CORRELATIONS AMONG OBSERVABLES IN TWO- AND THREE-NUCLEON SYSTEMS*

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(Received December 6, 2019)

We study the elastic nucleon–deuteron scattering process at incoming-nucleon laboratory energies up to $E = 200$ MeV working within the formalism of Faddeev equations. We focused here on the computation and systematic analysis of the correlation coefficients among various observables in nucleon-nucleon and nucleon–deuteron elastic scattering. As a result, we obtained pairs of correlated/uncorrelated observables or sets of very weakly correlated observables. The knowledge, if some observables are or are not correlated, would impact future methods of fixing free parameters of the two- and many-body potentials, and could possibly help determine observables which should be measured to increase the precision of potential parameters' determination.

DOI:10.5506/APhysPolB.51.273

1. Introduction

The *ab initio* theoretical studies of the three-nucleon ($3N$) observables in elastic nucleon–deuteron scattering are possible using modern models of nuclear forces [1–3]. The two-nucleon ($2N$) force models used in such investigations contain a number of free parameters whose values are fixed from the $2N$ data. In the case of some models, for instance the One-Pion-Exchange-Gaussian (OPE-Gaussian) force [4], obtained by the Granada group or the chiral force with the semilocal momentum-space regularization [5] derived by the Bochum group even beyond the fifth order of the chiral expansion (N^4LO), in addition to the central values of the parameters also their correlation matrix has been determined. The knowledge of the correlation matrix of the potential parameters opens new possibilities in studies of few-nucleon systems.

* Presented at the 3rd Jagiellonian Symposium on Fundamental and Applied Subatomic Physics, Kraków, Poland, June 23–28, 2019.

Recently, we successfully estimated the uncertainty of the $3N$ observables, which arises from uncertainties of the $2N$ potential parameters, see Refs. [6, 7]. Another possibility of using the covariance matrix of $2N$ potential parameters is to study systematically correlations among $2N$ observables between potential parameters (for a chosen model of the interaction) and $2N$ observables among $3N$ observables and, finally, between $2N$ and $3N$ observables. The aim of such a study would be to investigate to what extent the above-mentioned observables are correlated and to determine pairs of strongly/weakly correlated observables. Especially, the existence of correlations in the $3N$ system puts restrictions on data sets used during fitting the $3N$ potential parameters.

To achieve the goal of gathering information about the correlations among various quantities, it is necessary to collect a sufficiently large number of predictions for each observable. Using the covariance matrices of the above-mentioned models of the nucleon–nucleon interaction, we sampled 50 sets of potential parameters and obtained many versions of the corresponding potential. Next, we computed observables in the $2N$ system, solving, the Schrödinger equation for the deuteron as well as the Lippmann–Schwinger equation to obtain the t-matrix operator and the nucleon–nucleon scattering transition amplitude from which observables can be calculated. The same sets of potential parameters were subsequently used for the $3N$ system calculations.

To compute observables in the $2N$ system, we first solved the Lippmann–Schwinger equation for the t operator

$$t = V + VG_0 t \,, \tag{1}$$

with the $2N$ potential V and the free $2N$ propagator G_0. From the matrix elements of the t-operator, we obtained the nucleon–nucleon scattering transition amplitude and, finally, the observables. For $3N$ scattering, we work in the Faddeev approach neglecting the $3N$ force and solving the $3N$ scattering equation with the $2N$ off-shell t-operator

$$T|\phi\rangle = tP|\phi\rangle + tPG_0T|\phi\rangle \,. \tag{2}$$

Here, T is the $3N$ transition operator, P is a permutation operator composed from two-particle transpositions and $|\phi\rangle$ is the initial-channel state built as a product of the deuteron wave function and a momentum eigenstate of the projectile nucleon. We solve numerically Eq. (2) in the momentum-space partial wave based representation by generating the Neumann series and summing it by the Padé method, see [1].

2. Results

In Fig. 1, we present the angular dependence of selected correlation coefficients for pairs of $2N$ (top) and $3N$ (bottom) observables as functions of the center-of-mass scattering angle in the range of $\theta_{\mathrm{c.m.}} \in [12.5^\circ, 167.5^\circ]$. Correlations between two selected $2N$ observables, such as the differential cross section and the spin-correlation coefficient C_{NN} (Fig. 1 (a)), the polarization P and the depolarization A' parameter (Fig. 1 (b)) have been investigated with the semilocal momentum-space-regularized (SMS) chiral N^{2}LO, N^{4}LO and N^{4}LO+ potentials [5] with the value of the regulator parameter $\Lambda = 450$ MeV at the incoming-neutron laboratory energy $E = 10$ MeV. It can be observed that the differential cross section appears weakly anti-correlated with the spin-correlation coefficient in the case of the SMS N^{4}LO

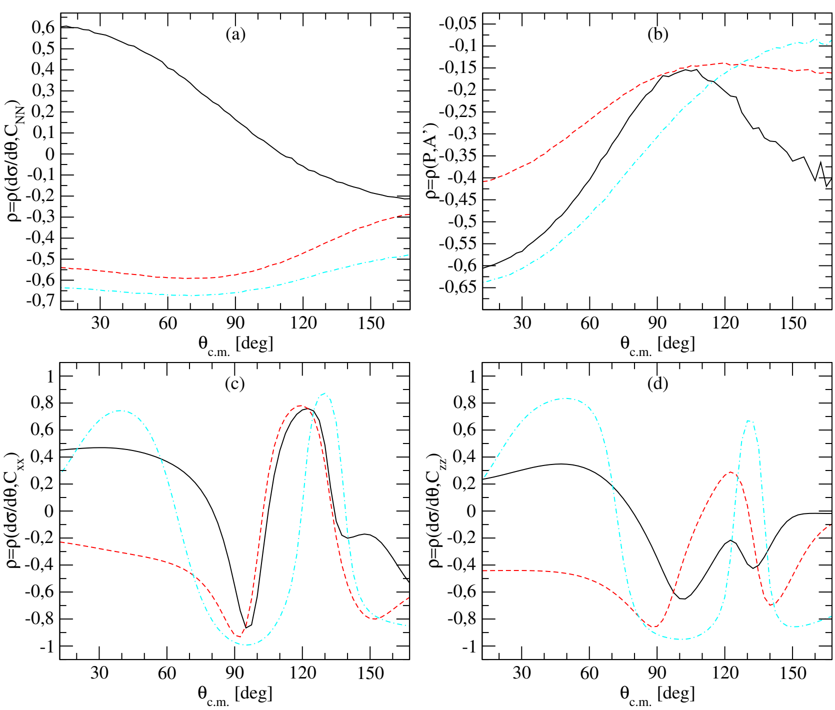

Fig. 1. Top panel: The angular dependence of correlations coefficients between selected $2N$ scattering observables: $\mathrm{d}\sigma/\mathrm{d}\Theta$ and C_{NN} (a), P and A' (b) for the incoming-neutron laboratory energy $E = 10$ MeV. Bottom panel: The angular dependence of correlations coefficients between selected elastic nucleon–deuteron scattering observables: $\mathrm{d}\sigma/\mathrm{d}\Theta$ and C_{xx} (c), $\mathrm{d}\sigma/\mathrm{d}\Theta$ and C_{zz} (d) at the incoming-neutron laboratory energy $E = 13$ MeV. The solid, dashed, dash-dotted lines represent predictions of the SMS chiral N^{2}LO, N^{4}LO and N^{4}LO+ forces, respectively.

and N^4LO+ potentials, while these observables are correlated at small scattering angles for the SMS N^2LO force. Another observation is that in the case of the SMS N^2LO force, the correlation coefficient undergoes faster changes with scattering angle than the correlation coefficient among these observables computed at the N^4LO and N^4LO+. For the (P, A') pair, we observe, regardless of the potentials, that these observables are anti-correlated at small scattering angles.

With regard to elastic neutron–deuteron scattering, we checked selected correlations, namely between the differential cross section and the spin-correlation coefficient C_{xx} (Fig. 1 (c)) as well as between the differential cross section and the spin-correlation coefficient C_{zz} (Fig. 1 (d)), respectively, at the incoming-neutron laboratory energy of $E = 13$ MeV. These $3N$ observables were calculated using the SMS chiral N^2LO, N^4LO and N^4LO+ forces with $\Lambda = 450$ MeV. Both plots 1 (c) and 1 (d) reveal a complex behavior of correlation coefficients with scattering angles and existence of regions with $|\rho| > 0.8$.

3. Summary and outlook

We investigated correlations among the chosen $2N$ and $3N$ observables for the first time in a statistically correct way. The results of this study should be taken into account before applying the fitting procedure to determine $2N$ potential parameters. In the future, we plan to investigate correlations among all $2N$ and $3N$ elastic scattering observables, between $2N$ and $3N$ bound states as well as between $2N$ and $3N$ scattering observables. Inclusion of the $3N$ force is also planned.

This work is a part of the LENPIC project and is supported by the National Science Centre, Poland (NCN) under grants No. 2016/22/M/ST2/00173 and No. 2016/21/D/ST2/01120. Numerical calculations were performed on the supercomputer cluster of the JSC, Jülich, Germany.

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