ANALYSIS OF THE VECTOR AND AXIALVECTOR $QQ\bar{Q}\bar{Q}$ TETRAQUARK STATES WITH QCD SUM RULES*

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In this article, we construct the axialvector-diquark–axialvector-antidiquark-type currents to study both the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states with the QCD sum rules, and obtain the masses $M_{Y(cc\bar{c}\bar{c},1^{+-})} = 6.05 \pm 0.08\,\mathrm{GeV}$, $M_{Y(cc\bar{c}\bar{c},1^{--})} = 6.11 \pm 0.08\,\mathrm{GeV}$, $M_{Y(bb\bar{b}\bar{b},1^{+-})} = 18.84 \pm 0.09\,\mathrm{GeV}$, $M_{Y(bb\bar{b}\bar{b},1^{--})} = 18.89 \pm 0.09\,\mathrm{GeV}$. The vector tetraquark states lie 40 MeV above the corresponding centroids of the 0^{++}, 1^{+-} and 2^{++} tetraquark states, which is a typical feature of the vector tetraquark states consisting of four heavy quarks.

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1. Introduction

The exotic charmonium-like and bottomonium-like states, such as the $Z_c(3900)$, $Z_c(4025)$, $Z_c(4200)$, $Z(4430)$, $Z_b(10610)$, $Z_b(10650)$, are excellent candidates for the multiquark states [1]. If they are really tetraquark states, their constituents are two heavy quarks and two light quarks. Up to now, no exotic tetraquark candidate composed of more than two heavy quarks has been reported. Theoretically, there have been several approaches to study the masses and widths of the exotic states Y_Q with $QQ\bar{Q}\bar{Q}$ quark composition, such as the non-relativistic potential models [2–7], the Bethe–Salpeter equation [8], the constituent diquark model with spin–spin interaction [9–11], the constituent quark model with color–magnetic interaction [12], the (moment) QCD sum rules [13, 14], *etc.* Experimentally, the ATLAS, CMS and LHCb collaborations have measured the cross section for double charmonium production [15], the CMS Collaboration has observed the Υ pair production [16]. Recently, the LHCb Collaboration studied the

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$\varUpsilon\,\mu^+\mu^-$ invariant-mass distribution for a possible exotic tetraquark state composed of two b quarks and two $\bar{b}$ quarks based on a data sample of pp collisions recorded with the LHCb detector at center-of-mass energies $\sqrt{s} = 7$, 8 and 13 TeV corresponding to an integrated luminosity of 6.3 fb^{-1}, and observed no significant excess [17]. The decays to the final states $\varUpsilon\,\mu^+\mu^-$ can take place through $Y_b(0^{++}/2^{++}) \to \varUpsilon\varUpsilon^*/\varUpsilon\varUpsilon \to \varUpsilon\,\mu^+\mu^-$ or $Y_b(1^{--}) \to \varUpsilon\varUpsilon^*/\varUpsilon\varUpsilon \to \varUpsilon\,\mu^+\mu^-$. In Ref. [11], Esposito and Polosa argue that the partial width for the $Y_b(2^{++}) \to \varUpsilon\,\mu^+\mu^-$ decay is too small to be currently observed at the LHC. However, if the barrier between the diquark and antidiquark is very narrow and the tetraquark width is sufficiently small, the detection of such a state is still possible.

In 2013, the BESIII Collaboration studied the $e^+e^- \to \pi^+\pi^- J/\psi$ process at a center-of-mass energy of 4.26 GeV, and observed a $Z_c^{\pm}(3900)$ structure in the $\pi^{\pm}J/\psi$ mass spectrum [18]. Recently, the BESIII Collaboration determined the spin and parity of the $Z_c^{\pm}(3900)$ state to be $J^P = 1^+$ with a statistical significance larger than 7σ over other quantum numbers [19]. Analogously, there may exist a tetraquark state $Y_{c/b}(1^{+-})$ which decays to the $\eta_c J/\psi$ or $\eta_b \varUpsilon$.

The diquarks (or diquark operators) $\varepsilon^{ijk}q_j^T C\varGamma q_k'$ have five structures in the Dirac spinor space, where the i, j and k are color indexes, $C\varGamma = C\gamma_5$, C, $C\gamma_\mu\gamma_5$, $C\gamma_\mu$ and $C\sigma_{\mu\nu}$ for the scalar, pseudoscalar, vector, axialvector and tensor diquarks, respectively. The favorite diquark configurations are the scalar ($C\gamma_5$) and axialvector ($C\gamma_\mu$) diquark states from the QCD sum rules [20–23]. The QCD sum rules have been extensively applied to study the tetraquark states and molecular states [24]. In Refs. [25, 26], we study the mass and width of the $Z_c^{\pm}(3900)$ with the $C\gamma_\mu\otimes\gamma_5 C$–$C\gamma_5\otimes\gamma_\mu C$-type current with the QCD sum rules in details, and reproduce the experimental data satisfactorily. In Ref. [27], we study both the vector and axialvector tetraquark states with the $C\gamma_\mu \otimes \gamma_\nu C$–$C\gamma_\nu \otimes \gamma_\mu C$-type currents, and reproduce the experimental values of the masses of the $Y(4660)$ and $Z_c(4020/4025)$ satisfactorily. The double-heavy diquark states $\varepsilon^{ijk}Q_j^T C\gamma_5 Q_k$ cannot exist due to the Pauli principle. In previous work, we took the double-heavy diquark $\varepsilon^{ijk}Q_j^T C\gamma_\mu Q_k$ states as basic constituents to construct the scalar and tensor tetraquark states with the QCD sum rules [14]. Now we extend our previous work to study the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states with the $C\gamma_\mu \otimes \gamma_\nu C$–$C\gamma_\nu \otimes \gamma_\mu C$-type currents, which are expected to couple potentially to the lowest tetraquark states, especially for the vector tetraquark states.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states in Section 2; in Section 3, we present the numerical results and discussions; Section 4 is reserved for our conclusion.

2. QCD sum rules for the vector and axialvector tetraquark states

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules

$$\Pi_{\mu\nu\alpha\beta}(p) \;=\; i\int \mathrm{d}^4x e^{ip\cdot x}\langle 0|T\left\{J_{\mu\nu}(x)J^{\dagger}_{\alpha\beta}(0)\right\}|0\rangle\,, \tag{1}$$

where

$$J_{\mu\nu}(x) = \varepsilon^{ijk}\varepsilon^{imn}$$
$$\times\left\{Q_j^T(x)C\gamma_\mu Q_k(x)\bar{Q}_m(x)\gamma_\nu C\bar{Q}_n^T(x) - Q_j^T(x)C\gamma_\nu Q_k(x)\bar{Q}_m(x)\gamma_\mu C\bar{Q}_n^T(x)\right\}, \tag{2}$$

where the i, j, k, m, n are color indexes and the C is the charge conjugation matrix. As the tetraquark states have many Fock states, we can study the mixing with the substitution

$$J_{\mu\nu}(x) \;\to\; \cos\theta J_{\mu\nu}(x) + \sin\theta \widetilde{J}_{\mu\nu}(x)\,, \tag{3}$$

where the θ is a mixing angle, the $\widetilde{J}_{\mu\nu}(x)$ is another (or any) tetraquark current with the same quantum numbers as the current $J_{\mu\nu}(x)$. We can also study the mixing between the two-quark and tetraquark components with the substitution

$$J_{\mu\nu}(x) \;\to\; \cos\theta J_{\mu\nu}(x) + \sin\theta\,\frac{i}{3}\,\left\langle\bar{Q}Q\right\rangle\,\bar{Q}(x)\sigma_{\mu\nu}Q(x)\,, \tag{4}$$

where the heavy quark condensate $\langle\bar{Q}Q\rangle = -\frac{1}{12m_Q}\langle\frac{\alpha_s GG}{\pi}\rangle + \cdots$ [28]. This may be our next work.

At the phenomenological side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{\mu\nu}(x)$ into the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ to obtain the hadronic representation [29, 30]. After isolating the ground state contributions of the axialvector and vector tetraquark states, we get the following results:

$$\begin{aligned}\Pi_{\mu\nu\alpha\beta}(p) &= \frac{\lambda_{Y+}^2}{M_{Y+}^2\left(M_{Y+}^2-p^2\right)}\\ &\times\left(p^2g_{\mu\alpha}g_{\nu\beta}-p^2g_{\mu\beta}g_{\nu\alpha}-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\right)\\ &+\frac{\lambda_{Y-}^2}{M_{Y-}^2\left(M_{Y-}^2-p^2\right)}\left(-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\right)+\cdots\,,\end{aligned} \tag{5}$$

where the Y^+ and Y^- denote the axialvector and vector tetraquark states respectively, and the pole residues $\lambda_{Y^\pm}$ are defined by

$$\begin{aligned}
\langle 0|J_{\mu\nu}(0)|Y^+(p)\rangle &= \frac{\lambda_{Y^+}}{M_{Y^+}}\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{\alpha}p^{\beta}\,, \\
\langle 0|J_{\mu\nu}(0)|Y^-(p)\rangle &= \frac{\lambda_{Y^-}}{M_{Y^-}}\left(\varepsilon_{\mu}p_{\nu}-\varepsilon_{\nu}p_{\mu}\right)\,, \qquad (6)
\end{aligned}$$

where the ε_μ are the polarization vectors of the vector and axialvector tetraquark states. We can rewrite the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ into the following form according to Lorentz covariance:

$$\begin{aligned}
&\Pi_{\mu\nu\alpha\beta}(p) = \Pi_{Y^+}\left(p^2\right) \\
&\times\left(p^2 g_{\mu\alpha}g_{\nu\beta}-p^2 g_{\mu\beta}g_{\nu\alpha}-g_{\mu\alpha}p_{\nu}p_{\beta}-g_{\nu\beta}p_{\mu}p_{\alpha}+g_{\mu\beta}p_{\nu}p_{\alpha}+g_{\nu\alpha}p_{\mu}p_{\beta}\right) \\
&+\Pi_{Y^-}\left(p^2\right)\left(-g_{\mu\alpha}p_{\nu}p_{\beta}-g_{\nu\beta}p_{\mu}p_{\alpha}+g_{\mu\beta}p_{\nu}p_{\alpha}+g_{\nu\alpha}p_{\mu}p_{\beta}\right)\,. \qquad (7)
\end{aligned}$$

Now, we project out the components $\Pi_{Y^+}(p^2)$ and $\Pi_{Y^-}(p^2)$ by introducing the operators $P_{Y^+}^{\mu\nu\alpha\beta}$ and $P_{Y^-}^{\mu\nu\alpha\beta}$

$$\begin{aligned}
\widetilde{\Pi}_{Y^+}\left(p^2\right) &= p^2\Pi_{Y^+}\left(p^2\right) = P_{Y^+}^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p)\,, \\
\widetilde{\Pi}_{Y^-}\left(p^2\right) &= p^2\Pi_{Y^-}\left(p^2\right) = P_{Y^-}^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p)\,, \qquad (8)
\end{aligned}$$

where

$$\begin{aligned}
P_{Y^+}^{\mu\nu\alpha\beta} &= \frac{1}{6}\left(g^{\mu\alpha}-\frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta}-\frac{p^\nu p^\beta}{p^2}\right)\,, \\
P_{Y^-}^{\mu\nu\alpha\beta} &= \frac{1}{6}\left(g^{\mu\alpha}-\frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta}-\frac{p^\nu p^\beta}{p^2}\right)-\frac{1}{6}g^{\mu\alpha}g^{\nu\beta}\,. \qquad (9)
\end{aligned}$$

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ in perturbative QCD. We contract the heavy quark fields with Wick theorem and obtain the results

$$\begin{aligned}
&\Pi_{\mu\nu\alpha\beta}(p) = 4i\varepsilon^{ijk}\varepsilon^{imn}\varepsilon^{i'j'k'}\varepsilon^{i'm'n'}\int \mathrm{d}^4x e^{ip\cdot x} \\
&\times\Big\{\mathrm{Tr}\left[\gamma_{\mu}S^{kk'}(x)\gamma_{\alpha}CS^{jj'T}(x)C\right]\mathrm{Tr}\left[\gamma_{\beta}S^{n'n}(-x)\gamma_{\nu}CS^{m'mT}(-x)C\right] \\
&-\mathrm{Tr}\left[\gamma_{\mu}S^{kk'}(x)\gamma_{\beta}CS^{jj'T}(x)C\right]\mathrm{Tr}\left[\gamma_{\alpha}S^{n'n}(-x)\gamma_{\nu}CS^{m'mT}(-x)C\right] \\
&-\mathrm{Tr}\left[\gamma_{\nu}S^{kk'}(x)\gamma_{\alpha}CS^{jj'T}(x)C\right]\mathrm{Tr}\left[\gamma_{\beta}S^{n'n}(-x)\gamma_{\mu}CS^{m'mT}(-x)C\right] \\
&+\mathrm{Tr}\left[\gamma_{\nu}S^{kk'}(x)\gamma_{\beta}CS^{jj'T}(x)C\right]\mathrm{Tr}\left[\gamma_{\alpha}S^{n'n}(-x)\gamma_{\mu}CS^{m'mT}(-x)C\right]\Big\}\,, \qquad (10)
\end{aligned}$$

where the $S_{ij}(x)$ is the full Q-quark propagator

$$
\begin{aligned}
S_{ij}(x) &= \frac{i}{(2\pi)^4}\int \mathrm{d}^4k e^{-ik\cdot x} \\
&\times\left\{\frac{\delta_{ij}}{\not{k}-m_Q}-\frac{g_sG^n_{\alpha\beta}t^n_{ij}}{4}\frac{\sigma^{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma^{\alpha\beta}}{\left(k^2-m_Q^2\right)^2}\right. \\
&\left.+\frac{g_s^2G^n_{\alpha\beta}G^{n\alpha\beta}}{12}\delta_{ij}m_Q\frac{k^2+m_Q\not{k}}{\left(k^2-m_Q^2\right)^4}+\cdots\right\}\,, \qquad (11)
\end{aligned}
$$

and $t^n=\frac{\lambda^n}{2}$, the λ^n is the Gell-Mann matrix [30]. Then we compute the integrals both in the coordinate and momentum spaces to obtain the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$, therefore, the QCD spectral densities through dispersion relation

$$
\begin{aligned}
\rho_{\mathrm{A}}(s) &= \frac{\mathrm{Im}\widetilde{\Pi}_{Y+}(s)}{\pi}\,, \\
\rho_{\mathrm{V}}(s) &= \frac{\mathrm{Im}\widetilde{\Pi}_{Y-}(s)}{\pi}\,, \qquad (12)
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{\Pi}_{Y+}\left(p^2\right) &= P^{\mu\nu\alpha\beta}_{Y+}\Pi_{\mu\nu\alpha\beta}(p)\,, \\
\widetilde{\Pi}_{Y-}\left(p^2\right) &= P^{\mu\nu\alpha\beta}_{Y-}\Pi_{\mu\nu\alpha\beta}(p)\,. \qquad (13)
\end{aligned}
$$

We take the quark–hadron duality below the continuum thresholds s_0 and perform the Borel transform with respect to the variable $P^2=-p^2$ to obtain the QCD sum rules

$$
\lambda_Y^2\exp\left(-\frac{M_Y^2}{T^2}\right)=\int\limits_{16m_Q^2}^{s_0}\mathrm{d}s\int\limits_{z_{\mathrm{i}}}^{z_f}\mathrm{d}z\int\limits_{t_{\mathrm{i}}}^{t_{\mathrm{f}}}\mathrm{d}t\int\limits_{r_{\mathrm{i}}}^{r_{\mathrm{f}}}\mathrm{d}r\,\rho_{\mathrm{A/V}}(s,z,t,r)\exp\left(-\frac{s}{T^2}\right)\,, \qquad (14)
$$

$$\rho_{\mathrm{A}}(s,z,t,r)=\frac{3m_Q^4}{16\pi^6}\left(s-\overline{m}_Q^2\right)^2+\frac{tzm_Q^2}{8\pi^6}\left(s-\overline{m}_Q^2\right)^2\left(4s-\overline{m}_Q^2\right)$$
$$+rtz(1-r-t-z)\frac{s}{16\pi^6}\left(s-\overline{m}_Q^2\right)^2\left(7s-4\overline{m}_Q^2\right)$$
$$+m_Q^2\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\left\{-\frac{1}{r^3}\frac{m_Q^4}{12\pi^4}\delta\left(s-\overline{m}_Q^2\right)-\frac{1-r-t-z}{r^2}\frac{m_Q^2}{12\pi^4}\right.$$
$$\times\left[1+s\,\delta\left(s-\overline{m}_Q^2\right)\right]-\frac{tz}{r^3}\frac{m_Q^2}{12\pi^4}\left[1+s\,\delta\left(s-\overline{m}_Q^2\right)\right]-\frac{tz(1-r-t-z)}{r^2}\frac{1}{12\pi^4}$$
$$\left.\times\left[4s+s^2\delta\left(s-\overline{m}_Q^2\right)\right]+\frac{1}{r^2}\frac{m_Q^2}{4\pi^4}+\frac{tz}{r^2}\frac{1}{4\pi^4}\left(2s-\overline{m}_Q^2\right)\right\}$$
$$+\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\left\{-\frac{m_Q^2}{48\pi^4}\left(4s-3\overline{m}_Q^2\right)-\frac{r(1-r-t-z)}{16\pi^4}\left(s-\overline{m}_Q^2\right)^2\right.$$
$$-\frac{r(1-r-t-z)}{48\pi^4}s\left(7s-6\overline{m}_Q^2\right)+\frac{1}{rz}\frac{m_Q^4}{48\pi^4}+\frac{t}{r}\frac{m_Q^2}{24\pi^4}\left(2s-\overline{m}_Q^2\right)$$
$$\left.+\frac{t(1-r-t-z)}{32\pi^4}\left(s-\overline{m}_Q^2\right)^2+\frac{t(1-r-t-z)}{48\pi^4}s\left(6s-5\overline{m}_Q^2\right)\right\}\,, \qquad (15)$$

$$\rho_{\mathrm{V}}(s,z,t,r)=-\frac{3m_Q^4}{16\pi^6}\left(s-\overline{m}_Q^2\right)^2-\frac{tzm_Q^2}{8\pi^6}\left(s-\overline{m}_Q^2\right)^3$$
$$+rtz(1-r-t-z)\frac{s}{16\pi^6}\left(s-\overline{m}_Q^2\right)^2\left(7s-4\overline{m}_Q^2\right)$$
$$+m_Q^2\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\left\{\frac{1}{r^3}\frac{m_Q^4}{12\pi^4}\delta\left(s-\overline{m}_Q^2\right)+\frac{1-r-t-z}{r^2}\frac{m_Q^2}{12\pi^4}\right.$$
$$+\frac{tz}{r^3}\frac{m_Q^2}{12\pi^4}-\frac{tz(1-r-t-z)}{r^2}\frac{1}{12\pi^4}\left[4s+s^2\delta\left(s-\overline{m}_Q^2\right)\right]$$
$$\left.-\frac{1}{r^2}\frac{m_Q^2}{4\pi^4}-\frac{tz}{r^2}\frac{1}{4\pi^4}\left(s-\overline{m}_Q^2\right)\right\}$$
$$+\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\left\{\frac{m_Q^2}{48\pi^4}\left(5s-3\overline{m}_Q^2\right)+\frac{r(1-r-t-z)}{16\pi^4}\left(s-\overline{m}_Q^2\right)^2\right.$$
$$+\frac{r(1-r-t-z)}{48\pi^4}s\left(7s-6\overline{m}_Q^2\right)-\frac{1}{rz}\frac{m_Q^4}{48\pi^4}-\frac{t}{r}\frac{m_Q^2}{24\pi^4}\left(s-\overline{m}_Q^2\right)$$
$$\left.-\frac{t(1-r-t-z)}{32\pi^4}\left(s-\overline{m}_Q^2\right)^2-\frac{t(1-r-t-z)}{48\pi^4}s\left(s-\overline{m}_Q^2\right)\right\}\,, \qquad (16)$$

where

$$
\begin{aligned}
\overline{m}_Q^2 &= \frac{m_Q^2}{r}+\frac{m_Q^2}{t}+\frac{m_Q^2}{z}+\frac{m_Q^2}{1-r-t-z}\,,\\
r_{\mathrm{f/i}} &= \frac{1}{2}\left\{1-z-t\pm\sqrt{(1-z-t)^2-4\frac{1-z-t}{\hat{s}-\frac{1}{z}-\frac{1}{t}}}\right\}\,,\\
t_{\mathrm{f/i}} &= \frac{1}{2\left(\hat{s}-\frac{1}{z}\right)}\\
&\times\left\{(1-z)\left(\hat{s}-\frac{1}{z}\right)-3\pm\sqrt{\left[(1-z)\left(\hat{s}-\frac{1}{z}\right)-3\right]^2-4(1-z)\left(\hat{s}-\frac{1}{z}\right)}\right\}\,,\\
z_{\mathrm{f/i}} &= \frac{1}{2\hat{s}}\times\left\{\hat{s}-8\pm\sqrt{(\hat{s}-8)^2-4\hat{s}}\right\}\,,
\end{aligned}
\tag{17}
$$

and $\hat{s}=\frac{s}{m_Q^2}$.

We derive Eq. (14) with respect to $\tau=\frac{1}{T^2}$, then eliminate the pole residues λ_Y, and obtain the QCD sum rules for the masses of the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states

$$
M_Y^2 = -\frac{\frac{\mathrm{d}}{\mathrm{d}\tau}\int_{16m_Q^2}^{s_0}\mathrm{d}s\int_{z_\mathrm{i}}^{z_\mathrm{f}}\mathrm{d}z\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t\int_{r_\mathrm{i}}^{r_\mathrm{f}}\mathrm{d}r\,\rho(s,z,t,r)\,\exp\left(-\tau s\right)}{\int_{16m_Q^2}^{s_0}\mathrm{d}s\int_{z_\mathrm{i}}^{z_\mathrm{f}}\mathrm{d}z\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t\int_{r_\mathrm{i}}^{r_\mathrm{f}}\mathrm{d}r\,\rho(s,z,t,r)\,\exp\left(-\tau s\right)}\,. \tag{18}
$$

3. Numerical results and discussions

We take the gluon condensate to be the standard value [29–31], and take the $\overline{\mathrm{MS}}$ masses $m_c(m_c)=(1.28\pm0.03)$ GeV and $m_b(m_b)=(4.18\pm0.03)$ GeV from the Particle Data Group [1]. We take into account the energy-scale dependence of the $\overline{\mathrm{MS}}$ masses from the renormalization group equation

$$
\begin{aligned}
m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_c)}\right]^{\frac{12}{25}}\,,\\
m_b(\mu) &= m_b(m_b)\left[\frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(m_b)}\right]^{\frac{12}{23}}\,,\\
\alpha_\mathrm{s}(\mu) &= \frac{1}{b_0 t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2\left(\log^2 t-\log t-1\right)+b_0 b_2}{b_0^4 t^2}\right]\,,
\end{aligned}
\tag{19}
$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_\mathrm{f}}{12\pi}$, $b_1 = \frac{153-19n_\mathrm{f}}{24\pi^2}$, $b_2 = \frac{2857-\frac{5033}{9}n_\mathrm{f}+\frac{325}{27}n_\mathrm{f}^2}{128\pi^3}$, $\Lambda = 210\,\mathrm{MeV}$, $292\,\mathrm{MeV}$ and $332\,\mathrm{MeV}$ for the flavors $n_\mathrm{f} = 5$, 4 and 3, respectively [1].

In Ref. [14], we study the energy-scale dependence of the predicated masses of the scalar and tensor $QQ\bar{Q}\bar{Q}$ tetraquark states with the QCD sum rules in details. The predicted tetraquark masses decrease monotonously and slowly with increase of the energy scales, the QCD sum rules are stable with variations of the Borel parameters at the energy scales of $1.2\,\mathrm{GeV} < \mu < 2.2\,\mathrm{GeV}$ and $2.5\,\mathrm{GeV} < \mu < 3.3\,\mathrm{GeV}$ for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquark states, respectively. At the energy scales of $\mu = 2.0\,\mathrm{GeV}$ and $3.1\,\mathrm{GeV}$, the relation $\sqrt{s_0} = M_\mathrm{gr} + 0.5\,\mathrm{GeV}$ is satisfied, where the 'gr' denotes the ground state tetraquark states, the optimal energy scales of the QCD spectral densities are $\mu = 2.0\,\mathrm{GeV}$ and $3.1\,\mathrm{GeV}$ for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquark states, respectively [14]. In this article, we choose the same energy scales for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquark states, respectively, which work well.

In the QCD sum rules, we usually take the continuum threshold parameters as $\sqrt{s_0} = M_\mathrm{gr} + (0.4 \sim 0.6)\,\mathrm{GeV}$ for the conventional mesons, where the 'gr' denotes the ground states. Experimentally, the energy gaps $M_{\psi'} - M_{J/\psi} = 589\,\mathrm{MeV}$, $M_{\eta_c'} - M_{\eta_c} = 656\,\mathrm{MeV}$, $M_{\Upsilon'} - M_{\Upsilon} = 563\,\mathrm{MeV}$ and $M_{\eta_b'} - M_{\eta_b} = 600\,\mathrm{MeV}$ from the Particle Data Group [1]. The QCD sum rules support assigning the $Z_c(3900)$ and $Z(4430)$ to be the ground state and the first radial excited state of the axialvector tetraquark states with $J^{PC} = 1^{+-}$, respectively, and assigning the $X(3915)$ and $X(4500)$ to be the ground state and the first radial excited state of the scalar $cs\bar{c}\bar{s}$ tetraquark states with $J^{PC} = 0^{++}$, respectively [32, 33]. The mass gaps are $M_{Z(4430)} - M_{Z_c(3900)} = 576\,\mathrm{MeV}$ and $M_{X(4500)} - M_{X(3915)} = 588\,\mathrm{MeV}$, which also satisfies the relation $\sqrt{s_0} = M_\mathrm{gr} + (0.4 \sim 0.6)\,\mathrm{GeV}$. In this article, we take the relation $\sqrt{s_0} = M_\mathrm{gr} + (0.4 \sim 0.6)\,\mathrm{GeV}$ as a constraint, and search for the optimal continuum thresholds s_0.

We search for the optimal Borel parameters T^2 and continuum threshold parameters s_0 to satisfy the two criteria of the QCD sum rules: pole dominance at the phenomenological side and convergence of the operator product expansion at the QCD side. The resulting Borel parameters, continuum threshold parameters, energy scales, pole contributions are shown explicitly in Table I. From the table, we can see that the pole contributions are about (45–60)%, the same as that for the scalar and tensor tetraquark states [14] and the pole dominance at the phenomenological side is well-satisfied.

In the Borel windows, the dominant contributions come from the perturbative terms, the contributions of the gluon condensate are about -15%, -3%, -8% and -2% for the tetraquark states $cc\bar{c}\bar{c}(1^{+-})$, $cc\bar{c}\bar{c}(1^{--})$, $bb\bar{b}\bar{b}(1^{+-})$ and $bb\bar{b}\bar{b}(1^{--})$, respectively, the operator product expansion is well-convergent. As the dominant contributions come from the perturbative

TABLE I

The Borel parameters, continuum threshold parameters, energy scales, pole contributions, masses and pole residues of the $QQ\bar{Q}\bar{Q}$ tetraquark states.

	T^2 [GeV^2]	s_0 [GeV^2]	μ [GeV]	Pole	M_Y [GeV]	λ_Y [GeV^5]
$cc\bar{c}\bar{c}(1^{+-})$	4.5–4.9	43 ± 1	2.0	(46–61)%	6.05 ± 0.08	$(2.97\pm0.44)\times10^{-1}$
$cc\bar{c}\bar{c}(1^{--})$	4.2–4.6	44 ± 1	2.0	(46–62)%	6.11 ± 0.08	$(1.82\pm0.33)\times10^{-1}$
$bb\bar{b}\bar{b}(1^{+-})$	13.3–13.9	374 ± 3	3.1	(48–60)%	18.84 ± 0.09	5.45 ± 1.01
$bb\bar{b}\bar{b}(1^{--})$	11.7–12.3	376 ± 3	3.1	(47–60)%	18.89 ± 0.09	1.64 ± 0.36

terms, perturbative $\mathcal{O}(\alpha_s)$ corrections amount to multiplying the perturbative terms by a factor κ, which can be absorbed into the pole residues and cannot impair the predicted masses remarkably. In the QCD sum rules for the tetraquark states, we usually carry out the operator product expansion to the vacuum condensates up to dimension 10 and assume vacuum saturation for the higher dimension vacuum condensates [25, 27]. As the vacuum condensates are vacuum expectations of the quark and gluon operators, we take the truncation $i \leq 1$ in a consistent way, the operators of the orders of $\mathcal{O}(\alpha_s^i)$ with $i > 1$ are discarded, *i.e.* we take into account the terms $\langle\bar{q}q\rangle$, $\langle\frac{\alpha_s GG}{\pi}\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{q}q\rangle^2$, $\langle\bar{q}q\rangle\langle\frac{\alpha_s GG}{\pi}\rangle$, $\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle^2$, $\langle\bar{q}q\rangle^2\langle\frac{\alpha_s GG}{\pi}\rangle$ [25, 27]. In this article, only gluon condensates have contributions. Now, the two criteria of the QCD sum rules are all satisfied, we expect to make reasonable predictions.

We take into account all uncertainties of the input parameters, and obtain the values of the ground state masses and pole residues, which are also shown explicitly in Table I. From Table I, we can see that the constraint $\sqrt{s_0} = M_{\rm gr} + (0.4 \sim 0.6)\,\text{GeV}$ is also satisfied. In Figs. 1–2, we plot the masses and pole residues with variations of the Borel parameters at larger intervals than the Borel windows shown in Table I. From Figs. 1–2, we can see that the predicted masses and pole residues are rather stable with variations of the Borel parameters. The uncertainties originate from the Borel parameters in the Borel windows are very small, there appear Borel platforms in the Borel windows.

In Fig. 3, we plot the predicted masses M_Y with variations of the energy scales μ for the central values of the input parameters shown in Table I. From the figure, we can see that the masses M_Y decrease monotonously and slowly with increase of the energy scales μ. In this article, we choose the same energy scales as the corresponding ones for the 0^{++} and 2^{++} $QQ\bar{Q}\bar{Q}$ tetraquark states, the uncertainties originate from the energy scales cannot impair the predicative ability remarkably.

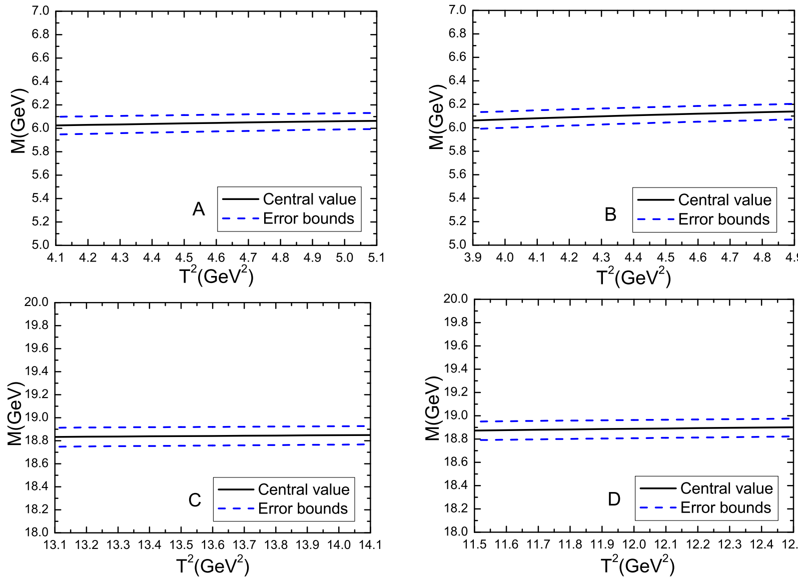

Fig. 1. The masses of the tetraquark states with variations of the Borel parameters T^2, where the A, B, C and D denote the $cc\bar{c}\bar{c}(1^{+-})$, $cc\bar{c}\bar{c}(1^{--})$, $bb\bar{b}\bar{b}(1^{+-})$ and $bb\bar{b}\bar{b}(1^{--})$, respectively.

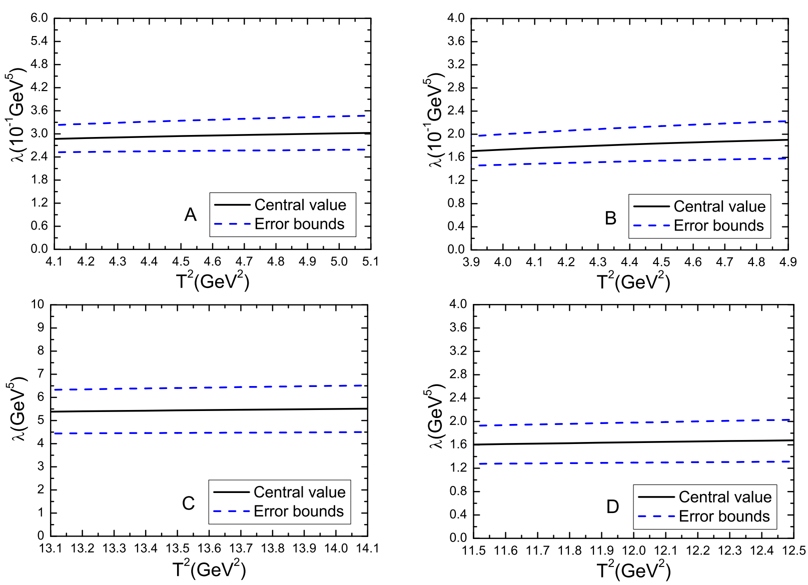

Fig. 2. The pole residues of the tetraquark states with variations of the Borel parameters T^2, where the A, B, C and D denote the $cc\bar{c}\bar{c}(1^{+-})$, $cc\bar{c}\bar{c}(1^{--})$, $bb\bar{b}\bar{b}(1^{+-})$ and $bb\bar{b}\bar{b}(1^{--})$, respectively.

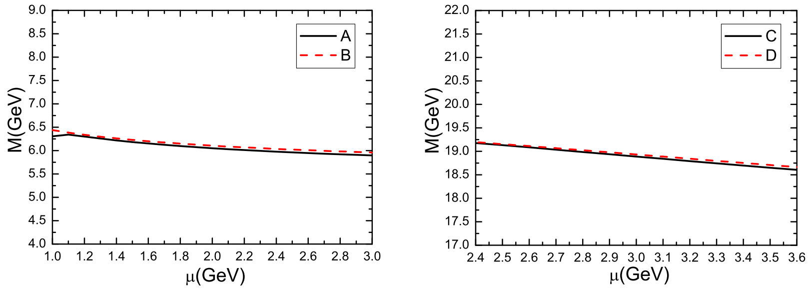

Fig. 3. The masses of the tetraquark states with variations of the energy scales μ, where the A, B, C and D denote the $cc\bar{c}\bar{c}(1^{+-})$, $cc\bar{c}\bar{c}(1^{--})$, $bb\bar{b}\bar{b}(1^{+-})$ and $bb\bar{b}\bar{b}(1^{--})$, respectively.

In Table II, we present all the masses of the 0^{++}, 1^{+-}, 2^{++} and 1^{--} $QQ\bar{Q}\bar{Q}$ tetraquark states from the QCD sum rules in Ref. [14] and in the present work. If we take the central values, the mass splittings among the spin multiplets are not consistent with the simple spin–spin interaction $C\,\vec{S}_1 \cdot \vec{S}_2$ between diquarks, where the C is a fitted constant, the $\vec{S}_1$ and $\vec{S}_2$ are the spins of the diquark and antidiquark, respectively. The spin–spin interactions among the quarks can be written as $\frac{C}{m_i m_j}\vec{s}_1 \cdot \vec{s}_2$, where the C is a fitted constant, the $\vec{s}_1$ and $\vec{s}_2$ are the spins of the quark and antiquark, respectively. Although the mass splittings (if the central values are taken) among the spin multiplets are also not consistent with the interaction $\frac{C}{m_i m_j}\vec{s}_1 \cdot \vec{s}_2$ quantitatively, they are reasonable qualitatively, the $cc\bar{c}\bar{c}$ tetraquark states have larger mass splittings according to the factor $\frac{1}{m_i m_j}$. Considering the uncertainties of the predicted tetraquark masses, we cannot draw the conclusion that the mass splittings are not consistent with the spin–spin interactions indeed.

TABLE II

The masses of the tetraquark states $QQ\bar{Q}\bar{Q}$ from the QCD sum rules.

	M_Y [GeV]	Centroids [GeV]
$cc\bar{c}\bar{c}(0^{++})$	5.99 ± 0.08	6.07 ± 0.08
$cc\bar{c}\bar{c}(1^{+-})$	6.05 ± 0.08	
$cc\bar{c}\bar{c}(2^{++})$	6.09 ± 0.08	
$bb\bar{b}\bar{b}(0^{++})$	18.84 ± 0.09	18.85 ± 0.09
$bb\bar{b}\bar{b}(1^{+-})$	18.84 ± 0.09	
$bb\bar{b}\bar{b}(2^{++})$	18.85 ± 0.09	
$cc\bar{c}\bar{c}(1^{--})$	6.11 ± 0.08	
$bb\bar{b}\bar{b}(1^{--})$	18.89 ± 0.09	

The vector tetraquark states lie 40 MeV above the corresponding centroids of the 0^{++}, $1^{+-}$and 2^{++} tetraquark states. Naively, we expect that an additional P-wave costs about 500 MeV, which is much larger than the energy gap 40 MeV. This may be a typical feature of the vector tetraquark states that consist of four heavy quarks. The calculations based on the QCD sum rules indicate that the $C\gamma_\mu \otimes \gamma_\nu C$–$C\gamma_\nu \otimes \gamma_\mu C$-type vector tetraquark state $cq\bar{c}\bar{q}$ has a mass 4.66 ± 0.09 GeV, the $C \otimes \gamma_\mu C$-type vector tetraquark state $cs\bar{c}\bar{s}$ has a mass 4.66 ± 0.09 GeV, which are all consistent with the $Y(4660/4630)$, the $C\gamma_5 \otimes \gamma_5\gamma_\mu C$-type vector tetraquark state $cq\bar{c}\bar{q}$ has a mass 4.34 ± 0.08 GeV, which is consistent with the $Y(4360/4320)$ [27, 34]. The energy gap between the vector and axialvector hidden-charm tetraquark states is about or larger than 440 MeV, which is much larger than 40 MeV.

In Ref. [27], we choose the current

$$
\begin{aligned}
\eta_{\mu\nu}(x) = {} & \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} \\
& \times \Big\{ u_j^T(x) C\gamma_\mu c_k(x) \bar{d}_m(x) \gamma_\nu C \bar{c}_n^T(x) - u_j^T(x) C\gamma_\nu c_k(x) \bar{d}_m(x) \gamma_\mu C \bar{c}_n^T(x) \Big\} , \quad (20)
\end{aligned}
$$

which has the same structure as the current $J_{\mu\nu}(x)$ in the present work. In the QCD sum rules for the vector tetraquark state $Y(4660)$, the pole contribution is (46–64)% [27], while in the QCD sum rules for the vector $QQ\bar{Q}\bar{Q}$ tetraquark states, the pole contributions are (46–62)% and (47–60)%, we can draw the conclusion tentatively that the same pole contributions lead to quite different mass splittings between the vector and axialvector tetraquark states. The four-heavy tetraquark states may have typical features due to absence of light quark contributions.

The values of the thresholds are $2M_{\eta_c}=5966.8$ MeV, $2M_{J/\psi}=6193.8$ MeV, $M_{\eta_c} + M_{J/\psi} = 6080.3$ MeV, $2M_{\eta_b} = 18798.0$ MeV, $2M_{\Upsilon} = 18920.6$ MeV, $M_{\eta_b} + M_{\Upsilon} = 18859.3$ MeV from the Particle Data Group [1]. The decays

$$
\begin{aligned}
Y\left(cc\bar{c}\bar{c}, 1^{+-}\right) &\to \eta_c J/\psi \to \mu^+\mu^- + \text{light hadrons}\,, \\
Y\left(bb\bar{b}\bar{b}, 1^{+-}\right) &\to \eta_b \Upsilon \to \mu^+\mu^- + \text{light hadrons}\,, \\
Y\left(bb\bar{b}\bar{b}, 1^{--}\right) &\to \Upsilon\Upsilon \to \mu^+\mu^-\mu^+\mu^- \qquad (21)
\end{aligned}
$$

can take place with very small phase spaces. The decays

$$
X\left(cc\bar{c}\bar{c}, 1^{--}\right) \to J/\psi J/\psi^* \to \mu^+\mu^-\mu^+\mu^- \qquad (22)
$$

can take place through the virtual J/ψ^*. We can search for the $Y(cc\bar{c}\bar{c}, 1^{+-}/1^{--})$ and $Y(bb\bar{b}\bar{b}, 1^{+-}/1^{--})$ in the mass spectrum of the $\mu^+\mu^-\mu^+\mu^-$ or $\mu^+\mu^-$ + light hadrons in the future.

4. Conclusion

In this article, we construct the axialvector-diquark–axialvector-antidiquark-type currents to study both the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states with the QCD sum rules, and obtain the predictions $M_{Y(cc\bar{c}\bar{c},1^{+-})} = 6.05 \pm 0.08\,\text{GeV}$, $M_{Y(cc\bar{c}\bar{c},1^{--})} = 6.11 \pm 0.08\,\text{GeV}$, $M_{Y(bb\bar{b}\bar{b},1^{+-})} = 18.84 \pm 0.09\,\text{GeV}$, $M_{Y(bb\bar{b}\bar{b},1^{--})} = 18.89 \pm 0.09\,\text{GeV}$. The vector tetraquark states lie 40 MeV above the corresponding centroids of the 0^{++}, $1^{+-}$and 2^{++} tetraquark states, which is a typical feature of the vector tetraquark states consisting of four heavy quarks. We can search for the $J^{PC} = 1^{+-}$ and 1^{--} $QQ\bar{Q}\bar{Q}$ tetraquark states in the mass spectrum of the $\mu^+\mu^- +$ light hadrons and $\mu^+\mu^-\mu^+\mu^-$, respectively, in the future.

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