PARTIAL DYNAMICAL SYMMETRY AND THE PHONON STRUCTURE OF CADMIUM ISOTOPES*

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The phonon structure and spectral properties of states in ^{110}Cd are addressed by including proton excitations in the phonon basis and exploiting a partial dynamical symmetry that mixes only certain classes of states and maintains the vibrational character in the majority of normal states.

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The cadmium isotopes since long have been considered as archetypal examples of spherical vibrators, manifesting the U(5) dynamical symmetry (DS) [1]. Recent studies, however, have cast doubt on the validity of this description [2, 3]. In the present contribution, we address this question from a symmetry-oriented perspective, focusing on ^{110}Cd [4].

The U(5)-DS limit of the interacting boson model (IBM) [1], corresponds to the chain of nested algebras: U(6) $\supset$ U(5) $\supset$ SO(5) $\supset$ SO(3). The basis states $|[N], n_d, \tau, n_\Delta, L\rangle$ have quantum numbers which are the labels of irreducible representations of the algebras in the chain. Here, N is the total number of monopole (s) and quadrupole (d) bosons, n_d and τ are the d-boson number and seniority, respectively, and L is the angular momentum. The multiplicity label n_Δ counts the maximum number of d-boson triplets coupled to $L = 0$. The U(5)-DS Hamiltonian has the form of [1]

$$\hat{H}_{\rm DS} = t_1\,\hat{n}_d + t_2\,\hat{n}_d^2 + t_3\,\hat{C}_{\rm SO(5)} + t_4\,\hat{C}_{\rm SO(3)}\,, \tag{1}$$

where $\hat{C}_{\rm G}$ is a Casimir operator of G, and $\hat{n}_d = \sum_m d^\dagger_m d_m = \hat{C}_{\rm U(5)}$. $\hat{H}_{\rm DS}$ is completely solvable for *any* choice of parameters t_i, with eigenstates $|[N], n_d, \tau, n_\Delta, L\rangle$ and energies $E_{\rm DS} = t_1 n_d + t_2 n_d^2 + t_3\tau(\tau+3) + t_4 L(L+1)$. A typical U(5)-DS spectrum exhibits n_d-multiplets of a spherical vibrator, with enhanced connecting $(n_d + 1 \rightarrow n_d)$ E2 transitions.

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The empirical spectrum of ^{110}Cd, shown in Fig. 1 (a), consists of both normal and intruder levels, the latter based on $2p$–$4h$ proton excitations across the $Z = 50$ closed shell. Experimentally known E2 rates are listed in Tables I, II. A comparison of the calculated spectrum [Fig. 1 (b)] and B(E2) values [Table I], obtained from $\hat{H}_{\rm DS}$ (1), demonstrates that most normal states have good spherical-vibrator properties, and conform well with the properties of U(5)-DS. However, the measured rates for E2 decays from the non-yrast states, 0^+_3 ($n_d = 2$) and [0^+_4, 2^+_5 ($n_d = 3$)], reveal marked deviations from this behavior. In particular, $B(\mathrm{E2};\, 0^+_3 \to 2^+_1) < 7.9$, $B(\mathrm{E2};\, 2^+_5 \to 4^+_1) < 5$, $B(\mathrm{E2};\, 2^+_5 \to 2^+_2) = 0.7^{+0.5}_{-0.6}$ W.u. are extremely small compared to the U(5)-DS values: 46.29, 19.84, 11.02 W.u., respectively. Absolute B(E2) values for transitions from the 0^+_4 state are not known, but its branching ratio to 2^+_2 is small.

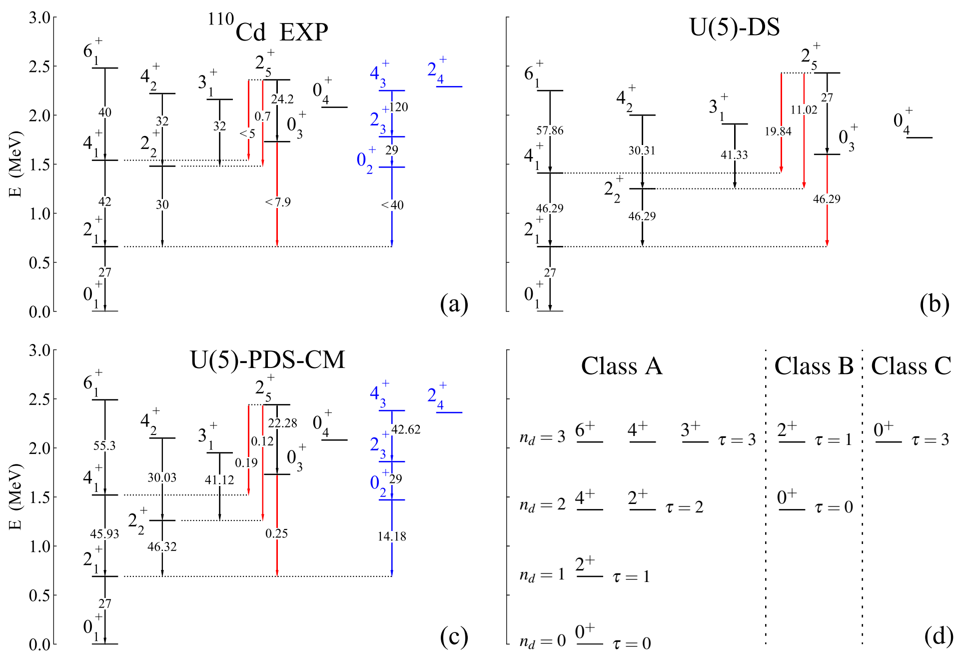

Fig. 1. (a) Experimental spectrum and representative E2 rates [3, 5] (in W.u.) of normal and intruder levels (0^+_2, 2^+_3, 4^+_3, 2^+_4) in ^{110}Cd. (b) Calculated U(5)-DS spectrum obtained from $\hat{H}_{\rm DS}$ (1) with parameters $t_1 = 715.75$, $t_2 = -t_3 = 42.10$, $t_4 = 11.38$ keV and $N = 7$. (c) Calculated U(5)-PDS-CM spectrum, obtained from $\hat{H}$ (3) with parameters $t_1 = 767.83$, $t_2 = -t_3 = 73.62$, $t_4 = 18.47$, $r_0 = 2.15$, $e_0 = -6.92$, $\kappa = -72.73$, $\mathit{\Delta} = 9978.86$, $\alpha = -42.78$ keV and $N = 7\,(9)$ in the normal (intruder) sector. (d) Classes of low-lying U(5)-DS states.

TABLE I

Absolute (relative in square brackets) B(E2) values in W.u. for E2 transitions from normal levels in ^{110}Cd. The experimental (EXP) values are taken from [3, 5]. The U(5)-DS [U(5)-PDS-CM] values are obtained for an E2 operator $e_B\,\hat{Q}$ $[e_B^{(N)}\hat{Q}^{(N)} + e_B^{(N+2)}\,\hat{Q}^{(N+2)}]$ with $e_B\!=\!1.964$ $[e_B^{(N)}\!=\!1.956$ and $e_B^{(N+2)} = 1.195]$ (W.u.)$^{1/2}$, where $\hat{Q} = d^\dagger s + s^\dagger\tilde{d}$ and $\hat{Q}^{(N)}$ denotes its projection onto the $[N]$ boson space. In both calculations, the boson effective charges were fixed by the empirical $2^+_1 \to 0^+_1$ rate. Intruder states $0^+_{2;\mathrm{i}}\, 2^+_{3;\mathrm{i}}$, $4^+_{3;\mathrm{i}}$, $2^+_{4;\mathrm{i}}$, are marked by a subscript 'i'. [a]From Ref. [3].

L_i	L_f	EXP	U(5)-DS	U(5)-PDS-CM
2^+_1	0^+_1	27.0 (8)	27.00	27.00
4^+_1	2^+_1	42 (9)	46.29	45.93
2^+_2	2^+_1	30 (5); 19 (4)[a]	46.29	46.32
	0^+_1	1.35 (20); 0.68 (14)[a]	0.00	0.00
0^+_3	2^+_2	$<$ 1680[a]	0.00	55.95
	2^+_1	$<$ 7.9[a]	46.29	0.25
6^+_1	4^+_1	40 (30); 62 (18)[a]	57.86	55.30
	4^+_2	$<$ 5[a]	0.00	0.00
	$4^+_{3;i}$	14 (10); 36 (11)[a]		2.39
4^+_2	4^+_1	12^{+4}_{-6}; [a]$10.7^{+4.9}_{-4.8}$	27.55	27.45
	2^+_2	32^{+10}_{-14}; 22 (10)[a]	30.31	30.03
	2^+_1	$0.20^{+0.06}_{-0.09}$; 0.14 (6)[a]	0.00	0.00
	$2^+_{3;i}$	$<$ 0.5[a]		0.005
3^+_1	4^+_1	$5.9^{+1.8}_{-4.6}$; [a]$2.4^{+0.9}_{-0.8}$	16.53	16.48
	2^+_2	32^{+8}_{-24}; 22.7 (69)[a]	41.33	41.12
	2^+_1	$1.1^{+0.3}_{-0.8}$; 0.85 (25)[a]	0.00	0.00
	$2^+_{3;i}$	$<$ 5[a]		0.012
0^+_4	2^+_2	[$<$ 0.65[a]]	57.86	1.24
	2^+_1	[0.010[a]]	0.00	31.76
	$2^+_{3;i}$	[100[a]]		16.32
2^+_5	0^+_3	24.2 (22)[a]	27.00	22.28
	4^+_1	$<$5[a]	19.84	0.19
	2^+_2	[a]$0.7^{+0.5}_{-0.6}$	11.02	0.12
	2^+_1	$2.8^{+0.6}_{-1.0}$	0.00	0.00
	$2^+_{3;i}$	$<$ 5[a]		0.002
	$0^+_{2;i}$	$<$ 1.9[a]		0.20

Attempts to explain the above deviations in terms of mixing between the normal spherical [U(5)-like] states and intruder deformed [SO(6)-like] states have been shown to be unsatisfactory [2, 3]. This has led to the conclusion that the normal-intruder strong-mixing scenario needs to be rejected, and

TABLE II

B(E2) values (in W.u.) for E2 transitions from intruder levels in ^{110}Cd. Notation and relevant information on the observables shown are as in Table I.

L_i	L_f	EXP	U(5)-PDS-CM
$0^+_{2;i}$	2^+_1	$< 40^{\mathrm{a}}$	14.18
$2^+_{3;i}$	$0^+_{2;i}$	$29\ (5)^{\mathrm{a}}$	29.00
	0^+_1	$0.31^{+0.08}_{-0.12}$; $0.28\ (4)^{\mathrm{a}}$	0.08
	2^+_1	$0.7^{+0.3}_{-0.4}$; ${}^{\mathrm{a}}0.32^{+0.10}_{-0.14}$	0.00
	2^+_2	$< 8^{\mathrm{a}}$	0.96
$2^+_{4;i}$	2^+_1	$0.019^{+0.020}_{-0.019}$	0.10
$4^+_{3;i}$	2^+_1	$0.22^{+0.09}_{-0.19}$; $0.14\ (4)^{\mathrm{a}}$	0.49
	2^+_2	$2.2^{+1.4}_{-2.2}$; $1.2(4)^{\mathrm{a}}$	0.00
	$2^+_{3;i}$	120^{+50}_{-110}; $115\ (35)^{\mathrm{a}}$	42.62
	4^+_1	$2.6^{+1.6}_{-2.6}$; ${}^{\mathrm{a}}1.8^{+1.0}_{-1.5}$	0.00

have raised serious questions on the appropriateness of the multi-phonon interpretation [2, 3]. In what follows, we consider a possible explanation for the "Cd problem", based on U(5) partial dynamical symmetry (PDS). The latter corresponds to a situation in which the U(5)-DS is obeyed by only a subset of states and is broken in other states [6]. Similar PDS-based approaches have been implemented in nuclear spectroscopy, in conjunction with the SU(3)-DS [7–9] and SO(6)-DS [10, 11] chains of the IBM.

As depicted in Fig. 1 (d), the lowest spherical-vibrator levels comprise three classes of states. Specifically, Class A: $n_d = \tau = 0, 1, 2, 3$ ($n_\Delta = 0$); Class B: $n_d = \tau + 2 = 2, 3$ ($n_\Delta = 0$); Class C: $n_d = \tau = 3$ ($n_\Delta = 1$). In the U(5)-DS calculation of Fig. 1 (b), applicable to normal states only, the "problematic" states [0^+_3 ($n_d = 2$) and 2^+_5 ($n_d = 3$)] belong to class B, and 0^+_4 ($n_d = 3$) belongs to class C. The remaining "good" spherical-vibrator states [0^+_1 ($n_d = 0$); 2^+_1 ($n_d = 1$); $4^+_1, 2^+_2$ ($n_d = 2$); $6^+_1, 4^+_2, 3^+_1$ ($n_d = 3$)] belong to class A. As mentioned, the spherical-vibrator interpretation is valid for most normal states in Fig. 1 (a), but not all. We are thus confronted with a situation in which some states in the spectrum (assigned to class A) obey the predictions of U(5)-DS, while other states (assigned to classes B and C) do not. These empirical findings signal the presence of U(5)-PDS.

The construction of a Hamiltonian with U(5)-PDS follows the general algorithm [6] and leads to the form of

$$\hat{H}_{\mathrm{PDS}} = \hat{H}_{\mathrm{DS}} + r_0\, G^\dagger_0 G_0 + e_0 \left(G^\dagger_0 K_0 + K^\dagger_0 G_0 \right) , \tag{2}$$

where $G^\dagger_0 = [(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(0)}$, $K^\dagger_0 = s^\dagger (d^\dagger d^\dagger)^{(0)}$. The last two terms in Eq. (2) annihilate the states $|[N], n_d{=}\tau, \tau, n_\Delta{=}0, L\rangle$ with $L{=}\tau, \tau{+}1, \ldots, 2\tau{-}2, 2\tau$.

These states, which include those of class A, form a subset of U(5) basis states, hence remain solvable eigenstates of $\hat{H}_{\rm PDS}$ (2) with good U(5) symmetry. It should be noted that while $\hat{H}_{\rm DS}$ (1) is diagonal in the U(5)-DS chain, the r_0 and e_0 terms can connect states with different n_d and/or τ. Accordingly, the remaining eigenstates of $\hat{H}_{\rm PDS}$ (2), in particular those of classes B and C, are mixed with respect to U(5) and SO(5). The U(5)-DS is thus preserved in a subset of eigenstates, for any choice of parameters in $\hat{H}_{\rm PDS}$, but is broken in others. By definition, $\hat{H}_{\rm PDS}$ exhibits U(5)-PDS.

The combined effect of normal and intruder states can be studied within the interacting boson model with configuration mixing (IBM-CM) [12]. The Hamiltonian for the two configurations has the form of [4]

$$\hat{H} = \hat{H}^{(N)}_{\rm PDS} + \hat{H}^{(N+2)}_{\rm intrud} + \hat{V}_{\rm mix}\,. \qquad (3)$$

For ^{110}Cd, the Hamiltonian in the normal sector is taken to be $\hat{H}_{\rm PDS}$ of Eq. (2), projected onto a space of $N=7$ bosons. The SO(6)-type of Hamiltonian in the intruder sector is $\hat{H}_{\rm intrud} = \kappa\hat{Q}\cdot\hat{Q} + \Delta$, projected onto a space of $N=9$ bosons. $\hat{V}_{\rm mix} = \alpha[(s^\dagger)^2 + (d^\dagger d^\dagger)^{(0)}] + {\rm H.c.}$ is a mixing term between the two spaces. In general, an eigenstate of $\hat{H}$, $|\Psi\rangle = a|\Psi_{\rm n}^{(N)}\rangle + b|\Psi_{\rm i}^{(N+2)}\rangle$, involves a mixture of normal (n) and intruder (i) components with N and $N+2$ bosons, respectively.

As seen in Fig. 1 (c) and Tables I, II, the IBM-PDS-CM calculation provides a good description of the empirical data in ^{110}Cd. The U(5) decomposition of the resulting eigenstates is shown in Fig. 2. The normal states of class A retain good U(5) symmetry to a good approximation. Their $|\Psi_{\rm n}^{(N)}\rangle$ part involves a single n_d-component. The mixing with the intruder states is weak (small b^2) of the order of a few percent. The high degree of purity is reflected in the calculated B(E2) values for transitions between class A states, which are very similar to those of U(5)-DS. In contrast, the structure of the non-yrast states assigned originally to classes B and C, whose decay properties show marked deviations from the U(5)-DS limit, changes dramatically. Specifically, the 0_3^+ and 0_4^+ states, which in the U(5)-DS classification are members of the two-phonon triplet and three-phonon quintuplet, interchange their character, and the U(5) decomposition of their $|\Psi_{\rm n}^{(N)}\rangle$ parts peaks at $n_d = 3$ and $n_d = 2$, respectively. Similarly, for the 2_5^+ state, which is originally a member of the three-phonon quintuplet, the $|\Psi_{\rm n}^{(N)}\rangle$ part now exhibits a peak at $n_d = 4$. The calculated $B({\rm E2};\, 0_3^+ \to 2_1^+) = 0.25$, $B({\rm E2};\, 2_5^+ \to 4_1^+) = 0.19$ and $B({\rm E2};\, 2_5^+ \to 2_2^+) = 0.12$ W.u. are consistent with the measured upper limits: 7.9, 5 and $0.7^{+0.5}_{-0.6}$ W.u., respectively. The vibrational interpretation is thus maintained in the majority of low-lying normal states in ^{110}Cd.

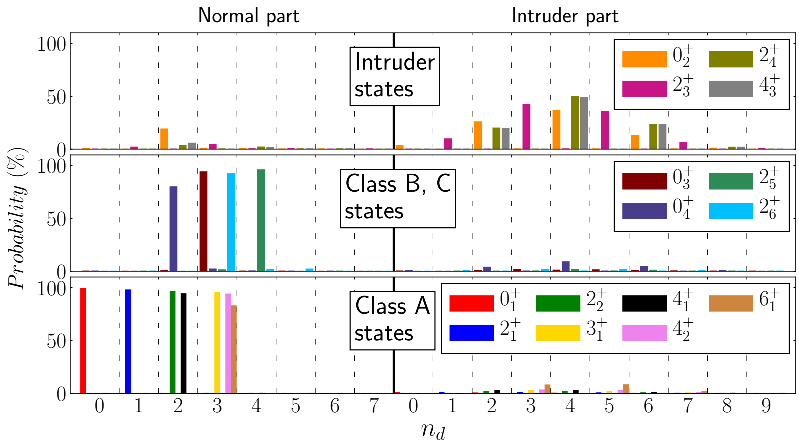

Fig. 2. U(5) n_d-decomposition of Class A (lower panel), Classes B, C (middle panel) and intruder (upper panel) states in the U(5)-PDS-CM calculation of Fig. 1 (c). In each panel, the left(right)-hand side displays the n_d-probabilities for the normal (intruder) components of the total wave function $|\Psi\rangle = a|\Psi_{\rm n}^{(N)}\rangle + b|\Psi_{\rm i}^{(N+2)}\rangle$.

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