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ON THE IMPORTANCE OF LEFT-HAND CUTS IN THE $\gamma\gamma^* \to \pi\pi$ PROCESS*

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We investigate the single virtual photon–photon scattering into two pions up to 1.5 GeV for the low spacelike virtualities in the dispersive formalism. In order to account for the rescattering effects in both S- and D-waves, we adopt the Omnès representation. The unsubtracted dispersion relations describe well the cross-section data and predicts charged pion dipole polarizability $(\alpha_1 - \beta_1)_{\pi^\pm} = 6.1 \times 10^{-4}$ fm^3 consistent with the recent COMPASS measurement and χPT. However, for the neutral pion, the dipole polarizability turns out to be far away from χPT value. In these proceedings, we show how a once-subtracted dispersion relation can potentially cure this problem. Besides, the preliminary error analysis is given.

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1. Introduction

The anomalous magnetic moment of the muon $a_\mu = (g-2)_\mu$ is one of the physical quantities known with a ppm precision. However, it shows a discrepancy of 3–4σ between theory [1] and experiment [2]. From the perspective of the ongoing programs at FERMILAB [3] and J-PARC [4], which aim to reduce the current experimental uncertainty by the factor of 4, the precise determination of the theoretical error becomes crucial. It results dominantly from the hadronic contributions, including hadronic vacuum polarization (HVP) and the hadronic light-by-light scattering (HLbL). The latter requires an approach based on the fundamental properties of the S-matrix, such as unitarity and analyticity to achieve a controllable reduction of the uncertainty. Apart from the dominant pseudo-scalar pole contributions, the next important contribution to HLbL comes from the two-particle intermediate states of $\pi\pi, \pi\eta$, and $K\bar{K}$. In the dispersive formalism,

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it is possible to account for the rescattering of the hadronic final states which results in the contribution from $f_0(500), f_0(980), a_0(980)$ and $f_2(1270)$ resonances.

The first measurement of the single tagged two-photon collisions with a neutral pion pair final state from the Belle Collaboration [5] covers the range of Q^2 from 3.5 to 30 GeV^2. The BESIII Collaboration is currently analyzing both $\pi^+\pi^-$ and $\pi^0\pi^0$ production in the $0.2 \lesssim Q^2 \lesssim 2.2$ GeV^2 region [6], which is relevant for the determination of the HLbL contribution to a_μ. In order to compare the previous results of [7] to the upcoming data, it is necessary to estimate the errors of the approach, which we address in this work.

2. Description of the method

Our approach is based on the partial wave (p.w.) dispersion relations, which implement the unitarity and analyticity constraints. The kinematically unconstrained p.w. amplitudes rely on a decomposition of the hadronic tensor $H^{\mu\nu}$ of $\gamma\gamma^* \to \pi\pi$ into a suitable set of invariant functions [8] $H^{\mu\nu} = \sum_{n=1}^{3} F_n(s,t)\, L_n^{\mu\nu}$, where

$$\begin{aligned}
L_1^{\mu\nu} &= q_1^\nu q_2^\mu - (q_1\cdot q_2) g^{\mu\nu}\,,\\
L_2^{\mu\nu} &= \left(\Delta^2 (q_1\cdot q_2) - 2(q_1\cdot\Delta)(q_2\cdot\Delta)\right) g^{\mu\nu} - \Delta^2 q_1^\nu q_2^\mu\\
&\quad -2(q_1\cdot q_2)\Delta^\mu\Delta^\nu + 2(q_2\cdot\Delta) q_1^\nu \Delta^\mu + 2(q_1\cdot\Delta) q_2^\mu \Delta^\nu\,,\\
L_3^{\mu\nu} &= -(q_1\cdot\Delta)\left(g^{\mu\nu}Q^2 + q_2^\mu q_2^\nu\right) + \Delta^\mu\left(q_2^\nu (q_1\cdot q_2) + q_1^\nu Q_2\right)
\end{aligned} \tag{1}$$

with $\Delta \equiv p_1 - p_2$, where q_1, q_2 stand for the virtual photon momenta and p_1, p_2 are the pion momenta. The first two terms coincide with the basis used in [9] for the real photon case. Since the invariant amplitudes are free from kinematic singularities and constraints, one can identify all kinematic constraints by analyzing projected amplitudes

$$h^{(J)}_{\lambda_1\lambda_2}\left(s,Q^2\right) = \int_{-1}^{1} \frac{\mathrm{d}\cos\theta}{2}\, \mathrm{d}^J_{\lambda_1-\lambda_2,0}(\theta)\, \epsilon_\mu(q_1,\lambda_1)\, \epsilon_\nu(q_2,\lambda_2)\, H^{\mu\nu}\, \mathrm{e}^{-\mathrm{i}\phi(\lambda_1-\lambda_2)}\,, \tag{2}$$

in terms of

$$A_n^J = \frac{1}{(p\,q)^J} \int_{-1}^{1} \frac{\mathrm{d}z}{2} P_J(z)\, F_n(s,t)\,, \tag{3}$$

which are good quantities due to the properties of the Legendre polynomials [10]. In (3), q, p denote the initial and final relative momenta in the c.m.

frame. The transformation from $h^{(J)}_{++}, h^{(J)}_{+-}, h^{(J)}_{+0}$ to the kinematically unconstrained amplitudes $h^{(J)}_{1,2,3}$ is given in [7]. We note that this transformation is valid only for the Born subtracted p.w. amplitudes, $\bar{h}^{(J)}_{\lambda_1\lambda_2} \equiv h^{(J)}_{\lambda_1\lambda_2} - h^{(J),\mathrm{Born}}_{\lambda_1\lambda_2}$, since Born invariant amplitudes posses an additional pole at the soft-photon point [11]. For the S-wave the kinematic constraint relates a Born subtracted amplitude to the generalized dipole polarizability of the pion $(\alpha_1 - \beta_1)_\pi$

$$\bar{h}^{(0)}_{I,++}\left(s, Q^2\right) = 2\,\pi\, m_\pi (\alpha_1 - \beta_1)^I_\pi \left(s + Q^2\right) + \ldots\,, \tag{4}$$

where we have explicitly written an isospin index I. In order to account for the hadronic final-state interactions, we implement the modified Muskhelishvili–Omnès method, which is based on writing a dispersion relation for $\bar{h}^{(J)}_i (\Omega^{(J)})^{-1}$ [12] , where $\Omega^{(J)}$ is the Omnès function. The resulting unsubtracted dispersion relation has the following form:

$$\begin{aligned} h^{(J)}_{I,i}(s) \;=\; & h^{(J),\mathrm{Born}}_{I,i}(s) + \Omega^{(J)}_I(s) \left[\int\limits_{-\infty}^{0} \frac{\mathrm{d}s'}{\pi} \frac{\left(\Omega^{(J)}_I(s')\right)^{-1} \operatorname{Disc} \bar{h}^{(J)}_{I,i}(s')}{s' - s} \right. \\ & \left. - \int\limits_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{\pi} \frac{\operatorname{Disc}\left(\Omega^{(J)}_I(s')\right)^{-1} h^{(J),\mathrm{Born}}_{I,i}(s')}{s' - s}\right]. \end{aligned} \tag{5}$$

In the $I = 0$, the S-wave $f_0(980)$ resonance is known to strongly couple to the $\{\pi\pi, K\bar{K}\}$ channels. The generalization of (5) to the coupled-channel case can be found in [7]. For the S-wave $I = 2$ as well as $J = 2$ amplitudes, we use the single-channel dispersion relations. Re-expressing Eq. (5) in terms of the helicity amplitudes leads to the following sum rule for the generalized dipole polarizability:

$$\begin{aligned} (\alpha_1 - \beta_1)^I_\pi \;=\; & \frac{\Omega^{(0)}_I(-Q^2)}{2\,\pi\, m_\pi} \left[\int\limits_{-\infty}^{0} \frac{\mathrm{d}s'}{\pi} \frac{\left(\Omega^{(0)}_I(s')\right)^{-1} \operatorname{Disc} \bar{h}^{(0)}_{I,++}(s')}{(s' + Q^2)^2} \right. \\ & \left. - \int\limits_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{\pi} \frac{\operatorname{Disc}\left(\Omega^{(0)}_I(s')\right)^{-1} h^{(0),\mathrm{Born}}_{I,++}(s')}{(s' + Q^2)^2}\right]. \end{aligned} \tag{6}$$

Before we discuss the results of the unsubtracted dispersion relation, let us first specify the left-hand cuts (l.h.c.) and the corresponding form factors that account for the finite virtuality of the photon. The Born l.h.c. are welldefined by the scalar QED which should be multiplied by the electromagnetic pion (kaon) form factors [13]

$$\begin{aligned}\left\langle \pi^+\left(p'\right)\right| j_\mu(0)\left|\pi^+(p)\right\rangle &= e\left(p+p'\right)_\mu f_\pi\left((p'-p)^2\right)\,,\\ \left\langle K^+\left(p'\right)\right| j_\mu(0)\left|K^+(p)\right\rangle &= e\left(p+p'\right)_\mu f_K\left((p'-p)^2\right)\,.\end{aligned}\tag{7}$$

To evaluate them in the region of $Q^2 \lesssim 1\ \mathrm{GeV}^2$, we improve the vector-meson dominance (VMD) prediction by performing the simple monopole fits with the following parameters: $\Lambda_\pi = 0.727(5)$ GeV with $\chi^2/\mathrm{d.o.f.} = 1.22$ and $\Lambda_K = 0.872(47)$ GeV with $\chi^2/\mathrm{d.o.f.} = 0.69$ (see Fig. 1). The l.h.c. contribution beyond the pion pole is approximated by the vector mesons ω and ρ, for which the vertex functions are expressed as

$$\langle V(k,\lambda)|j_\mu(0)|\pi(p)\rangle = 2\,e\,C_{\mathrm{VP}\gamma}\,f_{V,\pi}\left(Q^2\right)\,\epsilon_{\mu\alpha\beta\gamma}\,k^\alpha\,p^\beta\,\epsilon^{\gamma *}(k,\lambda)\,,\tag{8}$$

which can be justified by the observation that the coupling constant of photon, vector (V) and pseudoscalar (P) mesons effective interaction $g_{\mathrm{VP}\gamma} \simeq C_{\rho\pi\gamma} \simeq C_{\omega\pi^0\gamma}/3 = 0.33\ \mathrm{GeV}^{-1}$ describes well the cross sections [7], and this value lies within $\sim 10\%$ of the PDG average $g_{\mathrm{VP}\gamma}^{\mathrm{PDG}} = 0.37(2)\ \mathrm{GeV}^{-1}$ [2]. The small difference can be attributed to the contribution from other heavier resonances. For the vector-meson transition form factor, there is no data available in the spacelike region. For the transition form factor (TFF) $f_{\omega,\pi}(Q^2)$, we use the dispersive analysis from [14] (see also [15]), while for the TFF $f_{\rho,\pi}(Q^2)$, the VMD model is used.

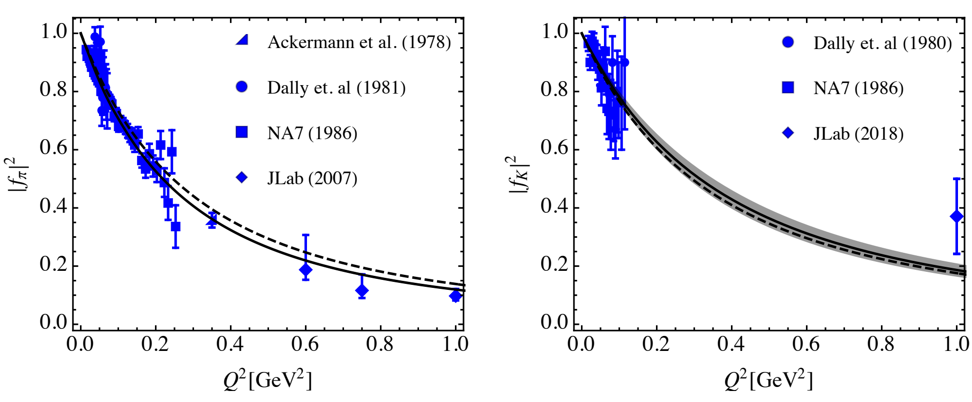

Fig. 1. Pion and kaon electromagnetic form factors in the space-like region. Monopole fits including the uncertainties (solid curves) are compared to the VMD predictions (dashed curves).

The Omnès functions for the $I = 0, 2$ D-wave $\Omega_I^{(2)}(s)$ are given by

$$\Omega_I^{(2)}(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_I^{(2)}(s')}{s'-s}\right), \tag{9}$$

where the corresponding phase shifts are taken from the Roy analysis [16]. For the S-wave $I = 0$ ($I = 2$) amplitude, we employ the coupled-channel (single channel) Omnès function from a dispersive resummation scheme [17, 18], which implements constraints from analyticity and unitarity. The method is based on the N/D Ansatz [19], where the set of coupled-channel (single-channel) integral equations for the N-function are solved numerically with the input from the left-hand cuts which we present in a model-independent form as an expansion in a suitably constructed conformal mapping variable. These coefficients can be matched to χPT at low energy [20]. Here, we use a data driven approach and determine them from fitting to Roy analyses for $\pi\pi \to \pi\pi$ [16], $\pi\pi \to K\bar{K}$ [21] and existing experimental data. In Fig. 2, we show the results of the N/D analysis for the $I = 0$ $\pi\pi$ phase shifts using single- or coupled-channel analyses as well as a comparison between the corresponding Omnès functions [22].

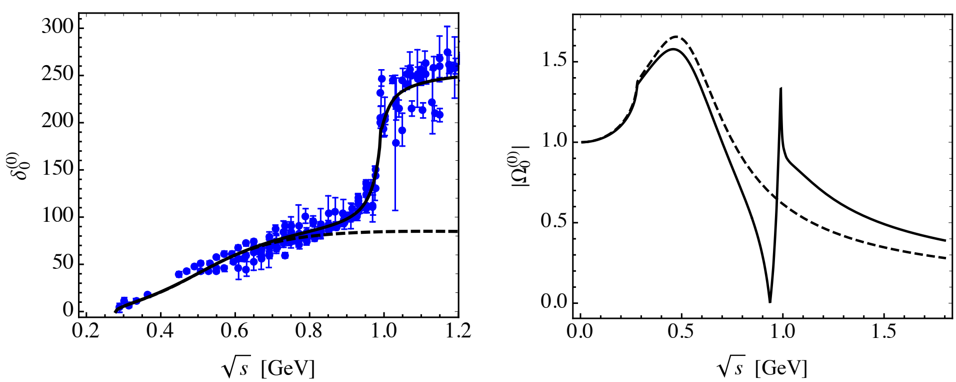

Fig. 2. Left panel: The result of the N/D analysis of the $I = 0$ $\pi\pi$ phase shifts for the S-wave in the single- (dashed) and coupled-channel (solid) cases. Right panel: Modulus of the corresponding Omnès functions.

3. Results and discussion

We start the discussion of the results with the S-wave contribution. We have found that account for the rescattering of the Born terms in the coupled-channel formalism is essential for describing both the $f_0(500)$ and $f_0(980)$

resonances in the total cross sections. This result is an extension of [23] to the coupled-channel case. Similarly to [23], we calculated the dipole polarizabilities of charged and neutral pions $(\alpha_1-\beta_1)_\pi$, as shown in (4). Unsubtracted dispersion relations allow us to extract the following charged pion dipole polarizability: $(\alpha_1-\beta_1)_{\pi^\pm}=6.1\times10^{-4}$ fm^3, which is consistent with NLO χPT $(\alpha_1-\beta_1)_{\pi^\pm}^{\chi\mathrm{PT}}=6.0\times10^{-4}$ fm^3 [24] and with the recent COMPASS measurement: $(\alpha_1-\beta_1)_{\pi^\pm}^{\exp}=4.0(1.2)_{\mathrm{stat}}(1.4)_{\mathrm{syst}}\times10^{-4}$ fm^3 [29]. For the neutral pion dipole polarizability, we obtain $(\alpha_1-\beta_1)_{\pi^0}=9.5\times10^{-4}$ fm^3, thus being far from the NLO χPT value of $(\alpha_1-\beta_1)_{\pi^0}^{\chi\mathrm{PT}}=-1.0\times10^{-4}$ fm^3 [25]. Even though the charged channel is the dominant one, the question might arise of how suitable the current input is when estimating $(g-2)_\mu$. The neutral pion dipole polarizability discrepancy can be cured by taking into account the correction from heavier l.h.c., *i.e.*, the first term in Eq. (6). The dominant l.h.c beyond the pion pole comes from vector-meson t- and u-channel exchanges. Since they are much stronger for the neutral channel due to ω-exchange, the π^0 polarizabilities are expected to get large corrections [23]. Even though the dispersive integral is formally convergent due to the asymptotically bounded behavior of our Omnès function and the discontinuity of the amplitude $\bar{h}^0_{I,++}$, it acquires significant corrections from the integration over large negative s. Therefore, the implementation of the dispersion relations with higher intermediate states beyond ρ and ω corresponds to introducing at least one subtraction, which can be fixed to the NLO χPT (with some adjustments as explained below) given by [26]

$$
\begin{aligned}
(\alpha-\beta)_{\pi^\pm} &= \frac{e^2}{4\,\pi\,m_\pi}\left\{\frac{8\,(L_9^r+L_{10}^r)}{F_0^2}+\frac{-Q^2}{F_0^2}\left[\bar{J}'_\pi\left(-Q^2\right)+\frac{1}{2}\,\bar{J}'_K\left(-Q^2\right)\right]\right\},\\
(\alpha-\beta)_{\pi^0} &= \frac{e^2}{2\,\pi\,m_\pi}\left\{\frac{-Q^2-m_\pi^2}{F_0^2}\,\bar{J}'_\pi\left(-Q^2\right)+\frac{-Q^2}{4\,F_0^2}\bar{J}'_K\left(-Q^2\right)\right\},\\
(\alpha-\beta)_{K^\pm} &= \frac{e^2}{4\,\pi\,m_K}\left\{\frac{8\,(L_9^r+L_{10}^r)}{F_0^2}+\frac{-Q^2}{F_0^2}\left[\frac{1}{2}\bar{J}'_\pi\left(-Q^2\right)+\bar{J}'_K\left(-Q^2\right)\right]\right\},\\
(\alpha-\beta)_{K^0} &= \frac{e^2}{8\,\pi\,m_K}\frac{-Q^2}{F_0^2}\left\{\bar{J}'_\pi\left(-Q^2\right)+\bar{J}'_K\left(-Q^2\right)\right\},
\end{aligned}
\tag{10}
$$

with $F_0\simeq F_\pi=92.4$ MeV and the following loop function:

$$
\bar{J}_i(s)=\frac{1}{16\pi^2}\left[2+\sigma_i(s)\log\left(\frac{\sigma_i(s)-1}{\sigma_i(s)+1}\right)\right],\qquad \sigma_i(s)=\sqrt{1-\frac{4\,m_i^2}{s}},
\tag{11}
$$

For $Q^2=0$, we fix $\pi^\pm$ polarizability to the COMPASS result [29], while for π^0 and K $(I=0)$ we used $(L_9^r+L_{10}^r)=(0.84\pm0.64)\times10^{-3}$ taken from [27] similar to [12]. The Q^2 dependence is fully governed by (10), where in the single-channel case we used the pion-loop contributions only. The comparison between unsubtracted and once-subtracted results is shown in Fig. 3. For $I = 0$, the single-channel descriptions coincide in the region of $f_0(500)$ both for $Q^2 = 0$, 0.2 GeV^2 cases. The coupled-channel description, in turn, shows a slight difference for $Q^2 = 0$, which becomes significant for

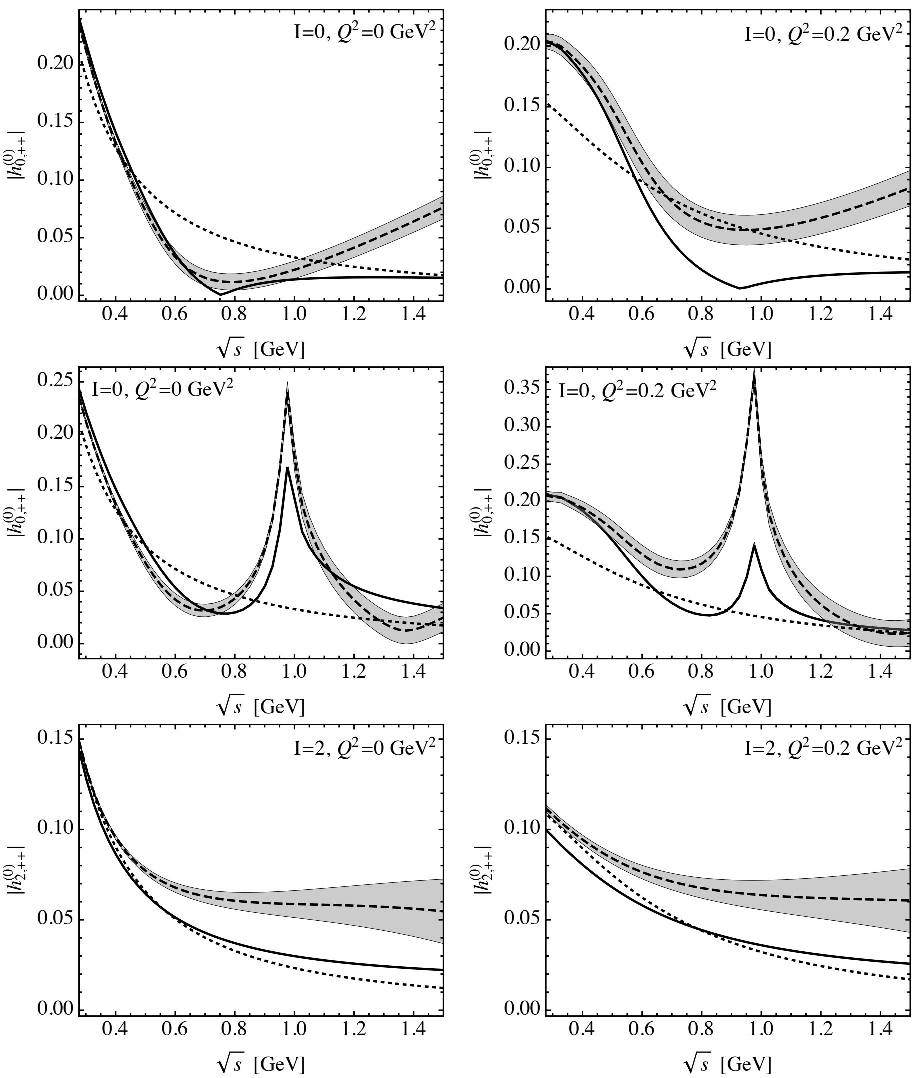

Fig. 3. Comparison of the p.w. amplitudes $|h^{(0)}_{I,++}|$ for once-subtracted (dashed line) and unsubtracted (solid line) dispersion relations; Born results are shown by the dotted lines. First row: $I = 0$, single channel; second row: $I = 0$, coupled channel; third row: $I = 2$, single channel.

the finite Q^2. This behavior can be ascribed to the lack of experimental information on the kaon polarizabilities and the poor convergence of SU(3) χPT. For $I = 2$, we note the discrepancy of about 10%–25% ($\sqrt{s} < 0.6$ GeV) at $Q^2 = 0.2$ GeV^2 which, however, does not affect strongly the total cross

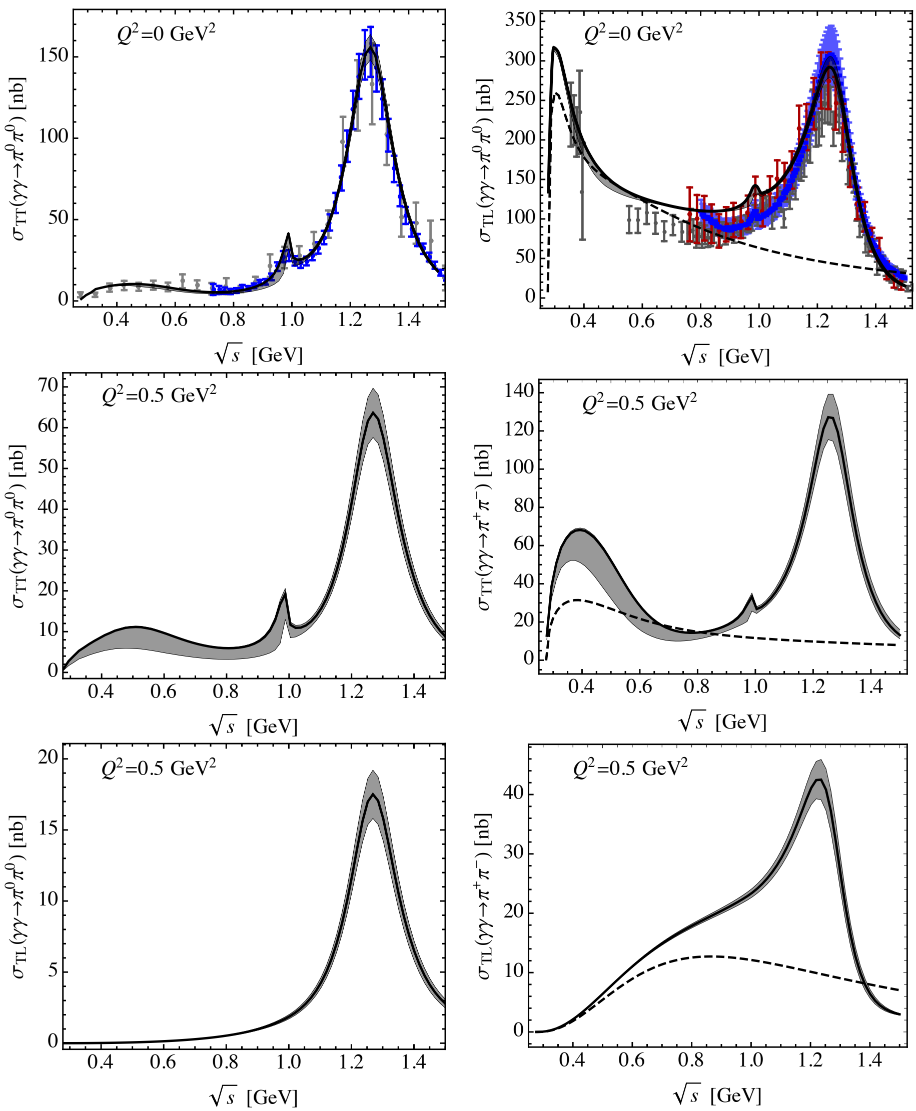

Fig. 4. Total sections for $\gamma\gamma \to \pi^0\pi^0$ (left column) and $\gamma\gamma \to \pi^+\pi^-$ (right column) processes. First row: cross sections for the real case in comparison to the data. Second (third) row: $\sigma_{\rm TT}$ ($\sigma_{\rm TL}$) cross sections for $Q^2 = 0.5$ GeV^2. The Born results are shown by dashed curves.

section. Since the NLO χPT is expected to be valid only in the region $Q^2 \lesssim 0.2$ GeV^2, the introduction of the additional subtraction reduces the predictive power of the dispersion relations. In the future, the upcoming data from the BESIII Collaboration in the range of $0.2 \lesssim Q^2 \lesssim 2.2$ GeV^2 [6] will allow to fix the subtraction constant directly from the data and hence, extract essential information on Q^2 dependence of the polarizabilities.

In contrast to the S-wave case, it is necessary to add the light vector-meson intermediate states to describe the $f_2(1270)$ region. The resulting cross sections are shown in the first row of Fig. 4, where the reasonable agreement with the data can be seen. These results have to be further confronted with the data and, therefore, we attempt to estimate the uncertainties of the given approach. For this purpose, we take into account the fitting error for $g_{\mathrm{VP}\gamma}$ which contributes to the D-wave. The uncertainties of the S-wave treatment originate mainly from the hadronic rescattering part, since the Born terms are well-known. Aiming for a conservative evaluation, we compare results using two different data driven coupled-channel Omnès functions: from our N/D analysis (see Fig. 2) and from [28]. It can be seen that for the real photons case this leads to the negligible difference, which can be also attributed to the numerical errors. However, the results for the single virtual process show a noticeable difference both in $f_0(500)$ and $f_0(980)$ regions. To further account for the uncertainties coming from the photon virtuality, we include the errors of the monopole fit for pion and kaon electromagnetic form factors. For the vector meson l.h.c., we include the dispersive estimation uncertainty of $f_{\omega\pi\gamma}(Q^2)$ and consider conservatively the error bar of the $f_{\rho\pi\gamma}(Q^2)$ at $Q^2 = 0.5$ GeV^2 in the VMD treatment to be at around 15%.

4. Summary

We have presented a dispersive analysis of the $\gamma\gamma^* \to \pi\pi$ reaction from the threshold up to 1.5 GeV in the $\pi\pi$-invariant mass. In order to capture the dynamical $\{\pi\pi, K\bar{K}\}$ origin of the $f_0(980)$ resonance, we used a coupled-channel dispersive approach for the S-wave. It was shown that unsubtracted dispersive formalisms, which account only for Born left-hand cuts perform similarly to the once-subtracted description with vector left-hand cuts in the region of $f_0(500)$, thus justifying the choice of the former. For the D-wave, we adopted the single-channel Omnès approach, that requires t- and u-channel vector-meson exchange contribution to the left-hand cut. The obtained results for the $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ cross sections are in the reasonable agreement with the experimental data. For the finite Q^2, we made a dispersive prediction of the cross section and provided preliminary error estimates.

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