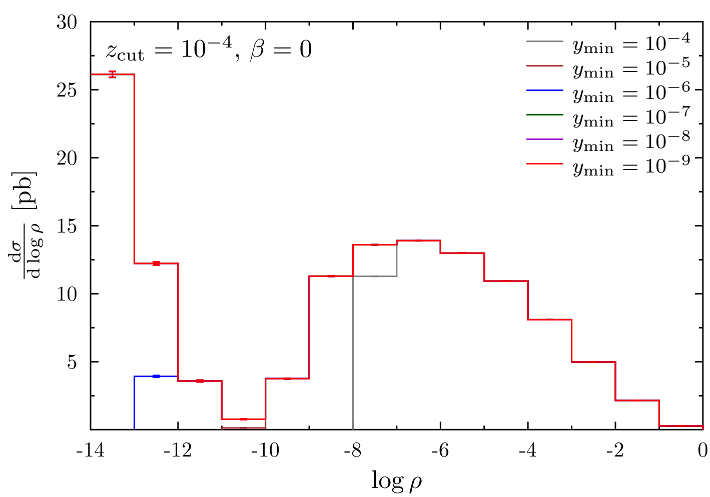

Fig. 1. The double-virtual (VV) contribution to the soft-dropped heavy-jet mass computed for various values of the technical cut parameter (y_{min}).

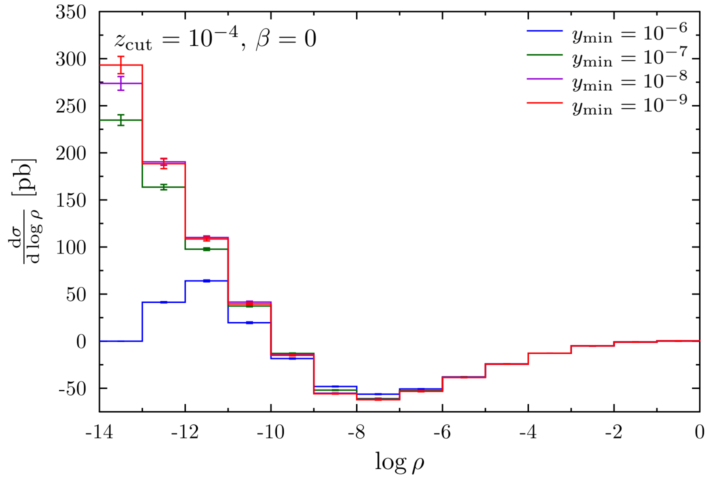

Fig. 2. The same as Fig. 1 but for the real-virtual (RV) contribution.

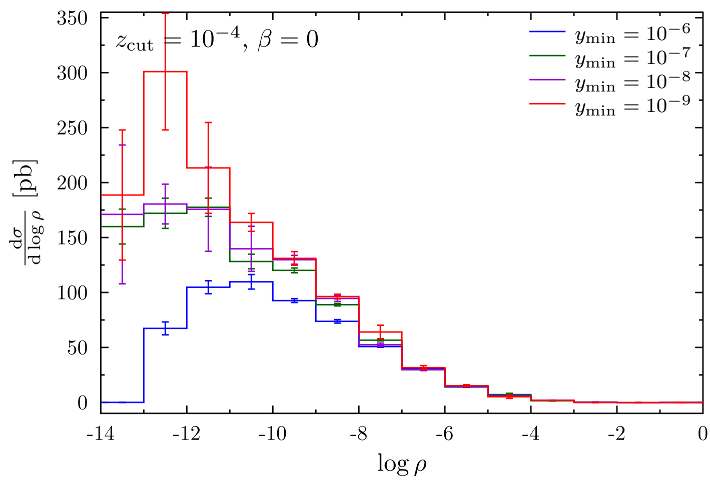

Fig. 3. The same as Fig. 1 but for the double-real (RR) contribution.

for $\log\rho \geq -10$. For smaller values of the heavy-jet mass, we also find independence of $y_{\rm min}$ but only within the large uncertainties of the numerical integration. We conclude that $y_{\rm min} \leq 10^{-8}$ is suitable for the real-virtual and $y_{\rm min} = 10^{-7}$ is so for the double-real contributions. Decreasing the value of $y_{\rm min}$ in the latter case does not give significantly different distributions due to the large uncertainties. In this case, the predictions with $y_{\rm min} = 10^{-7}$ and 10^{-9} are compatible apart from the second bin where the difference is roughly one sigma. The double-virtual contribution being finite does not require a very small technical cut.

In order to see the relative size of each of the contributions, we show those together in Fig. 4 using the optimal values for the technical cut parameter. Finally, the total NNLO QCD contribution to the soft-dropped heavy-jet mass distribution with $z_{\rm cut} = 10^{-4}$ is computed for the first time and shown in Fig. 5.

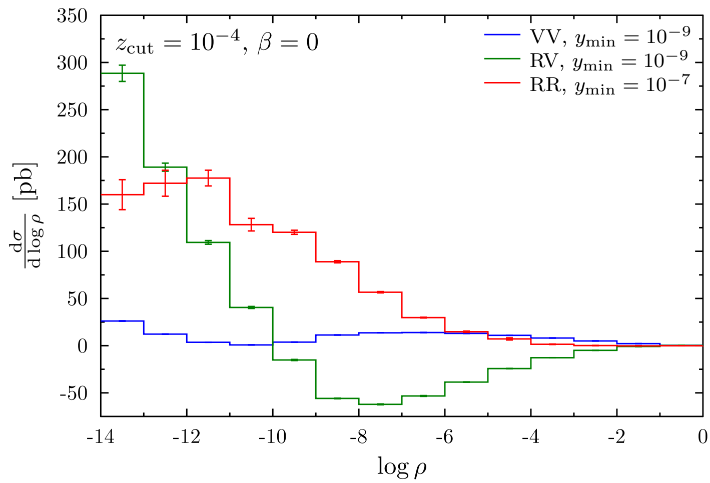

Fig. 4. The three contributions at NNLO QCD to the distribution of soft-dropped heavy-jet mass with the technical cut selected to be the optimal choice.

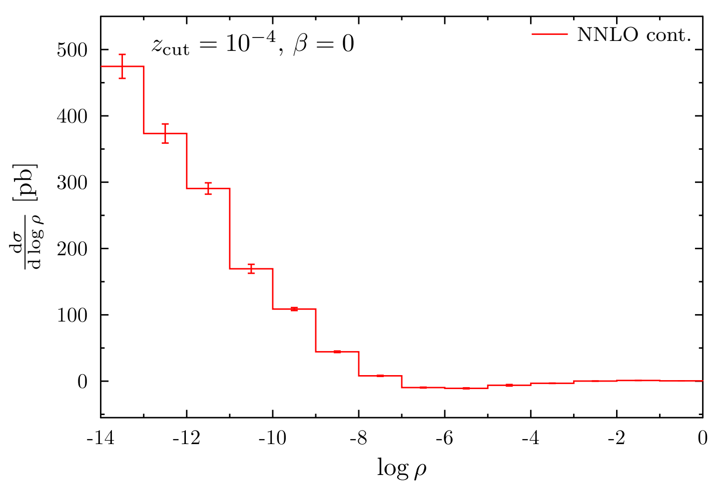

Fig. 5. The total NNLO QCD contribution to the soft-dropped heavy-jet mass.

5. Conclusions

We computed for the first time at fixed-order in perturbation theory the soft-dropped version of the heavy-jet mass observable defined in lepton collisions for small values at NNLO accuracy in QCD. As we were interested in the small values of the observable, we have to find the optimal value of the technical cut parameter for which the predictions are independent of it. We emphasize that a study of this should be performed whenever a distribution is to be obtained for an observable close to kinematic limit in order to truly obtain a physical value for the contribution. We used the CoLoRFuLNNLO subtraction scheme, hence it was sufficient to analyze the distribution for various values of the technical cut parameter. If a computation of this kind is performed with a slicing method, the saturation has to be studied not just as a function of the slicing parameter but also as a function of the technical cut.

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