BAGGER–LAMBERT–GUSTAVSSON MEMBRANE MODEL AS A CONSTRAINED SYSTEM AND DIRAC QUANTIZATION

Aruã M. de Aguiar[a,†], Albert C.R. Mendes[a,‡]
Everton M.C. Abreu[b,a,c,§], Jorge Ananias Neto[a,¶]

[a]Departamento de Física, Universidade Federal de Juiz de Fora
36036-330, Juiz de Fora, MG, Brazil
[b]Departamento de Física, Universidade Federal Rural do Rio de Janeiro
23890-971, Seropédica, RJ, Brazil
[c]Programa de Pós-Graduação Interdisciplinar em Física Aplicada
Instituto de Física, Universidade Federal do Rio de Janeiro
21941-972, Rio de Janeiro, RJ, Brazil

(Received September 12, 2020; accepted September 22, 2020)

The Dirac formalism allowing to deal with systems subject to constrains is applied to find the commutation relations that should be imposed on the canonical variables of the effective two-dimensional field theory of massless fields obtained by defining the recently introduced Bagger–Lambert–Gustavsson (BLG) three-dimensional theory of a membrane system on the compactified $\mathbb{R}^{1,1} \times S^1$space. The obtained set of constrains is of second class in the Dirac classification and should, therefore, be quantized through the introduction of the Dirac brackets which we write down in the explicit form.

DOI:10.5506/APhysPolB.51.2083

1. Introduction

The recently formulated [1–5] Bagger–Lambert–Gustavsson (BLG) model is a superconformal $\mathcal{N} = 8$ supersymmetric field theory. It is intended to capture the low-energy dynamics of the system of multiple M2-branes in eleven-dimensional space (the worldvolume). The special feature of the model is that its gauge fields take values in a Lie 3-algebra (instead in

† arua@fisica.ufjf.br
‡ albert@fisica.ufjf.br
§ evertonabreu@ufrrj.br
¶ jorge@fisica.ufjf.br

an ordinary Lie algebra). The model can be formulated on the $\mathcal{N} = 1$ superspace [6, 7], on the $\mathcal{N} = 2$ superspace [8, 9] as well as on the $\mathcal{N} = 8$ superspace [10, 11].

The interpretation of the BLG theory as well as its possible modifications, like introducing degenerate or indefinite metrics on the space of its scalar fields, describing the general dynamics of systems of multiple membranes, not only the low-energy one, *etc.* have been addressed in many works with the aim of formulating a more complete [12] theory of multiple membranes. In [13], the BRST quantization of mass-deformed BLG theory in $\mathcal{N} = 1$ superspace in Landau gauge was discussed. Recently, the BLG theory in $\mathcal{N} = 1$ superspace has been quantized in Lorentz gauge [14].

Owing to these works [6, 15–56], a considerable progress in the general understanding of the BLG theory structure, as well as its connection with the theory of n-Lie algebras has been achieved. Moreover, the general structure of the supersymmetric field theories reflecting the dynamics of condensates of M2-branes has been obtained in [57, 58] and a nonlinear extension of the BLG theory has been suggested in [59].

A useful characterization of the BLG theory can be obtained by performing its dimensional reduction [37, 60]. Originally, the formulation of the BLG model using the $\mathcal{N} = 8$ superspace in [10, 11] was obtained by conjecturing that 3-algebras can have positive definite metrics. In [25], it has been, however, demonstrated that only one type of 3-algebras admits such a metric. The definition of the Lorentzian 3-algebras has been clarified in works [27–30] and 3-algebras with Lorentzian metrics were explored in the compactification of D2 branes [15, 26]. The BLG theory can be also reduced to F1 strings of the type II string theory with a compactification along the direction of the M2-branes worldvolume [61, 62].

In this work, we analyzed the structure of the constraints of the effective field theory in two dimensions obtained by formulating the original BLG three-dimensional theory on $\mathbb{R}^{1,1} \times S^1$ space as in [61] and restricting it to its massless modes only.

After a brief review of the BLG theory and of its compactification used in this paper, we investigate the canonical structure of the two-dimensional theory, find the complete set of constraints its canonical variables are subjected to and apply to it the Dirac quantization prescription to determine the canonical commutation relations that they should satisfy.

2. A brief review of the BLG theory and of its compactification

As mentioned in Introduction, the model formulated by Bagger and Lambert [1–3] and by Gustavsson [4] which captures the low-energy dynamics of the system of M2 branes is based on a Lie 3-algebra in which the usual

Lie bracket is replaced by a more general structure called the Lie 3-bracket. Lie n-algebras were constructed by Filippov [63] as a generalization of the Nambu bracket introduced in [64].

Lie 3-algebras are a very peculiar type Lie n-algebraic structures which play various roles in mathematics and in theoretical physics [1–4]

$$\left[T^a, T^b, T^c\right] = f^{abc}{}_d T^d\,, \qquad a, b, c, d = 1, \ldots,\ \dim \mathcal{A}\,. \tag{1}$$

A Lie 3-algebra $\mathcal{A}$ can be viewed [63, 65] as a vector space spanned by the basis T^a of generators satisfying the rule in Eq. (1) in which the 3-bracket $[T^a, T^b, T^c]$ is completely antisymmetric in the indices a, b and c and so are, therefore, the structure constants $f^{abc}{}_d$. From Eq. (1) follows the generalization of the ordinary Jacobi identity [63]

$$\begin{aligned}&\left[T^g, T^d, \left[T^a, T^b, T^c\right]\right] = \\ &\left[\left[T^g, T^d, T^a\right], T^b, T^c\right] + \left[T^a, \left[T^g, T^d, T^b\right], T^c\right] + \left[T^a, T^b, \left[T^g, T^d, T^c\right]\right]\,. \end{aligned}\tag{2}$$

The relation in Eq. (2) implies that the structure constraints satisfy the relation

$$f^{abc}{}_e f^{gde}{}_f - f^{gda}{}_e f^{ebc}{}_f - f^{gdb}{}_e f^{aec}{}_f - f^{gdc}{}_e f^{abe}{}_f = 0\,. \tag{3}$$

The basic fields of the BLG theory dimensionally reduced to 3 dimensions are: the gauge fields taking values in a 3-Lie algebra, $\tilde{A}_\mu{}^a{}_b = f^{cda}{}_b A_{\mu cd}$, where the space-time indices $\mu = 0, 1, 2$ label the worldvolume variables, the set of scalars X^I_a, $I = 1, \ldots, 8$ transforming as the fundamental representation of the SO(8) R-symmetry group of which represent transverse coordinates and the 16-component 3-Lie algebra valued Majorana spinor field Ψ satisfying the chirality condition

$$\left(\Gamma^{012}\right)^A_B \Psi(x)^B = -\Psi(x)^A\,. \tag{4}$$

Considering the supersymmetric parameter ε, we can write that $\Gamma^{012}\varepsilon = \varepsilon$.

The SUSY transformations in this theory are given by

$$\delta X^I_a = i\bar{\epsilon}\Gamma^I \Psi_a\,, \tag{5}$$

$$\delta\Psi_a = D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \tfrac{1}{6} X^I_b X^J_c X^K_d f^{bcd}{}_a \Gamma^{IJK}\epsilon\,, \tag{6}$$

$$\delta\tilde{A}_\mu{}^b{}_a = i\bar{\epsilon}\Gamma_\mu \Gamma_I X^I_c \Psi_d f^{cdb}{}_a\,, \tag{7}$$

where $I, J, \ldots = 1, 2, \ldots, 8$ and Γ_I are the Dirac matrices. The covariant derivation D_μ has the form of

$$D_\mu X^I_a = \partial_\mu X^I_a - \tilde{A}_\mu{}^b{}_a X^I_b\,, \tag{8}$$

and

$$D_\mu \Psi_a = \partial_\mu \Psi_a - \tilde{A}_{\mu}{}^{b}{}_{a} \Psi_b \,. \tag{9}$$

The action S invariant with respect to the SUSY transformations in Eqs. (5)–(7), proposed by Bagger and Lambert, reads

$$S = \frac{1}{g_{\mathrm{BLG}}^2} \int \mathrm{d}^3x \Big[-\frac{1}{2} D_\mu X^{aI} D^\mu X_a^I + \frac{i}{2} \bar{\Psi}_a \Gamma^\mu D_\mu \Psi_a + \frac{i}{4} \bar{\Psi}_b \Gamma^{IJ} X_c^I X_d^J \Psi_a f^{abcd} \\ -V(X) + \frac{1}{2} \varepsilon^{\mu\nu\lambda} \Big(f^{abcd} A_{\mu ab} \partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_{g} f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef} \Big) \Big] \,, \tag{10}$$

where

$$\Gamma^{IJ} = \frac{1}{2} \Big(\Gamma^I \Gamma^J - \Gamma^J \Gamma^I \Big) \,. \tag{11}$$

The scalar potential $V(X)$ has the form of

$$V(X) = \frac{1}{12} f^{abcd} f^{efg}{}_{d} X_a^I X_b^J X_c^K X_d^I X_e^J X_f^K \,. \tag{12}$$

The Euler–Lagrange equations derived from Eq. (10) read

$$\Gamma^\mu D_\mu \Psi_a + \frac{1}{2} \Gamma_{IJ} X_c^I X_d^J \Psi_b f^{cdb}{}_{a} = 0 \,, \tag{13}$$

$$D^2 X_a^I - \frac{i}{2} \bar{\Psi}_c \Gamma_J^I X_d^J \Psi_b f^{cdb}{}_{a} + \frac{1}{2} f^{bcd}{}_{a} f^{efg}{}_{d} X_b^J X_c^K X_e^I X_f^J X_g^K = 0 \,, \tag{14}$$

$$\tilde{F}_{\mu\nu}{}^{b}{}_{a} + \epsilon_{\mu\nu\lambda} \Big(X_c^J D^\lambda X_d^J + \frac{i}{2} \bar{\Psi}_c \Gamma^\lambda \Psi_d \Big) f^{cdb}{}_{a} = 0 \,. \tag{15}$$

In these last equations $D^2 = D_\mu D^\mu$, the field strength-tensor of the gauge fields is given by

$$\tilde{F}_{\mu\nu}{}^{b}{}_{a} = \partial_\nu \tilde{A}_{\mu}{}^{b}{}_{a} - \partial_\mu \tilde{A}_{\nu}{}^{b}{}_{a} - \tilde{A}_{\mu}{}^{b}{}_{c} \tilde{A}_{\nu}{}^{c}{}_{a} + \tilde{A}_{\nu}{}^{b}{}_{c} \tilde{A}_{\mu}{}^{c}{}_{a} \,, \tag{16}$$

and the structure constants $f^{abc}{}_{d}$ must satisfy the fundamental identity given in [2, 4].

Important insight into the working of the BLG model can be obtained by reducing it to two-dimensional field theories [60–62, 66]. In this paper, we consider such a two-dimensional theory obtained by formulating the theory described above on the $\mathbb{R}^{1,1} \times S^1$ space and restricting it to its massless modes only. This corresponds to compacifying one of the directions of the

M2-brane worldvolume. This tedious procedure has been carried out in [61]. The action of the resulting two-dimensional theory is

$$
\begin{aligned}
S_0 = \int \mathrm{d}^2x\,\mathcal{L} = \frac{\pi R}{g^2}\int \mathrm{d}^2x \Big[&-\frac{1}{2} D_\alpha X^{aI} D^\alpha X_a^I - \frac{1}{R^3}\Phi_{cd} f^{cdba} X_b^I \Phi_{lp} f^{lps}{}_a X_s^I \\
&+ f^{abcd}\epsilon^{\alpha\beta}(A_{\alpha ab}\partial_\beta\Phi_{cd} + \Phi_{ab}\partial_\alpha A_{\beta cd}) + i\left(\bar{\Psi}^a\Gamma^\alpha D_\alpha\Psi_a - \frac{1}{R}\bar{\Psi}^a\Gamma^2\Phi_{cd}f^{cdb}{}_a\Psi_b\right) \\
&+ \frac{i}{2} f^{abcd}\bar{\Psi}_b\Gamma_{IJ}X_c^I X_d^J\Psi_a - \frac{1}{6} f^{abcd} f^{efg}{}_d X_a^I X_b^J X_c^K X_e^I X_f^J X_g^K \\
&+ \frac{2}{3} f^{abc}{}_g f^{defg}\epsilon^{\alpha\beta}\big(A_{\alpha ab}A_{\beta cd}\Phi_{ef} - A_{\alpha ab}\Phi_{cd}A_{\beta ef} + \Phi_{ab}A_{\alpha cd}A_{\beta ef}\big)\Big]\,. \qquad (17)
\end{aligned}
$$

The indices $\alpha, \beta = 0, 1$ are the two-dimensional worldsheet indices. The coupling constant g is related to the original coupling $g_{_{\rm BLG}}$ of the BLG theory by $g = g_{_{\rm BLG}} R^{-1/2}$, where R is the radius of the circle S^1. The original three-dimensional gauge fields have been split into a vector field A_α and scalars $A_{2ab} = \Phi_{ab}$.

3. The Dirac quantization of the model

We now consider canonical quantization of the two-dimensional theory defined at the end of the preceding section. To this end we, first determine the canonical momenta conjugated to the field variables X, A_0, A_1, Φ, Ψ and $\bar{\Psi}$

$$\pi^{nM} = \frac{\partial\mathcal{L}}{\partial\dot{X}_n^M} = \dot{X}^{nM} + X^{aM}\tilde{A}_a^{0n}\,, \qquad (18)$$

$$P_1^{nm} = \frac{\partial\mathcal{L}}{\partial\dot{A}_{1nm}} = -\frac{1}{2} f^{abnm}\Phi_{ab} = -\frac{1}{2}\tilde{\Phi}^{nm}\,, \qquad (19)$$

$$P_0^{nm} = \frac{\partial\mathcal{L}}{\partial\dot{A}_{0nm}} = X^{aI} f^{nmb}{}_a X_b^I\,, \qquad (20)$$

$$p_\Psi^n = \frac{\partial\mathcal{L}}{\partial\dot{\Psi}_n} = \frac{i}{2}\bar{\Psi}^a\Gamma^0\,, \qquad (21)$$

$$p_{\bar{\Psi}}^n = \frac{\partial\mathcal{L}}{\partial\dot{\bar{\Psi}}_n} = 0\,, \qquad (22)$$

$$p_\Phi^{nm} = \frac{\partial\mathcal{L}}{\partial\dot{\Phi}_{nm}} = \frac{1}{2} f^{abnm}A_{1ab} = \frac{1}{2}\tilde{A}_1^{nm}\,. \qquad (23)$$

From this, following the procedure of Dirac outlined clearly in [67], one can identify the primary constraints

$$\chi_1^{nm} = P_1^{nm} + \frac{1}{2}\tilde{\Phi}^{nm} \approx 0\,, \tag{24}$$

$$\chi_2^{nm} = P_0^{nm} - X^{aI} f^{nmb}{}_a X_b^I \approx 0\,, \tag{25}$$

$$\chi_3^{n} = p_{\Psi}^{n} - \frac{i}{2}\bar{\Psi}^a \Gamma^0 \approx 0\,, \tag{26}$$

$$\chi_4^{n} = p_{\bar{\Psi}}^{n} \approx 0\,, \tag{27}$$

$$\chi_5^{nm} = p_{\Phi}^{nm} - \frac{1}{2}\tilde{A}_1^{ab} \approx 0\,. \tag{28}$$

The Hamiltonian obtained along the lines of [67] is of the form of

$$\mathcal{H}_T = \mathcal{H} + \lambda^{(1)}\chi_1^{nm} + \lambda^{(2)}\chi_2^{nm} + \lambda^{(3)}\chi_3^{n} + \lambda^{(4)}\chi_4^{n} + \lambda^{(5)}\chi_5^{nm}\,, \tag{29}$$

with $\mathcal{H}$ which in the considered case reads

$$\begin{aligned}\mathcal{H} = {} & \frac{1}{2}\pi^{aI}\pi_a^I + \pi_a^I \tilde{A}_0{}^{ba} X_b^I + \frac{1}{2}\partial_1 X^{aI}\partial^1 X_a^I - \partial^1 X_a^I \tilde{A}_1{}^{ba} X_b^I + \frac{1}{2}\tilde{A}_1{}^a{}_b X^{bI}\tilde{A}^1{}_a{}^b X_b^I \\ & + \frac{1}{2R^3}\tilde{\Phi}^{ba} X_b^I \tilde{\Phi}^b{}_a X_b^I - \frac{1}{2}\bar{\Psi}^a \Gamma^1 \partial_1 \Psi_a + \frac{i}{2}\bar{\Psi}^a \Gamma^\alpha \tilde{A}_\alpha{}^b{}_a \Psi_b + \frac{i}{2R}\bar{\Psi}^a \Gamma^2 \tilde{\Phi}^b{}_a \Psi_b \\ & - \frac{i}{4} f^{abcd}\bar{\Psi}_b \Gamma_{IJ} X_c^I X_d^J \Psi_a + \frac{1}{2}\left(\tilde{A}_0{}^{cd}\partial_1 \Phi_{cd} - \tilde{\Phi}^{cd}\partial_1 A_{0cd}\right) \\ & - \frac{1}{3}\epsilon^{\alpha\beta}\left(\tilde{A}_\alpha{}^c{}_g A_{\beta cd}\tilde{\Phi}^{gd} - \tilde{A}_\alpha{}^c{}_g \Phi_{cd}\tilde{A}_\beta{}^{gd} - \tilde{\Phi}^c{}_g A_{\alpha cd}\tilde{A}_\beta{}^{gd}\right) \\ & - \frac{1}{6} f^{abcd} f^{efg}{}_d X_a^I X_b^J X_c^K X_e^I X_f^J X_g^K\,, \end{aligned} \tag{30}$$

and we have a new constraint

$$\begin{aligned}\chi_6^{qv} \equiv {} & \partial_1 \tilde{\Phi}^{qv} - \frac{i}{2}\bar{\Psi}^a \Gamma^0 f^{qvb}{}_a \Psi_b + f^{qvab}\pi_a^I X_b^I + 2 f^{qvab}\tilde{A}_{0ag} X_b^I X^{gI} \\ & - \frac{1}{3}\Big(f^{qvc}{}_g A_{1cd}\tilde{\Phi}^{vg} - \tilde{A}_1{}^q{}_g \tilde{\Phi}^{vg} - f^{qvc}{}_g \Phi_{cd}\tilde{A}_1{}^{dg} + \tilde{A}_1{}^c{}_g f^{dqvg}\Phi_{cd} \\ & + \tilde{\Phi}^q{}_g \tilde{A}_1{}^{vg} - \tilde{\Phi}^c{}_g f^{dqvg} A_{1cd}\Big) \\ \approx {} & 0\,. \end{aligned} \tag{31}$$

The next step consists of the successive checking whether the already identified constrains can be made (by giving the Lagrange multipliers the appropriate dependence on the canonical variables) consistent with the dynamics generated by the Hamiltonian in Eq. (29) and identify in this way all the (secondary) constraints.

We have checked that χ_6 is the only secondary constraint: its time derivative

$$\dot{\chi}_6^{qv} = \{\chi_6^{qv}, H_T\} \tag{32}$$

leads to no new constraints, and the similar derivatives of χ_1, χ_3, χ_4 and χ_5 can be made (weakly) vanishing by giving the Lagrange multipliers the appropriate dependence on the canonical variables.

Since the system of constraints in Eqs. (19)–(22) and (31) is second class, quantization of the theory requires using the Dirac brackets defined by the formulas

$$\begin{aligned}\{A(x,t), B(x,t)\}_{\rm DB} &= \{A(x,t), B(x,t)\} \\ &- \int \{A(x,t), \chi_i(z,t)\} C_{ij}^{-1}(z,w)\{\chi_j(y,t), B(x,t)\}\,,\end{aligned} \tag{33}$$

where $C_{ij}^{-1}(z,w) = \{\chi_i(z), \chi_j(z)\}$ is the inverse of the matrix formed out of the Poisson brackets of all the second class constraints. This matrix has the form

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & f^{nmqv} & F^{nmqv} \\ 0 & 0 & 0 & 0 & 0 & J^{nmqv} \\ 0 & 0 & 0 & iB^{nm} & 0 & i\bar{D}^{nmq} \\ 0 & 0 & -iB^{nm} & 0 & 0 & -iD^{nmq} \\ -f^{nmqv} & 0 & 0 & 0 & 0 & E^{nmqv} \\ -F^{nmqv} & -J^{nmqv} & -i\bar{D}^{nmq} & -iD^{nmq} & -E^{nmqv} & L^{nmqv} \end{bmatrix} \times \delta^2(x-y)\,, \tag{34}$$

where

$$F^{nmqv}\delta^2(x-y) = \{\chi_1^{nm}(x,t), \chi_6^{qv}(y,t)\} = -\frac{1}{3}\Big(f^{qvn}{}_g f^{efgm} - f^{nmq}{}_g f^{efgv}$$

$$-f^{qve}{}_g f^{nmgf} + f^{qvgf} f^{nme}{}_g + f^{nmgv} f^{efq}{}_g - f^{qvgm} f^{efn}{}_g\Big)\Phi_{ef}(y)\delta^2(x-y)\,,$$

$$J^{nmqv}\delta^2(x-y) = \{\chi_2^{nm}(x,t), \chi_6^{lq}(y,t)\} = 2f^{qvb}{}_a f^{nmac} X_b^I X_c^I \delta^2(x-y)\,,$$

$$iB^{nm}\delta^2(x-y) = \{\chi_3^n(x,t), \chi_4^m(y,t)\} = \frac{i}{2}\Gamma^0\delta^{nm}\delta^2(x-y)\,,$$

$$i\bar{D}^{nmq}\delta^2(x-y) = \{\chi_3^n(x,t), \chi_6^{mq}(y,t)\} = \frac{i}{2}\Gamma^0 f^{nmq}{}_a \bar{\Psi}^a \delta^2(x-y)\,,$$

$$-iD^{mq}{}_n\delta^2(x-y) = \{\chi_4^n(x,t), \chi_6^{mq}(y,t)\} = -\frac{i}{2}\Gamma^0 f^{mqb}{}_n \Psi^b \delta^2(x-y)\,,$$

$$E^{nmqv}\delta^2(x-y) = \{\chi_5^{nm}(x,t), \chi_6^{qv}(y,t)\} = -\partial_1\delta^2(x-y) - \frac{1}{3}\Big(f^{qvn}{}_g f^{efgm}$$
$$-f^{nmq}{}_g f^{efgv} - f^{qve}{}_g f^{nmgf} + f^{qvgf} f^{nme}{}_g + f^{nmgv} f^{efq}{}_g - f^{qvgm} f^{efn}{}_g\Big)$$
$$\times A_{1ef}(y)\delta^2(x-y)\,,$$
$$L^{nmqv}\delta^2(x-y) = \{\chi_6^{nm}(x,t), \chi_6^{qv}(y,t)\} = \left(f^{nmab}f^{qvbd} - f^{qvab}f^{nmbd}\right)$$
$$\times \pi_a^I X_d^I + 4f^{nmab}f^{qvcd}\tilde{A}_{0ac}X_b^I X_d^I + 2\left(f^{nmab}f^{qvca} - f^{qvab}f^{nmca}\right)$$
$$\times \tilde{A}_{0ae}X_b^I X_e^I\,. \tag{35}$$

It is important to note that matrix C in Eq. (34) is the so-called supermatrix which involves bosonic and fermionic elements (for the definition of Poisson brackets of anticommuting variables, see Ref. [67]). Obtaining its inverse, C^{-1}, which satisfies the relation

$$\int C_{ij}(x,z)C^{-1}_{jk}(z,y)\mathrm{d}^2 z = \delta_{ik}\delta^2(x-y) \tag{36}$$

requires tedious calculations. The result reads

$$C^{-1} = \begin{bmatrix} 0 & EJ^{-1}f^{-1} & 0 & 0 & -f^{-1} & 0 \\ -J^{-1}Ef^{-1} & J^{-1}LJ^{1} & 0 & 0 & J^{-1}Ff^{-1} & -J^{-1} \\ 0 & 0 & 0 & iB^{-1} & 0 & 0 \\ 0 & 0 & -iB^{-1} & 0 & 0 & 0 \\ f^{-1} & -FJ^{-1}f^{-1} & 0 & 0 & 0 & 0 \\ 0 & J^{-1} & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\times\delta^2(x-y)\,. \tag{37}$$

It is now straightforward to use the definition in Eq. (33) and write down the relevant nonzero Dirac brackets of the canonical variables

$$\left\{X_a^I(x,t), \pi^{bJ}(y,t)\right\}_{\mathrm{DB}} = \delta_a^b\delta^{IJ}\delta^2(x-y) + f^{nmc}{}_a X_c^I J_{nmqv}\Sigma^{qvb,J}\delta^2(x-y)\,,$$

$$\left\{A_{1ab}(x,t), P^{1cd}(y,t)\right\}_{\mathrm{DB}} = \delta_a^c\delta_b^d\delta^2(x-y) - \frac{1}{2}f_{abef}f^{efcd}\delta^2(x-y)\,,$$

$$\left\{\Phi_{ab}(x,t), p_\Phi^{cd}(y,t)\right\}_{\mathrm{DB}} = \delta_a^c\delta_b^d\delta^2(x-y) - \frac{1}{2}f_{abef}f^{efcd}\delta^2(x-y)\,,$$

$$\left\{A_{0cd}(x,t), P_0^{cd}(y,t)\right\}_{\mathrm{DB}} = \delta_a^c\delta_b^d\delta^2(x-y) - J_{abqv}J^{qvcd}\delta^2(x-y)\,,$$

$$\left\{\Psi_a(x,t), \bar{\Psi}_b(y,t)\right\}_{\mathrm{DB}} = -\frac{i}{2}\Gamma_0^{-1}\delta_{ab}\delta^2(x-y)\,, \tag{38}$$

where J is given in Eq. (35), and

$$\Sigma^{qvb,I} = -f^{qvab}\pi_a^I + 2f^{qvac}\tilde{A}_{0a}{}^{\ b}X_c^I + 2f^{nmab}\tilde{A}_{0ad}X^{dI}\,. \tag{39}$$

Hence, we have finished the final stage of the procedure, and to explain the situation we will sum up what we have done so far. After distinguishing between the first and second class constraints, the Poisson brackets were discarded. All the equations of the system were written in terms of the Dirac brackets. The second class constraints are just identities representing some canonical variables as functions of the others [67]. It is well-known that in simple scenarios, the second class constraints can be used to rule out completely some canonical variables from the formalism. However, what we have dealt with here was not a simple case, and in more complicated scenarios, the removal of some degrees of freedom instead of others can be a stumbling block, which can be a great difficulty for the next step of the process, *i.e.*, quantization.

It is easy to realize that in the Dirac method, we did not try to rule out the gauge degrees of freedom. As a matter of fact, we keep all the variables. No gauge condition was imposed, and all the dynamical variables were quantum mechanically realized. The constraints became operators that acted in a nontrivial manner on the Dirac representation space, which can carry unphysical information. We can remove this unphysical information by imposing conditions that select the physical states. This condition must be one such that it imposes gauge invariance in the quantum theory, *i.e.*, every physical state must remain unchanged as we provide a transformation generated by the constraints such that $\chi|\psi\rangle = 0$, which implies that the physical states are invariant under certain finite gauge transformations.

However, in our case, the second class constraints of BLG model are extremely complicated and it might be convenient to convert this second class system into one with the first class constraints, since it is easier to impose the first class constraints on the physical states. This constraints conversion procedure is out of the scope of this paper.

4. Conclusions

We have explored the model constructed by Bagger, Lambert and Gustavsson, in particular its compactification on $\mathbb{R}^{1,1} \times S^1$ into a new two-dimensional massless non-associative field theory, which shows an interesting behavior at weak and strong couplings.

To obtain a Hamiltonian formulation of the BLG structure, we have followed the method created by Dirac to deal with constrained dynamical systems. Hence, following this method, the BLG model is considered a second class system, since the whole group of constraints that appears there do not commute to each other considering the Poisson brackets algebra. This result means that the model is not gauge-invariant. After that, we have eliminated these constraints through the computation of the Dirac brackets. As a direct perspective, we can convert the second class constraints into the first class ones to obtain the gauge-invariant equivalent model together with its gauge transformations and it can be discussed if its properties are preserved. It is a current research project.

The authors thank Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazilian scientific support federal agency, for partial financial support, grants numbers 406894/2018-3 (E.M.C.A.) and 303140/2017-8 (J.A.N.).

REFERENCES

[1] J. Bagger, N. Lambert, «Modeling multiple M2-branes», *Phys. Rev. D* **75**, 045020 (2007), arXiv:hep-th/0611108.

[2] J. Bagger, N. Lambert, «Gauge symmetry and supersymmetry of multiple M2-branes», *Phys. Rev. D* **77**, 065008 (2008).

[3] J. Bagger, N. Lambert, «Comments on multiple M2-branes», *J. High Energy Phys.* **0802**, 105 (2008).

[4] A. Gustavsson, «Algebraic structures on parallel M2 branes», *Nucl. Phys. B* **811**, 66 (2009), arXiv:0709.1260 [hep-th].

[5] M. Cerdewal, «Superfield actions for $N = 8$ and $N = 6$ conformal theories in three dimensions», *J. High Energy Phys.* **0810**, 070 (2008).

[6] A. Mauri, A.C. Petkou, «An $\mathcal{N} = 1$ superfield action for M2 branes», *Phys. Lett. B* **666**, 527 (2008).

[7] S.V. Ketov, S. Kobayashi, «Higher-derivative gauge interactions of Bagger–Lambert–Gustavsson theory in superspace», *Phys. Rev. D* **83**, 045003 (2011).

[8] A. Benna, I. Klebanov, T. Klose, M. Smedback, «Superconformal Chern-Simons theories and AdS_4/CFT_3 correspondence», *J. High Energy Phys.* **0809**, 072 (2008).

[9] S. Cherskis, C. Saemann, «Multiple M2-branes and generalized 3-Lie algebras», *Phys. Rev. D* **78**, 066019 (2008).

[10] H. Samtleben, R. Wimmer, *J. High Energy Phys.* **1002**, 070 (2010).

[11] I.A. Bandos, *Phys. Lett. B* **669**, 193 (2008).

[12] M. Faizal, «Boundary effects in the BLG theory», *Mod. Phys. Lett. A* **29**, 1450154 (2014).

[13] S. Upadhyay, «A mass-deformed BLG theory in Gribov–Zwanziger framework», *Int. J. Mod. Phys. A* **31**, 1650148 (2016).

[14] M. Faizal, B.P. Mandal, S. Upadhyay, «Finite BRST transformations for the Bagger–Lambert–Gustavsson theory», *Phys. Lett. B* **721**, 159 (2013); M. Faizal, S. Upadhyay, B.P. Mandal, «Finite field-dependent BRST symmetry for ABJM theory in $\mathcal{N}=1$ superspace», *Phys. Lett. B* **738**, 201 (2014); «Anti-FFBRST transformations for the BLG theory in presence of a boundary», *Int. J. Mod. Phys. A* **30**, 1550032 (2015); M. Faizal, S. Upadhyay, «Spontaneous breaking of the BRST symmetry in the ABJM theory», *Phys. Lett. B* **736**, 288 (2014); S. Upadhyay, «Ward identities and gauge flow for M-theory in $\mathcal{N}=3$ superspace», *Phys. Rev. D* **92**, 065027 (2015); «Spontaneous breaking of nilpotent symmetry in boundary BLG theory», *Int. J. Mod. Phys. A* **30**, 1550150 (2015); S. Upadhyay, M. Faizal, P.A. Ganai, «Interpolating between different gauges in the ABJM theory», *Int. J. Mod. Phys. A* **30**, 1550185 (2015); S. Upadhyay, «Ward and Nielsen identities for ABJM theory in $\mathcal{N}=1$ superspace», *Int. J. Mod. Phys. A* **31**, 1650112 (2016); S. Upadhyay, D. Das, «ABJM theory in Batalin–Vilkovisky formulation», *Phys. Lett. B* **733**, 63 (2014).

[15] M.A. Bandres, A.E. Lipstein, J.H. Schwarz, «$N=8$ superconformal Chern–Simons theories», *J. High Energy Phys.* **0805**, 025 (2008), arXiv:0803.3242 [hep-th].

[16] M. Van Raamsdonk, «Comments on the Bagger–Lambert theory and multiple M2-branes», *J. High Energy Phys.* **0805**, 105 (2008), arXiv:0803.3803 [hep-th].

[17] J.P. Gauntlett, J.B. Gutowski, «Constraining maximally supersymmetric membrane actions», *J. High Energy Phys.* **0806**, 053 (2008), arXiv:0804.3078 [hep-th].

[18] D.S. Berman, L.C. Tadrowski, D.C. Thompson, «Aspects of multiple membranes», *Nucl. Phys. B* **802**, 106 (2008), arXiv:0803.3611 [hep-th].

[19] K. Hosomichi, K.M. Lee, S. Lee, «Mass-deformed Bagger–Lambert theory and its BPS objects», *Phys. Rev. D* **78**, 066015 (2008), arXiv:0804.2519 [hep-th].

[20] Y. Song, «Mass deformation of the multiple M2-branes theory», *Eur. Phys. J. C* **71**, 1599 (2011), arXiv:0805.3193 [hep-th].

[21] C. Ahn, «Holographic supergravity dual to three dimensional Script $N=2$ gauge theory», *J. High Energy Phys.* **0808**, 083 (2008), arXiv:0806.1420 [hep-th].

[22] C. Krishnan, C. Maccaferri, «Membranes on calibrations», *J. High Energy Phys.* **0807**, 005 (2008), arXiv:0805.3125 [hep-th].

[23] I. Jeon *et al.*, «Classification of the BPS states in Bagger–Lambert theory», *J. High Energy Phys.* **0807**, 056 (2008), arXiv:0805.3236 [hep-th].

[24] S. Banerjee, A. Sen, «Interpreting the M2-brane Action», *Mod. Phys. Lett. A* **24**, 721 (2009), arXiv:0805.3930 [hep-th].

[25] M.A. Bandres, A.E. Lipstein, J.H. Schwarz, «Ghost-free superconformal action for multiple M2-branes», *J. High Energy Phys.* **0807**, 117 (2008), arXiv:0806.0054 [hep-th].

[26] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk, H. Verlinde, «Supersymmetric Yang–Mills theory from lorentzian three-algebras», *J. High Energy Phys.* **0808**, 094 (2008), arXiv:0806.0738 [hep-th].

[27] J. Gomis, G. Milanesi, J.G. Russo, «Bagger–Lambert theory for general Lie algebras», *J. High Energy Phys.* **0806**, 075 (2008), arXiv:0805.1012 [hep-th].

[28] S. Benvenuti, D. Rodriguez-Gomez, E. Tonni, H. Verlinde, «$N = 8$ superconformal gauge theories and M2 branes», *J. High Energy Phys.* **0901**, 078 (2009), arXiv:0805.1087 [hep-th].

[29] P.M. Ho, Y. Imamura, Y. Matsuo, «M2 to D2 revisited», *J. High Energy Phys.* **0807**, 003 (2008), arXiv:805.1202 [hep-th].

[30] B. Ezhuthachan, S. Mukhi, C. Papageorgakis, «D2 to D2», *J. High Energy Phys.* **0807**, 041 (2008), arXiv:0806.1639 [hep-th].

[31] S. Cecotti, A. Sen, «Coulomb Branch of the Lorentzian Three Algebra Theory», arXiv:0806.1990 [hep-th].

[32] P.M. Ho, R.C. Hou, Y. Matsuo, «Lie 3-algebra and multiple M2-branes», *J. High Energy Phys.* **0806**, 020 (2008), arXiv:0804.2110 [hep-th].

[33] G. Papadopoulos, «M2-branes, 3-Lie algebras and Plücker relations», *J. High Energy Phys.* **0805**, 054 (2008), arXiv:0804.2662 [hep-th].

[34] G. Papadopoulos, «On the structure of k-Lie algebras», *Class. Quantum Grav.* **25**, 142002 (2008), arXiv:0804.3567 [hep-th].

[35] A. Morozov, «On the problem of multiple M2 branes», *J. High Energy Phys.* **0805**, 076 (2008), arXiv:0804.0913 [hep-th].

[36] U. Gran, B.E.W. Nilsson, C. Petersson, «On relating multiple M2 and D2-branes», *J. High Energy Phys.* **0810**, 067 (2008), arXiv:0804.1784 [hep-th].

[37] P.M. Ho, Y. Matsuo, «M5 from M2», *J. High Energy Phys.* **0806**, 105 (2008), arXiv:0804.3629 [hep-th].

[38] H. Shimada, «β-deformation for matrix model of M-theory», *Nucl. Phys. B* **813**, 283 (2009), arXiv:0804.3236 [hep-th].

[39] A. Morozov, «From simplified BLG action to the first-quantized M theory», *JETP Lett.* **87**, 659 (2008), arXiv:0805.1703 [hep-th].

[40] Y. Honma, S. Iso, Y. Sumitomo, S. Zhang, «Janus field theories from multiple M2 branes», *Phys. Rev. D* **78**, 025027 (2008), arXiv:0805.1895 [hep-th].

[41] H. Fuji, S. Terashima, M. Yamazaki, «A new $\mathcal{N} = 4$ membrane action via orbifold», *Nucl. Phys. B* **810**, 354 (2009), arXiv:0805.1997 [hep-th].

[42] M. Li, T. Wang, «M2-branes coupled to antisymmetric fluxes», *J. High Energy Phys.* **0807**, 093 (2008), arXiv:0805.3427 [hep-th].

[43] K. Hosomichi *et al.*, «$N = 4$ Superconformal Chern–Simons theories with hyper and twisted hyper multiplets», *J. High Energy Phys.* **0807**, 091 (2008), arXiv:0805.3662 [hep-th].

[44] J. Figueroa-O'Farrill, P. de Medeiros, E. Méndez-Escobar, «Lorentzian Lie 3-algebras and their Bagger–Lambert moduli space», *J. High Energy Phys.* **0807**, 111 (2008), arXiv:0805.4363 [hep-th].

[45] J.H. Park, C. Sochichiu, «Taking off the square root of Nambu–Goto action and obtaining Filippov–Lie algebra gauge theory action», *Eur. Phys. J. C* **64**, 161 (2009), arXiv:0806.0335 [hep-th].

[46] F. Passerini, «M2-brane superalgebra from Bagger–Lambert theory», *J. High Energy Phys.* **0808**, 062 (2008), arXiv:0806.0363 [hep-th].

[47] E.A. Bergshoeff, M. de Roo, O. Hohm, D. Roest, «Multiple membranes from gauged supergravity», *J. High Energy Phys.* **0808**, 091 (2008), arXiv:0806.2584 [hep-th].

[48] P. de Medeiros, J. Figueroa-O'Farrill, E. Méndez-Escobar, «Metric Lie 3-algebras in Bagger–Lambert theory», *J. High Energy Phys.* **0808**, 045 (2008), arXiv:0806.3242 [hep-th].

[49] M. Blau, M. O'Loughlin, «Multiple M2-branes and plane waves», *J. High Energy Phys.* **0809**, 112 (2008), arXiv:0806.3253 [hep-th].

[50] K. Furuuchi, S.Y. Shih, T. Takimi, «M-theory superalgebra from multiple membranes», *J. High Energy Phys.* **0808**, 072 (2008), arXiv:0806.4044 [hep-th].

[51] J. Figueroa-O'Farrill, «Metric Lie n-algebras, double extensions», arXiv:0806.3534 [math.RT].

[52] C. Sochichiu, «On Nambu-Lie 3-algebra representations», arXiv:0806.3520 [hep-th].

[53] I.A. Bandos, P.K. Townsend, «Light-cone M5 and multiple M2-branes», *Class. Quantum Grav.* **25**, 245003 (2008), arXiv:0806.4777 [hep-th].

[54] G. Bonelli, A. Tanzini, M. Zabzine, «Topological branes, p-algebras and generalized Nahm equations», *Phys. Lett. B* **672**, 390 (2009), arXiv:0807.5113 [hep-th].

[55] A. Hanany, N. Mekareeya, A. Zaffaroni, «Partition functions for membrane theories», *J. High Energy Phys.* **0809**, 090 (2008), arXiv:0806.4212 [hep-th].

[56] J. Bedford, D. Berman, «A note on quantum aspects of multiple membranes», *Phys. Lett. B* **668**, 67 (2008), arXiv:0806.4900 [hep-th].

[57] I.A. Bandos, P.K. Townsend, «SDiff gauge theory and the M2 condensate», *J. High Energy Phys.* **0902**, 013 (2009), arXiv:0808.1583 [hep-th].

[58] M. Cederwall, «$N = 8$ superfield formulation of the Bagger–Lambert–Gustavsson model», *J. High Energy Phys.* **0809**, 116 (2008), arXiv:0808.3242 [hep-th].

[59] R. Iengo, J.G. Russo, «Non-linear theory for multiple M2 branes», *J. High Energy Phys.* **0810**, 030 (2008), arXiv:0808.2473 [hep-th].

[60] S. Mukhi, C. Papageorgakis, «M2 to D2», *J. High Energy Phys.* **0805**, 085 (2008), arXiv:0803.3218 [hep-th].

[61] M.A. Santos, I.V. Vancea, «New Two-dimensional Massless Field Theory from Bagger–Lambert–Gustavsson Model», *Mod. Phys. Lett. A* **24**, 2275 (2009).

[62] P. Franche, «Reduction of the $N = 8$ BLG, $N = 6$ BL Theories to 2D Effective Field Theories», arXiv:0811.1443v1 [hep-th].

[63] V. Filippov, *Sibirisk. Mat. Zh.* **26**, 126140 (1985).

[64] Y. Nambu, «Generalized Hamiltonian Dynamics», *Phys. Rev. D* **7**, 2405 (1973).

[65] L. Takhatajan, «On foundation of the generalized Nambu mechanics», *Commun. Math. Phys.* **160**, 295 (1994).

[66] T. Li, Y. Liu, D. Xie, «Multiple D2-brane Action from M2-branes», *Int. J. Mod. Phys. A* **24**, 3039 (2009), arXiv:0807.1183 [hep-th].

[67] D.M. Gitman, I.V. Tyutin, «Quantization of Fields with Constraints», *Springer Berlin Heidelberg*, Berlin, Heidelberg 1990; P.A.M. Dirac, «Lectures on Quantum Mechanics», *Yeshiva University*, New York, USA 1964.