

CALCULATIONS OF  $\eta$  NUCLEI,  $K^-$  ATOMS  
AND  $K^-$  NUCLEI\*

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We report on recent progress in theoretical studies of mesic atoms and nuclei performed by the Jerusalem–Prague Collaboration. We present calculations of  $\eta$  few-nucleon systems within the stochastic variational method. Further, we discuss  $K^-$  multinucleon interactions in the nuclear medium and demonstrate their role in kaonic atoms and nuclei. Finally, we introduce a microscopic model for  $K^-NN$  absorption in nuclear matter, developed very recently by J. Hrtánková and À. Ramos.

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**1. Introduction**

Low-energy meson–baryon interactions are presently described within approaches based on chiral perturbation theory combined with coupled-channel T-matrix re-summations techniques. These approaches that generate dynamically the nearby s-wave resonance  $\Lambda(1405)$  ( $N^*(1535)$ ) give strongly energy-dependent  $\bar{K}N$  ( $\eta N$ ) scattering amplitudes near threshold. Models developed by different groups yield considerably different scattering amplitudes (except the  $K^-p$  amplitude at and above the  $K^-p$  threshold), nevertheless they predict the near-threshold  $K^-N$  and  $\eta N$  attraction strong enough to allow binding of the  $K^-$  and  $\eta$  meson in nuclei. In nuclear medium, the free-space scattering amplitudes are modified due to Pauli blocking and hadron self-energies. These medium modifications as well as the strong energy dependence of the amplitudes have to be thoroughly taken into account in relevant calculations. Having this in mind, we have performed calculations of  $\eta$  nuclei,  $K^-$  atoms and  $K^-$  nuclei.

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## 2. Light $\eta$ nuclei

Our theoretical studies of  $\eta$ -nuclear quasibound states were discussed thoroughly in Refs. [1–5]. Here, we present selected results of our recent calculation [6], performed within the Stochastic Variational Method (SVM) [7]. The  $NN$  interaction is described by the Minnesota and Argonne AV4' potentials, while the interaction of  $\eta$  with nucleons is given by a complex two-body energy-dependent potential

$$v_{\eta N}(\sqrt{s}, r) = -\frac{4\pi}{2\mu_{\eta N}} b(\sqrt{s}) \rho_A(r), \quad (1)$$

where  $\rho_A(r) = (\frac{A}{2\sqrt{\pi}})^3 \exp\left(-\frac{\Lambda^2 r^2}{4}\right)$ , and  $b(\sqrt{s})$  is fitted for a given  $\Lambda$  to phase shifts derived from the  $\eta N$  scattering amplitudes of the GW [8] and CS [9] models. Two different values of the scale parameter,  $\Lambda = 2$  and  $4 \text{ fm}^{-1}$  are considered (see Ref. [3] for details).

The conversion widths are calculated using the expression

$$\Gamma_\eta = -2 \langle \Psi_{\text{gs}} | \text{Im} V_{\eta N} | \Psi_{\text{gs}} \rangle, \quad (2)$$

where  $|\Psi_{\text{gs}}\rangle$  stands for the ground state obtained after variation. This approximation seems reasonable since  $|\text{Im} V_{\eta N}| \ll |\text{Re} V_{\eta N}|$ . Very recently,  $\Gamma_\eta$  has been calculated by solving a generalized eigenvalue problem for a complex Hamiltonian using variationally determined SVM states for  $\text{Re} V_{\eta N}$  [6]. This approach (denoted ‘cmplx’ in Table I) takes into account the effect of the non-zero  $\text{Im} V_{\eta N}$  on the  $\eta$  binding energy, acting as repulsion and thus making the  $\eta$  meson less bound.

No bound  $\eta NN$  system was found for the considered two-body interaction models. No  $\eta NNN$  bound state was found for the AV4'  $NN$  potential. The results of calculations of  $\eta^3\text{He}$ ,  $\eta^4\text{He}$ , and  $\eta^6\text{Li}$  using the Minnesota  $NN$  potential and GW model are summarized in Table I. The CS model (not shown here) gives noticeably less bound  $\eta$  in the considered nuclei.

TABLE I

The binding energy  $B_\eta$  and width  $\Gamma_\eta$  (in MeV) in light  $\eta$  nuclei, calculated using the Minnesota  $NN$  potential including Coulomb interaction and the GW  $\eta N$  interaction model.

		$\eta^3\text{He}$		$\eta^4\text{He}$		$\eta^6\text{Li}$	
		$B_\eta$	$\Gamma_\eta$	$B_\eta$	$\Gamma_\eta$	$B_\eta$	$\Gamma_\eta$
$\Lambda = 2 \text{ fm}^{-1}$	Eq. (2)	0.11	1.37	0.97	2.17	2.17	3.00
	cmplx	not bound		0.77	2.22		
$\Lambda = 4 \text{ fm}^{-1}$	Eq. (2)	1.01	3.32	4.62	4.38	6.40	4.90
	cmplx	0.36	3.44	4.40	4.41		

### 3. Kaonic atoms

The  $K^-$  optical potential  $V_{K^-}^{1N}$  constructed self-consistently from the in-medium chiral  $K^-N$  amplitudes fails heavily in calculations of kaonic atom characteristics, represented by strong-interaction level shifts and widths of lower states and relative yields of transitions from upper states to the lower ones [10]. In fact, in the nuclear medium,  $K^-$  multinucleon interactions take place and have to be included. The  $K^-$  single-nucleon potential  $V_{K^-}^{1N}$  was thus supplemented with a phenomenological optical potential  $V_{K^-}^{mN}$  describing the  $K^-$  multinucleon interactions

$$2\text{Re}(\omega_{K^-})V_{K^-}^{mN} = -4\pi B \left( \frac{\rho}{\rho_0} \right)^\alpha \rho, \quad (3)$$

with complex amplitude  $B$  and exponent  $\alpha$  fitted to kaonic atom data. The total  $K^-$  optical potential is then expressed as a sum  $V_{K^-} = V_{K^-}^{1N} + V_{K^-}^{mN}$ . Good fits to the data were then obtained for all considered models.

The  $K^-$  optical potentials  $V_{K^-}$  were then confronted with the fractions of single nucleon  $K^-$  absorption at rest measured in bubble chamber experiments (see Ref. [10] for details). Figure 1 shows very good agreement between experiment and the P [11], KM [12] and BCN [13] models and clear disagreement for the M1(2) [14] and B2(4) [15] models.

It is to be noted here that the P, KM and BCN models were found acceptable also in the analysis of the  $K^-n \rightarrow \Lambda\pi^-$  non-resonant transition amplitude below threshold [16].

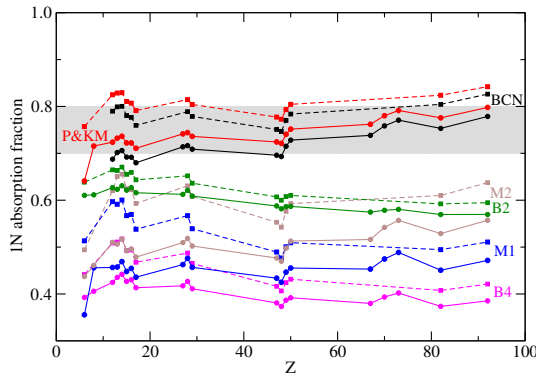


Fig. 1. Fractions of single-nucleon absorption for various  $K^-N$  interaction models, supplemented by  $V_{K^-}^{mN}$  fitted to atomic data. Solid lines and circles denote lower states, dashed lines and squares are for upper states, shaded band denotes the range of experimental values.

#### 4. Kaonic nuclei

The above  $K^-$  optical potentials were used in self-consistent calculations of  $K^-$ -nuclear quasi-bound states. Since the kaonic atom data probe the  $K^-$  optical potential up to at most  $\sim 50\%$  of the nuclear density [10], two scenarios were adopted in calculations of the  $K^-$ -nuclear states. In the full density (FD) option, the  $V_{K^-}^{mN} \sim B(\rho/\rho_0)^\alpha \rho$  form was used in the entire nucleus, while in the half density (HD) option, the  $mN$  term was fixed at a constant value equal to  $V_{K^-}^{mN}(0.5\rho_0)$  for densities  $\rho \geq 0.5\rho_0$ . The amplitude  $\text{Im } B$  in Eq. (3) was multiplied by a kinematical suppression factor to account for phase-space reduction. More details can be found in Ref. [17].

The role of the  $K^-$  multinucleon processes in the nuclear medium is demonstrated in Table II where we present the  $1s$   $K^-$  binding energies  $B_{K^-}$  and absorption widths  $\Gamma_{K^-}$  in selected  $K^-$ -nuclei, calculated within the KM model. In the  $KN$  column, we show the binding energies and widths obtained when only the  $1N$  term is used in the optical potential. The  $mN$  term causes a considerable increase of  $K^-$  widths, while  $K^-$  binding energies are affected only moderately. For most kaonic nuclei, the HD option of the  $V_{K^-}^{mN}$  potential yields  $K^-$  widths of about 100 MeV and the binding energies are much smaller than the widths. The FD version of the optical potential even does not predict any  $K^-$  bound state in the majority of nuclei. Our calculation for other interaction models confirmed that the widths of  $K^-$ -nuclear quasi-bound states in nuclei with  $A \geq 10$  are considerably larger than their binding energies. The identification of such states in experiment seems thus very unlikely.

TABLE II

$1s$   $K^-$  binding energies  $B_{K^-}$  and widths  $\Gamma_{K^-}$  (in MeV) in selected nuclei calculated within the KM model with the  $V_{K^-}^{1N}$  potential (denoted  $KN$ ); plus a phenomenological  $V_{K^-}^{mN}$  term for the HD $\alpha$  and FD $\alpha$  options ( $\alpha = 1, 2$ ).

		$KN$	HD1	FD1	HD2	FD2
$^{16}\text{O}$	$B_{K^-}$	45	34	not	48	not
	$\Gamma_{K^-}$	40	109	bound	121	bound
$^{40}\text{Ca}$	$B_{K^-}$	59	50	not	64	not
	$\Gamma_{K^-}$	37	113	bound	126	bound
$^{208}\text{Pb}$	$B_{K^-}$	78	64	33	80	53
	$\Gamma_{K^-}$	38	108	273	122	429

The phenomenological  $K^-$  multinucleon potential was found to be essential for a successful description of  $K^-$ -atomic data. It has a significant impact on the  $K^-$  absorption in the nuclear medium. However, its form in the nuclear interior is given by extrapolation or analytical continuation of

the empirical formula. It is thus desirable to develop a microscopic approach providing a unified description of the  $K^-$  single and multinucleon potential. Here, we briefly report on a very recent development of one such model describing the  $K^-$  absorption on two nucleons in nuclear matter, motivated by an approach applied in Ref. [18]. The  $K^-$  absorption on two nucleons is described within a meson-exchange picture with the  $K^-NN$  self-energy modeled using chirally motivated  $\bar{K}N$  amplitudes modified due to Pauli blocking. The details of the  $\bar{K}NN$  model including discussion of the results of calculations can be found in [19].

We illustrate applicability of the developed model on the evaluation of the  $\Lambda p$  to  $\Sigma^0 p$  production rate in  $K^-pp$  quasi-free absorption measured recently in the AMADEUS experiment [20]. In Fig. 2, we show this rate calculated using the  $\bar{K}NN$  model when either the free space (black lines) or Pauli blocked (gray/red lines) P amplitudes are employed, assuming the  $K^-$  binding energy in nuclear matter to be either  $B_{K^-} = 0$  or  $B_{K^-} = 50 \rho/\rho_0$  MeV. At the experimentally relevant densities, the ratio  $R$  calculated with Pauli blocked amplitudes is in good agreement with the measured value. This demonstrates importance of medium modifications of the scattering amplitudes.

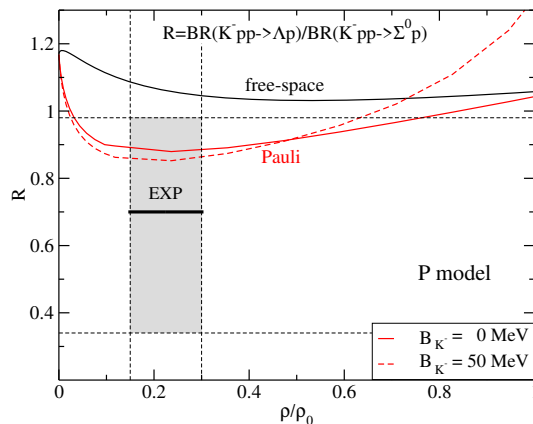


Fig. 2. (Color online) The ratio of branching ratios for the in-medium  $K^-pp \rightarrow \Lambda p$  and  $K^-pp \rightarrow \Sigma^0 p$  reactions calculated within the P model. The shaded area denotes the experimental error band and densities probed by low-energy antikaons.

## 5. Summary

We presented the results of our recent SVM calculations of lightest  $\eta$  nuclei. We discussed  $K^-$  multinucleon processes in the nuclear medium and demonstrated their decisive role in kaonic atoms and nuclei. Finally, we

introduced a currently developed microscopic model for  $\bar{K}NN$  interactions in nuclear matter, based on chiral  $\bar{K}N$  amplitudes, and showed the effect of medium modifications of the scattering amplitudes.

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