MICROSCOPIC OPTICAL MODEL ANALYSIS OF PROTON–NUCLEUS ELASTIC SCATTERING AT LOW ENERGY

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For the first time, a systematic calculation of low-energy proton–nucleus elastic scattering off a series of doubly closed-shell nuclei is carried out. The direct and exchange elements of the Coulomb interaction between protons have been included to the residual interaction of the particle-vibration coupling (PVC). The nucleon–nucleon (NN) effective interaction of the Skyrme-type is consistently used in the whole process to generate the microscopic optical potential (MOP). Within the present microscopic optical potential model, angular distributions and analyzing power could be reproduced without any adjustable parameters.

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1. Introduction

Optical model potential is still a hot topic in the nuclear physics community [1–3]. The nucleon–nucleus optical potentials are the most important inputs for the nuclear reactions codes [4–6]. Over the years, most of optical

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potentials are phenomenologically fitted from elastic scattering observables such as angular distributions and analyzing powers. These potentials are generally and successfully used in many nuclear reactions codes for nuclei in different regions of nuclear chart and for several energy ranges, for the situations where elastic scattering experimental data are available. However, it is well known that, for situations where experimental data are not yet available, *e.g.* for neutron-rich exotic nuclei, the use of phenomenological optical potentials leads to uncertainties. Microscopic optical potentials are expected to be more appropriate to investigate these situations.

At low energy (below 50 MeV), where the specific nuclear structure effects become more important, the nuclear structure approaches have proved their ability to extract reliable optical potentials directly from the effective phenomenological nucleon–nucleon interactions. Recently, the energy density functionals built from the NN Skyrme and Gogny effective interactions (even these interactions were not initially designed for scattering purpose) have been successfully applied to NA scattering at low energies. These calculations have succeeded to reproduce the experimental data on neutron elastic scattering [7] by ^{16}O, proton inelastic scattering [8] by ^{24}O, neutron and proton elastic scattering by ^{40}Ca and ^{48}Ca and the analyzing power [9–11], and neutron elastic scattering [12] by ^{16}O and ^{208}Pb. However, the calculations of Mizuyama *et al.* [7] are not fully self-consistent since the two-body spin-dependent terms, the spin–orbit terms, and the Coulomb term have been dropped in the cRPA as well as in the cPVC calculations. Only two calculations are fully self-consistent — of Blanchon *et al.* [9] with Gogny interaction and Nhan Hao *et al.* [12] with Skyrme interaction. Due to the finite-range of Gogny interaction, the calculations of Refs. [9, 11] are still limited in the medium region.

By using the zero-range Skyrme interaction (of course less realistic than the Gogny one), first systematic microscopic calculations [13] for neutron elastic scattering have been performed for a series of doubly closed-shell nuclei such as ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb. The goal of the present work is to extend the latter systematic calculations to the proton–nucleus elastic scattering as a part of our long-term project.

2. Formalism

The formalism for the microscopic potential has been extensively used to study the neutron elastic scattering [12, 13]. Thus, we will only sketch here the major improvements of this approach to take into account the Coulomb interaction between protons and discuss some of its relevant numerical aspects.

First, let us briefly recall some general features of the microscopic optical potentials. According to Refs. [12, 14], the MOPs are given as

$$V_{\rm opt} = V_{\rm HF} + \Delta\Sigma(\omega)\,, \tag{1}$$

where

$$\Delta\Sigma(\omega) = \Sigma(\omega) - \tfrac{1}{2}\Sigma^{(2)}(\omega)\,. \tag{2}$$

In Eqs. (1) and (2), $V_{\rm HF}$ is the Skyrme HF mean-field potential which is real, local, momentum-dependent, and energy-independent. The locality of $V_{\rm HF}$ is the limit of our model (due to the use of the zero-range interaction) since the non-locality has been proved to be very important as in the dispersive optical potential [15]. The polarization potential, $\Delta\Sigma(\omega)$, is non-local, complex, and energy-dependent, where ω is the nucleon incident energy. Thus, the obtained potential is non-local, energy-dependent, and complex. All these properties originate from the channel-coupling as shown in Ref. [16]. The imaginary part of $\Sigma(\omega)$ is responsible for the absorption of the optical potential due to the non-elastic channels. To take into account the issue of the Pauli principle correction, the second-order potential $\Sigma^{(2)}(\omega)$ has been generated from the uncorrelated particle–hole contributions.

The first order potential $\Sigma(\omega)$ is the contribution from the particle–hole correlations generated from a fully self-consistent particle-vibration coupling (PVC) calculations [12, 14, 17, 18]. In the PVC framework, the matrix elements are calculated by using the residual interaction, the RPA transition density, and the HF single-particle wave functions in order to describe the channel-coupling (particle-vibration coupling) at the intermediate states. Its reduced matrix elements read

$$\begin{aligned}\Sigma^{(lj)}_{\alpha\beta}(\omega) \;&\equiv\; \langle\epsilon_\alpha, lj||\Sigma(\omega)||\epsilon_\beta, lj\rangle \\ &=\; \hat{j}_\alpha^{-1}\hat{j}_\beta^{-1}\Bigg(\sum_{nL,A>F}\frac{\langle\alpha||V||A,nL\rangle\langle A,nL||V||\beta\rangle}{\omega-\epsilon_A-\omega_{nL}+i\eta} \\ &\quad+\sum_{nL,a<F}\frac{\langle\alpha||V||a,nL\rangle\langle a,nL||V||\beta\rangle}{\omega-\epsilon_a+\omega_{nL}-i\eta}\Bigg)\,,\end{aligned}$$

where ω_{nL} are the phonon energies with multipolarity L, and the symbol F denotes the Fermi level. The parameter η is introduced to perform the energy averaging on the potential $\Delta\Sigma(\omega)$. Regardless of the properties (finite or zero range) of residual interaction of the matrix element, the self-energy function is always given by the non-local complex function. The matrix elements $\langle i||V||j,nL\rangle$ are calculated by

$$\langle i||V||j,nL\rangle = \sqrt{2L+1}\sum_{\rm ph}X^{nL}_{\rm ph}V_L(ihjp) + (-)^{L+j_h-j_p}Y^{nL}_{\rm ph}V_L(ipjh)\,, \tag{3}$$

where i, j, p, h are HF single-particle (s.p.) states, nL are phonon states, and $X(Y)$ are the eigenvectors of the RPA secular matrix. The p–h coupled matrix elements V_L read

$$\begin{aligned} &V_L(ihjp) \\ &= \sum_{\text{all}m} (-)^{j_j - m_j + j_h - m_h} \langle j_i m_i j_j - m_j | LM \rangle \langle j_p m_p j_h - m_h | LM \rangle \\ &\times \langle j_i m_i, j_h m_h | V | j_j m_j, j_p m_p \rangle \,. \end{aligned} \tag{4}$$

To take into account the Coulomb interaction between protons, its particle–hole matrix elements [19] have been included into the residual interaction. The direct matrix element reads

$$\begin{aligned} V_L(ihjp) \;=\; & \frac{4\pi e^2}{(2L+1)^2} \langle i || Y_L || j \rangle \langle p || Y_L || h \rangle \\ & \times \int \mathrm{d}r_1 \mathrm{d}r_2 \frac{r_<^L}{r_>^{L+1}} u_i(r_1) u_h(r_2) u_j(r_1) u_p(r_2) \,, \end{aligned} \tag{5}$$

where $r_>(r_<)$ stands for the larger (smaller) among r_1, r_2, and $u_\alpha(r)$ are the radial function of the s.p. states. The p–h matrix element of the exchange term is

$$V_L(ihjp) = \frac{1}{2L+1} \langle i || Y_L || j \rangle \langle p || Y_L || h \rangle \int \frac{\mathrm{d}r}{r^2} v_{\mathrm{CX}}(r) u_i(r_1) u_h(r_2) u_j(r_1) u_p(r_2) \,, \tag{6}$$

where

$$V_{\mathrm{CX}} = -\frac{1}{3} e^2 \left(\frac{3}{\pi} \right)^{\frac{1}{3}} \rho_p(r)^{-\frac{2}{3}} \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) = v_{\mathrm{CX}}(r) \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \,. \tag{7}$$

In our calculations, the static exchange Coulomb has been approximated with a Slater approximation [20].

3. Results and conclusions

We briefly report the numerical aspect adopted in our calculations. First, we solve the radial HF equations in the coordinate space: the radial mesh size is 0.1 fm and the maximum value of the radial coordinate is set to be 15 fm. The NN effective interaction SLy5 has been adopted [21]. After the HF solutions are reached, the ground states and various excited states are then calculated within the fully self-consistent RPA framework [19]. All the natural parity RPA excited states with the multipolarity L from 0 to 5 whose energies are smaller than 50 MeV and fraction of the total isoscalar

or isovector strength larger than 5% are selected for the particle-vibration coupling calculations. It is rather well known that the divergence in the mean-field calculations (such as RPA and PVC) is expected for the energy or angular momentum cut-off. It is due to the zero-range interaction. If the finite-range interaction is used, this kind of problem never happens. In order to overcome this problem, the regularization scheme is necessary. There are some works devoted to solve this problem in nuclear matter calculation [22], but there is not straightforward way for a finite nuclei yet. Besides, no one still knows how the problem appears in the cross sections as the application of the PVC to the nuclear reaction. In order to check such a problem, firstly, the full self-consistency which means to include all of the terms of Skyrme interaction is necessary. Second, there are many types of NA scattering (or reaction) such as elastic, inelastic, capture, nucleon emission and so on. Is is also very important to check the applicability of the PVC of those types of reactions, because the results for each different type of scattering would show the applicability of the PVC model for reactions channels. Third, needless to say, the systematic calculation for various nuclei is also important. We have already published a systematic calculation for a series of doubly-closed shell targets at several incident energies below 50 MeV using this MOP in Ref. [13]. On top of that, each calculation and analysis are very time-consuming. Then we have to make the problem clear step by step. Finally, it is worth mentioning here that the above parameters are fixed for both neutron [13] and proton off all targets at all incident energies.

In this study, we focus on the angular distributions and analyzing power which are the most important nuclear reactions observables. In Fig. 1, we show the angular distributions for the proton elastic scattering on ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb at several incident energies below 50 MeV. Almost all the angular distributions are overall very well reproduced except the systematic disagreement at large scattering angles at all incident energies for all considered nuclei. It is already well known that these deviations are the inherent defects of both POP [23, 24] and MOP [12, 13, 25, 26]. The deviations from the experimental data at backward angles could be the consequence of several limits of our model such as: the effective Skyrme interaction itself, our RPA calculations based only on the first order of $1p1h$ excitations, the unnatural parity states having been dropped, the missing compound elastic contributions at energies below 10 MeV. It is also expected that the angular-distributions at large scattering angles could be sensitive to the treatment of the continuum. For ^{40}Ca, our results are almost coherent with the ones obtained from the calculations with Gogny interaction [11].

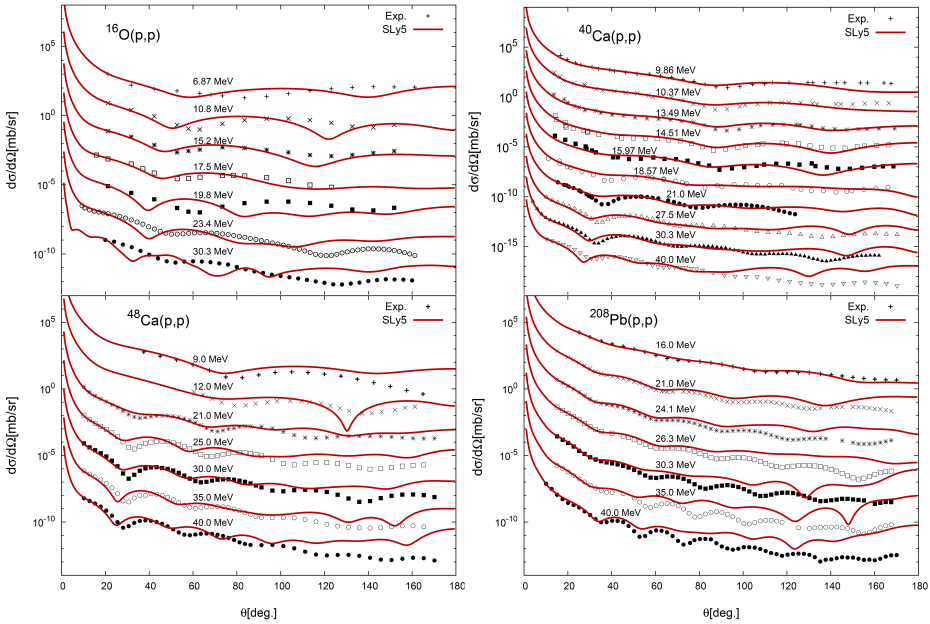

Fig. 1. Angular distributions of proton elastic scattering by ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb at different incident energies below 50 MeV. The solid curves show the results of the MOP calculations using the SLy5 interaction. The experimental data are the tabulated cross sections taken from Ref. [18].

In Fig. 2, we show the calculated analyzing powers for the proton elastic scattering on ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb at different incident energies. In general, our self-consistent mean-field model (SCMF) retains the correct spin–orbit behavior. However, the results for analyzing power for ^{16}O are bad compared to the experimental data. For ^{40}Ca and ^{48}Ca, the analyzing powers generated from almost the same framework with the finite-range Gogny interaction are better than our results. Together with the obtained angular distributions, these results indicate that the SCMF works very well for medium and heavy nuclei.

This work completes the process to generate the nucleon–nucleus microscopic optical potentials. It confirms the possibility to build a fully self-consistent MOP starting from any given Skyrme energy-density functional to reproduce the low-energy NA elastic experimental data with sufficient accuracy. In the near future, we will try to improve our models by fixing the limits mentioned above. The recent model could be extended by using the finite-range interaction such as DD-MY3. Recently, based on this self-consistent scheme, the role of each components of Skyrme interaction to the

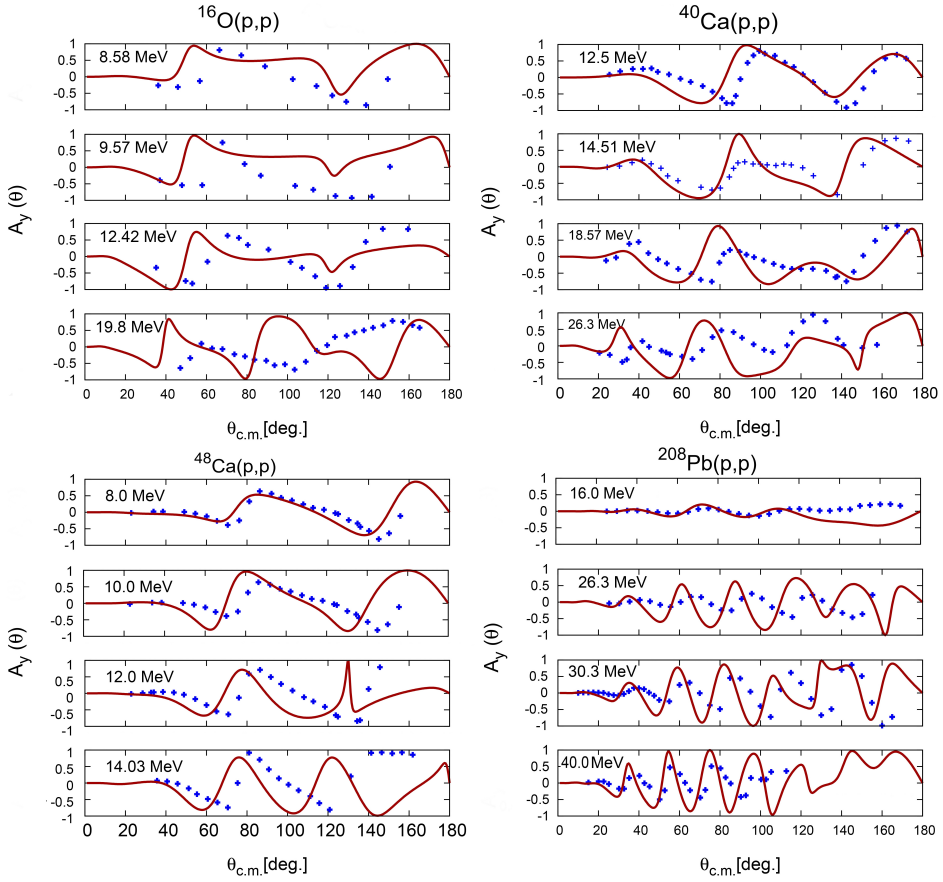

Fig. 2. The same as Fig. 1 but for analyzing power.

elastic scattering observables has been analyzed [27]. Since the calculations using the zero-range interactions are not too heavy, we intend to evaluate the sensitivity of the elastic scattering observables on each parameters of the effective interaction. This information is important for our long-term project to build a new generation of optical potentials.

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