GLUON TMDs FROM FORWARD pA COLLISIONS IN THE CGC*

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We calculate the production of a photon and two jets at forward rapidity in proton–nucleus collisions within the hybrid dilute-dense framework in the Color Glass Condensate (CGC) formalism. After obtaining the cross section for the quark-initiated channel, we consider the correlation limit, in which the vector sum of the transverse momenta of the three outgoing particles is small with respect to the individual transverse momenta. In this limit, the cross section simplifies considerably and can be written in a factorized form, sensitive to various unpolarized and linearly-polarized transverse-momentum-dependent gluon distribution functions (gluon TMDs).

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1. Introduction

Within the Color Glass Condensate (CGC) framework, the hybrid formalism [1] is used to study single inclusive particle production at next-to-leading order [2–10] and heavy-quark production [11] at forward rapidity. In this set-up, the wave function of the projectile proton is treated in the spirit

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of collinear factorization[1]. Perturbative corrections to this wave function are provided by the usual QCD splitting processes. On the other hand, the dense target is treated in the CGC, *i.e.* it is defined as a distribution of strong color fields, which during the scattering transfer transverse momentum to the propagating partonic configuration.

Recently, a lot of effort has been dedicated to the understanding of gluon TMDs in the CGC. In particular, it was shown in [13] that nonlinear corrections are intimately connected to the process dependence of the gluon TMDs. Various processes at small x which involve two scales have been studied, such as dijet, heavy-quark pair or photon–jet production in electron–nucleus or proton–nucleus collisions [14–20], see [21] for a recent review. The usual approach is to calculate the cross section in the CGC, after which one takes the small dipole or correlation limit which corresponds to a leading-power expansion in the ratio p_{t}^2/Q^2 of the hard scales with $\Lambda_{\mathrm{QCD}}^2 \ll p_{\mathrm{t}}^2 \ll Q^2$. On the one hand, the analysis of the CGC in terms of gluon TMDs allows for a better understanding of QCD dynamics at high energy. On the other hand, once a CGC expression is obtained for a gluon TMD in the small-x limit, its nonlinear evolution in rapidity can be computed with the help of the JIMWLK [13] or BK [22] equations. In addition, non-perturbative models such as the McLerran–Venugopalan (MV) [23] or Golec-Biernat–Wüsthoff (GBW) model [24] can be used to write down analytical expressions.

2. Production cross section

We are interested in the production of a hard photon with transverse momenta $q_1 \gg Q_{\mathrm{s}}$ and two hard jets with transverse momenta $q_2, q_3 \gg Q_{\mathrm{s}}$ in forward pA collisions. This process can be studied in two different channels: *(i)* quark-initiated channel where the incoming quark emits a photon and a gluon at $O(g_e g_s)$, and *(ii)* gluon-initiated channel where the incoming quark splits into quark–antiquark pair at the order of $O(g_s)$ and the final-state photon is emitted from either the quark or from the antiquark at $O(g_e g_s)$. Here, for the sake of simplicity, we restrict ourselves to the study of the quark-initiated channel. However, the set-up and the arguments used for this channel can be used for the study of the gluon channel (see [25] for the complete analysis).

The total production cross section for this process in the quark-initiated channel is written as convolution of the partonic cross section and the quark distribution function $f_q(x_p, \mu^2)$ inside the proton

$$\frac{\mathrm{d}\sigma^{pA\to\gamma qg+X}}{\mathrm{d}^3\underline{q}_1\,\mathrm{d}^3\underline{q}_2\,\mathrm{d}^3\underline{q}_3} = \int \mathrm{d}x_p\, f_q\left(x_p, \mu^2\right)\, \frac{\mathrm{d}\sigma^{qA\to\gamma qg+X}}{\mathrm{d}^3\underline{q}_1\,\mathrm{d}^3\underline{q}_2\,\mathrm{d}^3\underline{q}_3}\,, \tag{1}$$

[1] Multiple collinear scattering effects from the projectile protons have been studied in [12].

where x_p is the longitudinal momentum fraction carried by the incoming quark, μ^2 is the factorization scale, and the three-momenta $\underline{q}_i \equiv (q_i^+, q_i)$ are the longitudinal and transverse momenta of the produced jets. The partonic level cross section is formally defined as the expectation value of the number operator in the outgoing state

$$\begin{aligned}&(2\pi)^9 \frac{\mathrm{d}\sigma^{qA\to\gamma qg+X}}{\mathrm{d}^3\underline{q}_1\,\mathrm{d}^3\underline{q}_2\,\mathrm{d}^3\underline{q}_3}(2\pi)\,\delta\left(p^+ - \sum_{i=1}^{3} q_i^+\right)\\ &= \frac{1}{2N_\mathrm{c}}\sum_{s,\alpha}{}_{\mathrm{out}}\left\langle (\boldsymbol{q})\,\left[p^+,0\right]_s^\alpha \middle| O\left(\underline{q}_1,\underline{q}_2,\underline{q}_3\right) \middle| (\boldsymbol{q})\,\left[p^+,0\right]_s^\alpha \right\rangle_{\mathrm{out}}, \qquad (2)\end{aligned}$$

where the normalization $1/2N_\mathrm{c}$ comes from averaging over the spin and color indices of the outgoing state in the amplitude and in the complex conjugate amplitude. The number operator is built up in terms of creation/annihilation operators of the final-state particles. In the quark-initiated channel, the final-state particles are a quark, a gluon and a photon. Therefore, for this channel, the number operator reads

$$O\left(\underline{q}_1,\underline{q}_2,\underline{q}_3\right) = \gamma_\lambda^\dagger\left(\underline{q}_1\right)\gamma_\lambda\left(\underline{q}_1\right)\, a_i^{\dagger b}\left(\underline{q}_2\right) a_i^b\left(\underline{q}_2\right) b_t^{\dagger\beta}\left(\underline{q}_3\right) b_t^\beta\left(\underline{q}_3\right). \qquad (3)$$

In the above expression, $\gamma_\lambda(\underline{q}_1)$ is the annihilation operator of a dressed photon with polarization λ and three-momentum $\underline{q}_1$, $a_i^b(\underline{q}_2)$ is the one for a dressed gluon with color b, polarization i, and three-momentum $\underline{q}_2$ and, finally, $b_t^\beta(\underline{q}_3)$ is the annihilation operator ofthe dressed quark with color β, spin t, and three-momentum $\underline{q}_3$. The action of these creation and annihilation operators on the dressed states are defined in the standard way. For example, in the mixed longitudinal momentum and transverse coordinate space, the action of the gluon operators are

$$\begin{aligned} a_i^b\left(q_2^+, z_2\right)\left|(\boldsymbol{g})\left[k_2^+, x_2\right]_\eta^c\right\rangle_D &= 2\pi\,\delta^{bd}\delta_{i\eta}\,\delta\left(k_2^+ - q_2^+\right)\delta^{(2)}(x_2 - z_2)\,, &(4)\\ a_i^{\dagger b}\left(q_2^+, y_2\right)|0\rangle &= \left|(\boldsymbol{g})\left[q_2^+, y_2\right]_i^b\right\rangle_D\,. &(5)\end{aligned}$$

In order to evaluate the partonic cross section for the quark-initiated channel defined in Eq. (2), the crucial ingredient is to derive the outgoing state. The derivation of the outgoing state has been presented in [25, 26]. Here, we will only present the sketch of this derivation. The first step to compute the outgoing state is to calculate the dressed-quark state at $O(g_e g_s)$. In full momentum space, the dressed-quark state with longitudinal momentum p^+, vanishing transverse momenta, color α and spin s is given in terms of the bare states as

$$
\begin{aligned}
&\left|(\boldsymbol{q})\left[p^+,0\right]^{\alpha}_{s}\right\rangle_D = Z^q\left|(\boldsymbol{q})\left[p^+,0\right]^{\alpha}_{s}\right\rangle_0 \\
&+Z^{q\gamma}\,g_e\sum_{s',\lambda}\int\frac{\mathrm{d}k_1^+}{2\pi}\frac{\mathrm{d}^2k_1}{(2\pi)^2}\,F^{(1)}_{(\boldsymbol{q}\boldsymbol{\gamma})}\left[(\boldsymbol{\gamma})\left[k_1^+,k_1\right]^{\lambda},(\boldsymbol{q})\left[p^+-k_1^+,-k_1\right]_{ss'}\right] \\
&\times\left|(\boldsymbol{q})\left[p^+-k_1^+,-k_1\right]^{\alpha}_{s'};(\gamma)\left[k_1^+,k_1\right]_{\lambda}\right\rangle_0 \\
&+Z^{qg}\,g_s\sum_{s',\eta}\int\frac{\mathrm{d}k_2^+}{2\pi}\frac{\mathrm{d}^2k_2}{(2\pi)^2}\,t^c_{\alpha\beta}\,F^{(1)}_{(\boldsymbol{q}\boldsymbol{g})}\left[(\boldsymbol{g})\left[k_2^+,k_2\right]^{\eta},(\boldsymbol{q})\left[p^+-k_2^+,-k_2\right]_{ss'}\right] \\
&\times\left|(\boldsymbol{q})\left[p^+-k_2^+,-k_2\right]^{\beta}_{s'};(\boldsymbol{g})\left[k_2^+,k_2\right]^{c}_{\eta}\right\rangle_0 \\
&+Z^{qg\gamma}\,g_sg_e\sum_{s's''}\sum_{\lambda\eta}\int\frac{\mathrm{d}k_1^+}{2\pi}\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}k_2^+}{2\pi}\frac{\mathrm{d}^2k_2}{(2\pi)^2}\,t^c_{\alpha\beta} \\
&\times\Big\{F^{(2)}_{(\boldsymbol{q}\boldsymbol{\gamma}-\boldsymbol{q}\boldsymbol{g})}\left[(\gamma)\left[k_1^+,k_1\right]^{\lambda},(\boldsymbol{g})\left[k_2^+,k_2\right]^{\eta},(\boldsymbol{q})\left[p^+-k_1^+-k_2^+,-k_1-k_2\right]_{ss''}\right] \\
&+F^{(2)}_{(\boldsymbol{q}\boldsymbol{g}-\boldsymbol{q}\boldsymbol{\gamma})}\left[(\boldsymbol{g})\left[k_2^+,k_2\right]^{\eta},(\gamma)\left[k_1^+,k_1\right]^{\lambda},(\boldsymbol{q})\left[p^+-k_2^+-k_1^+,-k_2-k_1\right]_{ss''}\right]\Big\} \\
&\times\left|(\boldsymbol{q})\left[p^+-k_1^+-k_2^+,-k_1-k_2\right]^{\beta}_{s''},(\boldsymbol{g})\left[k_2^+,k_2\right]^{c}_{\eta},(\gamma)\left[k_1^+,k_1\right]^{\lambda}\right\rangle_0\,. \qquad (6)
\end{aligned}
$$

Here, Z^q, $Z^{q\gamma}$, Z^{qg} and $Z^{qg\gamma}$ are the normalization functions. In the wave function approach, they provide the virtual contributions to the production process, and are determined by using the orthogonality conditions of the states. Since we are focused on the tree-level production of a hard photon and two hard jets, the explicit expressions for these normalization functions are not relevant for our purposes, and they can be set to identity.

The functions $F^{(1)}_{(\boldsymbol{q}\boldsymbol{\gamma})}$ and $F^{(1)}_{(\boldsymbol{q}\boldsymbol{g})}$ define the momentum structure of the quark–photon and quark–gluon splittings. The quark–photon splitting function reads (see, for example, [26])

$$
\begin{aligned}
&F^{(1)}_{(\boldsymbol{q}\boldsymbol{\gamma})}\left[(\gamma)[k_1^+,k_1]^{\lambda},(\boldsymbol{q})\left[p^+-k_1^+,p-k_1\right]_{ss'}\right] \\
&=\left[\frac{-1}{\sqrt{2\xi_1p^+}}\right]\phi^{\lambda\bar{\lambda}}_{ss'}(\xi_1)\frac{(\xi_1p-k_1)^{\bar{\lambda}}}{(\xi_1p-k_1)^2} \qquad (7)
\end{aligned}
$$

with

$$
\phi^{\lambda\bar{\lambda}}_{ss'}(\xi_1)=\left[(2-\xi_1)\delta^{\lambda\bar{\lambda}}\delta_{ss'}-i\epsilon^{\lambda\bar{\lambda}}\sigma^3_{ss'}\xi_1\right], \qquad (8)
$$

where σ^3 is the third Pauli matrix and where we have defined the longitudinal momentum ratio $\xi_1\equiv k_1^+/p^+$. The function $F^{(1)}_{(\boldsymbol{q}\boldsymbol{g})}$ has the same structure as $F^{(1)}_{(\boldsymbol{q}\boldsymbol{\gamma})}$, and its explicit expression can be read off from Eq. (7) by exchanging $(1\to 2)$.

The functions $F^{(2)}_{(\boldsymbol{q}\gamma-\boldsymbol{qg})}$ and $F^{(2)}_{(\boldsymbol{qg}-\boldsymbol{q}\gamma)}$ in Eq. (6) define the momentum structure of successive quark–photon and quark–gluon splittings, and differ in the sequence of the emissions (see Fig. 1). The explicit expression for $F^{(2)}_{(\boldsymbol{q}\gamma-\boldsymbol{qg})}$, in which the photon is emitted before the gluon, reads (see, for example, [26])

$$\begin{aligned} & F^{(2)}_{(\boldsymbol{q}\gamma-\boldsymbol{qg})}\left[(\boldsymbol{p})\left[k_1^+,k_1\right]^\lambda,(\boldsymbol{g})\left[k_2^+,k_2\right]^\eta,(\boldsymbol{q})\left[p^+-k_1^+-k_2^+,p-k_1-k_2\right]_{ss''}\right] \\ & = \sum_{s'}\left[\frac{-1}{\sqrt{2\xi_1 p^+}}\phi^{\lambda\bar{\lambda}}_{ss'}(\xi_1)\right]\left[\frac{-1}{\sqrt{2\xi_2 p^+}}\tilde{\phi}^{\eta\bar{\eta}}_{s's''}(\xi_1,\xi_2)\right]\frac{(\xi_1 p-k_1)^{\bar{\lambda}}}{(\xi_1 p-k_1)^2} \\ & \times\frac{\left[\xi_2(p-k_1)-\bar{\xi}_1 k_2\right]^{\bar{\eta}}}{\xi_2(\xi_1 p-k_1)^2+\xi_1(\xi_2 p-k_2)^2-(\xi_2 k_1-\xi_1 k_2)^2} \end{aligned} \tag{9}$$

with

$$\tilde{\phi}^{\eta\bar{\eta}}_{s's''}(\xi_1,\xi_2)=\frac{\xi_1}{\bar{\xi}_1}\left[\left(2\bar{\xi}_1-\xi_2\right)\delta^{\eta\bar{\eta}}\delta_{s's''}-i\epsilon^{\eta\bar{\eta}}\sigma^3_{s's''}\xi_2\right], \tag{10}$$

where the ratios of longitudinal momenta are defined as

$$\xi_1\equiv k_1^+/p^+\,,\qquad \xi_2\equiv k_2^+/p^+\,,\qquad \bar{\xi}_1\equiv 1-\xi_1\,,\qquad \bar{\xi}_2\equiv 1-\xi_2\,. \tag{11}$$

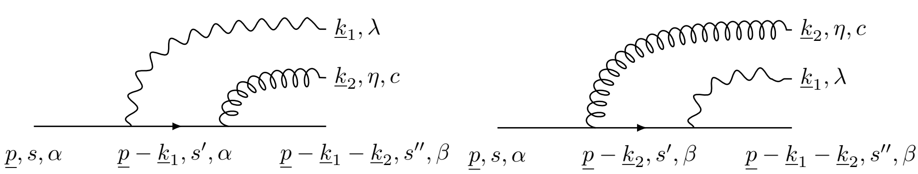

Fig. 1. The dressed-quark state to order $O(g_e g_s)$, with the two possible orderings of the photon respective gluon emission by the quark.

Equation (6) is the dressed-quark state at $O(g_e g_s)$. Computing the outgoing state from the dressed-quark state requires three more steps. The first step is to perform the two-dimensional Fourier transform of the dressed-quark state, and write it in the mixed longitudinal momentum and the transverse coordinate space. Then, the second step is to take into account the interaction of the dressed-quark state written in the mixed space. This interaction occurs in the eikonal approximation and, as a result, the quark and gluon inside the dressed-quark state undergo a rotation in the color space, which can be encoded in a Wilson line in the fundamental respective adjoint representation at the transverse position of the quark or gluon. The Wilson lines are defined in the standard way in terms of the color fields α^- of the target as

$$S_{\mathrm{F,A}}(x) = \mathcal{P}\, e^{ig\int \mathrm{d}x^+ \tau^a \alpha_a^-(x^+,x)}\,, \tag{12}$$

where τ^a is the $\mathrm{SU}(N_\mathrm{c})$ generator either in the fundamental or the adjoint representation, as indicated in the subscript 'F' or 'A'. This procedure provides the outgoing state in terms of the bare components. The final step is to rewrite the outgoing state in terms of dressed components. After performing these three steps, one gets the relevant part of the final outgoing state[2] as

$$\begin{aligned}
&\big|(\boldsymbol{q})\,[p^+,0]_s^\alpha\big\rangle_{\mathrm{out}} = g_s g_e \sum_{s's''}\sum_{\lambda\eta}\int \frac{\mathrm{d}^2k_1^+}{2\pi}\frac{\mathrm{d}^2k_2^+}{2\pi}\int_{wvx_1x_2x_3}\\
&\times\Bigg(\delta^{(2)}\Big[\omega-\big(\xi_1x_1+\bar{\xi}_1v\big)\Big]\delta^{(2)}\left[v-\left\{\left(1-\frac{\xi_2}{\bar{\xi}_1}\right)x_3+\frac{\xi_2}{\bar{\xi}_1}x_2\right\}\right]\left[\frac{(-i)}{\sqrt{2\xi_1p^+}}\phi_{ss'}^{\lambda\bar{\lambda}}(\xi_1)\right]\\
&\times\left[\frac{(-i)}{\sqrt{2\xi_2p^+}}\phi_{s's''}^{\eta\bar{\eta}}\left(\frac{\xi_2}{\bar{\xi}_1}\right)\right]A^{\bar{\eta}}(x_3-x_2)\Big\{\left[t^c_{\alpha\beta}S_\mathrm{F}^{\beta\sigma}(x_3)S_\mathrm{A}^{cd}(x_2)-S_\mathrm{F}^{\alpha\beta}(\omega)t^d_{\beta\sigma}\right]\\
&\times\mathcal{A}^{\bar{\lambda}}\left(\xi_1,v-x_1;\frac{\xi_2}{\bar{\xi}_1},x_3-x_2\right)-\left[S_\mathrm{F}^{\alpha\beta}(v)-S_\mathrm{F}^{\alpha\beta}(\omega)\right]t^d_{\beta\sigma}A^{\bar{\lambda}}(v-x_1)\Big\}\\
&+\delta^{(2)}\Big[\omega-\big(\xi_2x_2+\bar{\xi}_2v\big)\Big]\delta^{(2)}\left[v-\left\{\left(1-\frac{\xi_1}{\bar{\xi}_2}\right)x_3+\frac{\xi_1}{\bar{\xi}_2}x_1\right\}\right]\left[\frac{(-i)}{\sqrt{2\xi_2p^+}}\phi_{ss'}^{\eta\bar{\eta}}(\xi_2)\right]\\
&\times\left[\frac{(-i)}{\sqrt{2\xi_1p^+}}\phi_{s's''}^{\lambda\bar{\lambda}}\left(\frac{\xi_1}{\bar{\xi}_2}\right)\right]A^{\bar{\lambda}}(x_3-x_1)\Big\{\left[t^c_{\alpha\beta}S_\mathrm{F}^{\beta\sigma}(x_3)S_\mathrm{A}^{cd}(x_2)-S_\mathrm{F}^{\alpha\beta}(\omega)t^d_{\beta\sigma}\right]\\
&\times\mathcal{A}^{\bar{\eta}}\left(\xi_2,v-x_2;\frac{\xi_1}{\bar{\xi}_2},x_3-x_1\right)-\left[t^c_{\alpha\beta}S_\mathrm{F}^{\beta\sigma}(v)S_\mathrm{A}^{cd}(x_2)-S_\mathrm{F}^{\alpha\beta}(\omega)t^d_{\beta\sigma}\right]A^{\bar{\eta}}(v-x_2)\Big\}\Bigg)\\
&\times\Big|(\boldsymbol{q})\,\big[p^+-k_1^+-k_2^+,x_3\big]^\sigma_{s''},(\boldsymbol{g})\,\big[k_2^+,x_2\big]^d_\eta,(\gamma)\,\big[k_1^+,x_1\big]^\lambda\Big\rangle_D\,.
\end{aligned}\tag{13}$$

Here, $A^\lambda(x-y)$ is the standard Weizsäcker–Williams field and $\mathcal{A}^\lambda(\xi_1,v-x_1;\frac{\xi_2}{\bar{\xi}_1},x_3-x_2)$ is the modified Weizsäcker–Williams field for two successive splittings, which we defined as

$$\mathcal{A}^\lambda\left(\xi_1,v-x_1;\frac{\xi_2}{\bar{\xi}_1},x_3-x_2\right) = -\frac{1}{2\pi}\frac{\xi_1(v-x_1)^\lambda}{\xi_1(v-x_1)^2+\frac{\xi_2}{\bar{\xi}_1}\left(1-\frac{\xi_2}{\bar{\xi}_1}\right)(x_3-x_2)^2}\,. \tag{14}$$

Moreover, the first term on the right-hand side of Eq. (13) corresponds to the case where the photon is emitted before the quark–gluon splitting and the second term corresponds to photon emission after the quark–gluon splitting.

[2] For the production of a dijet and a photon, the only relevant component of the outgoing state is the dressed quark–photon–gluon one.

After all said and done, the final expression for the partonic level cross section can be organized as follows:

$$(2\pi)^9 \frac{\mathrm{d}\sigma^{qA\to\gamma gq+X}}{\mathrm{d}^3\underline{q}_1 \mathrm{d}^3\underline{q}_2 \mathrm{d}^3\underline{q}_3} = \frac{1}{2N_\mathrm{c}} g_s^2 g_e^2 (2\pi)\delta\left(p^+ - \sum_{i=1}^{3} q_i^+\right)\frac{1}{2q_1^+}\frac{1}{2q_2^+} \times \left\langle I_{\mathrm{bef-bef}} + I_{\mathrm{aft-aft}} + I_{\mathrm{bef-aft}} + I_{\mathrm{aft-bef}}\right\rangle . \quad (15)$$

The explicit expression for each of the contribution in Eq. (15) can be found in [25]. Here, for the sake of brevity, let us just give the expression of the before–before contribution which reads

$$\begin{aligned} I_{\mathrm{bef-bef}} = & \int\limits_{ww'vv'y_1z_1y_2z_2y_3z_3} e^{iq_1(y_1-z_1)+iq_2(y_2-z_2)+iq_3(y_3-z_3)}\delta^{(2)}\left[w' - \left(\bar{\xi}_1 v' + \xi_1 y_1\right)\right] \\ & \times\delta^{(2)}\left[w - \left(\bar{\xi}_1 v + \xi_1 z_1\right)\right]\delta^{(2)}\left[v' - \left\{\left(1-\frac{\xi_2}{\bar{\xi}_1}\right)y_3 + \frac{\xi_2}{\bar{\xi}_1}y_2\right\}\right] \\ & \times\delta^{(2)}\left[v - \left\{\left(1-\frac{\xi_2}{\bar{\xi}_1}\right)z_3 + \frac{\xi_2}{\bar{\xi}_1}z_2\right\}\right] 8\,\mathcal{M}_q^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'}\left(\xi_1, \frac{\xi_2}{\bar{\xi}_1}\right) A^{\bar{\eta}}(z_3-z_2) A^{\bar{\eta}'}(y_3-y_2) \\ & \times\Bigg(A^{\bar{\lambda}'}(v'-y_1)\, A^{\bar{\lambda}}(v-z_1)\, \mathrm{W}^{(AA)}_{\mathrm{bef-bef}} \\ & + \mathcal{A}^{\bar{\lambda}'}\left(\xi_1, v'-y_1; \frac{\xi_2}{\bar{\xi}_1}, y_3-y_2\right)\mathcal{A}^{\bar{\lambda}}\left(\xi_1, v-z_1; \frac{\xi_2}{\bar{\xi}_1}, z_3-z_2\right) W^{(\mathcal{A}\mathcal{A})}_{\mathrm{bef-bef}} \\ & - \mathcal{A}^{\bar{\lambda}'}\left(\xi_1, v'-y_1; \frac{\xi_2}{\bar{\xi}_1}, y_3-y_2\right) A^{\bar{\lambda}}(v-z_1)\, W^{(\mathcal{A}A)}_{\mathrm{bef-bef}} \\ & - A^{\bar{\lambda}'}(v'-y_1)\, \mathcal{A}^{\bar{\lambda}}\left(\xi_1, v-z_1; \frac{\xi_2}{\bar{\xi}_1}, z_3-z_2\right) W^{(A\mathcal{A})}_{\mathrm{bef-bef}}\Bigg) , \end{aligned} \quad (16)$$

where the function $\mathcal{M}_q^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'}$ is the product of the splitting amplitudes and the functions $W_{\mathrm{bef-bef}}$ define the dipole and quadrupole structures of the before–before contribution accompanied by the standard or the modified Weizsäcker–Williams fields as indicated by the superscripts. The explicit expression of these functions can be found in [25].

3. Correlation limit and gluon TMDs

It was shown in [13] that in the so-called correlation limit $|q_1 + q_2| \ll |q_1|, |q_2|$ (corresponding to nearly back-to-back jets), the dilute-dense CGC expression for forward dijet production can be factorized into a hard factors and transverse-momentum-dependent (TMD) gluon distributions. In the case of production of three final-state particles, this regime corresponds to

$|q_1+q_2+q_3| \ll |q_1|, |q_2|, |q_3|$. In this limit, the production cross section can be simplified by defining the dipole sizes as

$$r_g = z_3 - z_2\,, \qquad r'_g = y_3 - y_2\,, \tag{17}$$
$$r_\gamma = v - z_1\,, \qquad r'_\gamma = v' - y_1 \tag{18}$$

with corresponding conjugate momenta

$$(y_3 - z_3) \;\to\; P = q_1 + q_2 + q_3\,, \tag{19}$$
$$\left(r_\gamma - r'_\gamma\right) \;\to\; q_1\,, \tag{20}$$
$$\left(r_g - r'_g\right) \;\to\; Q = q_2 + \frac{\xi_2}{\bar{\xi}_1} q_1\,. \tag{21}$$

In this limit, we can utilize a small dipole approximation and expand the cross section in powers of the dipole sizes. This expansion can be performed for all the contributions in the quark-initiated channel as well as the all the contributions in the gluon-initiated channel[3]. Again, for the sake of simplicity, here we only present the result for the before–before contribution in the quark-initiated channel. After all said and done, we get a factorized expression for this contribution which can be written as

$$\begin{aligned}\langle I_{\mathrm{bef-bef}}\rangle_{x_{\mathrm{A}}} =& \mathcal{M}_q^{\bar\lambda,\bar\lambda';\bar\eta,\bar\eta'}\left(\xi_1,\frac{\xi_2}{\bar\xi_1}\right) g_s^2 (2\pi)^3 N_{\mathrm{c}} \left\{ \left[H_{qg}^{(1)}\right]_{\mathrm{bef-bef}}^{\bar\lambda,\bar\lambda';\bar\eta\bar\eta';ij} \frac{P^iP^j}{P^2}\mathcal{F}_{qg}^{(1)}(x_{\mathrm{A}},P)\right.\\ &\left.+\left[H_{qg}^{(2)}\right]_{\mathrm{bef-bef}}^{\bar\lambda,\bar\lambda';\bar\eta\bar\eta';ij}\left[\frac{1}{2}\delta^{ij}\mathcal{F}_{qg}^{(2)}(x_{\mathrm{A}},P) - \frac{1}{2}\left(\delta^{ij} - 2\frac{P^iP^j}{P^2}\right)\mathcal{H}_{qg}^{(2)}(x_{\mathrm{A}},P)\right]\right\}.\end{aligned} \tag{22}$$

Here, $\langle\cdots\rangle_{x_{\mathrm{A}}}$ denote CGC averaging with x_{A} referring to the small longitudinal momentum fraction of the gluons in the target wave function. Moreover, gluon TMDs are defined as

$$\int\limits_{y_3z_3} e^{iP\,(y_3-z_3)}\left\langle \mathrm{tr}\left[\left(\partial^i S_{z_3}\right)\left(\partial^j S^\dagger_{y_3}\right)\right]\right\rangle_{x_{\mathrm{A}}} = g_s^2(2\pi)^3\frac{P^iP^j}{4P^2}\mathcal{F}_{qg}^{(1)}(x_{\mathrm{A}},P)\,, \tag{23}$$

$$\begin{aligned}&\int\limits_{y_3z_3} e^{iP\,(y_3-z_3)}\left\langle \mathrm{tr}\left[S^\dagger_{z_3}\left(\partial^i S_{z_3}\right)S^\dagger_{y_3}\left(\partial^j S_{y_3}\right)\right]\mathrm{tr}\left[S_{z_3}S^\dagger_{y_3}\right]\right\rangle_{x_{\mathrm{A}}} = -g_s^2(2\pi)^3N_{\mathrm{c}}\frac{1}{4}\\ &\times\left[\frac{1}{2}\delta^{ij}\mathcal{F}_{qg}^{(2)}(x_{\mathrm{A}},P) - \frac{1}{2}\left(\delta^{ij}-2\frac{P^iP^j}{P^2}\right)\mathcal{H}_{qg}^{(2)}(x_{\mathrm{A}},P)\right].\end{aligned} \tag{24}$$

The gluon TMD defined in Eq. (24) consists of two parts, corresponding the to unpolarized ($\mathcal{F}_{qg}^{(2)}$) and linearly-polarized ($\mathcal{H}_{qg}^{(2)}$) distributions inside

[3] The expansion of each contribution in both quark and gluon channels can be found in [25].

the unpolarized target. For the so-called (fundamental) dipole gluon TMD defined in Eq. (23), the simpler Wilson line structure implies that $\mathcal{H}^{(1)_{qg}} = \mathcal{F}^{(1)}_{qg}$. Finally, the hard factors that accompany the TMDs are defined as

$$\begin{aligned}\left[H^{(1)}_{qg}\right]^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}';ij}_{\mathrm{bef-bef}} &= \left(\xi_1 \mathit{\Pi}^{\bar{\eta}';\bar{\lambda}'j}\left[Q;c_0^{-1},q_1\right] + \left(1-\frac{\xi_2}{\bar{\xi}_1}\right)\mathit{\Pi}^{\bar{\lambda}';\bar{\eta}'j}\left[q_1;c_0,Q\right]\right)\\ &\quad\times\left(\xi_1 \mathit{\Pi}^{\bar{\eta};\bar{\lambda}j}\left[Q;c_0^{-1},q_1\right] + \left(1-\frac{\xi_2}{\bar{\xi}_1}\right)\mathit{\Pi}^{\bar{\lambda};\bar{\eta}j}\left[q_1;c_0,Q\right]\right)\\ &\quad-\frac{1}{N_c^2}\left(\xi_1 \mathit{\Pi}^{\bar{\eta}';\bar{\lambda}'j}\left[Q;c_0^{-1},q_1\right] - \frac{\xi_2}{\bar{\xi}_1}\mathit{\Pi}^{\bar{\lambda}';\bar{\eta}'j}\left[q_1;c_0,Q\right]\right)\\ &\quad\times\left(\xi_1 \mathit{\Pi}^{\bar{\eta};\bar{\lambda}j}\left[Q;c_0^{-1},q_1\right] - \frac{\xi_2}{\bar{\xi}_1}\mathit{\Pi}^{\bar{\lambda};\bar{\eta}j}\left[q_1;c_0,Q\right]\right)\end{aligned} \tag{25}$$

and

$$\left[\mathcal{H}^{(2)}_{qg}\right]^{\bar{\lambda},\bar{\lambda}';\bar{\eta}\bar{\eta}';ij}_{\mathrm{bef-bef}} = \left(\mathit{\Pi}^{\bar{\lambda}';\bar{\eta}'j}\left[q_1;c_0,Q\right]\right)\left(\mathit{\Pi}^{\bar{\lambda};\bar{\eta}i}\left[q_1;c_0,Q\right]\right), \tag{26}$$

where we have introduced the compact notation $\mathit{\Pi}^{i;jk}[p;c_0,q]$ given by

$$\mathit{\Pi}^{i;jk}[p;c_0,q] \equiv \left(\frac{p^i}{p^2}\right)\left\{\frac{1}{q^2+c_0p^2}\left[\delta^{jk} - 2\frac{q^jq^k}{q^2+c_0p^2}\right]\right\} \tag{27}$$

and $c_0 = \frac{1}{\xi_1}\frac{\xi_2}{\bar{\xi}_1}(1-\frac{\xi_2}{\bar{\xi}_1})$.

4. Conclusions

In conclusion, we have computed the production cross section of a hard photon and two hard jets in forward pA collisions. The computation is performed adopting the hybrid formalism which is suitable for forward collisions. Here, we have only considered the quark-initiated channel. In this channel, the quark coming form the dilute projectile emits a gluon and a photon which then scatter off the target via eikonal interactions, producing a photon together with a quark jet and a gluon jet. For this channel, we have taken into account the two possible cases, depending on whether the photon is emitted before or after the gluon in the amplitude and in the complex conjugate amplitude.

In the correlation limit, the transverse momenta of the three final-state particles are much larger than their total transverse momentum P. This kinematic regime allowed us to perform a small dipole approximation, and the cross section can be simplified and cast in a factorized form involving TMD gluon distributions.

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