REVISIT THE $X(4274)$ AS THE AXIALVECTOR TETRAQUARK STATE*

Zhi-Gang Wang

Department of Physics, North China Electric Power University
Baoding 071003, P.R. China
zgwang@aliyun.com

(Received July 31, 2019; accepted December 3, 2019)

In this article, we construct the $[sc]_{\rm A}[\bar{s}\bar{c}]_{\rm V}-[sc]_{\rm V}[\bar{s}\bar{c}]_{\rm A}$-type tensor current to study the mass and width of the $X(4274)$ with the QCD sum rules in detail. The predicted mass $M_X = (4.27 \pm 0.09)$ GeV for the $J^{PC}=1^{++}$ tetraquark state is in excellent agreement with the experimental data $4273.3 \pm 8.3^{+17.2}_{-3.6}$ MeV from the LHCb Collaboration. The central value of the width $\Gamma(X(4274) \to J/\psi\phi) = 47.9$ MeV is in excellent agreement with the experimental data $56 \pm 11^{+8}_{-11}$ MeV from the LHCb Collaboration. The present work supports assigning the $X(4274)$ to be the $J^{PC}=1^{++}$ $[sc]_{\rm A}[\bar{s}\bar{c}]_{\rm V}-[sc]_{\rm V}[\bar{s}\bar{c}]_{\rm A}$ tetraquark state with a relative P-wave between the diquark and antidiquark constituents. Furthermore, we obtain the mass of the $[sc]_{\rm A}[\bar{s}\bar{c}]_{\rm V}-[sc]_{\rm V}[\bar{s}\bar{c}]_{\rm A}$-type tetraquark state with $J^{PC}=1^{-+}$ as a byproduct.

DOI:10.5506/APhysPolB.51.435

1. Introduction

In 2011, the CDF Collaboration confirmed the $X(4140)$ in the $B^\pm \to J/\psi\,\phi K^\pm$ decays produced in $p\bar{p}$ collisions at $\sqrt{s}=1.96$ TeV with a statistical significance greater than 5σ, and observed an evidence for the $X(4274)$ with approximate significance of 3.1σ. The measured mass and width are $4274.4^{+8.4}_{-6.7} \pm 1.9$ MeV and $32.3^{+21.9}_{-15.3} \pm 7.6$ MeV, respectively [1]. In 2013, the CMS Collaboration observed an evidence for a second peaking structure (which is consistent with the $X(4274)$) besides the $X(4140)$ with the mass $4313.8 \pm 5.3 \pm 7.3$ MeV and width $38^{+30}_{-15} \pm 16$ MeV, respectively, in the $B^\pm \to J/\psi\,\phi K^\pm$ decays produced in pp collisions at $\sqrt{s}=7$ TeV collected with the CMS detector at the LHC [2].

* Funded by SCOAP^3 under Creative Commons License, CC-BY 4.0.

In 2016, the LHCb Collaboration performed the first full amplitude analysis of the $B^+ \to J/\psi\phi K^+$ decays with a data sample of 3 fb^{-1} of pp collision data collected at $\sqrt{s} = 7$ and 8 TeV with the LHCb detector, and confirmed the two old particles $X(4140)$ and $X(4274)$ in the $J/\psi\phi$ mass spectrum with statistical significances 8.4σ and 6.0σ, respectively. The measured masses and widths are

$$\begin{aligned} X(4140) : M_X &= 4146.5 \pm 4.5^{+4.6}_{-2.8}\ \mathrm{MeV}\,, \qquad \Gamma_X = 83 \pm 21^{+21}_{-14}\ \mathrm{MeV}\,, \\ X(4274) : M_X &= 4273.3 \pm 8.3^{+17.2}_{-3.6}\ \mathrm{MeV}\,, \qquad \Gamma_X = 56 \pm 11^{+8}_{-11}\ \mathrm{MeV}\,. \end{aligned} \tag{1}$$

Furthermore, the LHCb Collaboration determined for the first time the spin-parity-charge-conjugation of the $X(4140)$ and $X(4274)$ to be $J^{PC} = 1^{++}$ with statistical significances 5.7σ and 5.8σ, respectively [3, 4], which rules out the 0^{-+} molecule assignment, and it is consistent with our previous work [5].

There have been several possible assignments, such as the color sextet–sextet-type $cs\bar{c}\bar{s}$ tetraquark state [6–8], the conventional orbitally excited state $\chi_{c1}(3P)$ [9, 10], the color triplet–triplet-type $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})c\bar{c}$ tetraquark state [11], *etc.* In Ref. [12], Maiani, Polosa and Riquer take the mass of the $X(4140)$ as an input parameter, and obtain the mass spectrum of the $sc\bar{s}\bar{c}$ tetraquark states with positive parity based on the effective Hamiltonian with the spin–spin and spin–orbit interactions. However, they observe that there is no room for the $X(4274)$, and suggest that the $X(4274)$ corresponds to two, almost degenerate, unresolved lines with $J^{PC} = 0^{++}$ and 2^{++}.

In Ref. [10], we construct the color octet–octet-type axialvector current to study the mass and width of the $X(4274)$ with the QCD sum rules in detail. The predicted mass strongly favors assigning the $X(4274)$ to be the color octet–octet-type tetraquark molecule-like state, but the predicted width disfavors assigning the $X(4274)$ to be the color octet–octet-type tetraquark molecule-like state.

In Ref. [13], we study in detail the masses of the $[sc]_{\mathrm{S}}[\bar{s}\bar{c}]_{\mathrm{A}} \pm [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{S}}$-type and $[sc]_{\mathrm{P}}[\bar{s}\bar{c}]_{\mathrm{V}} \mp [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{P}}$-type tetraquark states with $J^{PC} = 1^{+\pm}$, respectively, with the QCD sum rules, where the subscripts S, P, A and V denote the scalar, pseudoscalar, axialvector and vector diquark constituents, respectively. The numerical results $M_X = 3.95 \pm 0.09$ GeV and 5.00 ± 0.10 GeV disfavor assigning the $X(4140/4274)$ to be the $J^{PC} = 1^{++}$ $[sc]_{\mathrm{S}}[\bar{s}\bar{c}]_{\mathrm{A}} + [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{S}}$-type and $[sc]_{\mathrm{P}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{P}}$-type tetraquark states.

In Ref. [14], we construct both the $[sc]_{\mathrm{T}}[\bar{s}\bar{c}]_{\mathrm{A}} + [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{T}}$-type and $[sc]_{\mathrm{T}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{T}}$-type axialvector currents with $J^{PC} = 1^{++}$ to study the mass and width of the $X(4140)$ with the QCD sum rules, where the subscript T denotes the tensor diquark operator. The predicted masses support assigning the $X(4140)$ to be the $[sc]_{\mathrm{T}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{T}}$-type axialvector tetraquark state; the predicted decay width $\Gamma(X(4140) \to J/\psi\phi) = 86.9 \pm 22.6\,\mathrm{MeV}$ is in excellent agreement with the experimental data $83 \pm 21^{+21}_{-14}$ MeV from the LHCb Collaboration, which also supports assigning the $X(4140)$ to be the $[sc]_{\mathrm{T}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{T}}$-type axialvector tetraquark state.

In Refs. [8, 15], the $[sc]_{\mathrm{S}}[\bar{s}\bar{c}]_{\mathrm{A}} + [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{S}}$-type and $[sc]^{6}_{\mathrm{S}}[\bar{s}\bar{c}]^{\bar{6}}_{\mathrm{A}} + [sc]^{6}_{\mathrm{A}}[\bar{s}\bar{c}]^{\bar{6}}_{\mathrm{S}}$-type tetraquark states with $J^{PC} = 1^{++}$ are studied with the QCD sum rules — the criteria for choosing the Borel windows are different from our previous works [10, 13, 14], see Section 2 for the technical details. The quark structures, predicted masses and widths are all shown explicitly in Table I.

TABLE I

The structures, masses and widths of the $sc\bar{s}\bar{c}$ tetraquark states with $J^{PC} = 1^{++}$ from the QCD sum rules, where the superscript P denotes the parity, the S denotes the spin, the L denotes the relative angular momentum, the J denotes the total angular momentum, the superscript 8 denotes the color octet, the superscripts 6 and $\bar{6}$ denote the color sextet and antisextet, respectively. The superscripts 3 and $\bar{3}$ for the color triplet and antitriplet are neglected for simplicity.

$\lvert S^P_{sc}, S^P_{\bar{s}\bar{c}}; L; J\rangle$	Structures	M [GeV]	Γ [MeV]	Refs.
$\lvert 0^+,1^+;0;1\rangle + \lvert 1^+,0^+;0;1\rangle$	$[sc]_{\mathrm{S}}[\bar{s}\bar{c}]_{\mathrm{A}} + [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{S}}$	3.95 ± 0.09		[13]
$\lvert 0^-,1^-;0;1\rangle - \lvert 1^-,0^-;0;1\rangle$	$[sc]_{\mathrm{P}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{P}}$	5.00 ± 0.10		[13]
$\lvert 1^+,1^+;0;1\rangle + \lvert 1^+,1^+;0;1\rangle$	$[sc]_{\mathrm{T}}[\bar{s}\bar{c}]_{\mathrm{A}} + [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{T}}$	5.20 ± 0.11		[14]
$\lvert 1^-,1^-;0;1\rangle - \lvert 1^-,1^-;0;1\rangle$	$[sc]_{\mathrm{T}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{T}}$	4.14 ± 0.10	86.9 ± 22.6	[14]
$\lvert 0^+,1^+;0;1\rangle + \lvert 1^+,0^+;0;1\rangle$	$[sc]_{\mathrm{S}}[\bar{s}\bar{c}]_{\mathrm{A}} + [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{S}}$	4.07 ± 0.10		[15]
$\lvert 0^+,1^+;0;1\rangle + \lvert 1^+,0^+;0;1\rangle$	$[sc]^{6}_{\mathrm{S}}[\bar{s}\bar{c}]^{\bar{6}}_{\mathrm{A}} + [sc]^{6}_{\mathrm{A}}[\bar{s}\bar{c}]^{\bar{6}}_{\mathrm{S}}$	4.22 ± 0.10		[15]
$\lvert 0^+,1^+;0;1\rangle + \lvert 1^+,0^+;0;1\rangle$	$[sc]_{\mathrm{S}}[\bar{s}\bar{c}]_{\mathrm{A}} + [sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{S}}$	4.18 ± 0.12	80 ± 29	[8]
$\lvert 0^+,1^+;0;1\rangle + \lvert 1^+,0^+;0;1\rangle$	$[sc]^{6}_{\mathrm{S}}[\bar{s}\bar{c}]^{\bar{6}}_{\mathrm{A}} + [sc]^{6}_{\mathrm{A}}[\bar{s}\bar{c}]^{\bar{6}}_{\mathrm{S}}$	4.26 ± 0.12	272 ± 81	[8]
	$[\bar{s}c]^{8}_{\mathrm{P}}[\bar{c}s]^{8}_{\mathrm{V}} - [\bar{s}c]^{8}_{\mathrm{V}}[\bar{c}s]^{8}_{\mathrm{P}}$	4.27 ± 0.09	1800	[10]
$\lvert 1^+,1^-;1;1\rangle - \lvert 1^-,1^+;1;1\rangle$	$[sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{A}}$			This work

In this article, we extend our previous works [10, 13, 14], construct the $[sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{V}} - [sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{A}}$-type tensor current to study the mass and decay width of the $X(4274)$ with the QCD sum rules, and explore the possible assignment of the $X(4274)$ as the diquark–antidiquark-type axialvector tetraquark state once more.

The article is arranged as follows: we derive the QCD sum rules for the mass and width of the diquark–antidiquark-type axialvector tetraquark state $X(4274)$ in Section 2 and in Section 3 respectively; Section 4 is reserved for our conclusion.

2. The mass of the $X(4274)$ as the axialvector tetraquark state

In the following, we write down the two-point correlation function $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules

$$\Pi_{\mu\nu\alpha\beta}(p) \;=\; i\int \mathrm{d}^4x \mathrm{e}^{ip\cdot x}\langle 0|T\left\{J_{\mu\nu}(x)J^{\dagger}_{\alpha\beta}(0)\right\}|0\rangle\,, \tag{2}$$

where

$$\begin{aligned}J_{\mu\nu}(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\\ &\times\Big\{s_j^{\mathrm{T}}(x)C\gamma_\mu c_k(x)\bar{s}_m(x)\gamma_5\gamma_\nu C\bar{c}_n^{\mathrm{T}}(x)-s_j^{\mathrm{T}}(x)C\gamma_\nu\gamma_5 c_k(x)\bar{s}_m(x)\gamma_\mu C\bar{c}_n^{\mathrm{T}}(x)\Big\},\end{aligned} \tag{3}$$

the i, j, k, m, n are color indexes, the C is the charge conjugation matrix. Under charge conjugation (parity) transform $\widehat{C}$ ($\widehat{P}$), the current $J_{\mu\nu}(x)$ has the properties

$$\begin{aligned}\widehat{C}J_{\mu\nu}(x)\widehat{C}^{-1} &= +J_{\mu\nu}(x)\,,\\ \widehat{P}J_{\mu\nu}(x)\widehat{P}^{-1} &= -J^{\mu\nu}(\tilde{x})\,,\end{aligned} \tag{4}$$

where $x^\mu=(t,\vec{x})$ and $\tilde{x}^\mu=(t,-\vec{x})$. The current has definite charge conjugation, and couples potentially to the tetraquark states with positive charge conjugation. The component $J_{0i}(x)$ has the positive parity, while the component $J_{ij}(x)$ has the negative parity, where the space indexes i, j = 1, 2, 3. The current $J_{\mu\nu}$ couples potentially to both the spin-parity-charge-conjugation $J^{PC}=1^{++}$ and 1^{-+} tetraquark states

$$\begin{aligned}\langle 0|J_{\mu\nu}(0)|X^-(p)\rangle &= \frac{\lambda_{X^-}}{M_{X^-}}\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{\alpha}p^{\beta}\,,\\ \langle 0|J_{\mu\nu}(0)|X^+(p)\rangle &= \frac{\lambda_{X^+}}{M_{X^+}}\left(\varepsilon_\mu p_\nu-\varepsilon_\nu p_\mu\right)\,,\end{aligned} \tag{5}$$

where ε_μ are the polarization vectors of the vector and axialvector tetraquark states, the $M_{X^\pm}$ and $\lambda_{X^\pm}$ are the masses and pole residues, respectively.

The scattering amplitude for one-gluon exchange is proportional to

$$\left(\frac{\lambda^a}{2}\right)_{ij}\left(\frac{\lambda^a}{2}\right)_{kl} = -\frac{N_\mathrm{c}+1}{4N_\mathrm{c}}t^{\mathrm{A}}_{ik}t^{\mathrm{A}}_{lj}+\frac{N_\mathrm{c}-1}{4N_\mathrm{c}}t^{\mathrm{S}}_{ik}t^{\mathrm{S}}_{lj}\,, \tag{6}$$

where

$$
\begin{aligned}
t^{\mathrm{A}}_{ik}t^{\mathrm{A}}_{lj} &= \delta_{ij}\delta_{kl}-\delta_{il}\delta_{kj}=\varepsilon_{mik}\varepsilon_{mjl}\,,\\
t^{\mathrm{S}}_{ik}t^{\mathrm{S}}_{lj} &= \delta_{ij}\delta_{kl}+\delta_{il}\delta_{kj}\,,
\end{aligned}
\tag{7}
$$

the λ^a is the Gell-Mann matrix, the i, j, k, m and l are color indexes, the $N_{\rm c}$ is the color number. The negative sign in front of the antisymmetric antitriplet $\bar{3}_{\rm c}$ indicates the interaction is attractive, which favors formation of the diquarks in color antitriplet, while the positive sign in front of the symmetric sextet $6_{\rm c}$ indicates the interaction is repulsive, which disfavors formation of the diquarks in color sextet. We prefer the diquarks in color antitriplet $\bar{3}_{\rm c}$ to the diquarks in color sextet $6_{\rm c}$ in constructing the tetraquark current operators.

The spin-dependent hypersplitting chromomagnetic interactions H_{cs} can be expressed in terms of Pauli spin matrices $\vec{\sigma}$ and $\mathrm{SU}_c(3)$ generators λ^a as

$$
\begin{aligned}
&H_{cs}=-\sum_{a}^{8}\sum_{i>j}\vec{\sigma}_i\cdot\vec{\sigma}_j\lambda^a_i\lambda^a_j=8N+\tfrac{1}{2}C_6(\mathrm{tot})-\tfrac{4}{3}S_{\mathrm{tot}}(S_{\mathrm{tot}}+1)\\
&+C_3(Q)+\tfrac{8}{3}S_Q(S_Q+1)-C_6(Q)+C_3\left(\bar{Q}\right)+\tfrac{8}{3}S_{\bar{Q}}(S_{\bar{Q}}+1)-C_6\left(\bar{Q}\right)\;,
\end{aligned}
\tag{8}
$$

where N is the total number of quarks, Q and $\bar{Q}$ are the diquark and antidiquark respectively, and C_3 and C_6 are quadratic Casimir operators of $\mathrm{SU}_c(3)$ and $\mathrm{SU}_{cs}(6)$, respectively. The chromomagnetic interaction H_{cs} favors taking the scalar diquarks or "good" diquarks in color antitriplet as the most stable building blocks of the tetraquark states [16, 17]. However, it cannot exclude taking the axialvector diquarks or "bad" diquarks in color antitriplet and other diquarks as the building blocks of the tetraquark states because the dominant contributions to the tetraquark masses do not originate from the chromomagnetic interaction H_{cs}. We need those "bad" diquarks besides the "good" diquarks in studying the higher tetraquark states. The calculations based on the QCD sum rules indicate that the favored configurations are the scalar and axialvector diquark states [18, 19], and the heavy-light scalar and axialvector diquark states have almost degenerate masses [18], the heavy-light axialvector (or "bad") diquark states are not "bad".

In fact, we can obtain the four-quark interactions T from the one-gluon exchange, then perform Fierz re-arrangement both in the color and Dirac-spinor spaces to obtain the result

$$
\begin{aligned}
T &= \bar{Q}\gamma_\mu \frac{\lambda^a}{2} Q\, \bar{q}\gamma^\mu \frac{\lambda^a}{2} q \\
&= -\frac{N_c+1}{4N_c}\Big\{ - q^{\mathrm{T}}C\gamma_5 t^{\mathrm{A}} Q\, \bar{Q}\gamma_5 C t^{\mathrm{A}}\bar{q}^{\mathrm{T}} + q^{\mathrm{T}}C t^{\mathrm{A}} Q\, \bar{Q} C t^{\mathrm{A}}\bar{q}^{\mathrm{T}} \\
&\quad -\frac{1}{2} q^{\mathrm{T}}C\gamma_\mu\gamma_5 t^{\mathrm{A}} Q\, \bar{Q}\gamma^\mu\gamma_5 C t^{\mathrm{A}}\bar{q}^{\mathrm{T}} - \frac{1}{2} q^{\mathrm{T}}C\gamma_\mu t^{\mathrm{A}} Q\, \bar{Q}\gamma^\mu C t^{\mathrm{A}}\bar{q}^{\mathrm{T}}\Big\} \\
&\quad +\frac{N_c+1}{4N_c}\Big\{ - q^{\mathrm{T}}C\gamma_5 t^{\mathrm{S}} Q\, \bar{Q}\gamma_5 C t^{\mathrm{S}}\bar{q}^{\mathrm{T}} + q^{\mathrm{T}}C t^{\mathrm{S}} Q\, \bar{Q} C t^{\mathrm{S}}\bar{q}^{\mathrm{T}} \\
&\quad -\frac{1}{2} q^{\mathrm{T}}C\gamma_\mu\gamma_5 t^{\mathrm{S}} Q\, \bar{Q}\gamma^\mu\gamma_5 C t^{\mathrm{S}}\bar{q}^{\mathrm{T}} - \frac{1}{2} q^{\mathrm{T}}C\gamma_\mu t^{\mathrm{S}} Q\, \bar{Q}\gamma^\mu C t^{\mathrm{S}}\bar{q}^{\mathrm{T}}\Big\} . \qquad (9)
\end{aligned}
$$

We can obtain the diquark operators $q^{\mathrm{T}}C\gamma_5 t^{\mathrm{A}}Q$, $q^{\mathrm{T}}Ct^{\mathrm{A}}Q$, $q^{\mathrm{T}}C\gamma_\mu\gamma_5 t^{\mathrm{A}}Q$, $q^{\mathrm{T}}C\gamma_\mu t^{\mathrm{A}}Q$ in the attractive channels from the QCD indeed. Although we cannot obtain the tensor diquark operators $q^{\mathrm{T}}C\sigma_{\alpha\beta}\gamma_5 t^{\mathrm{A}}Q$ and $q^{\mathrm{T}}C\sigma_{\alpha\beta}t^{\mathrm{A}}Q$ from the one-gluon exchange, they play an important role in building the tetraquark currents. In the QCD sum rules, we can take the scalar, pseudoscalar, vector, axialvector and tensor diquark and antidiquark operators as basic constituents to construct the tetraquark currents, then calculate the two-point and three-point correlation functions in full QCD (not just for the chromomagnetic interaction) to study the masses and partial decay widths, respectively, and finally, we confront the predictions to the experimental data to examine the structures of the tetraquark states.

We can also construct the following currents to interpolate the axialvector tetraquark states with $J^{PC} = 1^{++}$

$$
\begin{aligned}
J^1_\mu(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big[s^{Tj}(x)C\gamma_5 c^k(x)\bar{s}^m(x)\gamma_\mu C\bar{c}^{Tn}(x) \\
&\quad + s^{Tj}(x)C\gamma_\mu c^k(x)\bar{s}^m(x)\gamma_5 C\bar{c}^{Tn}(x)\Big] , \\
J^2_\mu(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big[s^{Tj}(x)C c^k(x)\bar{s}^m(x)\gamma_5\gamma_\mu C\bar{c}^{Tn}(x) \\
&\quad - s^{Tj}(x)C\gamma_\mu\gamma_5 c^k(x)\bar{s}^m(x) C\bar{c}^{Tn}(x)\Big] , \\
J^3_\mu(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big[s^{Tj}(x)C\sigma_{\mu\nu}\gamma_5 c^k(x)\bar{s}^m(x)\gamma^\nu C\bar{c}^{Tn}(x) \\
&\quad + s^{Tj}(x)C\gamma^\nu c^k(x)\bar{s}^m(x)\gamma_5\sigma_{\mu\nu} C\bar{c}^{Tn}(x)\Big] , \\
J^4_\mu(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big[s^{Tj}(x)C\sigma_{\mu\nu} c^k(x)\bar{s}^m(x)\gamma_5\gamma^\nu C\bar{c}^{Tn}(x) \\
&\quad - s^{Tj}(x)C\gamma^\nu\gamma_5 c^k(x)\bar{s}^m(x)\sigma_{\mu\nu} C\bar{c}^{Tn}(x)\Big] .
\end{aligned}
\qquad (10)
$$

The predicted masses are not consistent with the experimental value of the mass of the $X(4274)$ [13, 14], see Table I. In Table I, we also present the results from the diquark–antidiquark-type interpolating currents with the color sextet–sextet structure [8, 15].

At the hadron side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J_{\mu\nu}(x)$ into the correlation function $\Pi_{\mu\nu\alpha\beta}(p)$ to obtain the hadronic representation [20, 21]. After isolating the ground state contributions of the lowest axialvector and vector tetraquark states, we get the following results:

$$\begin{aligned}\Pi_{\mu\nu\alpha\beta}(p) &= \frac{\lambda_{X^-}^2}{M_{X^-}^2\left(M_{X^-}^2-p^2\right)} \\ &\times\left(p^2 g_{\mu\alpha}g_{\nu\beta}-p^2 g_{\mu\beta}g_{\nu\alpha}-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\right) \\ &+\frac{\lambda_{X^+}^2}{M_{X^+}^2\left(M_{X^+}^2-p^2\right)}\left(-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\right)+\dots \qquad (11)\end{aligned}$$

We can rewrite the correlation function $\Pi_{\mu\nu\alpha\beta}(p)$ in the following form according to Lorentz covariance:

$$\begin{aligned}\Pi_{\mu\nu\alpha\beta}(p) &= \Pi_{X^-}\left(p^2\right)\left(p^2 g_{\mu\alpha}g_{\nu\beta}-p^2 g_{\mu\beta}g_{\nu\alpha}-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha\right. \\ &\left.+g_{\nu\alpha}p_\mu p_\beta\right)+\Pi_{X^+}\left(p^2\right)\left(-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\right). \qquad (12)\end{aligned}$$

We project out the components $\Pi_{X^\pm}(p^2)$ by introducing the operators $P_{X^\pm}^{\mu\nu\alpha\beta}$

$$\widetilde{\Pi}_{X^\pm}\left(p^2\right) = p^2\Pi_{X^\pm}\left(p^2\right) = P_{X^\pm}^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p)\,, \qquad (13)$$

where

$$\begin{aligned}P_{X^-}^{\mu\nu\alpha\beta} &= \frac{1}{6}\left(g^{\mu\alpha}-\frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta}-\frac{p^\nu p^\beta}{p^2}\right), \\ P_{X^+}^{\mu\nu\alpha\beta} &= \frac{1}{6}\left(g^{\mu\alpha}-\frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta}-\frac{p^\nu p^\beta}{p^2}\right)-\frac{1}{6}g^{\mu\alpha}g^{\nu\beta}\,. \qquad (14)\end{aligned}$$

Now, we carry out the operator product expansion for the correlation function $\Pi_{\mu\nu\alpha\beta}(p)$ up to the vacuum condensates of dimension 10. We contract the quark fields s and c in the correlation function $\Pi_{\mu\nu\alpha\beta}(p)$ with the Wick theorem, and obtain the result

$$
\begin{aligned}
\Pi_{\mu\nu\alpha\beta}(p) &= -\frac{i}{2}\varepsilon^{ijk}\varepsilon^{imn}\varepsilon^{i'j'k'}\varepsilon^{i'm'n'}\int \mathrm{d}^4x\, \mathrm{e}^{ip\cdot x} \\
&\times \Big\{ \mathrm{Tr}\Big[\gamma_\mu S_c^{kk'}(x)\gamma_\alpha C S^{Tjj'}(x)C\Big]\mathrm{Tr}\Big[\gamma_\beta\gamma_5 S_c^{n'n}(-x)\gamma_5\gamma_\nu C S^{Tm'm}(-x)C\Big] \\
&+\mathrm{Tr}\Big[\gamma_\nu\gamma_5 S_c^{kk'}(x)\gamma_\alpha C S^{Tjj'}(x)C\Big]\mathrm{Tr}\Big[\gamma_\beta\gamma_5 S_c^{n'n}(-x)\gamma_\mu C S^{Tm'm}(-x)C\Big] \\
&+\mathrm{Tr}\Big[\gamma_\mu S_c^{kk'}(x)\gamma_5\gamma_\beta C S^{Tjj'}(x)C\Big]\mathrm{Tr}\Big[\gamma_\alpha S_c^{n'n}(-x)\gamma_5\gamma_\nu C S^{Tm'm}(-x)C\Big] \\
&+\mathrm{Tr}\Big[\gamma_\nu\gamma_5 S_c^{kk'}(x)\gamma_5\gamma_\beta C S^{Tjj'}(x)C\Big]\mathrm{Tr}\Big[\gamma_\alpha S_c^{n'n}(-x)\gamma_\mu C S^{Tm'm}(-x)C\Big]\Big\}\,, \qquad (15)
\end{aligned}
$$

where

$$
\begin{aligned}
S^{ij}(x) &= \frac{i\delta_{ij}\,\not{x}}{2\pi^2x^4} - \frac{\delta_{ij}m_s}{4\pi^2x^2} - \frac{\delta_{ij}\langle\bar{s}s\rangle}{12} + \frac{i\delta_{ij}\,\not{x}m_s\langle\bar{s}s\rangle}{48} - \frac{\delta_{ij}x^2\langle\bar{s}g_s\sigma Gs\rangle}{192} \\
&+\frac{i\delta_{ij}x^2\,\not{x}m_s\langle\bar{s}g_s\sigma Gs\rangle}{1152} - \frac{ig_sG^a_{\alpha\beta}t^a_{ij}\left(\not{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\,\not{x}\right)}{32\pi^2x^2} \\
&-\frac{\delta_{ij}x^4\langle\bar{s}s\rangle\langle g_s^2GG\rangle}{27648} - \frac{1}{8}\langle\bar{s}_j\sigma^{\mu\nu}s_i\rangle\sigma_{\mu\nu} + \dots\,, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
S_c^{ij}(x) &= \frac{i}{(2\pi)^4}\int \mathrm{d}^4k\, \mathrm{e}^{-ik\cdot x} \\
&\times\left\{\frac{\delta_{ij}}{\not{k}-m_c} - \frac{g_sG^n_{\alpha\beta}t^n_{ij}}{4}\frac{\sigma^{\alpha\beta}(\not{k}+m_c)+(\not{k}+m_c)\sigma^{\alpha\beta}}{(k^2-m_c^2)^2}\right. \\
&\left.-\frac{g_s^2\left(t^at^b\right)_{ij}G^a_{\alpha\beta}G^b_{\mu\nu}\left(f^{\alpha\beta\mu\nu}+f^{\alpha\mu\beta\nu}+f^{\alpha\mu\nu\beta}\right)}{4\left(k^2-m_c^2\right)^5}+\dots\right\}, \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
f^{\lambda\alpha\beta} &= (\not{k}+m_c)\gamma^\lambda(\not{k}+m_c)\gamma^\alpha(\not{k}+m_c)\gamma^\beta(\not{k}+m_c)\,, \\
f^{\alpha\beta\mu\nu} &= (\not{k}+m_c)\gamma^\alpha(\not{k}+m_c)\gamma^\beta(\not{k}+m_c)\gamma^\mu(\not{k}+m_c)\gamma^\nu(\not{k}+m_c)\,, \qquad (18)
\end{aligned}
$$

and $t^n = \frac{\lambda^n}{2}$ [21]. Then we project out the components

$$\widetilde{\Pi}_{X^\pm}\left(p^2\right) = P^{\mu\nu\alpha\beta}_{X^\pm}\Pi_{\mu\nu\alpha\beta}(p) \qquad (19)$$

and compute the integrals both in the coordinate space and momentum space, and obtain the correlation function at the QCD side, therefore, the QCD spectral densities through the dispersion relation

$$\rho_\pm(s) = \frac{\mathrm{Im}\widetilde{\Pi}_{X^\pm}(s)}{\pi}\,. \qquad (20)$$

For the technical details, see Ref. [22].

Now we take the quark–hadron duality below the continuum thresholds s_0 and perform the Borel transform with respect to the variable $P^2 = -p^2$ to obtain two QCD sum rules

$$\lambda^2_{X^\pm} M^2_{X^\pm} \exp\left(-\frac{M^2_{X^\pm}}{T^2}\right) = \int_{4m_c^2}^{s_0} \mathrm{d}s\,\rho_\pm(s)\exp\left(-\frac{s}{T^2}\right), \tag{21}$$

where

$$\rho_\pm(s) = \rho_0^\pm(s) + \rho_3^\pm(s) + \rho_4^\pm(s) + \rho_5^\pm(s) + \rho_6^\pm(s) + \rho_7^\pm(s) + \rho_8^\pm(s) + \rho_{10}^\pm(s), \tag{22}$$

$$\rho_0^+(s) = \frac{1}{1536\pi^6}\int \mathrm{d}y\mathrm{d}z\, yz\,(1-y-z)^2\,\bar{m}_c^2\left(s-\bar{m}_c^2\right)^3$$
$$+\frac{1}{6144\pi^6}\int \mathrm{d}y\mathrm{d}z\, yz\,(1-y-z)^3\left(s-\bar{m}_c^2\right)^2\left(33s^2-18s\bar{m}_c^2+\bar{m}_c^4\right), \tag{23}$$

$$\rho_3^+(s) = -\frac{m_s\langle\bar{s}s\rangle}{96\pi^4}\int \mathrm{d}y\mathrm{d}z\, yz\left(s-\bar{m}_c^2\right)\left(s-2\bar{m}_c^2\right)$$
$$+\frac{m_s\langle\bar{s}s\rangle}{192\pi^4}\int \mathrm{d}y\mathrm{d}z\, yz\,(1-y-z)\left(35s^2-30s\bar{m}_c^2+3\bar{m}_c^4\right)$$
$$-\frac{7m_s m_c^2\langle\bar{s}s\rangle}{48\pi^4}\int \mathrm{d}y\mathrm{d}z\,\left(s-\bar{m}_c^2\right), \tag{24}$$

$$\rho_4^+(s) = +\frac{m_c^2}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\frac{z\,(1-y-z)^2}{y^2}\left(3s-4\bar{m}_c^2\right)$$
$$-\frac{m_c^2}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\frac{z\,(1-y-z)^3}{y^2}\left[5s-\bar{m}_c^2+\frac{4}{3}s^2\delta\left(s-\bar{m}_c^2\right)\right]$$
$$+\frac{1}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\, z\,(1-y-z)\left(s-\bar{m}_c^2\right)\left(4s-5\bar{m}_c^2\right)$$
$$+\frac{1}{9216\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\left(2y-y^2-8zy-5z^2+6z-1\right)\left(s-\bar{m}_c^2\right)\left(2s-s\bar{m}_c^2\right)$$
$$+\frac{1}{110592\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,(1-y-z)\left(2y-y^2-26zy-19z^2+20z-1\right)$$
$$\times\left(35s^2-30s\bar{m}_c^2+3\bar{m}_c^4\right), \tag{25}$$

$$\rho_5^+(s) = \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{576\pi^4}\int \mathrm{d}y\, y\,(1-y)\left(3s-4\widetilde{m}_c^2\right)$$
$$-\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{192\pi^4}\int \mathrm{d}y\mathrm{d}z\, yz\left[5s-\bar{m}_c^2+\frac{4}{3}s^2\delta\left(s-\bar{m}_c^2\right)\right]$$
$$+\frac{7m_s m_c^2\langle\bar{s}g_s\sigma Gs\rangle}{192\pi^4}\int \mathrm{d}y-\frac{m_s m_c^2\langle\bar{s}g_s\sigma Gs\rangle}{384\pi^4}\int \mathrm{d}y\mathrm{d}z\,\frac{1}{y}\,, \tag{26}$$

$$\rho_6^+(s) = \frac{7m_c^2\langle\bar{s}s\rangle^2}{72\pi^2}\int \mathrm{d}y\,, \tag{27}$$

$$\rho_7^+(s) = \frac{m_s m_c^2\langle\bar{s}s\rangle}{864\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\frac{z}{y^2}\left(5-\frac{s}{T^2}\right)\delta\left(s-\bar{m}_c^2\right)$$
$$+\frac{m_s m_c^2\langle\bar{s}s\rangle}{216\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\frac{z\,(1-y-z)}{y^2}\left(\frac{1}{4}+\frac{s}{T^2}-\frac{s^2}{T^4}\right)\delta\left(s-\bar{m}_c^2\right)$$
$$+\frac{7m_s m_c^2\langle\bar{s}s\rangle}{432\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\frac{1}{y^2}\left(y-2+y\frac{s}{T^2}\right)\delta\left(s-\bar{m}_c^2\right)$$
$$+\frac{m_s\langle\bar{s}s\rangle}{3456\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\,\left[5-\frac{s}{2}\delta\left(s-\widetilde{m}_c^2\right)\right]$$
$$+\frac{m_s\langle\bar{s}s\rangle}{3456\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\,\left[1+\frac{s}{2}\delta\left(s-\widetilde{m}_c^2\right)\right]$$
$$-\frac{m_s\langle\bar{s}s\rangle}{1728\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\left[1+\frac{s}{2}\delta\left(s-\bar{m}_c^2\right)\right]$$
$$+\frac{m_s\langle\bar{s}s\rangle}{2304\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\left(y+4z-1\right)\left[1+\frac{4}{3}\left(s+\frac{s^2}{T^2}\right)\delta\left(s-\bar{m}_c^2\right)\right]$$
$$+\frac{m_s m_c^2\langle\bar{s}s\rangle}{576\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\mathrm{d}z\,\frac{1}{yz}\delta\left(s-\bar{m}_c^2\right)$$
$$-\frac{7m_s m_c^2\langle\bar{s}s\rangle}{1728\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int \mathrm{d}y\,\left(1+\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_c^2\right)\,, \tag{28}$$

$$\rho_8^+(s) = -\frac{7m_c^2\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{144\pi^2}\int \mathrm{d}y\,\left(1+\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_c^2\right)$$
$$+\frac{m_c^2\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{288\pi^2}\int \mathrm{d}y\,\frac{1}{y}\delta\left(s-\widetilde{m}_c^2\right)\,, \tag{29}$$

$$\begin{aligned}
\rho_{10}^{+}(s) &= \frac{7m_c^2\langle\bar{s}g_s\sigma Gs\rangle^2}{1152\pi^2T^6}\int dy\, s^2\delta\left(s-\widetilde{m}_c^2\right) \\
&-\frac{7m_c^2\langle\bar{s}s\rangle^2}{324T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y^2}\left(1-y\frac{s}{2T^2}\right)\delta\left(s-\widetilde{m}_c^2\right) \\
&-\frac{m_c^2\langle\bar{s}s\rangle^2}{864T^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\frac{1}{y(1-y)}\delta\left(s-\widetilde{m}_c^2\right) \\
&-\frac{m_c^2\langle\bar{s}g_s\sigma Gs\rangle^2}{1152\pi^2T^4}\int dy\,\frac{1}{y}s\delta\left(s-\widetilde{m}_c^2\right) \\
&-\frac{11m_c^2\langle\bar{s}g_s\sigma Gs\rangle^2}{9216\pi^2T^2}\int dy\,\frac{1}{y(1-y)}\delta\left(s-\widetilde{m}_c^2\right) \\
&+\frac{7m_c^2\langle\bar{s}s\rangle^2}{1296T^6}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\, s^2\delta\left(s-\widetilde{m}_c^2\right)\,, \qquad (30)
\end{aligned}$$

$$\begin{aligned}
\rho_0^{-}(s) &= \frac{1}{1536\pi^6}\int dydz\, yz\,(1-y-z)^2\left(s-\bar{m}_c^2\right)^3\left(6s-\bar{m}_c^2\right) \\
&+\frac{1}{6144\pi^6}\int dydz\, yz\,(1-y-z)^3\left(s-\bar{m}_c^2\right)^2\left(33s^2-18s\bar{m}_c^2+\bar{m}_c^4\right)\,, \qquad (31)
\end{aligned}$$

$$\begin{aligned}
\rho_3^{-}(s) &= \frac{m_s\langle\bar{s}s\rangle}{96\pi^4}\int dydz\, yz\left(s-\bar{m}_c^2\right)\left(7s-2\bar{m}_c^2\right) \\
&+\frac{m_s\langle\bar{s}s\rangle}{192\pi^4}\int dydz\, yz\,(1-y-z)\left(35s^2-30s\bar{m}_c^2+3\bar{m}_c^4\right) \\
&+\frac{3m_s m_c^2\langle\bar{s}s\rangle}{16\pi^4}\int dydz\,\left(s-\bar{m}_c^2\right)\,, \qquad (32)
\end{aligned}$$

$$\begin{aligned}
\rho_4^{-}(s) &= -\frac{m_c^2}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{z(1-y-z)^2}{y^2}\left(9s-4\bar{m}_c^2\right) \\
&-\frac{m_c^2}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{z(1-y-z)^3}{y^2}\left[5s-\bar{m}_c^2+\frac{4}{3}s^2\delta\left(s-\bar{m}_c^2\right)\right] \\
&-\frac{1}{2304\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\, z\,(1-y-z)\left(s-\bar{m}_c^2\right)\left(20s-7\bar{m}_c^2\right) \\
&+\frac{1}{4608\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\left(y^2+(8z-2)y+5z^2-6z+1\right)\left(s-\bar{m}_c^2\right)\left(2s-\bar{m}_c^2\right) \\
&+\frac{1}{110592\pi^4}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\left(1-y-z\right)\left(2y-y^2-26zy-19z^2+20z-1\right) \\
&\times\left(35s^2-30s\bar{m}_c^2+3\bar{m}_c^4\right)\,, \qquad (33)
\end{aligned}$$

$$\rho_5^-(s) = -\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{576\pi^4}\int dy\, y\,(1-y)\left(9s-4\widetilde{m}_c^2\right)$$
$$-\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{192\pi^4}\int dydz\, yz\left[5s-\bar{m}_c^2+\frac{4}{3}s^2\delta\left(s-\bar{m}_c^2\right)\right]$$
$$-\frac{3m_s m_c^2\langle\bar{s}g_s\sigma Gs\rangle}{64\pi^4}\int dy+\frac{m_s m_c^2\langle\bar{s}g_s\sigma Gs\rangle}{384\pi^4}\int dydz\,\frac{1}{y}\,, \tag{34}$$

$$\rho_6^-(s) = -\frac{m_c^2\langle\bar{s}s\rangle^2}{8\pi^2}\int dy\,, \tag{35}$$

$$\rho_7^-(s) = +\frac{m_s m_c^2\langle\bar{s}s\rangle}{864\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{z}{y^2}\left(1-\frac{5s}{T^2}\right)\delta\left(s-\bar{m}_c^2\right)$$
$$+\frac{m_s m_c^2\langle\bar{s}s\rangle}{216\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{z\,(1-y-z)}{y^2}\left(\frac{1}{4}+\frac{s}{T^2}-\frac{s^2}{T^4}\right)\delta\left(s-\bar{m}_c^2\right)$$
$$+\frac{m_s m_c^2\langle\bar{s}s\rangle}{48\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1}{y^2}\left(2-y-y\frac{s}{T^2}\right)\delta\left(s-\bar{m}_c^2\right)$$
$$-\frac{m_s\langle\bar{s}s\rangle}{3456\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\left[7+\frac{13}{2}s\delta\left(s-\widetilde{m}_c^2\right)\right]$$
$$-\frac{m_s\langle\bar{s}s\rangle}{1728\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\left[1+\frac{s}{2}\delta\left(s-\widetilde{m}_c^2\right)\right]$$
$$+\frac{m_s\langle\bar{s}s\rangle}{864\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\left[1+\frac{s}{2}\delta\left(s-\bar{m}_c^2\right)\right]$$
$$+\frac{m_s\langle\bar{s}s\rangle}{2304\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\left(y+4z-1\right)\left[1+\frac{4}{3}\left(s+\frac{s^2}{T^2}\right)\delta\left(s-\bar{m}_c^2\right)\right]$$
$$-\frac{m_s m_c^2\langle\bar{s}s\rangle}{576\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dydz\,\frac{1}{yz}\delta\left(s-\bar{m}_c^2\right)$$
$$+\frac{m_s m_c^2\langle\bar{s}s\rangle}{192\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int dy\,\left(1+\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_c^2\right)\,, \tag{36}$$

$$\rho_8^-(s) = +\frac{m_c^2\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{16\pi^2}\int dy\,\left(1+\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_c^2\right)$$
$$-\frac{m_c^2\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{288\pi^2}\int dy\,\frac{1}{y}\delta\left(s-\widetilde{m}_c^2\right)\,, \tag{37}$$

$$
\begin{aligned}
\rho_{10}^{-}(s) &= -\frac{m_c^2\langle\bar{s}g_s\sigma Gs\rangle^2}{128\pi^2T^6}\int \mathrm{d}y\, s^2\delta\left(s-\widetilde{m}_c^2\right) \\
&+\frac{m_c^2\langle\bar{s}s\rangle^2}{36T^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int \mathrm{d}y\,\frac{1}{y^2}\left(1-y\frac{s}{2T^2}\right)\delta\left(s-\widetilde{m}_c^2\right) \\
&+\frac{m_c^2\langle\bar{s}s\rangle^2}{864T^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int \mathrm{d}y\,\frac{1}{y(1-y)}\delta\left(s-\widetilde{m}_c^2\right) \\
&+\frac{m_c^2\langle\bar{s}g_s\sigma Gs\rangle^2}{1152\pi^2T^4}\int \mathrm{d}y\,\frac{1}{y}s\delta\left(s-\widetilde{m}_c^2\right) \\
&+\frac{11m_c^2\langle\bar{s}g_s\sigma Gs\rangle^2}{9216\pi^2T^2}\int \mathrm{d}y\,\frac{1}{y(1-y)}\delta\left(s-\widetilde{m}_c^2\right) \\
&-\frac{m_c^2}{144T^6}\langle\bar{s}s\rangle^2\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int \mathrm{d}y\, s^2\delta\left(s-\widetilde{m}_c^2\right)\,, \qquad (38)
\end{aligned}
$$

where $\int \mathrm{d}y\mathrm{d}z = \int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z$, $\int\mathrm{d}y = \int_{y_i}^{y_f}\mathrm{d}y$, $y_f = \frac{1+\sqrt{1-4m_c^2/s}}{2}$, $y_i = \frac{1-\sqrt{1-4m_c^2/s}}{2}$, $z_i = \frac{ym_c^2}{ys-m_c^2}$, $\overline{m}_c^2 = \frac{(y+z)m_c^2}{yz}$, $\widetilde{m}_c^2 = \frac{m_c^2}{y(1-y)}$, $\int_{y_i}^{y_f}\mathrm{d}y \to \int_0^1\mathrm{d}y$, $\int_{z_i}^{1-y}\mathrm{d}z \to \int_0^{1-y}\mathrm{d}z$, when the δ functions $\delta\left(s-\overline{m}_c^2\right)$ and $\delta\left(s-\widetilde{m}_c^2\right)$ appear.

We derive Eq. (21) with respect to $\tau = \frac{1}{T^2}$, then eliminate the pole residues $\lambda_{X^\pm}$ to obtain the QCD sum rules for the tetraquark masses

$$
M^2_{X^\pm} = -\frac{\int_{4m_c^2}^{s_0}\mathrm{d}s\,\frac{\mathrm{d}}{\mathrm{d}\tau}\,\rho_\pm(s)\,\mathrm{e}^{-\tau s}}{\int_{4m_c^2}^{s_0}\mathrm{d}s\rho_\pm(s)\,\mathrm{e}^{-\tau s}}\,. \qquad (39)
$$

At the QCD side, we take the vacuum condensates to be the standard values $\langle\bar{q}q\rangle = -(0.24\pm 0.01\text{ GeV})^3$, $\langle\bar{s}s\rangle = (0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $m_0^2 = (0.8\pm0.1)\text{ GeV}^2$, $\langle\frac{\alpha_sGG}{\pi}\rangle = (0.33\text{ GeV})^4$ at the energy scale $\mu = 1$ GeV [20, 21, 23], and take the $\overline{\text{MS}}$ masses $m_c(m_c) = (1.275\pm0.025)$ GeV and $m_s(\mu = 2\text{ GeV}) = (0.095\pm0.005)$ GeV from the Particle Data Group [24]. Moreover, we take into account the energy-scale dependence of the quark condensate, mixed quark condensate and $\overline{\text{MS}}$ masses according to the renormalization group equation

$$
\begin{aligned}
\langle\bar{s}s\rangle(\mu) &= \langle\bar{s}s\rangle(1\text{ GeV})\left[\frac{\alpha_s(1\text{ GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}\,, \\
\langle\bar{s}g_s\sigma Gs\rangle(\mu) &= \langle\bar{s}g_s\sigma Gs\rangle(1\text{ GeV})\left[\frac{\alpha_s(1\text{ GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}\,, \\
m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}}\,,
\end{aligned}
$$

$$\begin{aligned}
m_s(\mu) &= m_s(2\text{ GeV})\left[\frac{\alpha_{\rm s}(\mu)}{\alpha_{\rm s}(2\text{ GeV})}\right]^{\frac{12}{33-2n_{\rm f}}}\,,\\
\alpha_{\rm s}(\mu) &= \frac{1}{b_0 t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2(\log^2 t-\log t-1)+b_0 b_2}{b_0^4 t^2}\right]\,,
\end{aligned} \tag{40}$$

where $t = \log\frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_{\rm f}}{12\pi}$, $b_1 = \frac{153-19n_{\rm f}}{24\pi^2}$, $b_2 = \frac{2857-\frac{5033}{9}n_{\rm f}+\frac{325}{27}n_{\rm f}^2}{128\pi^3}$, $\Lambda = 210$ MeV, 292 MeV and 332 MeV for the flavors $n_{\rm f} = 5$, 4 and 3, respectively [24, 25], and evolve all the input parameters to the ideal energy scales μ to extract the masses of the tetraquark states. In this article, we choose the flavor $n_{\rm f} = 4$.

The hidden-charm (and hidden-bottom) tetraquark states $qQ\bar{q}'\bar{Q}$ can be described by a double-well potential. In the tetraquark states $qQ\bar{q}'\bar{Q}$, the Q-quark serves as a static well potential and attracts the light quark q to form a heavy diquark in color antitriplet, the $\bar{Q}$-quark serves as another static well potential and attracts the light antiquark $\bar{q}'$ to form a heavy antidiquark in color triplet [26–28]. The diquark and antidiquark attract each other to form a compact tetraquark state [26–28], the two heavy quarks Q and $\bar{Q}$ stabilize the tetraquark state $qQ\bar{q}'\bar{Q}$, just as in the case of the $(\mu^-e^+)(\mu^+e^-)$ molecule in QED [29].

We can divide the tetraquark states $qQ\bar{q}'\bar{Q}$ into the heavy and light degrees of freedom. The heavy degree of freedom is characterized by the effective heavy quark masses $\mathbb{M}_Q$, the light degree of freedom is characterized by the virtuality $V = \sqrt{M^2_{X/Y/Z}-(2\mathbb{M}_Q)^2}$ which includes the interactions among the light quarks and gluons. If there exists a P-wave between the light quark and heavy quark in the heavy diquark or between the light antiquark and heavy antiquark in the heavy antidiquark, the P-wave effect can be taken as the light degree of freedom, the virtuality $V = \sqrt{M^2_{X/Y/Z}-(2\mathbb{M}_Q)^2}$. On the other hand, if there exists a P-wave between the heavy diquark and heavy antidiquark, the P-wave effect can be taken as the heavy degree of freedom, the virtuality $V = \sqrt{M^2_{X/Y/Z}-(2\mathbb{M}_Q+P_{\rm E})^2}$, the energy exciting a P-wave costs about 0.5 GeV, *i.e.* $P_{\rm E} \approx 0.5$ GeV.

In this article, we study the heavy-diquark–heavy-antidiquark-type tetraquark states, in other words, the color $\bar{3}_{\rm c}\otimes 3_{\rm c}$-type tetraquark states, just like the charmonium and bottomnium states, where the $\bar{Q}Q$ states are of the color $\bar{3}_{\rm c}\otimes 3_{\rm c}$-type. For the charmonium states, the energy exciting a P-wave costs 457 MeV [24]

$$P_{\rm E} = \frac{5m_{\chi_{c2}}+3m_{\chi_{c1}}+m_{\chi_{c0}}}{9}-\frac{3m_{J/\psi}+m_{\eta_c}}{4} = 457\text{ MeV}. \tag{41}$$

If we take the updated value $\mathbb{M}_c = 1.82$ GeV [30], then $2\mathbb{M}_c + P_{\rm E} = 4.10$ GeV, the energy of the heavy degree of freedom of the diquark–andidiquark-type tetraquark states $qc\bar{q}'\bar{c}$ is estimated to be 4.10 GeV.

We set the energy scale $\mu = V$ to obtain the ideal energy scales of the QCD spectral densities [26–28, 31]. The energy scale formula works well for the hidden-charm (and hidden-bottom) tetraquark states, for example, $X^*(3860)$, $X(3872)$, $Z_c(3900/3885)$, $X(3915)$, $Z_c(4020/4025)$, $X(4140)$, $Z_c(4250)$, $X(4360)$, $Z_c(4430)$, $X(4500)$, $X(4660/4630)$, $X(4700)$, $Z_b(10610)$, $Z_b(10650)$, and also works well for the hidden-charm pentaquark states, for example, $P_c(4380)$ and $P_c(4450)$ [32, 33]. The energy scale formula can remarkably enhance the pole contributions, and can improve the convergent behaviors of the operator product expansion. In 2015, we studied the scalar–diquark–scalar–diquark–antiquark-type pentaquark states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 10 [33]. In calculations, we took the energy scale formula $\mu = \sqrt{M_P^2 - (2\mathbb{M}_c)^2}$ to determine the ideal energy scales of the QCD spectral densities with the old value $\mathbb{M}_c = 1.80$ GeV and obtained the mass $M_P = 4.29 \pm 0.13$ GeV for the pentaquark state with $J^P = \frac{1}{2}^-$, which is in excellent agreement with the value of the mass of the new pentaquark candidate $P_c(4312)$, $4311.9 \pm 0.7^{+6.8}_{-0.6}$ MeV, observed by the LHCb Collaboration this year [34]. Recently, we restudied the scalar–diquark–scalar–diquark–antiquark-type pentaquark states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 13 and took the updated value $\mathbb{M}_c = 1.82$ GeV [30], and obtained even better pentaquark mass 4.31 ± 0.11 GeV [35].

In Ref. [31], we introduce the relative P-wave between the diquark and antidiquark constituents explicitly to construct the vector tetraquark currents and take the modified energy scale formula

$$\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c + P_{\rm E})^2} = \sqrt{M_{X/Y/Z}^2 - (4.10\text{ GeV})^2}$$

to determine the optimal energy scales of the QCD spectral densities, and study the vector tetraquark states with the QCD sum rules systematically. The predictions support assigning the $Y(4220/4260)$, $Z_c(4250)$, $Y(4320/4360)$ and $Y(4390)$ to be the vector tetraquark states.

The axialvector diquark operator $\varepsilon^{ijk}s_j^{\rm T}(x)C\gamma_\mu c_k(x)$ has the $J^P = 1^+$, while the vector diquark operator $\varepsilon^{ijk}s_j^{\rm T}(x)C\gamma_\mu\gamma_5 c_k(x)$ has the $J^P = 1^-$. The tetraquark quark states X^- and X^+ have negative parity and positive parity respectively. The parity conservation requires that $1^+ + 1^- \to 1^-$ for the X^- and $1^+ + 1^- + 1^- \to 1^+$ for the X^+, there should exist an additional P-wave (or 1^-) between the diquark and antidiquark constituents in the tetraquark state X^+. We choose the energy scale formula

$$\mu = \sqrt{M_X^2 - (3.64\ \text{GeV})^2} \tag{42}$$

for the tetraquark state X^-, where we have taken the updated value $\mathbb{M}_c = 1.82$ GeV [30]

$$\mu = \sqrt{M_X^2 - (4.10\ \text{GeV})^2} \tag{43}$$

for the tetraquark state X^+ as there exists a P-wave between the heavy diquark and heavy antidiquark constituents. If the $X(4274)$ can be assigned to be the X^+, the optimal energy scale of the QCD spectral density is $\mu = 1.2$ GeV. At the energy scale $\mu = 1.2$ GeV, the flavor SU(3) breaking effects are sizeable. In calculations, we take into account the effect of the finite s quark mass, and take the energy scale $\mu = \sqrt{M_X^2 - (4.10\ \text{GeV})^2} - 2m_s(\mu) \approx 1$ GeV. We evolve all the input parameters in the QCD spectral densities to the special energy scales determined by the energy scale formula, as the integrals

$$\int_{4m_c^2(\mu)}^{s_0} \mathrm{d}s \rho_\pm(s,\mu) \exp\left(-\frac{s}{T^2}\right) \tag{44}$$

are sensitive to the heavy quark mass m_c or the energy scale μ. In calculations, we observe that the predicted masses decrease monotonously and quickly with increase of the energy scales. If we abandon the energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c)^2}$ or modified energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c + P_\mathrm{E})^2}$, we are puzzled about which energy scale should be chosen. With the help of the (modified) energy scale formula, we can choose the acceptable or optimal energy scales of the QCD spectral densities in a consistent way. The values of the effective heavy quark masses $\mathbb{M}_Q$ are universal for all the diquark–antidiquark-type hidden-charm and hidden-bottom tetraquark states [26, 27, 30, 31].

Now, we search for the optimal Borel parameters T^2 and continuum threshold parameters s_0 to satisfy the following four criteria:

1. Pole dominance at the phenomenological side;
2. Convergence of the operator product expansion;
3. Appearance of the Borel platforms;
4. Satisfying the energy scale formula

via trial and error, and obtain the Borel windows T^2, continuum threshold parameters s_0, optimal energy scales of the QCD spectral densities, and pole contributions of the ground states, which are shown explicitly in Table II.

TABLE II

The Borel windows, continuum threshold parameters, ideal energy scales, pole contributions, masses and pole residues for the axialvector and vector tetraquark states.

	T^2 [GeV^2]	$\sqrt{s_0}$ [GeV]	μ [GeV]	Pole	M [GeV]	λ [GeV^5]
X^+	2.9–3.3	4.80±0.10	1.0	(38–60)%	4.27±0.09	$(1.52\pm0.25)\times10^{-2}$
X^-	3.7–4.3	5.15±0.10	2.9	(39–61)%	4.66±0.08	$(7.94\pm1.00)\times10^{-2}$

From Table II, we can see that the pole contributions are about (40–60)% and the pole dominance criterion is well-satisfied. In calculations, we observe that the contributions of the vacuum condensates of dimension 10 are about 1% and $\ll$ 1% for the tetraquark states X^+ and X^-, respectively, and the operator product expansion is well-convergent. The first two criteria or the basic criteria of the QCD sum rules are satisfied.

We take into account all uncertainties of the input parameters and obtain the values of the masses and pole residues of the $sc\bar{s}\bar{c}$ tetraquark states, which are shown explicitly in Figs. 1–2 and Table II

$$\begin{aligned} M_{X^+} &= (4.27\pm0.09)\ \text{GeV}\,, \\ M_{X^-} &= (4.66\pm0.08)\ \text{GeV}\,, \end{aligned} \tag{45}$$

$$\begin{aligned} \lambda_{X^+} &= (1.52\pm0.25)\times10^{-2}\ \text{GeV}^5\,, \\ \lambda_{X^-} &= (7.94\pm1.00)\times10^{-2}\ \text{GeV}^5\,. \end{aligned} \tag{46}$$

In Figs. 1–2, we plot the masses and pole residues of the tetraquark states with variations of the Borel parameters T^2 at larger intervals than the Borel windows. From the figure, we can see that there appear platforms in the Borel windows, the criterion 3. is also satisfied. From Table II, we can see that the energy scale formula is satisfied. Now, the four criteria are all satisfied and it is reliable to extract the ground-state masses. The predicted mass $m_{X^+} = (4.27\pm0.09)$ GeV is in excellent agreement with the experimental data $4273.3\pm8.3^{+17.2}_{-3.6}$ MeV from the LHCb Collaboration [3, 4], which supports assigning the $X(4274)$ to be the $[sc]_A[\bar{s}\bar{c}]_V-[sc]_V[\bar{s}\bar{c}]_A$-type axialvector tetraquark state X^+ with a relative P-wave between the diquark and antidiquark constituents.

In the non-relativistic quark models, we naively expect that the wave functions of the P-wave excitations vanish at the origin. In the present case, the pole residues have the relation $\lambda_{X^+}\ll\lambda_{X^-}$, the effect of the P-wave between the diquark and antidiquark constituents manifests itself, which is consistent with our naive expectation.

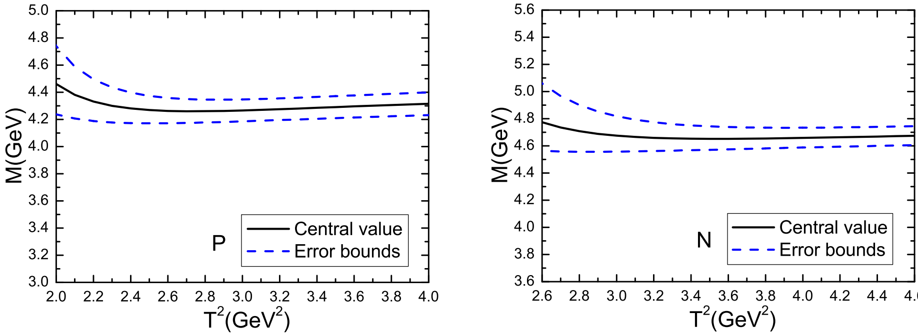

Fig. 1. The masses of the tetraquark states with variations of the Borel parameters T^2, where the P and N denote to the positive parity and negative parity, respectively.

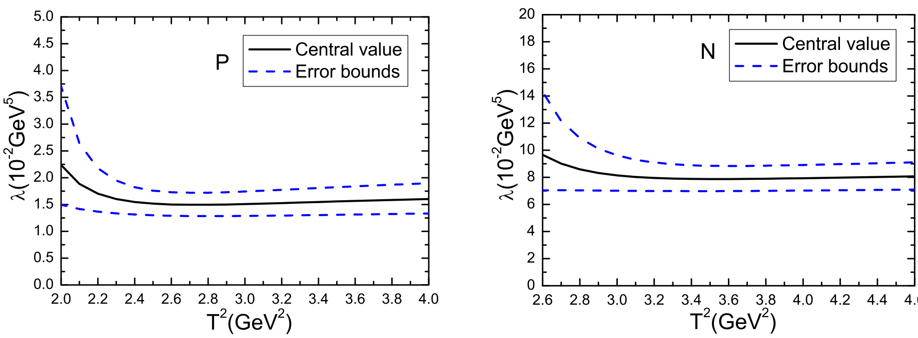

Fig. 2. The pole residues of the tetraquark states with variations of the Borel parameters T^2, where the P and N denote to the positive parity and negative parity, respectively.

From Eq. (45), we can see that the relation between masses is $M_{X^+} < M_{X^-}$. If we use the $\vec{S}_{\rm A}$ and $\vec{S}_{\rm V}$ to represent the spins of the axialvector and vector diquarks (or antidiquarks), respectively, the effective Hamiltonian contains a term $\frac{1}{2}b\vec{L}\cdot\vec{L}+2a\vec{L}\cdot\vec{S}$, where $\vec{S}=\vec{S}_{\rm A}+\vec{S}_{\rm V}$, the $\vec{L}$ is the relative angular momentum [36]. In the case of X^-, $L=0$ and $\frac{1}{2}b\vec{L}\cdot\vec{L}+2a\vec{L}\cdot\vec{S}=0$. In the case of X^+, the total spin $\vec{J}=\vec{L}+\vec{S}$, $J=1$ and $L=1$, the term $\frac{1}{2}b\vec{L}\cdot\vec{L}+2a\vec{L}\cdot\vec{S}=b+a\left[J(J+1)-L(L+1)-S(S+1)\right]=b-aS(S+1)=b$, $b-2a$ and $b-6a$ for $S=0$, 1 and 2, respectively. If the spin–orbit coupling is strong enough, the $b-2a$ and $b-6a$ can have negative values and the effect of the additional P-wave leads to a smaller tetraquark mass. At the present time, we have rare experimental data to fit the parameters a and b if the vector diquarks are involved, the calculations based on the QCD sum rules indicate that $M_{X^+} < M_{X^-}$.

3. The width of the $X(4274)$ as the axialvector tetraquark state

We can study the hadronic coupling constant $g_{X^+J/\psi\phi}$ with the three-point correlation function $\Pi_{\alpha\beta\mu\nu}(p,q)$

$$\Pi_{\alpha\beta\mu\nu}(p,q) = i^2 \int \mathrm{d}^4x\mathrm{d}^4y\, \mathrm{e}^{ipx}\, \mathrm{e}^{iqy}\langle 0|T\left\{J_\alpha^{J/\psi}(x)J_\beta^{\phi}(y)J_{\mu\nu}^{\dagger}(0)\right\}|0\rangle\,, \quad (47)$$

where the currents

$$\begin{aligned} J_\alpha^{J/\psi}(x) &= \bar{c}(x)\gamma_\alpha c(x)\,, \\ J_\beta^{\phi}(y) &= \bar{s}(y)\gamma_\beta s(y) \end{aligned} \quad (48)$$

interpolate the mesons J/ψ and $\phi(1020)$, respectively,

$$\begin{aligned} \langle 0|J_\alpha^{J/\psi}(0)|J/\psi(p)\rangle &= f_{J/\psi}m_{J/\psi}\xi_\alpha\,, \\ \langle 0|J_\beta^{\phi}(0)|\phi(q)\rangle &= f_\phi m_\phi \zeta_\beta\,, \end{aligned} \quad (49)$$

and where $f_{J/\psi}$ and f_ϕ are the decay constants, ξ_α and ζ_β are polarization vectors of the mesons J/ψ and $\phi(1020)$, respectively.

At the phenomenological side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_\alpha^{J/\psi}(x)$, $J_\beta^{\phi}(y)$, $J_{\mu\nu}^{\dagger}(0)$ into the three-point correlation function $\Pi_{\alpha\beta\mu\nu}(p,q)$ [20, 21], and isolate the ground state contributions to obtain the result

$$\begin{aligned}
\Pi_{\alpha\beta\mu\nu}(p,q) =& \frac{f_\phi m_\phi f_{J/\psi}m_{J/\psi}\bar{\lambda}_{X^+}g_{X^+J/\psi\phi}}{\left(m_{X^+}^2-p'^2\right)\left(m_{J/\psi}^2-p^2\right)\left(m_\phi^2-q^2\right)}\,\varepsilon^{\lambda\tau\rho\theta}p'_\lambda\left(-g_{\alpha\rho}+\frac{p_\alpha p_\rho}{p^2}\right) \\
&\times\left(-g_{\beta\theta}+\frac{q_\beta q_\theta}{q^2}\right)\left[\left(-g_{\mu\tau}+\frac{p'_\mu p'_\tau}{p'^2}\right)p'_\nu-\left(-g_{\nu\tau}+\frac{p'_\mu p'_\tau}{p'^2}\right)p'_\mu\right] \\
&+\frac{f_\phi m_\phi f_{J/\psi}m_{J/\psi}\bar{\lambda}_{X^-}\,\langle J/\psi(p,\xi)\phi(q,\zeta)|X^-(p',\varepsilon)\rangle}{\left(m_{X^-}^2-p'^2\right)\left(m_{J/\psi}^2-p^2\right)\left(m_\phi^2-q^2\right)}\,\xi_\alpha\zeta_\beta\,\varepsilon_{\mu\nu\rho\sigma}\,\varepsilon^{*\rho}p'^{\sigma}+\ldots \\
=&\left\{\frac{f_\phi m_\phi f_{J/\psi}m_{J/\psi}\bar{\lambda}_{X^+}g_{X^+J/\psi\phi}}{\left(m_{X^+}^2-p'^2\right)\left(m_{J/\psi}^2-p^2\right)\left(m_\phi^2-q^2\right)}+\frac{1}{\left(m_{X^+}^2-p'^2\right)\left(m_{J/\psi}^2-p^2\right)}\right. \\
&\times\int_{s_\phi^0}^{\infty}\mathrm{d}t\frac{\rho_{X^+\phi'}\left(p'^2,p^2,t\right)}{t-q^2}+\frac{1}{\left(m_{X^+}^2-p'^2\right)\left(m_\phi^2-q^2\right)}\int_{s_{J/\psi}^0}^{\infty}\mathrm{d}t\frac{\rho_{X^+\psi'}\left(p'^2,t,q^2\right)}{t-p^2}
\end{aligned}$$

$$+\frac{1}{\left(m_{J/\psi}^2-p^2\right)\left(m_\phi^2-q^2\right)}\int_{s_X^0}^{\infty}\mathrm{d}t\frac{\rho_{X^{+\prime}J/\psi}\left(t,p^2,q^2\right)+\rho_{X^{+\prime}\phi}\left(t,p^2,q^2\right)}{t-p'^2}+\dots\Bigg\}$$
$$\times\left(\varepsilon_{\alpha\beta\mu\lambda}p^\lambda p_\nu-\varepsilon_{\alpha\beta\nu\lambda}p^\lambda p_\mu+\dots\right)+\dots$$
$$=\Pi\left(p'^2,p^2,q^2\right)\left(\varepsilon_{\alpha\beta\mu\lambda}p^\lambda p_\nu-\varepsilon_{\alpha\beta\nu\lambda}p^\lambda p_\mu\right)+\dots\,, \tag{50}$$

where $p'=p+q$, $\bar{\lambda}_{X^\pm}=\frac{\lambda_{X^\pm}}{m_{X^\pm}}$, $m_{X^+}=M_{X^+}$ and $g_{X^+J/\psi\phi}$ is the hadronic coupling constant defined by

$$\left\langle J/\psi(p,\xi)\phi(q,\zeta)|X^+\left(p',\varepsilon\right)\right\rangle = ig_{X^+J/\psi\phi}\,\varepsilon^{\lambda\tau\rho\theta}p'_\lambda\varepsilon_\tau\xi^*_\rho\zeta^*_\theta\,, \tag{51}$$

the four functions $\rho_{X^+\phi'}(p'^2,p^2,t)$, $\rho_{X^+\psi'}(p'^2,t,q^2)$, $\rho_{X^{+\prime}J/\psi}(t',p^2,q^2)$ and $\rho_{X^{+\prime}\phi}(t',p^2,q^2)$ have complex dependence on the transitions between the ground states and the higher resonances or the continuum states.

In this article, we choose the tensor structure $\varepsilon_{\alpha\beta\mu\lambda}p^\lambda p_\nu-\varepsilon_{\alpha\beta\nu\lambda}p^\lambda p_\mu$ to study the hadronic coupling constant $g_{X^+J/\psi\phi}$ to avoid the contamination from the vector tetraquark state X^-, as the tetraquark state X^- is associated with the tensor structure $\varepsilon_{\mu\nu\bullet\bullet}$, where the $\bullet\bullet$ denotes some functions of the p, p', q. Furthermore, the contaminations originate from the scalar meson $\chi_{c0}(3414)$ and scalar meson $f_0(980)$ are also avoided

$$\begin{aligned}\langle 0|J_\alpha^{J/\psi}(0)|\chi_{c0}(p)\rangle &= f_{\chi_{c0}}p_\alpha\,,\\ \langle 0|J_\beta^{\phi}(0)|f_0(q)\rangle &= f_{f_0}q_\beta\,,\end{aligned} \tag{52}$$

where $f_{\chi_{c0}}$ and f_{f_0} are the decay constants of $\chi_{c0}(3414)$ and $f_0(980)$, respectively. Thereafter, we will smear the superscript $+$ in the X^+ for simplicity.

The correlation function $\Pi(p'^2,p^2,q^2)$ at the phenomenological side can be written as

$$\Pi_H\left(p'^2,p^2,q^2\right)=\int_{(m_{J/\psi}+m_\phi)^2}^{s_X^0}\mathrm{d}s'\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\int_0^{u_\phi^0}\mathrm{d}u\frac{\rho_H\left(s',s,u\right)}{\left(s'-p'^2\right)\left(s-p^2\right)\left(u-q^2\right)}+\dots \tag{53}$$

through the dispersion relation, where the $\rho_H(s',s,u)$ is the hadronic spectral density

$$\rho_H\left(s',s,u\right)=\lim_{\epsilon_3\to 0}\lim_{\epsilon_2\to 0}\lim_{\epsilon_1\to 0}\frac{\mathrm{Im}_{s'}\,\mathrm{Im}_s\,\mathrm{Im}_u\,\Pi_H\left(s'+i\epsilon_3,s+i\epsilon_2,u+i\epsilon_1\right)}{\pi^3}\,. \tag{54}$$

We introduce the subscript H to denote the hadron side.

We carry out the operator product expansion for the correlation function $\Pi_{\alpha\beta\mu\nu}(p,q)$ up to the vacuum condensates of dimension 5 and neglect the tiny contributions of the gluon condensate. We contract the quark fields s and c in the correlation function $\Pi_{\alpha\beta\mu\nu}(p,q)$ with the Wick theorem, and obtain the result

$$\begin{aligned}
\Pi_{\alpha\beta\mu\nu}(p,q) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\int d^4x d^4y\, e^{ip\cdot x}e^{iq\cdot y} \\
&\times\Big\{\mathrm{Tr}\Big[\gamma_\alpha S_c^{ak}(x)\gamma_5\gamma_\nu C S^{Tbj}(y)C\gamma_\beta C S^{Tmb}(-y)C\gamma_\mu S_c^{na}(-x)\Big] \\
&+\mathrm{Tr}\Big[\gamma_\alpha S_c^{ak}(x)\gamma_\mu C S^{Tbj}(y)C\gamma_\beta C S^{Tmb}(-y)C\gamma_\nu\gamma_5 S_c^{na}(-x)\Big]\Big\}\,, \qquad (55)
\end{aligned}$$

where the a, b, i, j, k, m and n are color indexes, the $S_c^{ak}(x)$ and $S^{mb}(x)$ are the full c and s quark propagators, respectively, see Eqs. (16)–(17). Then we compute the integrals both in the coordinate space and in the momentum space, and obtain the correlation function at the QCD side, therefore, the QCD spectral density through dispersion relation

$$\Pi_{\mathrm{QCD}}\left(p'^2,p^2,q^2\right) = \int_{4m_c^2}^{s^0_{J/\psi}} ds \int_0^{u^0_\phi} du \frac{\rho_{\mathrm{QCD}}\left(p'^2,s,u\right)}{\left(s-p^2\right)\left(u-q^2\right)} + \dots\,, \qquad (56)$$

where the $\rho_{\mathrm{QCD}}(p'^2,s,u)$ is the QCD spectral density,

$$\rho_{\mathrm{QCD}}\left(p'^2,s,u\right) = \lim_{\epsilon_2\to 0}\lim_{\epsilon_1\to 0}\frac{\mathrm{Im}_s\,\mathrm{Im}_u\,\Pi_{\mathrm{QCD}}\left(p'^2,s+i\epsilon_2,u+i\epsilon_1\right)}{\pi^2}\,, \qquad (57)$$

we introduce the subscript QCD to denote the QCD side. However, the QCD spectral density $\rho_{\mathrm{QCD}}(s',s,u)$ does not exist

$$\begin{aligned}
\rho_{\mathrm{QCD}}(s',s,u) &= \lim_{\epsilon_3\to 0}\lim_{\epsilon_2\to 0}\lim_{\epsilon_1\to 0}\frac{\mathrm{Im}_{s'}\,\mathrm{Im}_s\,\mathrm{Im}_u\,\Pi_{\mathrm{QCD}}\left(s'+i\epsilon_3,s+i\epsilon_2,u+i\epsilon_1\right)}{\pi^3} \\
&= 0\,, \qquad (58)
\end{aligned}$$

because

$$\lim_{\epsilon_3\to 0}\frac{\mathrm{Im}_{s'}\,\Pi_{\mathrm{QCD}}\left(s'+i\epsilon_3,p^2,q^2\right)}{\pi} = 0\,. \qquad (59)$$

We match the hadron side with the QCD side of the correlation function, and carry out the integral over ds' firstly to obtain the solid duality [37]

$$\int_{\Delta_s^2}^{s^0} ds \int_{\Delta_u^2}^{u^0} du \frac{\rho_{\mathrm{QCD}}\left(p'^2,s,u\right)}{\left(s-p^2\right)\left(u-q^2\right)} = \int_{\Delta_s^2}^{s^0} ds \int_{\Delta_u^2}^{u^0} du \frac{1}{\left(s-p^2\right)\left(u-q^2\right)}\left[\int_{\Delta^2}^{\infty} ds' \frac{\rho_H\left(s',s,u\right)}{s'-p'^2}\right]. \qquad (60)$$

$\varDelta_s^2$ and $\varDelta_u^2$ are the thresholds $4m_c^2$ and 0, respectively, $\varDelta^2$ is the threshold $(m_{J/\psi}+m_\phi)^2$. Now, we explicitly write the quark–hadron duality

$$\int\limits_{4m_c^2}^{s^0_{J/\psi}} \mathrm{d}s \int\limits_0^{u^0_\phi} \mathrm{d}u \frac{\rho_{\rm QCD}\left(p'^2,s,u\right)}{(s-p^2)\,(u-q^2)} = \int\limits_{4m_c^2}^{s^0_{J/\psi}} \mathrm{d}s \int\limits_0^{u^0_\phi} \mathrm{d}u \int\limits_{(m_{J/\psi}+m_\phi)^2}^{\infty} \mathrm{d}s' \frac{\rho_H\left(s',s,u\right)}{(s'-p'^2)\,(s-p^2)\,(u-q^2)}$$
$$= \frac{f_\phi m_\phi f_{J/\psi} m_{J/\psi} \bar{\lambda}_X g_{XJ/\psi\phi}}{\left(m_X^2-p'^2\right)\left(m_{J/\psi}^2-p^2\right)\left(m_\phi^2-q^2\right)} + \frac{C_{X'J/\psi}+C_{X'\phi}}{\left(m_{J/\psi}^2-p^2\right)\left(m_\phi^2-q^2\right)}\,, \tag{61}$$

and introduce the parameters $C_{X'\phi}$ and $C_{X'J/\psi}$ to parameterize the net effects

$$\begin{aligned} C_{X'\phi} &= \int\limits_{s^0_X}^{\infty} \mathrm{d}t \frac{\rho_{X'\phi}\left(t,p^2,q^2\right)}{t-p'^2}\,, \\ C_{X'J/\psi} &= \int\limits_{s^0_X}^{\infty} \mathrm{d}t \frac{\rho_{X'J/\psi}\left(t,p^2,q^2\right)}{t-p'^2}\,. \end{aligned} \tag{62}$$

No approximation is needed, we do not need the continuum threshold parameter s^0_X in the s' channel. The present approach was introduced in Ref. [37].

In numerical calculations, we take the unknown functions $C_{X'\phi}$ and $C_{X'J/\psi}$ as free parameters, and choose the suitable values to eliminate the contaminations from the higher resonances (*i.e.* X' *etc.*) and continuum states to obtain the stable QCD sum rules with the variations of the Borel parameters. We set $p'^2=p^2$ and perform the double Borel transform with respect to the variables $P^2=-p^2$and $Q^2=-q^2$, respectively, to obtain the QCD sum rules

$$\frac{f_\phi m_\phi f_{J/\psi} m_{J/\psi} \bar{\lambda}_X g_{XJ/\psi\phi}}{m_X^2-m_{J/\psi}^2}\left[\exp\left(-\frac{m_{J/\psi}^2}{T_1^2}\right)-\exp\left(-\frac{m_X^2}{T_1^2}\right)\right]\exp\left(-\frac{m_\phi^2}{T_2^2}\right)$$
$$+\left(C_{X'J/\psi}+C_{X'\phi}\right)\exp\left(-\frac{m_{J/\psi}^2}{T_1^2}-\frac{m_\phi^2}{T_2^2}\right)$$
$$= -\frac{1}{48\sqrt{2}\pi^4}\int\limits_{4m_c^2}^{s^0_{J/\psi}} \mathrm{d}s \int\limits_0^{s^0_\phi} \mathrm{d}u u\sqrt{1-\frac{4m_c^2}{s}}\left(1+\frac{2m_c^2}{s}\right)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right)$$

$$
+\frac{m_s\langle\bar{s}s\rangle}{6\sqrt{2}\pi^2}\int_{4m_c^2}^{s^0_{J/\psi}} ds\sqrt{1-\frac{4m_c^2}{s}}\left(1+\frac{2m_c^2}{s}\right)\exp\left(-\frac{s}{T_1^2}\right)
$$

$$
-\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{72\sqrt{2}\pi^2T_2^2}\int_{4m_c^2}^{s^0_{J/\psi}} ds\sqrt{1-\frac{4m_c^2}{s}}\left(1+\frac{2m_c^2}{s}\right)\exp\left(-\frac{s}{T_1^2}\right)\,. \quad (63)
$$

The hadronic parameters are taken as $m_\phi = 1.019461$ GeV, $m_{J/\psi} = 3.0969$ GeV [24], $f_{J/\psi} = 0.418$ GeV [38], $f_\phi = 0.253$ GeV, $\sqrt{s^0_\phi} = 1.5$ GeV [10], $\sqrt{s^0_{J/\psi}} = 3.6$ GeV, $M_X = 4273.3$ MeV [3, 4], $\lambda_X = 1.52\times10^{-2}$ GeV^5. At the QCD side, we can take the energy scale of the QCD spectral density to be $\mu = 1$ GeV, just like in the two-point QCD sum rules. However, at the energy scale $\mu = 1.0$ GeV, $2m_c(1\text{ GeV}) = 2.8$ GeV, $\sqrt{s^0_{J/\psi}} - 2m_c(1\text{ GeV}) = 0.8$ GeV, the integral interval $4m_c^2 - s^0_{J/\psi}$ is too small to obtain stable QCD sum rules; the interval $\sqrt{s^0_{J/\psi}} - 2m_c(\mu)$ should be larger than 1 GeV to obtain stable QCD sum rules. At the energy scale $\mu = m_c(m_c) = 1.275 \pm 0.025$ GeV, $\sqrt{s^0_{J/\psi}} - 2m_c(m_c) = 1.05 \pm 0.05$ GeV, the lower bound is 1.0 GeV, the uncertainty is out of control. Thus, in this article, we choose the typical energy scale $\mu = m_c(m_c)$ and neglect the uncertainties of the quark masses. It is the shortcoming of the present QCD sum rules, we can only obtain qualitative conclusion, as rigorous uncertainty analysis is missing. For simplicity, we set the Borel parameters to be $T_1^2 = T_2^2 = T^2$. The unknown parameters are chosen as $C_{X'J/\psi} + C_{X'\phi} = -0.00145$ GeV^6 to obtain platform in the Borel window $T^2 = (2.8{-}3.8)$ GeV^2. In calculations, we observe that the predicted hadronic coupling constant $g_{XJ/\psi\phi}$ increases monotonously with the increase of the energy scale. The energy scale $\mu = m_c(m_c) = 1.275$ GeV is an acceptable energy scale in the QCD sum rules for the J/ψ and $\phi(1020)$, although it slightly deviates from the optimal energy scale $\mu = 1$ GeV in the QCD sum rules for the $X(4274)$; the deviation leads to unavoidable uncertainty in the hadronic coupling constant $g_{XJ/\psi\phi}$, *i.e.* we slightly underestimate the value of the $g_{XJ/\psi\phi}$.

In Fig. 3, we plot the hadronic coupling constant $g_{XJ/\psi\phi}$ with variation of the Borel parameter T^2. From the figure, we can see that there indeed appears a platform in the Borel window, where the uncertainty originating from the Borel parameter T^2 is small and can be safely neglected. The central value of the hadronic coupling constant $g_{XJ/\psi\phi}$ is

$$g_{XJ/\psi\phi} = -1.05\,, \tag{64}$$

which corresponds to the central values of all the input parameters. We obtain the decay width

$$\begin{aligned}\Gamma(X(4274)\to J/\psi\phi) &= \frac{p\left(m_X, m_{J/\psi}, m_\phi\right)}{24\pi m_X^2} g_{XJ/\psi\phi}^2 \\ &\times \left\{ \frac{\left(m_X^2-m_\phi^2\right)^2}{2m_{J/\psi}^2} + \frac{\left(m_X^2-m_{J/\psi}^2\right)^2}{2m_\phi^2} + 4m_X^2 - \frac{m_{J/\psi}^2+m_\phi^2}{2} \right\} \\ &= 47.9\,\mathrm{MeV} \sim 56\pm 11^{+8}_{-11}\ \mathrm{MeV} \qquad (\text{experimental value [3, 4]})\,,\end{aligned} \tag{65}$$

where $p(a,b,c)=\frac{\sqrt{[a^2-(b+c)^2][a^2-(b-c)^2]}}{2a}$. The width $\Gamma(X(4274)\to J/\psi\phi) = 47.9$ MeV is in excellent agreement with the experimental data $56\pm 11^{+8}_{-11}$ MeV from the LHCb Collaboration [3, 4]. The present work supports assigning the $X(4274)$ to be the $[sc]_{\mathrm{A}}[\bar{s}\bar{c}]_{\mathrm{V}}-[sc]_{\mathrm{V}}[\bar{s}\bar{c}]_{\mathrm{A}}$-type tetraquark state with a relative P-wave between the diquark and antidiquark constituents.

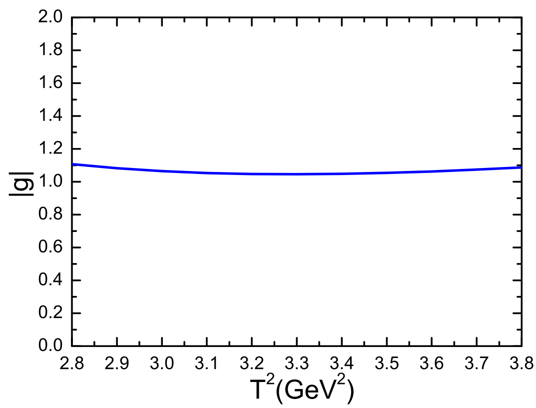

Fig. 3. The hadronic coupling constant $g_{XJ/\psi\phi}$ with variation of the Borel parameter T^2.

In Ref. [10], we construct the color octet–octet-type axialvector current $\eta_\mu(x)$ to study the mass and width of the $X(4274)$

$$\eta_\mu(x) = \frac{1}{\sqrt{2}}\left[\bar{s}(x)i\gamma_5\lambda^a c(x)\,\bar{c}(x)\gamma_\mu\lambda^a s(x) - \bar{s}(x)\gamma_\mu\lambda^a c(x)\,\bar{c}(x)i\gamma_5\lambda^a s(x)\right]\,. \tag{66}$$

Now, we perform the Fierz re-arrangement both in the color and Dirac-spinor spaces and obtain the result

$$
\begin{aligned}
\eta_\mu &= -\frac{N_c+1}{N_c}\left\{\frac{i}{2\sqrt{2}}\left(s^TC\gamma_5t^Ac\,\bar{s}\gamma_\mu Ct^A\bar{c}^T+s^TC\gamma_\mu t^Ac\,\bar{s}\gamma_5Ct^A\bar{c}^T\right)\right.\\
&\quad\left.+\frac{1}{2\sqrt{2}}\left(s^TC\gamma^\alpha\gamma_5t^Ac\,\bar{s}\sigma_{\alpha\mu}Ct^A\bar{c}^T-s^TC\sigma_{\alpha\mu}t^Ac\,\bar{s}\gamma_5\gamma^\alpha Ct^A\bar{c}^T\right)\right\}\\
&\quad+\frac{N_c-1}{N_c}\left\{\frac{i}{2\sqrt{2}}\left(s^TC\gamma_5t^Sc\,\bar{s}\gamma_\mu Ct^S\bar{c}^T+s^TC\gamma_\mu t^Sc\,\bar{s}\gamma_5Ct^S\bar{c}^T\right)\right.\\
&\quad\left.+\frac{1}{2\sqrt{2}}\left(s^TC\gamma^\alpha\gamma_5t^Sc\,\bar{s}\sigma_{\alpha\mu}Ct^S\bar{c}^T-s^TC\sigma_{\alpha\mu}t^Sc\,\bar{s}\gamma_5\gamma^\alpha Ct^S\bar{c}^T\right)\right\}\\
&= -\frac{N_c+1}{N_c}\left\{\frac{i}{2}J^1_\mu+\frac{1}{2}J^4_\mu\right\}+\frac{N_c-1}{N_c}\left\{\frac{i}{2}\hat{J}^1_\mu+\frac{1}{2}\hat{J}^4_\mu\right\}, \qquad (67)
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{J}^1_\mu &= \frac{1}{\sqrt{2}}\left(s^TC\gamma_5t^Sc\,\bar{s}\gamma_\mu Ct^S\bar{c}^T+s^TC\gamma_\mu t^Sc\,\bar{s}\gamma_5Ct^S\bar{c}^T\right),\\
\hat{J}^4_\mu &= \frac{1}{\sqrt{2}}\left(s^TC\gamma^\alpha\gamma_5t^Sc\,\bar{s}\sigma_{\alpha\mu}Ct^S\bar{c}^T-s^TC\sigma_{\alpha\mu}t^Sc\,\bar{s}\gamma_5\gamma^\alpha Ct^S\bar{c}^T\right). \qquad (68)
\end{aligned}
$$

The current $J^1_\mu(x)$ couples potentially to the $[sc]_S[\bar{s}\bar{c}]_A+[sc]_A[\bar{s}\bar{c}]_S$-type axialvector tetraquark state with a mass 3.95 ± 0.09 GeV [13], the current $J^4_\mu(x)$ couples potentially to the $[sc]_T[\bar{s}\bar{c}]_V-[sc]_V[\bar{s}\bar{c}]_T$-type axialvector tetraquark state with a mass 4.14 ± 0.10 GeV [14], while the currents $\hat{J}^1_\mu(x)$ and $\hat{J}^4_\mu(x)$ couple potentially to the $[sc]^6_S[\bar{s}\bar{c}]^{\bar{6}}_A+[sc]^6_A[\bar{s}\bar{c}]^{\bar{6}}_S$-type and $[sc]^6_T[\bar{s}\bar{c}]^{\bar{6}}_V-[sc]^6_V[\bar{s}\bar{c}]^{\bar{6}}_T$-type axialvector tetraquark states, respectively. The current $\eta_\mu(x)$ is a special superposition of the currents $J^1_\mu(x)$, $J^4_\mu(x)$, $\hat{J}^1_\mu(x)$ and $\hat{J}^4_\mu(x)$, and embodies the net effects. The ideal energy scales of the QCD spectral densities of the correlation functions for the currents $J^1_\mu(x)$ and $J^4_\mu(x)$ are $\mu=1.5$ GeV and 2.0 GeV, respectively [13, 14], while the ideal energy scale of the QCD spectral density of the correlation function for the current $\eta_\mu(x)$ is $\mu=1.45$ GeV [10]. The energy scale for the lowest tetraquark state is consistent with that for the color octet–octet-type tetraquark molecule-like state, although the two energy scales are determined by very different c-quark mass $\mathbb{M}_c$. There does not exist a $[sc]_A[\bar{s}\bar{c}]_V-[sc]_V[\bar{s}\bar{c}]_A$-type component in the current $\eta_\mu(x)$, the current $J_{\mu\nu}(x)$ chosen in the present work differs from the current chosen in Ref. [10] completely. Furthermore, the $[sc]_A[\bar{s}\bar{c}]_V-[sc]_V[\bar{s}\bar{c}]_A$-type and $[\bar{s}\lambda^ac]_P[\bar{c}\lambda^as]-[\bar{s}\lambda^ac]_V[\bar{c}\lambda^as]$-type axialvector four-quark states have completely different widths, which originate from the completely different quark structures.

4. Conclusion

In this article, we construct the $[sc]_{\rm A}[\bar{s}\bar{c}]_{\rm V}-[sc]_{\rm V}[\bar{s}\bar{c}]_{\rm A}$-type tensor current to study the $X(4274)$ with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 10. The tensor current couples potentially to both the $J^{PC}=1^{++}$ and 1^{-+} tetraquark states. We separate those contributions unambiguously by introducing suitable projectors. In calculations, we use the energy scale formula to determine the optimal energy scales of the QCD spectral densities, and extract the masses of the $J^{PC}=1^{++}$ and 1^{-+} tetraquark states at different energy scales. The predicted mass $M_X=(4.27\pm 0.09)\,{\rm GeV}$ for the $J^{PC}=1^{++}$ tetraquark state is in excellent agreement with the experimental value $4273.3\pm 8.3^{+17.2}_{-3.6}$ MeV from the LHCb Collaboration. Then we study the two-body strong decay $X(4274)\to J/\psi\phi$ with the QCD sum rules based on the solid quark–hadron duality introduced in our previous work. The central value of the predicted width $\Gamma(X(4274)\to J/\psi\phi)=47.9\,{\rm MeV}$ is in excellent agreement with the experimental value $56\pm 11^{+8}_{-11}$ MeV from the LHCb Collaboration. In summary, the present work supports assigning the $X(4274)$ to be the $J^{PC}=1^{++}$ $[sc]_{\rm A}[\bar{s}\bar{c}]_{\rm V}-[sc]_{\rm V}[\bar{s}\bar{c}]_{\rm A}$ tetraquark state with a relative P-wave between the diquark and antidiquark constituents. Furthermore, we obtain the mass of the $[sc]_{\rm A}[\bar{s}\bar{c}]_{\rm V}-[sc]_{\rm V}[\bar{s}\bar{c}]_{\rm A}$-type tetraquark state with $J^{PC}=1^{-+}$ as a byproduct.

This work is supported by the National Natural Science Foundation, grant number 11775079.

REFERENCES

[1] T. Aaltonen *et al.*, *Observation of the Y(4140) structure in the $J/\psi\phi$ mass spectrum in $B^\pm\to J/\psi\phi K^\pm$ decays*, Mod. Phys. Lett. A **32**, 1750139 (2017).

[2] S. Chatrchyan *et al.*, *Observation of a peaking structure in the $J/\psi\phi$ mass spectrum from $B^\pm\to J/\psi\phi K^\pm$ decays*, Phys. Lett. B **734**, 261 (2014).

[3] R. Aaij *et al.*, *Observation of $J/\psi\phi$ structures consistent with exotic states from amplitude analysis of $B^+\to J/\psi\phi K^+$ decays*, Phys. Rev. Lett. **118**, 022003 (2017).

[4] R. Aaij *et al.*, *Amplitude analysis of $B^+\to J/\psi\phi K^+$ decays*, Phys. Rev. D **95**, 012002 (2017).

[5] Z.G. Wang, *Analysis of the Y(4274) with QCD sum rules*, Int. J. Mod. Phys. A **26**, 4929 (2011).

[6] H.X. Chen *et al.*, *Understanding the internal structures of X(4140), X(4274), X(4500) and X(4700)*, Eur. Phys. J. C **77**, 160 (2017).

[7] J. Wu *et al.*, *X(4140), X(4270), X(4500), and X(4700) and their $cs\bar{c}\bar{s}$ tetraquark partners*, Phys. Rev. D **94**, 094031 (2016).

[8] S.S. Agaev, K. Azizi, H. Sundu, *Exploring the resonances X(4140) and X(4274) through their decay channels*, Phys. Rev. D **95**, 114003 (2017).

[9] X.H. Liu, *How to understand the underlying structures of X(4140), X(4274), X(4500) and X(4700)*, Phys. Lett. B **766**, 117 (2017); Q.F. Lu, Y.B. Dong, *X(4140), X(4274), X(4500), and X(4700) in the relativized quark model*, Phys. Rev. D **94**, 074007 (2016); D.Y. Chen, *Where are $\chi_{cJ}(3P)$?*, Eur. Phys. J. C **76**, 671 (2016).

[10] Z.G. Wang, *Analysis of the mass and width of Y(4274) as axialvector molecule-like state*, Eur. Phys. J. C **77**, 174 (2017).

[11] R. Zhu, *Hidden charm octet tetraquarks from a diquark–antidiquark model*, Phys. Rev. D **94**, 054009 (2016).

[12] L. Maiani, A.D. Polosa, V. Riquer, *Interpretation of axial resonances in $J/\psi-\phi$ at the LHCb*, Phys. Rev. D **94**, 054026 (2016).

[13] Z.G. Wang, *Reanalysis of X(4140) as axial-vector tetraquark state with QCD sum rules*, Eur. Phys. J. C **76**, 657 (2016).

[14] Z.G. Wang, Z.Y. Di, *Analysis of the mass and width of the X(4140) as axialvector tetraquark state*, Eur. Phys. J. C **79**, 72 (2019).

[15] W. Chen, S.L. Zhu, *Vector and axial-vector charmoniumlike states*, Phys. Rev. D **83**, 034010 (2011).

[16] R.L. Jaffe, *Multiquark hadrons. II. Methods*, Phys. Rev. D **15**, 281 (1977).

[17] R.L. Jaffe, *Exotica*, Phys. Rep. **409**, 1 (2005) and references therein.

[18] Z.G. Wang, *Analysis of the scalar and axial-vector heavy diquark states with QCD sum rules*, Eur. Phys. J. C **71**, 1524 (2011); R.T. Kleiv, T.G. Steele, A. Zhang, I. Blokland, *Heavy-light diquark masses from QCD sum rules and constituent diquark models of tetraquarks*, Phys. Rev. D **87**, 125018 (2013).

[19] Z.G. Wang, *Analysis of the light-flavor scalar and axial-vector diquark states with QCD sum rules*, Commun. Theor. Phys. **59**, 451 (2013).

[20] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *QCD and resonance physics. Theoretical foundations*, Nucl. Phys. B **147**, 385 (1979); *QCD and resonance physics. applications*, Nucl. Phys. B **147**, 448 (1979).

[21] L.J. Reinders, H. Rubinstein, S. Yazaki, *Hadron properties from QCD sum rules*, Phys. Rep. **127**, 1 (1985).

[22] Z.G. Wang, T. Huang, *Analysis of the X(3872), Z_c(3900), and Z_c(3885) as axial-vector tetraquark states with QCD sum rules*, Phys. Rev. D **89**, 054019 (2014).

[23] P. Colangelo, A. Khodjamirian, *QCD sum rules, a modern perspective*, in: *At the frontier of particle physics*, 2001, pp. 1495–1576, DOI:10.1142/9789812810458_0033 [`arXiv:hep-ph/0010175`].

[24] M. Tanabashi *et al.*, *Review of particle physics*, Phys. Rev. D **98**, 030001 (2018).

[25] S. Narison, R. Tarrach, *Higher dimensional renormalization group invariant vacuum condensates in quantum chromodynamics*, Phys. Lett. B **125**, 217 (1983).

[26] Z.G. Wang, *Analysis of the $Z_c(4020)$, $Z_c(4025)$, $Y(4360)$, and $Y(4660)$ as vector tetraquark states with QCD sum rules*, Eur. Phys. J. C **74**, 2874 (2014).

[27] Z.G. Wang, T. Huang, *The $Z_b(10610)$ and $Z_b(10650)$ as axial-vector tetraquark states in the QCD sum rules*, Nucl. Phys. A **930**, 63 (2014); Z.G. Wang, Y.F. Tian, *Tetraquark state candidates: $Y(4140)$, $Y(4274)$ and $X(4350)$*, Int. J. Mod. Phys. A **30**, 1550004 (2015); Z.G. Wang, *Analysis of $X(5568)$ as scalar tetraquark state in diquark–antidiquark model with QCD sum rules*, Commun. Theor. Phys. **66**, 335 (2016).

[28] Z.G. Wang, T. Huang, *Possible assignments of the $X(3872)$, $Z_c(3900)$, and $Z_b(10610)$ as axial-vector molecular states*, Eur. Phys. J. C **74**, 2891 (2014); Z.G. Wang, *Reanalysis of the $Y(3940)$, $Y(4140)$, $Z_c(4020)$, $Z_c(4025)$, and $Z_b(10650)$ as molecular states with QCD sum rules*, Eur. Phys. J. C **74**, 2963 (2014).

[29] S.J. Brodsky, D.S. Hwang, R.F. Lebed, *Dynamical picture for the formation and decay of the exotic XYZ mesons*, Phys. Rev. Lett. **113**, 112001 (2014).

[30] Z.G. Wang, *Tetraquark state candidates: $Y(4260)$, $Y(4360)$, $Y(4660)$, and $Z_c(4020/4025)$*, Eur. Phys. J. C **76**, 387 (2016).

[31] Z.G. Wang, *Lowest vector tetraquark states: $Y(4260/4220)$ or $Z_c(4100)$*, Eur. Phys. J. C **78**, 933 (2018); *Analysis of the vector tetraquark states with P-waves between the diquarks and antidiquarks via the QCD sum rules*, Eur. Phys. J. C **79**, 29 (2019).

[32] Z.G. Wang, *Analysis of $P_c(4380)$ and $P_c(4450)$ as pentaquark states in the diquark model with QCD sum rules*, Eur. Phys. J. C **76**, 70 (2016); *Analysis of the $\frac{3}{2}^{\pm}$ pentaquark states in the diquark–diquark–antiquark model with QCD sum rules*, Nucl. Phys. B **913**, 163 (2016).

[33] Z.G. Wang, T. Huang, *Analysis of the $\frac{1}{2}^{\pm}$ pentaquark states in the diquark model with QCD sum rules*, Eur. Phys. J. C **76**, 43 (2016).

[34] R. Aaij *et al.*, *Observation of a narrow pentaquark state, $P_c(4312)^+$, and of the two-peak structure of the $P_c(4450)^+$*, Phys. Rev. Lett. **122**, 222001 (2019).

[35] Z.G. Wang, *Analysis of the $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and related hidden-charm pentaquark states with QCD sum rules*, `arXiv:1905.02892 [hep-ph]`.

[36] L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, *$Z(4430)$ and a new paradigm for spin interactions in tetraquarks*, Phys. Rev. D **89**, 114010 (2014).

[37] Z.G. Wang, J.X. Zhang, *The decay width of the $Z_c(3900)$ as an axialvector tetraquark state in solid quark–hadron duality*, Eur. Phys. J. C **78**, 14 (2018).

[38] D. Bečirević *et al.*, *Lattice QCD and QCD sum rule determination of the decay constants of η_c, J/ψ and h_c states*, Nucl. Phys. B **883**, 306 (2014).