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NEW CONCEPTS IN TESTS OF THE PAULI EXCLUSION PRINCIPLE IN BULK MATTER*

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The standard scheme of several tests of the Pauli Exclusion Principle in bulk matter — both in the experiment and in the subsequent data analysis — has long been based on the seminal paper by E. Ramberg, G.A. Snow [*Phys. Lett. B* **238**, 438 (1990)]. The ideas exposed in that paper are so simple and immediate that they have long gone unchallenged. However, while some of the underlying approximations are still valid, other parts of the article must be reconsidered. Here, we discuss some new concepts that

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are related to the motion of the electrons in the test metal (the “target” of the experiment) and which have been recently studied in the framework of the VIP-2 Collaboration.

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1. Introduction

In 1990, Ramberg and Snow (RS) [1] carried out a very simple and clean experiment to test the validity of the Pauli exclusion principle. They used a direct current to force conduction electrons to flow in a copper strip (the “target”), and they conjectured that if any one of these electrons approached an atom where another electron had a “wrong” pairing with it, it could undergo radiative capture and emit an X-ray as it settled into an anomalous atomic ground state. Due to the non-standard electronic configuration, the emitted X-rays should differ from the usual characteristic X-rays of copper, and their detection would provide a signature of a violation of the principle.

The VIP-2 Collaboration [2] has set up a highly improved version of the original RS experiment. The optimized experimental setup includes state-of-the-art solid-state detectors, and it operates in a low-background environment to achieve a very high sensitivity of any violation of the Pauli exclusion principle.

In its effort to improve the upper limit of the violation probability, the collaboration has also questioned some of the simple assumptions that RS used in their original search of anomalous X-rays. Here, we briefly discuss some aspects that are related to the diffusion of conduction electrons in the metal target.

2. Diffusion of electrons in bulk metal

In their analysis, RS relied on an extremely simplified view of electron motion in the copper target. They assumed that each conduction electron performs a simple straight motion from the entrance to the exit, and from this they computed the number of scatterings as determined from the elementary theory of conduction in metals. However, electrons do not move along a straight line, instead each electron follows a complex path determined both by random scattering events and by the drift due to the superimposed electric field. Moreover, in conduction theory, the scatterings that actually matter are not those between electrons and atoms, but rather those between conduction electrons and phonons, and those with the irregularities (impurities and dislocations) of the crystal lattice.

In [3], we made an attempt at overcoming these problems. We treated the motion of the conduction electrons in metals with the formalism of classical (non-quantum) random walks, and we showed that the total number of scatterings observed for a single electron that drifts from the entrance to the exit is

$$S(\Delta t_{\mathrm{M}}, I) = \frac{1}{\tau} \int_0^{\Delta t_{\mathrm{M}}} P(t) \mathrm{d}t\,, \tag{1}$$

where Δt_{M} is the total measurement time, $1/\tau$ is the individual electron's scattering rate, which is related to the mean free path and to the Fermi velocity, and I is the current. The function $P(t)$ is given by the integral

$$P(t) = \frac{1}{2}\left[\mathrm{erf}\left((L - v_{\mathrm{d}}t)/\sqrt{2Dt}\right) - \mathrm{erf}\left(-v_{\mathrm{d}}t/\sqrt{2Dt}\right)\right]\,, \tag{2}$$

where L is the target length, v_{d} is the drift speed, and D is the diffusion constant for the conduction electrons in copper. The error function appears in the formula because the Green function of simple diffusion has a Gaussian shape. The other target properties are included in the expression of the drift speed

$$v_{\mathrm{d}} \approx \frac{1}{n_{0\,\mathrm{K}}\, ezw} I\,, \tag{3}$$

where e is the elementary charge, z is the target thickness, w is the target width, and $n_{0\,\mathrm{K}}$ is the electron density in copper at 0 K.

It is important to note that in this approach, the scattering time τ is not related to conduction theory. The time τ should rather be taken to be the mean time between successive "close encounters" with atoms, where a conduction electron's wave function has a considerable overlap with the atomic electron wave functions. We find a rough estimate of τ starting from the electron wavelength $\lambda_e \approx h/m_e v_{\mathrm{F}} \approx 6.1 \times 10^{-10}$ m, where $v_{\mathrm{F}} \approx 1.18 \times 10^6$ m/s is the Fermi speed of conduction electrons in copper. The radiative capture probability per close encounter r can be estimated from the measured width of the naturally occurring K_α line complex $\mathit{\Gamma} \approx 2.73$ eV (weighted value for the whole K_α complex, values taken from [4]), and from the transit time $\lambda_e/v_{\mathrm{F}} \approx 5.3 \times 10^{-16}$ s. A rough approximation for the radiative capture probability r yields $r \approx (\hbar/\mathit{\Gamma}) \times (\lambda_e/v_{\mathrm{F}}) \approx 1.6 > 1$; this means that we can assume that every close encounter leads to a capture with a probability that is only limited by the Pauli violating probability $\beta^2/2$. Now, using the computed electron wavelength and the electron density in copper, we find the average distance between close encounters with atoms, $\ell \approx 1/n\pi(\lambda/2)^2 \approx 41$ pm and the corresponding mean time between close encounters $\tau = \ell/v_{\mathrm{F}} \approx 3.5 \times 10^{-17}$ s.

Taking these numbers, we find that in the case of VIP-2 ($I = 100$ A) with a mean traversal time of about 10 s, there are on average at least 2.8×10^{17} close encounters. When we repeat the RS analysis with these new estimates, we find the bound on the violation parameter $\beta^2/2 < 2.6 \times 10^{-40}$.

For further details on this analysis, see Ref. [3]. However, this is not the end of the story, this detailed view of electron motion leads to several other interesting byways. In the next section, we give a schematic introduction to one of them.

3. Remnant signals

In addition to the data collected with the VIP-2 apparatus, we had the opportunity to study the X-ray emission from a large lead block using a high purity germanium detector. The block is made of extremely pure Roman lead, it has a cylindrical shape and completely surrounds the Ge detector. It is composed of three 5 cm thick sections, with a total volume $V \sim 2.17 \times 10^3$ cm^3 (a total of 108 mole). With this arrangement, the thick lead block also acts as an additional radiation shield for the Ge detector. Further details on the experimental layout are given in [5].

When we assume that a violation of the Pauli exclusion principle stems from the existence of electron pairs with the wrong symmetry — as in the work of Rahal and Campa [6] — then the emission of anomalous X-rays takes place only when the pairs finally meet. In a large block of lead like the one available to us, this process may take a very long time. When we consider Pb and take the average scattering time from conduction theory $t \approx 1.3 \times 10^{-15}$ s, we find about 2.42×10^{22} scatterings per year and, therefore, it would take at least 25 years to total one mole of scatterings (2700 years for the 108-mole Roman lead block). This simple argument hints at the possibility of detecting a remnant signal of anomalous X-rays in blocks of metal that were forged as long as several hundred years ago.

As a conduction electron performs its random walk and wanders through the lead block, it may meet an atomic electron which is wrongly paired with it. Then, it can be shown that the probability that a given conduction electron survives capture during the time interval T is

$$p = \exp\left[-T/(N\tau/P_{\mathrm{cpt}})\right] , \tag{4}$$

where N is the number of atoms in the block, τ is the mean time between successive electron–atom encounters, and P_{cpt} is the capture probability in a single interaction (if the wrong pairing is verified; for further details, see [5]).

In our case, the situation is somewhat more complex, because the lead block is actually subdivided in three smaller blocks. Moreover, we must take into account the fact that the Roman lead [7] was used to forge the blocks from smaller bricks that were originally cast in Roman times, and where it is reasonable to expect that there is no remaining signal. However, the probabilistic capture model can be extended to cover the case of several smaller blocks forged from many individual bricks [5], so that the estimate of the relic signal N_{X} (total number of detected anomalous X-rays) is

$$N_{\mathrm{X}} = \frac{\beta^2}{2} \times (N_{\mathrm{int}} P_{\mathrm{cpt}} \epsilon_{\mathrm{tot}}) \times \left[\sum_{i=1}^{i=\nu} N_{\mathrm{free}}^{i} \exp(-T_i P_{\mathrm{cpt}}/N_i \tau) \right] , \qquad (5)$$

where ν is the number of blocks, T_i is the time since the original forging of the i^{th} block, N_i is the number of atoms in the i^{th} block, $N_{\mathrm{int}} = \Delta t/\tau$ is the number of interactions per electron during the measurement time Δt, and ϵ_{tot} is the experimental detection efficiency. Finally, the number of free conduction electrons in each block N_{free}^{i} can be replaced by the effective number computed with the assumption that the original bricks are nearly identical, taking into account the depletion of the useful electrons

$$N_{\mathrm{free,eff}}^{i} \approx [(n-1) + \exp(-T_{\mathrm{brick}}^{i} P_{\mathrm{cpt}}/N_{\mathrm{brick}} \tau)] N_{\mathrm{free}}^{\mathrm{brick}} \approx (n-1) N_{\mathrm{free}}^{\mathrm{brick}} . \qquad (6)$$

The reduction of the number of useful conduction electrons is largest for two same-size bricks, and when we assume, conservatively, $n = 2$, we find

$$N_{\mathrm{X}} = \frac{\beta^2}{2} \times \frac{N_{\mathrm{int}} P_{\mathrm{cpt}} \epsilon_{\mathrm{tot}}}{2} \times \left[\sum_{i=1}^{i=\nu} N_{\mathrm{free}}^{i} \exp(-T_i P_{\mathrm{cpt}}/N_i \tau) \right] , \qquad (7)$$

and finally we can give the experimental 3σ-bound for the violation probability

$$\frac{\beta^2}{2} < \frac{2 N_{3\sigma}}{(N_{\mathrm{int}} P_{\mathrm{cpt}} \epsilon_{\mathrm{tot}}) \times \left[\sum_{i=1}^{i=\nu} N_{\mathrm{free}}^{i} \exp(-T_i P_{\mathrm{cpt}}/N_i \tau) \right]} . \qquad (8)$$

The experiment was carried out in the low-background environment of the Gran Sasso underground laboratory of the Italian Institute for Nuclear Physics (INFN). The Roman lead was chosen because of its very low intrinsic radioactivity [7]. Finally, using the model described in the previous section, we found the following p-value for the violation of the null hypothesis (no violation) $p = 0.0118$, corresponding to 2.26σ — too large to claim any discovery. Moreover, from equation (8), we obtained $\beta^2/2 < 2.7 \times 10^{-40}$.

4. Conclusions

It is interesting to remark that taking the estimate of the observable mass of the whole Universe $M \approx 1.6 \times 10^{55}$ g (roughly equivalent to 10^{28} Earths), and assuming that nearly all of this matter is composed of hydrogen atoms, we find that the total number of electrons in the observable universe is about 10^{79}. As a consequence, the nearly equal bounds reported here mean that less than about 10^{36} electron pairs in the universe can actually have a wrong symmetry pairing (a total of about 10^8 on Earth, about 1 for every 6×10^{16} kg of the Earth's mass).

Although the two bounds are very similar in magnitude, they have been obtained with significantly different methods. The first one — which is an improved version of the original RS method — is obtained with a direct current in a stationary physical system. On the contrary, the second one is based on a non-stationary system. This consideration and all the developments that we have briefly described in this paper are now spawning further ideas for possible improvements in the VIP-2 experimental setup, and may lead to even more stringent bounds on the violation of the Pauli exclusion principle.

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