MASSES OF THE DOUBLY-HEAVY TETRAQUARKS IN A CONSTITUENT QUARK MODEL*

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We analyze the masses of the doubly-heavy tetraquark states T_{QQ} using the variational method. We also find that our full model calculations give, in general, smaller binding energy, compared to those from the simplified quark model that treats quark dynamics inside the tetraquark the same as that inside a baryon. We investigate the main origin of this weaker binding energy. The original work is described in W. Park, S. Noh, S.H. Lee, *Nucl. Phys. A* **983**, 1 (2019).

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1. Introduction

With the recent discovery of the doubly-charmed baryon Ξ_{cc}^{++} [1, 2] and its decay [3], there is a special interest in the doubly-heavy tetraquark $T_{QQ}(QQ\bar{u}\bar{d})$ state with isospin zero and, in particular, with the quantum number $I(J^P) = 0(1^+)$ [4–6]. First of all, this particle is a flavor exotic tetraquark, which has never been observed before. Second, the recent discovery raises the chances of observing a similar hadron such as T_{QQ}. Finally, this particle is the only candidate for a compact configuration. This is so because the proposed quark structure of T_{QQ} state, $QQ\bar{q}\bar{q}$, favors a compact tetraquark configuration as the additional $\bar{q}\bar{q}$ in the isospin zero channel provides an attraction larger than that for the two separated meson configuration [7–10]. In this work, we will perform a detailed quark model analysis of the tetraquark state after we fix the fitting parameters.

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2. Formalism

We use a nonrelativistic Hamiltonian for the constituent quarks of the following form [11]:

$$H=\sum_{i=4}^{4}\left(m_i+\frac{\boldsymbol{p}_i^2}{2m_i}\right)-\frac{3}{4}\sum_{i<j}^{4}\frac{\lambda_i^{\rm c}}{2}\frac{\lambda_j^{\rm c}}{2}\left(V_{ij}^{\rm C}+V_{ij}^{\rm CS}\right), \tag{1}$$

where m_i are the quark masses and $\lambda_i^{\rm c}/2$ is the color operator of the $i^{\rm th}$ quark for color SU(3). For the internal quark potentials $V_{ij}^{\rm C}$ and $V_{ij}^{\rm CS}$, we adopt the following forms [11, 12]:

$$V_{ij}^{\rm C}=-\frac{\kappa}{r_{ij}}+\frac{r_{ij}}{a_0^2}-D\,,\qquad V_{ij}^{\rm CS}=\frac{\hbar^2c^2\kappa'}{m_im_jc^4}\frac{e^{-(r_{ij})^2/(r_{0ij})^2}}{(r_{0ij})r_{ij}}\sigma_i\cdot\sigma_j\,. \tag{2}$$

Here

$$r_{0ij}=1/\left(\alpha+\beta\frac{m_im_j}{m_i+m_j}\right),\qquad \kappa'=\kappa_0\left(1+\gamma\frac{m_im_j}{m_i+m_j}\right), \tag{3}$$

where $r_{ij}=|\boldsymbol{r}_i-\boldsymbol{r}_j|$ is the distance between the quarks i and j of masses m_i and m_j, respectively, and σ_i is the spin operator. Here, we introduce additional mass dependence as in Eq. (3) so the hyperfine splitting between heavy quarks are larger than that between lighter quarks obtained with the constituent quark masses appearing only as an overall factor in the denominator of $V_{ij}^{\rm CS}$ in Eq. (2).

For the parameters appearing in Eqs. (2)–(3), we fix them to fit the masses of hadrons involving heavy quarks relevant to the stability of T_{QQ}. Up to now, no hadrons with two heavy quarks were found so fitting the mass of Ξ_{cc}^{++} provides a crucial input to study other configurations involving two heavy quarks [13]. The parameters are referred to Eq. (7) in Ref. [13], and the fitting hadrons are listed in Tables 1, 2 in Ref. [13].

The basis of the Hamiltonian is determined to satisfy the symmetry constraint due to the Pauli principle, and constructed by combining the color–spin basis with the spatial part. The results are presented in Table 3 in Ref. [13].

3. Comparison with a simple quark model

It is essential to perform a full constituent quark model analysis to calculate the mass and binding energy of a multiquark configuration. In a simple quark model, as discussed in Ref. [14], the mass of a hadron is typically composed of the sum of effective constituent quark masses, the hyperfine interaction, and a possible binding energy for heavier quarks. We want to identify the origin of the effective constituent quark mass and the binding energy used in the simple model from our full model calculation, and then investigate whether it is sensible to extrapolate these concepts to higher multiquark configurations.

In the simple constituent quark model, the mass of Λ_c, Ξ_{cc} can be obtained from the following formula [14]:

$$\begin{aligned} M_{\Lambda_c} &= 2m_q^b + m_c^b - \frac{3a}{\left(m_q^b\right)^2}, \\ M_{\Xi_{cc}} &= 2m_c^b + B(cc) + m_q^b + \frac{a_{cc}}{\left(m_c^b\right)^2} - \frac{4a}{m_q^b m_c^b}, \end{aligned} \tag{4}$$

where $m_{c,q}^b$ are the constituent quark masses for the charm and light quark inside a baryon, $B(cc)$ is the binding between the charm quarks, and $a(a_{cc})$ are the multiplicative constants for the color–spin interaction. Treating $B(cc)$ as part of the two charm quark system, one can dive the energy into the charm quark, light quark and color–spin (CS) interaction parts.

The constant $-D$ term appearing in Eq. (2) is divided into each quark by multiplying a factor of 1/2. This is so because when the total hadron is a color singlet, the total color factors contribute equally for all quarks involved. For the kinetic terms, when they involve the quark pairs, it is included in the corresponding quark pairs. For the relative kinetic energy involving $\boldsymbol{p}_{\boldsymbol{\lambda}}$, it is divided according to their relative contribution depending on the mass of either the quark pair or the single quark.

The comparisons of the values for $\Xi_{cc}(\Lambda_c)$, which are in the third and fifth column in Tables I and II, can be summarized into the following important conclusion: The constituent quark masses and the binding energy as needed in Eq. (4) should be the sum of the quark mass, the relevant kinetic term, and all the relevant interaction terms in the full model, which indeed seems to approximately reproduce the simple constituent quark mass value; compare the subtotal values in q or c quark part in each table.

Now, let us see what happens when we try to build up a similar table for T_{cc}. According to the simple constituent quark model, the mass is given as [15]

$$M_{T_{cc}} = 2m_c^b + B(cc) + 2m_q^b + \frac{a_{cc}}{\left(m_c^b\right)^2} - \frac{3a}{\left(m_q^b\right)^2}. \tag{5}$$

Apart from the color–spin interaction part, it is the sum of the subtotal mass of (cc) pair in Ξ_{cc} and (qq) pair in Λ_c. This is so because the color spin state of (cc) pair in Ξ_{cc} is the same as that of the charge conjugated $(\bar{c}\bar{c})$ pair in the lowest energy component of $ud\bar{c}\bar{c}$ state. Similarly, the color spin

TABLE I

Division of the Ξ_{cc}^{++} mass. Here, (i,j) denotes the i and j quarks, where $i = 1,2$ are for c quark, and 3 for the light quark. V^{C} = Coulomb+linear interaction, $\sum V^{\mathrm{C}}(i,j) \equiv [V^{\mathrm{C}}(1,3) + V^{\mathrm{C}}(2,3)]$, $m'_1 = m_c$, $m'_2 = \frac{3m_q m_c}{m_q+2m_c}$, $\boldsymbol{p_\sigma} = m'_1\dot{\boldsymbol{\sigma}}$, $\boldsymbol{p_\lambda} = m'_2\dot{\boldsymbol{\lambda}}$, and $\boldsymbol{\sigma} = \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$, $\boldsymbol{\lambda} = \frac{1}{\sqrt{6}}(\boldsymbol{r}_1 + \boldsymbol{r}_2 - 2\boldsymbol{r}_3)$. All the values are expressed in MeV unit with $c = 1$.

Overall	Present Work		Karliner and Rosner [14]	
	Contribution	Value	Contribution	Value
c-quark	$2m_c$	3836.0	$2m_c^b$	3421.0
	$\frac{\boldsymbol{p}_{\boldsymbol{\sigma}}^2}{2m'_1}$	243.6		
	$\frac{m_q}{m_q+2m_c}\frac{\boldsymbol{p}_{\boldsymbol{\lambda}}^2}{2m'_2}$	32.5		
	$V^{\mathrm{C}}(1,2)$	5.6	$B(cc)$	-129.0
	$\frac{1}{2}\sum V^{\mathrm{C}}(i,j)$	156.0		
	$-D$	-983.0		
Subtotal		3290.7		3292.0
q-quark	m_q	326.0	m_q^b	363.0
	$\frac{2m_c}{m_q+2m_c}\frac{\boldsymbol{p}_{\boldsymbol{\lambda}}^2}{2m'_2}$	382.2		
	$\frac{1}{2}\sum V^{\mathrm{C}}(i,j)$	156.0		
	$-\frac{1}{2}D$	-491.5		
Subtotal		372.7		363.0
CS	$-4a/(m_q m_c)$	-58.8	$-4a/\left(m_q^b m_c^b\right)$	-42.4
	$a_{cc}/(m_c)^2$	7.8	$a_{cc}/\left(m_c^b\right)^2$	14.2
Subtotal		-51.0		-28.2
Total		3612.4		3626.8

state of (ud) pair is the same as that of the light quark pair in the lowest energy component of $ud\bar{c}\bar{c}$ state. Table III shows the numbers in our model calculations. The estimate in the fourth column is obtained by taking such prescription in our model. As can be seen in the table, the estimate is much closer to the simple quark model result of Ref. [15]. On the other hand, the value from the full model calculation in column three, except for $\frac{\boldsymbol{p}_{\boldsymbol{\sigma}'}^2}{2m'_2}$, is systematically larger than the other estimates. The change comes from the slight differences in the potentials. The dominant change comes from the kinetic energy of the relative momentum $\boldsymbol{p}_{\boldsymbol{\lambda}}$. The magnitude of this relative

TABLE II

Division of the Λ_c^+ mass from our work. Notations are similar as in Table I. $i = 1, 2$ labels the light quarks, and 3 c. $m_1' = m_q$, $m_2' = \frac{3m_q m_c}{2m_q + m_c}$, $\boldsymbol{p_\sigma} = m_1'\dot{\boldsymbol{\sigma}}$, $\boldsymbol{p_\lambda} = m_2'\dot{\boldsymbol{\lambda}}$ and $\boldsymbol{\sigma} = \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$, $\boldsymbol{\lambda} = \frac{1}{\sqrt{6}}(\boldsymbol{r}_1 + \boldsymbol{r}_2 - 2\boldsymbol{r}_3)$. All the values are expressed in MeV unit with $c = 1$.

Overall	Present Work		Karliner and Rosner [14]	
	Contribution	Value	Contribution	Value
q-quark	$2m_q$	652.0	$2m_q^b$	726.0
	$\frac{\boldsymbol{p}_{\boldsymbol{\sigma}}^2}{2m_1'}$	501.7		
	$\frac{m_c}{2m_q+m_c}\frac{\boldsymbol{p}_{\boldsymbol{\lambda}}^2}{2m_2'}$	221.0		
	$V^{\mathrm{C}}(1,2)$	219.9		
	$\frac{1}{2}\sum V^{\mathrm{C}}(i,j)$	176.0		
	$-D$	−983.0		
Subtotal		787.6		726.0
c-quark	m_c	1918.0	m_c^b	1710.5
	$\frac{2m_q}{2m_q+m_c}\frac{\boldsymbol{p}_{\boldsymbol{\lambda}}^2}{2m_2'}$	75.1		
	$\frac{1}{2}\sum V^{\mathrm{C}}(i,j)$	176.0		
	$-\frac{1}{2}D$	−491.5		
Subtotal		1677.6		1710.5
CS	$-3a/(m_q)^2$	−181.8	$-3a/\left(m_q^b\right)^2$	−150.0
Total		2283.4		2286.5

kinetic energy and its contribution to each pair depend on the masses of each pair and cannot be extrapolated from an estimate obtained within a hadron of different quark numbers. This is related to the changes of the effective constituent quark mass in a simple quark model, which indeed shows the need for different values for the quark masses depending on whether they are inside a meson or a baryon. Reference [14] finds that the constituent quark masses (with $c = 1$) to be $m_q^m = 310$ MeV, $m_s^m = 483$ MeV, $m_c^m = 1663.3$ MeV to fit the meson spectrum, while they are $m_q^b = 363$ MeV, $m_s^b = 538$ MeV, $m_c^b = 1710.5$ MeV to fit the baryon spectrum. The trend of needing larger masses when it is inside configurations with larger constituent seems also to be true when one goes to the tetraquark configuration as can be seen in our full model calculation shown in Table III. Our estimates are systematically larger than the simple quark model estimates.

TABLE III

Division of the $T_{cc}(ud\bar{c}\bar{c})$ mass. (i,j) denotes the i and j quarks, where $i = 1,2$ labels the light quarks, and 3, 4 are for $\bar{c}$. $\sum V^{\mathrm{C}}(i,j) \equiv \left[V^{\mathrm{C}}(1,3)+V^{\mathrm{C}}(1,4)+V^{\mathrm{C}}(2,3)+V^{\mathrm{C}}(2,4)\right]$, $m'_1 = m_q$, $m'_2 = m_c$, $m'_3 = \frac{2m_q m_c}{m_q+m_c}$, $\boldsymbol{p_\sigma} = m'_1\dot{\boldsymbol{\sigma}}$, $\boldsymbol{p_{\sigma'}} = m'_2\dot{\boldsymbol{\sigma}}'$, $\boldsymbol{p_\lambda} = m'_3\dot{\boldsymbol{\lambda}}$ and $\boldsymbol{\sigma} = \frac{1}{\sqrt{2}}(\boldsymbol{r}_1-\boldsymbol{r}_2)$, $\boldsymbol{\sigma}' = \frac{1}{\sqrt{2}}(\boldsymbol{r}_3-\boldsymbol{r}_4)$, $\boldsymbol{\lambda} = \frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2-\boldsymbol{r}_3-\boldsymbol{r}_4)$. The estimate in the fourth column is obtained by taking the c-quark pair value of Ξ_{cc} and q-quark pair value of Λ_c from our model calculation. All the values are expressed in MeV unit with $c=1$.

Overall	Present Work			Karliner and Rosner [15]	
	Contribution	Value	Estimate	Contribution	Value
c-quark	$2m_c$	3836.0	3836.0	$2m_c^b$	3421.0
	$\frac{\boldsymbol{p}^2_{\boldsymbol{\sigma}'}}{2m'_2}$	231.4	243.6		
	$\frac{m_q}{m_c+m_q}\frac{\boldsymbol{p}^2_{\boldsymbol{\lambda}}}{2m'_3}$	41.1	32.5		
	$V^{\mathrm{C}}(3,4)$	15.6	5.6	$B(cc)$	−129.0
	$\frac{1}{2}\sum V^{\mathrm{C}}(i,j)$	187.0	156.0		
	$-D$	−983.0	−983.0		
Subtotal		3328.1	3290.7		3292.0
q-quark	$2m_q$	652.0	652.0	$2m_q^b$	726.0
	$\frac{\boldsymbol{p}^2_{\boldsymbol{\sigma}}}{2m'_1}$	501.7	501.7		
	$\frac{m_c}{m_c+m_q}\frac{\boldsymbol{p}^2_{\boldsymbol{\lambda}}}{2m'_3}$	241.9	221.0		
	$V^{\mathrm{C}}(1,2)$	219.9	219.9		
	$\frac{1}{2}\sum V^{\mathrm{C}}(i,j)$	187.0	176.0		
	$-D$	−983.0	−983.0		
Subtotal		819.5	787.6		726.0
CS	$a_{cc}/(m_c)^2$	7.5	7.8	$a_{cc}/\left(m_c^b\right)^2$	14.2
	$-3a/(m_q)^2$	−181.8	−181.8	$-3a/\left(m_q^b\right)^2$	−150.0
Subtotal		−174.3	−174.0		−135.8
Total		3973.3	3904.3		3882.2

4. Summary

The simplified model was based on approximating the tetraquark mass to be the sum of the constituent quark masses and the hyperfine interaction. In this work, we have performed a full constituent quark model calculation with specific potentials, and using the variational method with one Gaussian to fit the mass of Ξ_{cc} and other related hadrons. With the conclusion of comparing the results of the two approaches for the masses of Λ_c and Ξ_{cc} in Ref. [13], we have found that the tetraquark masses calculated in our model are systematically larger than those estimated in the simple model calculations. Comparing our full model calculations for $ud\bar{c}\bar{c}$ to that from the simplified model approach, we also found that when using a simplified quark model, it is necessary to introduce a slightly larger constituent quark mass in the tetraquark configuration than in the baryon. This is consistent with the trend where one needs a larger constituent quark mass in the baryon than in the meson.

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