EVALUATION OF THE INFLUENCE OF NEIGHBORING RADIOACTIVE SOURCES PLACED ON A ROTATING DISK ON THE PHOTON ENERGY SPECTRUM

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The energy spectra of numerous radioactive samples placed on a rotating disk can be measured with a single-gamma radiation detector. This technique provides also lifetime information when the same sample returns several times close to the radiation detector. However, the presence of neighboring sources may affect the primary measurement. Assuming equal activity of all sources, analytical formulas for the relative contribution of neighboring sources are developed for typical configurations of 2, 3, 4, 6, 8, 12, 16 samples on the rotating disk. The calculated values compare well to the experimental results obtained for the configuration of 16 samples. The resulting analytical formulas help to decid, whether a passive shield around the detector is needed to reduce the contribution from neighboring samples, prior to the precise Monte Carlo simulation of the experimental set-up.

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1. Introduction

Measurement of the γ energy spectra of multiple samples with a single spectrometric device (HPGe or scintillation detector) is a common experimental procedure and dedicated set-ups are commercially available or constructed in laboratories. These measurements typically aim to determine low-level radioactivity of long-lived isotopes, so the measurement time is long and the time needed to exchange the samples is of minor importance. In such devices, the actually measured sample and the detector are well-separated from the remaining samples, so their influence on the running measurement is none. This is not the case when the lifetime of the measured radioactivity is short, but long enough for out-of-beam spectroscopy (typically 100 s–1000 s). The positioning of the samples in the experimental set-up has to be easy. The set-up should be easily transportable to be placed relatively close to the irradiation hall. Then, the radioactive samples have to be exchanged quickly to obtain sufficient spectroscopic data for any of them. A set-up consisting of a stepping motor driven rotating disk, where the samples are placed, and one or more scintillation detectors (see Fig. 1) fulfills these constraints. In the past, rotating sample holders were used mainly to irradiate samples [1–3].

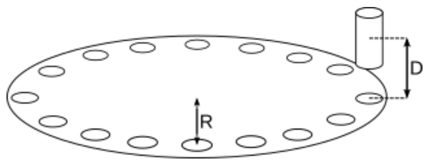

Fig. 1. Schematic view of the experimental apparatus composed of a rotating disk (samples placed at radius R) and γ-radiation detector placed at (effective) distance D from the surface of the disk.

In a compact set-up, all samples are relatively close to the scintillation detector(s), so their activity influences the spectrum measured for the sample placed in front of the detector. Two scenarios can be considered:

— The development of an algorithm that would cope with the appropriate correction of the spectrum, basing on the complete set of spectra for all measured configurations. The decay of produced activities within the measurement time has to be accounted for. Precise knowledge of the detection efficiency for each configuration of the sample with respect to the detector is essential for reliable correction procedure.

— The construction of a passive shield around the scintillating crystal that would reduce the probability of reaching the active volume by photons emitted from neighboring samples.

The knowledge of the level of activity from neighboring samples, even approximate, can help to make a decision about the construction of a passive shield. Analytical formulas for this contribution were obtained (Section 2) considering typical cases of a rotating disk with 2, 3, 4, 6, 8, 12 and 16 samples. The experimental results (Section 3) were obtained with a single radioactive source placed in one cell of the rotating disk with 16 cells. Photons were registered in $LaBr_3$:Ce scintillator read out by a photomultiplier. The measurements were done for ^{137}Cs and ^{60}Co calibration sources. The obtained results were compared (Section 4) to the analytical model for some configurations, namely 2, 4, 8 and 16. The paper is summarized by the conclusions (Section 5).

2. Analytical model

The model assumes point-like radioactive sources of equal activity A and a point-like detector. It is also assumed that the efficiency of the detector does not depend on the direction of the incident photon. It means the model might be applied approximately to the case of a typical cylindrical scintillation detector, which has the same height and diameter, but with restrictions to flat or elongated ones.

Considering the disk, where K ($K = 2, 3, 4, 6, 8, 12, 16$) samples can be equally distributed at radius R, their centers can be written as

$$r_k = \left[R\cos\left(\frac{2k\pi}{K}\right), R\sin\left(\frac{2k\pi}{K}\right), 0\right] , \tag{1}$$

where $k = 0, 1, \ldots, K-1$. The position indexed as 0 is the position above (or below) the scintillation detector. The position of the (effective) center of the scintillation detector is then the following:

$$r_D = [R, 0, \pm D] . \tag{2}$$

Variable D is the sum of two components, the distance from the disk plane to the surface of the detector and the average distance the incoming photon interacts with the detector material (the mean free path, which depends on photon energy). The signal S registered in the detector is inversely proportional to the square of the distance between the sample and the detector, summed over all samples

$$\begin{aligned} S &\sim \frac{A}{D^2} + \sum_{k=1}^{K-1} \frac{A}{(r_k - r_D)^2} \\ &= \frac{A}{D^2} + \sum_{k=1}^{K-1} \frac{A}{R^2\left(\left(\cos\left(\frac{2k\pi}{K}\right) - 1\right)^2 + \sin^2\left(\frac{2k\pi}{K}\right)\right) + D^2} . \end{aligned} \tag{3}$$

In the above equation, the first term denotes the signal generated by the sample placed close to the detector. Introducing the dimensionless variable

$$r = \frac{R}{D} , \tag{4}$$

Eq. (3) can be written as

$$S \sim \frac{A}{D^2}\left(1 + f_K(r)\right) , \tag{5}$$

where the function $f_K(r)$ describes the additional contribution to the spectrum due to the neighboring samples. This function can be written as

$$f_K(r) = \sum_{k=1}^{K-1} \frac{1}{2r^2\left(1-\cos\left(\frac{2k\pi}{K}\right)\right)+1}\,. \tag{6}$$

For all even values of K, the function $f_K(r)$ can be simplified: the first term is the signal from the sample being opposite to the detector and the number of remaining elements in the sum is reduced twice due to the symmetry

$$f_K(r) = \frac{1}{4r^2+1} + 2\sum_{k=1}^{\frac{K-2}{2}} \frac{1}{2r^2\left(1-\cos\left(\frac{2k\pi}{K}\right)\right)+1}\,. \tag{7}$$

The exact formulas for the cases of $K = 2, 3, 4, 6, 8, 12, 16$ are in Appendix A. All functions $f_K(r)$ are decreasing, all in high r limit like r^{-2} (Fig. 2). The increasing number of sources leads naturally to the increase of their total contribution. For the situation, where the separation between samples is 5 times larger than the distance of the disk to the effective center of the detector (parameter $r = 5$), the contribution coming from additional samples is at the significant level of 10% already for the case of 6 samples on the disk. For more samples, this contribution is expected to be stronger, what suggests preventive measures. In this case, a passive shield around the detector should be installed.

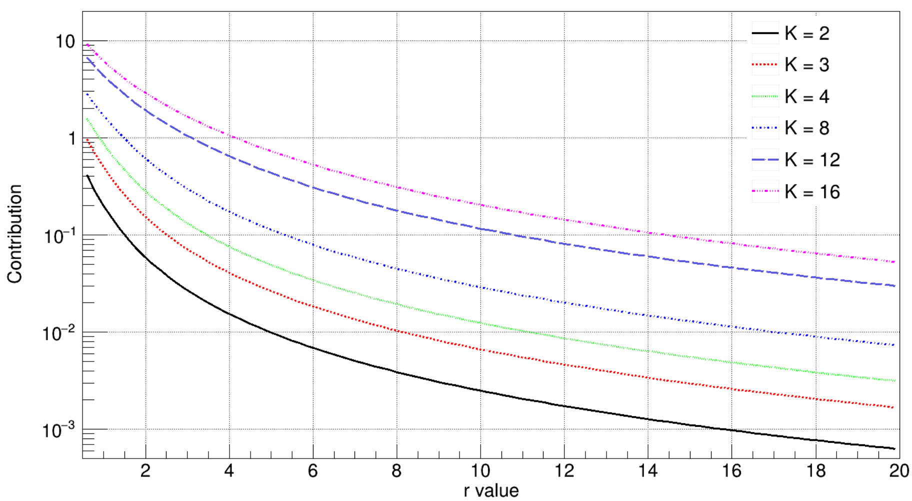

Fig. 2. The relative excess contribution of neighboring samples with respect to that close to the detector, for selected numbers of samples on the rotating disk ($K = 2, 3, 4, 6, 8, 12, 16$). The line for K_n is above K_m for $n > m$, as the contribution is additive.

3. Experiment

The measurements were done using a set-up with a rotating disk with 16 holes to place radioactive samples. The diameter of the disk is 300 mm. The radius, at which the samples ($r_{\rm sample} = 3$ mm) are placed, is $R = 143$ mm. Stepping motor (model 85BYGH450C) rotates the disk. The same computer, which is used for data acquisition, steers the motor. The set-up provides space for three pairs of scintillation detectors. In the measurements, we used one truncated cone ($\phi(1''/1.5'') \times 1.5''$) $LaBr_3$:Ce scintillator considered for the FATIMA Collaboration [4]. The signal from the photomultiplier (Hamamatsu R9779) was amplified and digitized. Tukan8k [5] system was used for the data acquisition. Two radioactive sources, namely ^{137}Cs and ^{60}Co, were used. The energy resolution of our detector was 4% at $E_\gamma = 662$ keV. Each measurement lasted for 1 hour, then the disk was rotated and a new measurement was done automatically. The measurement, when the source returned to the initial position, was done to check the stability. The measurements were performed in a low-background area surrounded by lead bricks. The deadtime correction was about 2%, when the source was placed above the detector. For other positions, it was around 0.1% to 0.3%. The energy spectra (Fig. 3) show, apart from well-known peaks due to the

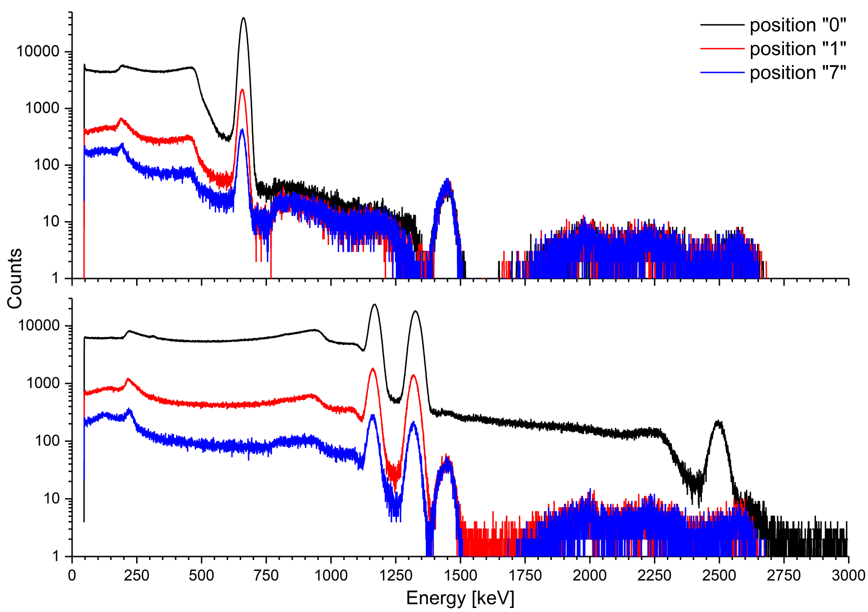

Fig. 3. Energy spectra measured for three positions of the source: position 0 just above the detector, position 2 corresponding to the rotation of the disk by 45°, position 7 corresponding to the rotation of the disk by 157.5°. The top (bottom) panel is for ^{137}Cs (^{60}Co) source.

calibration sources, another structures. The most prominent is the signal at 1436 keV, which results from beta decay of ^{138}La ($T_{1/2}=1.02\times10^{11}$ y) [6–8]. The structures at around 2 MeV are attributed to the α decay of ^{227}Ac.

The signals measured in subsequent positions of the source with respect to the detector (Table I) have been normalized to the position 0, where the source was just above the detector. The results for symmetric positions, corresponding to positive and negative rotation angles, were averaged in order to reduce uncertainty related to misalignment of the position 0 at 0°. The intensity of the 1436 keV line, which is due to the intrinsic detector radioactivity, should be insensitive to the position of the source. Indeed, the signal intensity of the 1436 keV line is very stable and proves the stability of the measurements. Significant error of the measurement at 0° for ^{60}Co source is due to the increased background from the radiation emitted by the source of more energetic photons than for the case of ^{137}Cs.

TABLE I

Relative yields in the photon energy spectrum due to calibration sources and originating from the intrinsic activity of $LaBr_3$:Ce scintillator (1436 keV) for different positions of the source on the rotating disk. The yield obtained at 0° (the source directly above the detector) was used for normalization of the activity stemming from the radioactive source, while the yield of the background line at 1436 keV was normalized to the most distant position (180°).

Source	^{137}Cs		^{60}Co		
E_γ	662 keV	1436 keV	1173 keV	1332 keV	1436 keV
Angle	Yield	Yield	Yield	Yield	Yield
0°	100.0±0.1	98±4	100.0±0.2	100.0±0.1	112±32
22.5°	22.42±0.05	97±3	21.3±0.2	20.66±0.06	100±8
45°	6.52±0.03	99±4	6.50±0.09	6.37±0.03	109±7
67.5°	2.95±0.02	96±3	3.16±0.07	3.03±0.02	99±6
90°	1.80±0.02	97±3	1.94±0.06	1.84±0.02	102±6
112.5°	1.37±0.02	96±4	1.38±0.03	1.33±0.01	100±6
135°	1.14±0.01	103±4	1.14±0.05	1.07±0.01	103±6
157.5°	1.04±0.01	94±3	0.99±0.04	0.97±0.01	100±6
180°	0.62±0.01	100±4	0.65±0.03	0.64±0.01	100±8

4. Results and discussion

The experimental results (Table I) show that the relative yield is rather insensitive to the photon energy. For the comparison with detailed Geant4 simulations [9, 10], we selected the 662 keV photons. The results presented in Fig. 4 show very good agreement between the analytical formula, experimental results, and Geant simulations.

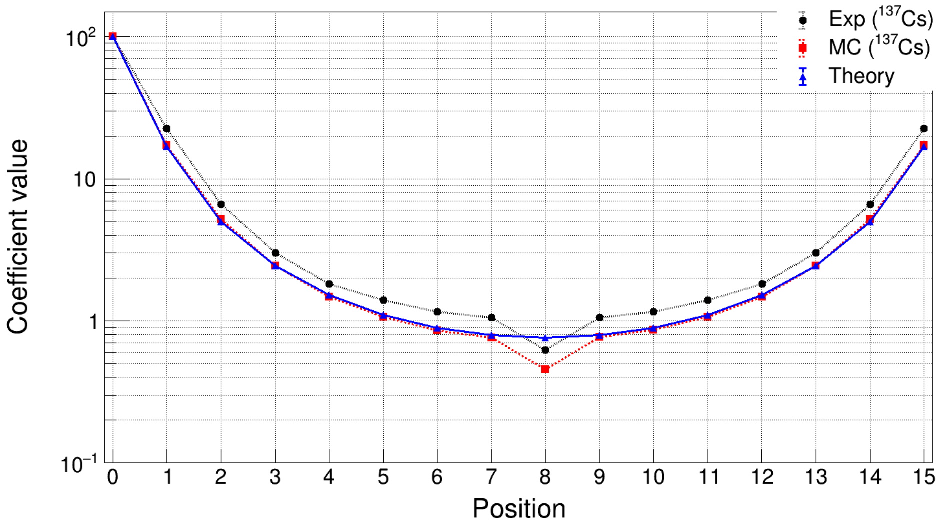

Fig. 4. The influence of other sources to the strength of principal signal for a rotating wheel with 16 sources, normalized to 100 at 0 position, is obtained from the experiment, the theoretical model f_{16} (Eq. (14)), and Geant simulations.

The results measured for the number of slots on rotating disk equal 16 allow also to make a comparison to the analytical formulas (Section 2) not only for $K = 16$, but also for $K = 8$, $K = 4$ and $K = 2$ by appropriate summing of the contributions. The experimental values of f_K^{EXP} allow to determine the parameter $r_K^{\mathrm{EXP}} = f_K^{-1}(f_K^{\mathrm{EXP}})$ using the inverse functions of equations in Appendix A. The results are summarized in Table II.

TABLE II

Experimental relative contribution f_K^{EXP} of neighboring sources on rotating disk to the signal from the source located close to the detector, for 3 photon energies and 4 cases of sources on rotating disk in the current experiment. The relative radius r_K^{EXP} is extracted (last column) from the average value of f_K^{EXP}, as a numerical result of inverse function of appropriate formulas in Appendix A.

	f_K^{EXP}			$\langle f_K^{\mathrm{EXP}} \rangle$	r_K^{EXP}
K	662 keV	1173 keV	1332 keV		
2	0.0062	0.0065	0.0064	0.0064	6.25
4	0.0422	0.0453	0.0432	0.044	5.30
8	0.1954	0.1981	0.1920	0.195	5.05
16	0.7510	0.7347	0.7118	0.73	5.00

The values of r_K^{EXP} are consistent for $K = 4$, 8 and 16, while for $K = 2$ it strongly deviates. It can be explained by the fact that the source rotated by 180° is partly screened from the detector by the axis of the rotating disk, what reduces the value of f_2^{EXP}. This effect is important for $K = 2$, but has minor influence for higher values of K. The consistency of the results of r_K^{EXP} justifies the next step: determination of the effective distance D from the disk plane to the detector, $D = R/r_K^{\mathrm{EXP}} \simeq 143$ mm$/5.1 = 28$ mm. This distance is consistent with the sum of the separation between the disk plane and the surface of the scintillating crystal (about 10 mm), and half thickness of the detector (19 mm).

5. Conclusions

The γ energy spectra of numerous radioactive samples placed on a rotating disk can be measured with a single-gamma radiation detector. This technique provides also lifetime information, provided the same sample returns several times close to the detector. The presence of neighboring sources may affect the primary measurement. Assuming equal activity of all sources, analytical formulas for the relative contribution of neighboring sources were developed for typical configurations of 2, 3, 4, 6, 8, 12, 16 samples on the rotating disk. Point-like sources and point-like detector were assumed. The calculated values compare surprisingly well to the experimental results obtained for a disk with 16 samples (from which results for 2, 4 and 8 samples were also extracted) and with Geant4 simulations. The resulting analytical formulas, which appeared to be in very good agreement with the data and simulations, may provide a reasonable estimate of the effect. Thus, they may help to decide whether a passive shield around the detector is needed to reduce the contribution from neighboring samples, prior to the precise Monte Carlo simulation of the experimental set-up.

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Appendix A

Formulas of relative contribution

For the considered range of values of the number K (2, 3, 4 6, 8, 12, 16) of samples in the disk, the additional relative signal due to all neighboring samples can be written as

$$f_2 = \frac{1}{4r^2+1}\,, \tag{8}$$

$$f_3 = \frac{2}{3r^2+1}\,, \tag{9}$$

$$f_4 = \frac{1}{4r^2+1} + \frac{2}{2r^2+1}\,, \tag{10}$$

$$f_6 = \frac{1}{4r^2+1} + \frac{2}{r^2+1} + \frac{2}{3r^2+1}\,, \tag{11}$$

$$f_8 = \frac{1}{4r^2+1} + \frac{2}{2r^2+1} + \frac{4(2r^2+1)}{(2r^2+1)^2-2r^4}\,, \tag{12}$$

$$f_{12} = \frac{1}{4r^2+1} + \frac{2}{r^2+1} + \frac{2}{2r^2+1} + \frac{2}{3r^2+1} + \frac{4(2r^2+1)}{(2r^2+1)^2-2r^4}\,, \tag{13}$$

$$f_{16} = \frac{1}{4r^2+1} + \frac{2}{2r^2+1} + \frac{4(2r^2+1)}{(2r^2+1)^2-2r^4} + \frac{8(2r^2+1)\left((2r^2+1)^2-2r^4\right)}{(2r^2+1)^4-4r^2(2r^2+1)^2+2r^8}\,. \tag{14}$$

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