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ASYMPTOTIC EXPANSIONS THROUGH THE LOOP–TREE DUALITY*

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Accurate theoretical predictions in the Standard Model (SM) are vital to disentangle possible new physics effects. The loop–tree duality (LTD) formalism transforms the integration domain of loop scattering amplitudes to a Euclidean space where asymptotic expansions of the integrand are well defined. The effectiveness of LTD for making asymptotic expansions has been shown in the large-mass and small-mass limits for Higgs production through gluon fusion. In this paper, we present a preliminary study aimed at generalising the method of asymptotic expansions in the LTD formalism. We use a toy amplitude and derive general guidelines.

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1. Introduction

After the successful discovery of the Higgs boson at CERN's Large Hadron Collider (LHC) almost a decade ago, the goal of describing the fundamental laws of nature with unprecedented accuracy remains a priority in high-energy physics. In the pursuit of this objective comparing measurements with theoretical predictions at increasing precision is at the forefront of current research. Consequently, it is necessary to include higher order contributions in perturbative Quantum Field Theory (pQFT) which quickly reaches its limits in the classical approach of Dimensional Regularization (DREG): divergent expressions appearing in the loop calculations of Feynman diagrams therein are regularized by working in $d = 4 - 2\varepsilon$ space-time dimensions — only to take the limit of $\varepsilon \to 0$ after both infrared (IR) and ultraviolet (UV) singularities have been cancelled and/or renormalized.

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Generally, the difficulty posed by the integral in a Feynman amplitude scales with the number of loops, external legs and mass scales. For two mass scales, the state of art is surpassed already at the two-loop level for four-point functions.

In recent years, an alternative regularization method based on the loop–tree duality (LTD) has been developed and applied both at one loop and beyond [1–17]. Other alternative methods to DREG are summarized in Ref. [18]. LTD is based on applying the Cauchy residue theorem in order to solve one component of the loop momentum. As a consequence, loop amplitudes can be expressed as a sum of residues which can be reformulated as so-called dual amplitudes. These consist of tree-level-like objects to be integrated in what essentially is a phase-space integral.

A general one-loop amplitude with N external legs is given by

$$\mathcal{A}_N^{(1)} = \int_\ell \left(\prod_{i=1}^{N} G_\mathrm{F}(q_i) \right) \mathcal{N}\left(\ell, \{p_k\}\right) , \tag{1}$$

where the integral measure in $d = 4 - 2\varepsilon$ is $\int_\ell = -\mathrm{i} \int \mathrm{d}^d\ell/(2\pi)^d$ and the Feynman propagator $G_\mathrm{F}(q_i) = (q_i^2 - m_i^2 + \mathrm{i}0)^{-1}$. Applying the loop–tree duality theorem, this amplitude is written as

$$\mathcal{A}_N^{(1)} = -\int_\ell \sum_{i=1}^{N} \tilde{\delta}\left(q_i\right) \left(\prod_{j \neq i} G_\mathrm{D}\left(q_i; q_j\right) \right) \mathcal{N}\left(\ell, \{p_k\}\right) , \tag{2}$$

where $G_\mathrm{D}(q_i; q_j) = (q_j^2 - m_j^2 - \mathrm{i}0\, \eta \cdot k_{ji})^{-1}$, with $k_{ji} = q_j - q_i$, is called the dual propagator and η is an arbitrary timelike vector. The dual propagator differs from the Feynman propagator only in its imaginary part, whose sign depends on the external momenta to take into account the imaginary part introduced when taking the residue and evaluating the non-singular part of the amplitude on the complex pole. Another consequence of the residue theorem is the on-shell delta functional $\tilde{\delta}\left(q_i\right) = 2\pi i\, \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$. Since in each term in the sum of Eq. (2) a different internal line is set on-shell, these cuts only depend on the spatial part of the loop momentum (for the customary choice $\eta = (1, \mathbf{0})$) and the remaining part of the loop integral has the structure of a phase-space integral.

This three-dimensional structure of the dual integrand leads to the arguably most striking achievement of the LTD since it allows the local cancellation of IR singularities as done in the Four-dimensional Unsubtraction method (FDU) [7–9]. At the same time, it allows us to explore the topic of asymptotic expansions since the presence of only Euclidean momenta allows the direct comparison between the size of scalar products and external scales. LTD has already been used successfully to derive integrand-level expansions in $H \to \gamma\gamma$ at one loop [10] allowing for optimism concerning the development of a general method.

Asymptotic expansions in pQFT are of interest since they would facilitate analytic results in specific kinematic situations even for amplitudes where the full analytic calculation in DREG is not (yet) possible due to the absence of solutions for certain master integrals. There is a variety of situations where an analytic result is not necessary for every set of kinematics and where specific limits are the window to test potential discrepancies between experiments and SM calculation caused by new physics.

There are already well-developed methods available for simplifying the integrands of Feynman amplitudes through asymptotic expansions, notably among them Expansion by Regions [19, 20]. This method has been shown to produce correct results, though it still lacks a general proof [21]. Furthermore, every term in the expansion has a higher degree of UV divergence which can be considered problematic.

Here, we present the starting point for developing the general method of asymptotic expansions in the context of LTD by considering the simplest diagram. This easy toy amplitude allows to study the behaviour of the dual propagator under different expansions and how these expansions must be defined. We aim towards obtaining an expansion that is well-defined also at integrand level and simplifies integrands sufficiently to obtain loop analytic results at higher orders and multiple scales.

2. Generalized expansion of the dual propagator

The behaviour of Feynman amplitudes is mostly determined by its analytic structure which follows from the appearing propagators. Therefore, these are the starting point for developing a general method for expansions at integrand level. The objectives are for the expansion to converge fast, *i.e.* a small number of terms in the expansion should be sufficient to achieve acceptable accuracy, and the expanded amplitude should be easily integrable analytically.

Necessary groundwork is understanding the position of physical and removable non-causal singularities. The singular behaviour of a dual propagator can be efficiently examined through a reparametrization [14]

$$\frac{\tilde{\delta}\left(q_i\right)}{\pi i}\, G_{\mathrm{D}}(q_i;q_j) = \frac{\delta\left(q_{i,0}-q_{i,0}^{(+)}\right)}{q_{i,0}^{(+)}\lambda_{ij}^{+-}\lambda_{ij}^{++}}\,, \qquad \lambda_{ij}^{\pm\pm} = \pm q_{i,0}^{(+)} \pm q_{j,0}^{(+)} + k_{ji,0}\,, \quad (3)$$

where a causal unitarity threshold appears for $\lambda_{ij}^{++} \to 0$ and an unphysical threshold for $\lambda_{ij}^{+-} \to 0$. The latter always appears in two paired dual amplitudes at once and cancels due to the prescription of the imaginary part. This notation is convenient as it allows to easily derive the kinematic conditions for these limits to occur.

Having understood its analytic structure, one may rewrite the dual propagator in terms of properly chosen parameters $\mathit{\Gamma}_{ji}$ and $\mathit{\Delta}_{ji}$ and it may be expanded

$$\begin{aligned} G_{\mathrm{D}}(q_i;q_j) &= \frac{1}{2q_i\cdot k_{ji}+\mathit{\Gamma}_{ji}+\mathit{\Delta}_{ji}-\mathrm{i}0\,\eta\cdot k_{ji}} \\ &= \sum_{n=0}^{\infty}\frac{(-\mathit{\Delta}_{ji})^n}{(2q_i\cdot k_{ji}+\mathit{\Gamma}_{ji}-\mathrm{i}0\,\eta\cdot k_{ji})^{n+1}}\,, \qquad \mathit{\Gamma}_{ji}+\mathit{\Delta}_{ji}=m_i^2-m_j^2+k_{ji}^2\,. \end{aligned} \tag{4}$$

If the choice $\boldsymbol{k_{ji}}=0$ is possible, the denominator shall be rewritten as

$$2q_i\cdot k_{ji}+\mathit{\Gamma}_{ji}=\frac{Q_i^2}{x_i}\left(r_{ij}+x_i\right)\left(r_{ij}x_i+1\right)\,, \qquad x_i=\frac{|\boldsymbol{\ell}|+\sqrt{\boldsymbol{\ell}^2+m_i^2}}{m_i}\,. \tag{5}$$

In order to achieve the form above, it is necessary that the parameters $\mathit{\Gamma}_{ji}$ and $\mathit{\Delta}_{ji}$ appearing in the expansion fulfill the following conditions:

$$\mathit{\Gamma}_{ji}=Q_i^2\left(1+r_{ij}^2\right)\,, \qquad \mathit{\Delta}_{ji}=m_i^2-m_j^2+k_{ji}^2-\mathit{\Gamma}_{ji}\,, \qquad r_{ij}=\frac{m_ik_{ji,0}}{Q_i^2}\,. \tag{6}$$

As it will be demonstrated in the following, integrals of dual propagators reshaped in this form are easily solvable analytically, based on

$$\int_1^{\infty}\frac{\mathrm{d}x}{(r+x)(rx+1)\pm\mathrm{i}0}=\frac{\log(r\pm\mathrm{i}0)}{r^2-1}\,, \qquad |r|<1\,. \tag{7}$$

For any limit, the expansion parameter r_{ij}, and thus $\mathit{\Gamma}_{ji}$ and $\mathit{\Delta}_{ji}$, must be chosen such that the propagators may be simplified as in Eq. (5). The analytic behaviour of the dual propagator may only be changed in a smooth way through the expansion and the Q_i^2 must be chosen such that the expansion converges both at integrand and at integral level. To achieve this, one should demand that

$$|-\mathit{\Delta}_{ji}|\ll|2q_i\cdot k_{ji}+\mathit{\Gamma}_{ji}| \tag{8}$$

for all of the integration space expect for possibly a small region around physical divergences.

One might think that it should generally be possible to directly derive the optimal expansion parameters from maximizing Eq. (8). Indeed, for the types of limits where one large scale Q is available, this is true: choosing

$Q_i^2 = \pm Q^2$ and determining the sign in order to respect the second condition directly gives correct expansion parameters. Alternatively, the sign may be deduced from the expected behaviour of the logarithm: if a physical threshold appears in the cut, the respective r should be negative which leads to an imaginary part coming from the logarithm.

Considering the scenario of approaching a physical threshold, this method is not sufficient, though. It appears that when a propagator's behaviour is strongly influenced by the pole, which clearly is the case in the threshold expansion, one must consider the trajectory of the pole more carefully. For this goal, it is necessary to rewrite the dual propagator in terms of its singularities (again, if the choice $\boldsymbol{k_{ji}} = 0$ is possible) as

$$\begin{aligned} G_{\mathrm{D}}\left(q_i; q_j\right) &= \frac{1}{2q_i \cdot k_{ji} + m_i^2 - m_j^2 + k_{ji}^2 - \mathrm{i}0\,\eta \cdot k_{ji}} \\ &= \frac{x}{k_{ji,0} m_i\,(x - x_1)(x - x_2)}\,, \qquad |\boldsymbol{q_i}| = \frac{m_i}{2}\left(x - \frac{1}{x}\right)\,, \end{aligned} \tag{9}$$

where the poles are given in terms of the Källén function $\lambda(x^2, y^2, z^2) = (x^2 - (y+z)^2)(x^2 - (y-z)^2)$ as

$$x_{1/2} = -\frac{k_{ji,0}^2 + m_i^2 - m_j^2 \pm \sqrt{\lambda\left(k_{ji,0}^2, m_i^2, m_j^2\right)}}{2k_{ji,0} m_i}\,. \tag{10}$$

The full propagator can thus be brought into the form desired for the integration by identifying $r = -x_1$ or $r = -x_1^{-1} = -x_2$, even in its non-expanded form. The ideal r to be used in the expansion can thus be obtained by expanding the divergences of the propagator $x_{1/2}$. Between x_1 and x_2, the appropriate choice is the one whose absolute value is smaller than 1. In this way, asymptotic limits such as approaching the threshold both from below and from above can be obtained and also the parameters for the expansion with a large scale can be reproduced.

3. Bubble diagram

The general rules above can be applied to the bubble diagram, given by the amplitude

$$\mathcal{A}^{(1)} = \int_\ell G_{\mathrm{F}}(\ell; m)\, G_{\mathrm{F}}(q_1; M)\,. \tag{11}$$

Note that the choice of integration momenta with $q_1 = \ell - p$ instead of $\ell + p$ avoids the appearance of artificial singularities at integrand level. While these, being unphysical, always cancel, they would make the calculation unnecessarily complicated. With $L = \log(M^2/m^2)$, the full renormalized result as obtained through FeynCalc is [22, 23]

$$\mathcal{A}^{(1,R)} = \mathcal{A}^{(1)} - \left[\mathcal{A}^{(1)}\right]_{\mathrm{UV}}^{\mathrm{cnt}} = \frac{1}{16\pi^2}\left[2 - \log\left(\frac{M^2}{\mu_{\mathrm{UV}}^2}\right) - \frac{M^2 - p^2 - m^2}{2p^2} L \right.$$
$$\left. + \frac{\lambda^{1/2}\left(p^2, m^2, M^2\right)}{p^2} \log \frac{m^2 + M^2 - p^2 + \lambda^{1/2}\left(p^2, m^2, M^2\right)}{2mM}\right], \tag{12}$$

where the appropriate counterterm in the Feynman and the LTD representation, respectively, is given by

$$\left[\mathcal{A}^{(1)}\right]_{\mathrm{UV}}^{\mathrm{cnt}} = \int_\ell G_{\mathrm{F}}(\ell; \mu_{\mathrm{UV}})^2 = \int_\ell \frac{\tilde{\delta}\left(\ell; \mu_{\mathrm{UV}}\right)}{2\left(\ell_0^{(+)}\right)^2}. \tag{13}$$

This is an $\overline{\mathrm{MS}}$ counterterm when identifying μ_{UV} with the DREG renormalization scale.

The dual representation for the amplitude in Eq. (11), when assuming $p = (p_0, \mathbf{0})$ with $p_0 > 0$, is given by

$$\mathcal{A}^{(1)} = -\int_\ell \left[\tilde{\delta}\left(\ell; m\right) G_{\mathrm{D}}(\ell; q_1) + \tilde{\delta}\left(q_1; M\right) G_{\mathrm{D}}(q_1; \ell)\right], \tag{14}$$

with the dual propagators

$$G_{\mathrm{D}}(\ell; q_1) = \frac{1}{-2\ell\cdot p + p^2 + m^2 - M^2 + \mathrm{i}0}, \tag{15}$$
$$G_{\mathrm{D}}(q_1; \ell) = \frac{1}{2q_1\cdot p + p^2 - m^2 + M^2 - \mathrm{i}0}, \tag{16}$$

and the on-shell energies and scalar products $\ell_0^{(+)} = \sqrt{\boldsymbol{\ell}^2 + m^2}$, $q_{1,0}^{(+)} = \sqrt{\boldsymbol{\ell}^2 + M^2}$, $\ell\cdot p = \ell_0^{(+)} p_0$, and $q_1 \cdot p = q_{1,0}^{(+)} p_0$.

Applying the general propagator expansion of Eqs. (4) and (5), the amplitude of the bubble diagram takes the form of

$$\mathcal{A}^{(1)} = -\frac{1}{16\pi^2}\left[\frac{m^2}{Q_1^2}\sum_{n=0}^{\infty}\frac{(-\Delta_{21})^n}{Q_1^{2n}} I\left(r_{12}, \Lambda_1, n, -Q_1^2\right)\right.$$
$$\left. + \frac{M^2}{Q_2^2}\sum_{n=0}^{\infty}\frac{(-\Delta_{12})^n}{Q_2^{2n}} I(r_{21}, \Lambda_2, n, 1)\right], \tag{17}$$

where the functions I contain the remaining integration as

$$I(r,\varLambda,n,a)=\int\limits_{1}^{\varLambda}\mathrm{d}x\;\frac{(x^2-1)^2\;x^{n-2}}{\left[(r+x)\,(rx+1)-a\cdot\mathrm{i}0\right]^{n+1}}\,. \tag{18}$$

Since the threshold divergence can only appear in the first cut, we already know that $Q_2^2>0$ and, in fact, the imaginary part of the propagator in the second cut can be dropped. Since the integrals over the two cuts do not converge separately, they have to either be integrated numerically together with the counterterm or the separate analytic integrations must be performed up to the cutoff $\varLambda_i=(\varLambda+(\varLambda^2+m_i^2)^{1/2})/m_i$. The limit $\varLambda\to\infty$ is only to be taken after summing both cuts and cancelling the UV divergence with the appropriate counterterm. The results of the integrals are functions of logarithms of the cutoff and the parameters r

$$\begin{aligned}
I(|r|<1,\varLambda,n,0)&\overset{n=0}{=}\frac{\varLambda}{r}-\left(1+\frac{1}{r^2}\right)\log(\varLambda)+\left(1-\frac{1}{r^2}\right)\log(r)+\mathcal{O}\left(\varLambda^{-1}\right)\\
&\overset{n=1}{=}\frac{1}{r^2}\log(\varLambda)+\frac{1+r^2}{r^2\,(1-r^2)}\log(r)-\frac{1}{r^2}+\mathcal{O}\left(\varLambda^{-1}\right)\\
&\overset{n=2}{=}\frac{2}{(1-r^2)^3}\log(r)-\frac{1+r^2}{2r^2\,(1-r^2)^2}+\mathcal{O}\left(\varLambda^{-1}\right)\,.
\end{aligned} \tag{19}$$

It is apparent that the UV divergence lessens with every order in the expansion. Thus for renormalization, only the first two terms are necessary. Therein, the linearly divergent terms cancel between the two cuts and the logarithmic dependence on the UV cutoff $\varLambda$ is canceled by the counterterm of Eq. (13). Any further precision improvement by including more orders of the expansion will not affect the UV behaviour.

Independently of the chosen limit, the renormalized amplitude including the expansion terms up to $n=1$ is thus given by

$$\begin{aligned}
\mathcal{A}^{(1,R)}_{n=1}&=\frac{1}{16\pi^2}\Bigg[2-\log\left(\frac{M^2}{\mu_{\mathrm{UV}}^2}\right)-\frac{M^2-m^2-p^2}{2p^2}L\\
&-\frac{m^2}{Q_1^2}\left(\left(c^{(0)}_{r_{12}}+c^{(1)}_{r_{12}}\right)\log(r_{12})+c^{(1)}_{1}\right)-\frac{M^2}{Q_2^2}\left(\left(c^{(0)}_{r_{21}}+c^{(1)}_{r_{21}}\right)\log(r_{21})+c^{(1)}_{2}\right)\Bigg]\,.
\end{aligned} \tag{20}$$

Further terms in the expansion do not affect the first line of this result and only increase the precision in the coefficients of the logarithms of the r_{ij} as well as adding more terms without logarithms. The coefficients needed in the result for the first few orders can be read off Eqs. (17) and (19).

In the limit of one large mass, $M \gg m, p^2$, following the rules described above leads to $Q_1^2 = -M^2$, $r_{12} = m\sqrt{p^2}/M^2$, $Q_2^2 = M^2$ and $r_{21} = \sqrt{p^2}/M$. Using the values $M/m = 10$ and $p^2/m^2 = 3$, the relative error of the result is 2.7% at first renormalized order in the expansion ($n = 1$) and decreases to 0.03% for $n = 2$. At integrand level, the relative error lies around 10^{-4} (10^{-5}) for $n = 1$ ($n = 2$) for all the range of the loop momentum. Similarly, with the limit of large external momentum, a scenario above threshold has been successful.

For the case of the threshold limit with $\beta = 1 - p^2/(m+M)^2 \to 0^{\pm}$, the expansion parameters must be obtained by expanding the position of the propagator's singularity given in Eq. (10). This leads to $r_1 = -1 + \sqrt{M/m}\sqrt{-\beta}$ and Q_1^2 follows according to the conditions in Eq. (6). Since no singularity determines the behaviour of the second cut there, the simple choice $Q_2^2 = p^2$ may be used. The expression provided for r_1 is to be used both when approaching the threshold from above and from below — even though in the second case no divergence appears in the propagator, the complex singularity approaches the real axis and is thus the most important feature of the propagator. The imaginary part arising from the complex logarithm is canceled by the coefficients leading to an amplitude that is real, except for a tiny imaginary part that goes to zero quickly when increasing the precision of the expansion: the ratio of imaginary part over real part of the result below threshold ($\beta > 0$) is 0.02% for $n = 1$ and 2×10^{-6} for $n = 2$. The relative error of the result for $\beta = -0.1$ ($\beta = 0.1$) is 0.06% (0.2%) for $n = 1$ and 4×10^{-6} (2×10^{-5}) for $n = 2$.

4. Conclusions and outlook

Motivated by previously successful results in the LTD calculation of Higgs production through gluon fusion at one loop, we started the development of a general method for asymptotic expansions in the context of LTD by exploiting the Euclidean structure of dual integrands. In this work, we considered the bubble diagram with two massive particles in the loop as a starting point. For this process, we developed a general formula for the expanded amplitude as well as rules on how to apply it for different types of limits. The result is fully renormalized when including only two terms of the integrand-level expansion and the following terms are finite in the UV. We tested the convergence for the limit of one large mass in the loop, for a large external momentum and for the threshold limit. In all cases, the expansion converged very fast such that the relative error is acceptable even at leading order. The result obtained is simpler than the full version and thus allows an intuitive understanding of the amplitude in the respective limits.

We have developed successful expansions also for the scalar three-point function. Embedding these in the general formalism is still in progress. In addition, a systematic comparison with the results obtained for comparable cases in Expansion by Regions is still outstanding. Finally, we are looking forward to applying this expansion method to highly boosted Higgs production and extending it to the two-loop case.

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