NEUTRON–PROTON PAIRING CORRELATIONS AND DEFORMATION FOR $N = Z$ NUCLEI IN sd- AND pf-SHELL BY DEFORMED BCS AND DEFORMED QRPA *

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We investigate neutron–proton (np) pairing correlation effect on the shell evolution of ground-state energies along with the deformation for $N = Z$ nuclei in sd- and pf-shell, such as ^{24}Mg and ^{72}Kr. We start from a simple shell-filling model constructed by a deformed Woods–Saxon potential characterized as β_2 deformation, and then we include all kinds of pairing correlations in the residual interaction. In this work, like- and unlike-pairing correlation decomposed as isoscalar (IS) $T = 0$ and isovector (IV) $T = 1$ component are explicitly taken into account to estimate the ground-state energies. It turns out that the IS condensation can explain the oblate deformation for ^{72}Kr. We also test those effects on the Gamow–Teller (GT) transition for another $N = Z$ nucleus, ^{56}Ni, which explicitly exhibits the effects by the IS condensation and the deformation.

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1. Introduction

Deformation in nuclei is one of the key ingredients understanding nuclear structure. In particular, weakly-bound or neutron-rich nuclei show many interesting features related to the deformation [1, 2]. For instance, a single particle spectrum obtained by a deformed WS (DWS) potential is sensitive on the deformation parameter β_2 [3]. Beyond the mean field, pairing correlations play important roles in the nuclear structure. In particular, for $N = Z$

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nuclei, the np pairing may become significant because protons and neutrons occupy the same orbitals and have the maximum spatial overlap. The nn and pp pairing correlations have an IV spin-singlet $(T = 1, J = 0)$ mode, while the np pairing correlations have the IS spin-triplet $(T = 0, J = 1)$ as well as the IV spin-singlet mode. Over the last few decades, many discussions [4–8] have been done regarding the np pairing correlations, in particular, the coexistence of IS and IV correlation and their competition in some specific nuclear observables [9].

Recently, more interesting experimental data were reported, which reveal IV quenching in the M1 spin transition for $N = Z$ nuclei in sd-shell [10]. Since then, many interesting papers discussed the IV quenching and argued that the IS mode in the np pairing may become significant inside nuclei, similarly to the tensor force in the deuteron structure [11, 12], and give rise to IS condensation in nuclear symmetric matter.

The aim of the present work is to study the possibility whether we can understand the nuclear shape evolution of sd- and pf-shell nuclei in a deformed BCS (DBCS) approach by including all kinds of pairing correlations with the IV quenching phenomena [13, 14]. We also investigate how the IS condensation and the deformation affect the GT strength distributions for ^{56}Ni in pf-shell.

2. Formalism

We perform the DBCS calculation by using the deformed single particle (s.p.) wave functions obtained from a DWS potential [15]. Since the theoretical framework for the DBCS approach has been detailed in our previous papers [13, 16], we explain briefly some essential points in the formalism:

(i) The np pairing correlations change the conventional quasi-particle concept, *i.e.*, quasi-neutron and quasi-proton, to quasi-particle 1 and 2 which have mixed properties of the quasi-proton and quasi-neutron in the DBCS.

(ii) In a deformed basis representation, the quasi-particle states are mixed with different particle states because each deformed state (basis) is expanded by a linear combination of the spherical state (basis) [3]. In this respect, the DBCS is another representation of the Hartree–Fock–Bogoliubov (HFB) transformation in the spherical basis.

(iii) The pairing potentials for each state in the DBCS are calculated in the deformed basis by using G-matrix calculated from the realistic Bonn CD potential for nucleon–nucleon (N–N) interaction and adjusted to reproduce the empirical pairing gaps deduced from a five-point mass formula as shown in Table I.

TABLE I

Deformation parameter β_2^{E2} from the experimental E2 transition data [18] and theoretical β_2 by the Relativistic Mean Field (RMF) [19] and FRDM model [20] for ^{24}Mg and ^{72}Kr.

Nucleus	β_2^{E2} [18]	β_2^{RMF} [19]	β_2^{FRDM} [20]	Δ_p^{emp}	Δ_n^{emp}	$\delta_{np}^{\mathrm{emp}}$
^{24}Mg	0.605	0.416	0	3.123	3.193	1.844
^{72}Kr	0.330	−0.358	−0.366	2.001	1.985	1.353

(iv) We include $n\bar{p}$ and $p\bar{n}$ pairings in addition to the usual $p\bar{p}$ and $n\bar{n}$ pairing correlations. The np and $\bar{n}\bar{p}$ pairings in the same orbital (*e.g.* $|np, T = 0\rangle$ and $|\bar{n}\bar{p}, T = 0\rangle$) are taken into account implicitly by multiplying a factor 2 to the $T = 0$ matrices by the $n\bar{p}$ and $p\bar{n}$ pair.

(v) Because the experimental M1 spin data for $N = Z$ nuclei in *sd*-shell show that the IV contribution is much more quenched than the IS spin strength [10], we introduce a factor 1.5 to consider the evident enhanced IS pairing in the ground state of $N = Z$ nuclei [17]. As a result, we multiply a weighting factor $1.5 \times 2 = 3$ for the enhanced $T = 0$ pairing strength.

3. Results and discussion

The single particle states (SPSs) were obtained by the DWS potential with the optimal parameter set [15]. The particle model space for all the nuclei was exploited up to $N_0 = 5\hbar\omega$ for a deformed basis and up to $N = 10\hbar\omega$ for a spherical basis. In a simple shell-filling model, which means the occupation probability $v^2 = 1$ or 0 by no smearing, all particles are filled up to the outermost shell by allocating two particles in each deformed SPS. Figures 1 and 2 show ground-state energies (GSEs) by the simple shell-filling model for ^{24}Mg and ^{72}Kr, respectively[13, 14]. The shell evolution in a mean field Fig. 1 (a) was largely changed by the pairing correlations Fig. 1 (b), which makes a wide smearing of the Fermi surface. The GSE denoted as red circles in Fig. 1 (b) show two minima in each prolate and oblate region, contrary to the case without pairings in Fig. 1 (a). The lower energy than the GSE by the simple-shell model comes from change of occupation probabilities due to the smearing. Pairing energy, E_{pair}, without the np in Fig. 1 (b) becomes weaker along the prolate deformation, but becomes more or less stronger with the np pairing in Fig. 1 (c) and leads to more bound nuclei.

However, the pairing energy contribution to the total energy is relatively rather small comparing to the mean-field energy, so the total energy locates still in the oblate region. One remarkable point is a role of the enhanced $T = 0$ np pairing correlations shown in Fig. 1 (d), which makes the bounding a bit

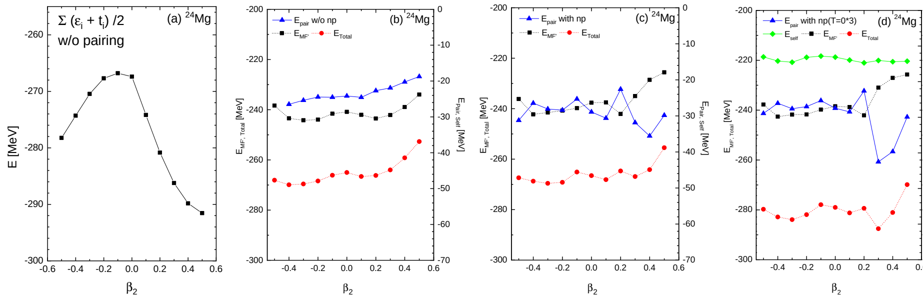

Fig. 1. (Color online) Ground-state energy (GSE) for ^{24}Mg by the DBCS model based on a deformed Woods–Saxon potential. E_{MF} is the mean-field energy with respect to the Fermi energy and E_{pair} is the pairing energy indicated in the right axis label. The pairing energies are estimated by three different cases, (b) without and (c) with the *np* pairing and (d) with the enhanced $T = 0$ pairing and the self-energy.

stronger due to its attractive property and leads to the prolate deformation in ^{24}Mg. The nucleon self-energy denoted as a light gray/green color does not affect so much the evolution of GSEs.

The energy minimum for ^{72}Kr in Fig. 2 (a) has a spherical shape without the pairing correlations. The unlike-pairing correlations do not affect so much the shell evolution by the like-pairing correlations and make only the nucleus more bound as shown in Fig. 2 (b) and (c). However, the enhanced $T = 0$ pairing shifts the deformation to some oblate deformation regions as confirmed in Fig. 2 (d). Our results of two nuclei give reasonable deformation minima, prolate for ^{24}Mg and oblate for ^{72} Kr.

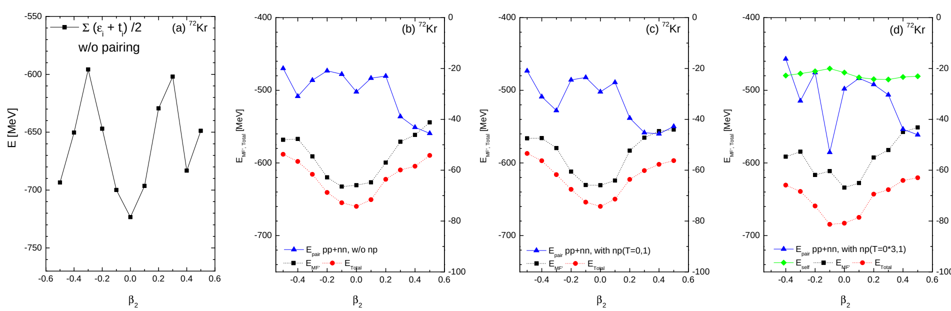

Fig. 2. (Color online) The same as Fig. 1, but for ^{72}Kr.

3.1. The GT transition strength distribution for ^{56}Ni

Figure 3 presents GT strength distributions for ^{56}Ni(p,n) reaction by our Deformed QRPA (DQRPA) [3]. In the left panel, the more deformation scatters the distribution to a bit higher energy regions because of the repulsive particle–hole (p–h) interaction. However, the two peaks peculiar to this GT distribution data were not well-reproduced by the deformation. That is, the 2$^{\text{nd}}$ high energy peak does not appear enough to explain the data. In the right panel, the np pairing effects are shown to push the distribution to the higher energy region even without the deformation. Contrary to the p–h repulsive force, the np pairing is mainly attractive, by which the Fermi energy difference of protons and neutrons is reduced by its attractive interaction and, consequently, gives rise to the high-lying GT transition between more deeply bound neutrons and protons SPSs [22]. As a result, the two peaks and their magnitudes appear explicitly by the np pairing.

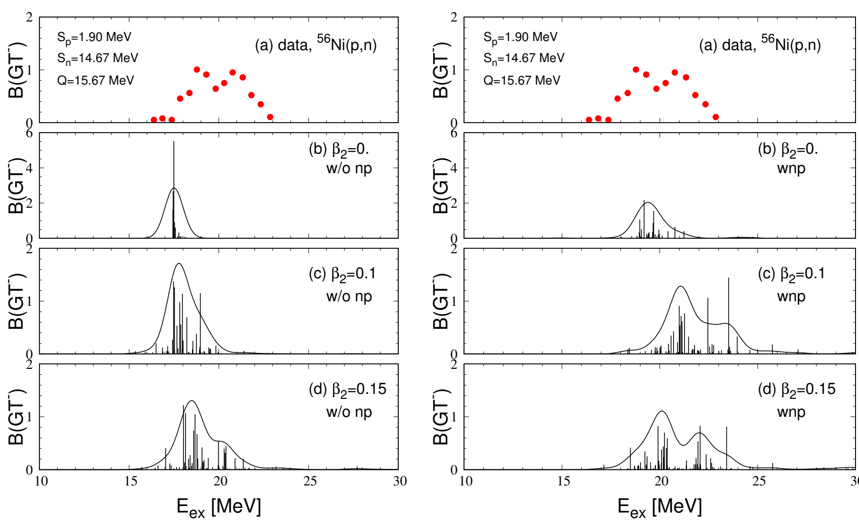

Fig. 3. (Color online) The Gamow–Teller (GT(−)) transition strength distribution B(GT(−)) of ^{56}Ni. Experimental data by ^{56}Ni(p,n) in panels (a) are from Ref. [21]. Results of (b)–(d) in the left- (right-) hand side are without (with) the np pairing. Results are presented by the excitation energy from the parent nucleus.

4. Summary and conclusion

We study the shape evolution of two $N = Z$ nuclei, ^{24}Mg and ^{72}Kr in the DWS and DBCS approximation taking into account both $T = 0$ and $T = 1$ pairing correlations. The shape evolution by a simple-filling model depends mainly on the shell structure of SPS energies near to the last occupied orbit. However, the pairing interactions by the residual interaction changes significantly the evolution, particularly, with the enhanced $T = 0$ pairing. We find a coexistence of two types of superconductivities, IS and IV, at the large deformation region in ^{24}Mg and ^{72}Kr with the enhanced $T = 0$ pairing [13, 14]. In order to further discuss effects of the IS and IV np-pairing correlations and the deformation, we calculated the GT strength distribution of ^{56}Ni, which is another $N = Z$ nucleus. The np pairing effects turn out to be able to properly explain the GT strength although the deformation is also another important property.

In conclusion, the IS spin-singlet mode, which contributes more or less to the deformation property due to its coupling to odd-J states, may give rise to more microscopic deformation features which cannot be included in the deformed mean-field approach and play important role in the ground-state properties. The GT transition as well as the M1 spin transition strengths are shown to be also useful for investigating the IS and IV pairing properties inside nuclei.

REFERENCES

[1] I. Hamamoto, *Phys. Rev. C* **93**, 054328 (2016).
[2] I. Hamamoto, *Phys. Scr.* **91**, 023004 (2016).
[3] Eunja Ha, Myung-Ki Cheoun, *Nucl. Phys. A* **934**, 73 (2015).
[4] H.T. Chen, A. Goswami, *Phys. Lett. B* **24**, 257 (1967).
[5] H.H. Wolter, A. Faessler, P.U. Sauer, *Phys. Lett. B* **31**, 516 (1970).
[6] A.L. Goodman, *Phys. Rev. C* **58**, R3051 (1998).
[7] F. Šimkovic *et al.*, *Phys. Rev. C* **68**, 054319 (2003).
[8] S. Frauendorf, A.O. Miacchiavelli, *Prog. Part. Nucl. Phys.* **78**, 24 (2014).
[9] A. Gezerlis *et al.*, *Phys. Rev. Lett.* **106**, 252502 (2011).
[10] H. Matsubara *et al.*, *Phys. Rev. Lett.* **115**, 102501 (2015).
[11] E. Garrido *et al.*, *Phys. Rev. C* **60**, 064312 (1999).
[12] E. Garrido *et al.*, *Phys. Rev. C* **63**, 037304 (2001).
[13] Eunja Ha, Myung-Ki Cheoun, H. Sagawa, *Phys. Rev. C* **97**, 024320 (2018).
[14] Eunja Ha, Myung-Ki Cheoun, H. Sagawa, W.Y. So, *Phys. Rev. C* **97**, 064322 (2018).
[15] S. Cwiok *et al.*, *Comput. Phys. Commun.* **46**, 379 (1987).
[16] Eunja Ha, Myung-Ki Cheoun, F. Šimkovic, *Phys. Rev. C* **92**, 044315 (2015).
[17] H. Sagawa, T. Suzuki, M. Sasano, *Phys. Rev. C* **94**, 041303(R) (2016).
[18] S. Raman *et al.*, *At. Data Nucl. Data Tables* **78**, 1 (2001).
[19] G.A. Lalazissis *et al.*, *At. Data Nucl. Data Tables* **71**, 1 (1999).
[20] P. Möller, A.J. Sierk, T. Ichikawa, H. Sagawa, *At. Data Nucl. Data Tables* **109–110**, 1 (2016).
[21] M. Sasano *et al.*, *Phys. Rev. Lett.* **107**, 202501 (2011).
[22] Eunja Ha, Myung-Ki Cheoun, *Eur. Phys. J. A* **53**, 26 (2017).