

MEASURING INTENSITY INTERFERENCE IN A LOW MULTIPLICITY SYSTEM $\pi\pi X$ WITH A NEW OBSERVING METHOD

Q. HE[†], X. HE, T. LI

Department of Nuclear Science and Technology
Nanjing University of Aeronautics and Astronautics (NUAA)
29 Yudao St., Nanjing 210016, China

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Prior proposed observing approaches using event mixing technique for the Bose–Einstein correlations (BEC) measurements in exclusive reactions with very low multiplicities are still unsatisfactory due to the problems of sample reduction and introducing extra and unnecessary fitting parameters. We propose here an event mixing method with a new mixing cut, named energy sum range (ESR) cut, to investigate the two-pion Bose–Einstein correlations in reactions with only two identical pions among three final-state particles. This mixing method employs two-pion energy sum characteristic to control the mixing procedures, with no requirement on eliminating any original events. Numerical simulations are performed to show the viability of this new BEC observing method.

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1. Introduction

Intensity interference between identical bosons, generally known as the Bose–Einstein correlations [1–4], is widely used to provide insights into the dynamics process and space-time structure of the particle emitting source created via hadron collisions [5–15] or heavy-ion collisions [16–23]. Using this method to measure the spatial size of nucleon resonances excited by hadronic or electromagnetic probes in the non-perturbative QCD energy region (assuming these resonances decay via emitting identical bosons, *e.g.* $\gamma p \rightarrow N^* \rightarrow \pi^0 \pi^0 p$) is, however, challenging because a proper BEC observing method at low energies with low multiplicities is still lacking.

One key technique involved in observing intensity correlations is how to construct a valid reference sample for measuring the correlation function which is constructed from the detection probabilities I_1 and I_2 :

[†] Corresponding author: heqh@nuaa.edu.cn

$$g^{(2)}(p_1, p_2) = \frac{\langle I_{12} \rangle}{\langle I_1 \rangle \langle I_2 \rangle}, \quad (1)$$

where $\langle I_{12} \rangle$ is the joint probability for the emission of two identical bosons with momenta of p_1 and p_2 , respectively, subject to the Bose–Einstein symmetry (BES), while $\langle I_1 \rangle \langle I_2 \rangle$ corresponds to the emission probability in the absence of BES and generally is known as “reference sample”. If a Gaussian density profile of the boson-emission source is assumed, Eq. (1) is written as

$$g^{(2)}(p_1, p_2) = g^{(2)}(Q) = N \left(1 + \lambda_2 e^{-r_0^2 Q^2} \right), \quad (2)$$

where N is the normalization factor, Q the relative momentum of two bosons defined by $Q^2 = -(p_1 - p_1)^2$, and r_0 the Gaussian radius of the source. The parameter λ_2 is introduced as a measure of the BEC strength ranging from 0 to 1, where 0 and 1 correspond to completely coherent and totally chaotic emission, respectively.

A primary method for the reference sample construction is the event mixing technique [24, 25], which produces “un-correlated” samples from the original sample through making artificial events by randomly selecting two bosons’ momenta from different original events. The event mixing method works well for BEC observations in high-energy reactions with sufficiently large multiplicities [4]. However, its applications in exclusive reactions with very low multiplicities is still a big challenge. The main reason is that the event mixing is strongly obscured by non-BEC factors such as global conservation laws and decays of resonances [26, 27]. Conservation laws induce significant kinematical correlations between final-state particles and complicate the BEC analysis [28, 29].

To develop a proper event mixing method for observing particle correlations in exclusive reactions with low multiplicities, one needs to explore appropriate constraints to manage the mixing process and ensure that the produced reference sample is free of BEC effects but preserves all other kind of correlations arising from global conservation laws and decays of resonances. In the work of Ref. [30], authors tried to develop an event mixing method for $\pi\pi$ BEC observations in a three-particle final-state system $\pi\pi X$, taking the reaction $\gamma p \rightarrow \pi^0 \pi^0 p$ at incident photon energies around 1 GeV (a non-QCD region) as an example. The effects of kinematical correlations due to energy-momentum conservation were investigated, and an event mixing method which contains two mixing cuts was proposed. The first cut, named missing mass consistency (MMC) cut is adopted to conserve the energy momentum of the mixed events and to make them physically meaningful as the original events. The second cut, named pion energy (PE) cut, is used to adjust the slope of the correlation function to extract correct BEC parameters. Although this mixing method works well for $\pi\pi X$ events in

pure phase space, a disadvantage of the PE cut is that it needs to eliminate a large portion (about 40%) of original events and hence reduces the statistics. In order to solve the sample reduction problem, two new mixing methods, energy sum order and invariant-mass/energy hierarchy correspondence cuts, were proposed later [31, 32] to replace the PE cut. Although the new proposed methods solved the problem of sample reduction, they introduced extra and unnecessary fitting parameters, which hence leads to a worse analysis accuracy.

In this work, we propose a new mixing cut constraint, named energy sum range (ESR) cut, in order to solve both the sample reduction and extra fitting parameters issue. This new cut employs the two-pion energy sum to control the mixing procedure. Numerical tests using $\gamma p \rightarrow \pi^0 \pi^0 p$ events are performed to test the ability of this cut to observe BEC effects.

2. Event mixing with ESR constraint

We search for suitable cut conditions in event mixing according to such a criterion that the cut should affect correlations arising from energy-momentum conservations so strongly that the mixed events still retain the original pure phase-space distribution but, on the other hand, it should be weakly sensitive to BEC correlations so that no BEC correlations in the mixed sample are preserved. By investigating several cut conditions, the two-boson energy sum, E_{sum} , is selected empirically as a cut condition in event mixing. Since in the low-energy case the fitting range of the BEC correlation function is very limited, a flat background correlation function from appropriate mixing cut conditions is required for extraction proper BEC parameters.

The new event mixing method is composed of two constraints. The MMC cut [30] is still included in the mixing method, which requires $|m_X^{\text{mix}} - m_X^{\text{ori}}| < M_{\text{cut}}$, where M_{cut} is the cut window, and m_X^{mix} and m_X^{ori} are the missing particle masses for the mixed event and that for the original event, respectively, in order to force the mixed events to be physically equal to original events and to be located in the allowed phase-space region. In addition to the MMC cut, a new constraint, named energy sum range cut (ESR), is introduced. It requires that two events can be mixed only when the following relation is satisfied:

$$\left| E_{\text{sum}}^{(\text{ori},1)} - E_{\text{sum}}^{(\text{ori},2)} \right| < E_{\text{cut}}, \quad \text{and} \quad (3)$$

$$\min \left(E_{\text{sum}}^{(\text{ori},1)}, E_{\text{sum}}^{(\text{ori},2)} \right) < E_{\text{sum}}^{\text{mix}} < \max \left(E_{\text{sum}}^{(\text{ori},1)}, E_{\text{sum}}^{(\text{ori},2)} \right), \quad (4)$$

where $E_{\text{sum}}^{(\text{ori},1)}$ and $E_{\text{sum}}^{(\text{ori},2)}$ are the two-boson energy sums in the two original events, $E_{\text{sum}}^{\text{mix}}$ the two-boson energy sum in the mixed event. E_{cut} is a cut window and its optimum value is determined empirically.

3. Numerical test

We adopt here the reaction $\gamma p \rightarrow \pi^0 \pi^0 p$, which has only three final state particles including two identical bosons among them, to demonstrate the event mixing method employing the MMC and ESR constraints. Both pure phase-space events and BEC-effect events of the $\gamma p \rightarrow \pi^0 \pi^0 p$ reaction are used to validate the effectiveness of the ESR cut. The Monte Carlo events generation employs a ROOT utility named TGenPhaseSpace developed by CERN [33] based on the GENBOD function, which employs the Raubold and Lynch method [34] and has already been implemented in the CERN library. Details of generating a non-BEC sample can be found in Ref. [30].

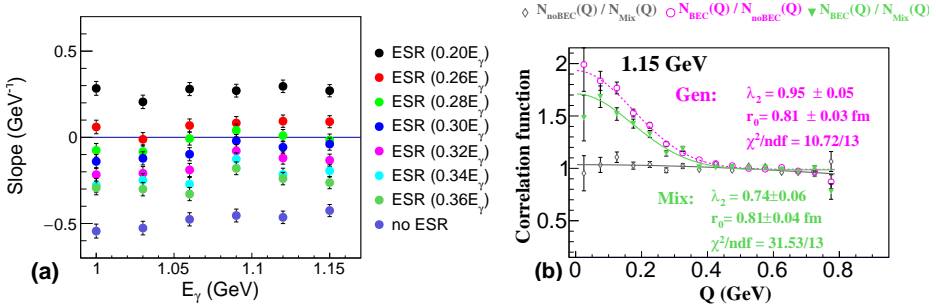


Fig. 1. (a) Slope values of the correlation functions obtained with different E_{cut} values for the ESR cut (indicated in parenthesis on the right) at six incident photon energies (E_γ) for the reaction $\gamma p \rightarrow \pi^0 \pi^0 p$. The MMC cut is also used. (b) A typical correlation function obtained by the event mixing with the MMC cut and the ESR cut with $E_{\text{cut}} = 0.32 E_\gamma$ for $\gamma p \rightarrow \pi^0 \pi^0 p$ events in the presence of BEC effects at the incident photon energy of 1.15 GeV. Equation (2) is used to fit the data to get BEC parameters.

A valid cut should produce a flat correlation function. To satisfy this requirement, the cut window parameter E_{cut} of the ESR cut is adjusted through finding the optimum value which can make a correlation function the closest to a flat line. Six pure phase-space $\gamma p \rightarrow \pi^0 \pi^0 p$ event samples free of BEC effects at typical incident photon energies of 1.0, 1.03, 1.06, 1.09, 1.12, 1.15 GeV are generated and used to make mixed sample via the event mixing method using the MMC cut and the ESR cut with different E_{cut} values. Figure 1 (a) shows the slope values of correlation functions obtained with different E_{cut} values. The slope value is obtained by fitting a linear function $f(Q) = aQ + b$ to the correlation function. $E_{\text{cut}} = 0.32 E_\gamma$ is selected as the optimum value for the ESR constraint owing to a trade-off between flat correlation function for non-BEC sample and effective BEC parameters measurement for BEC sample. With the optimum ESR cut, the correlation function for the non-BEC effect $\gamma p \rightarrow \pi^0 \pi^0 p$ events exhibits a good flat feature as shown in Fig. 1 (b).

To investigate the ability of the proposed event mixing method to measure BEC effects, event mixing is also performed for BEC samples of $\gamma p \rightarrow \pi^0 \pi^0 p$ events, which are constructed from the prepared pure phase-space samples using the following procedures based on the fact that Eq. (2) has a maximum value $g_{\max}^{(2)} = N(1 + \lambda_2)$ when $Q = 0$. The events in the phase-space sample are selected to compose the BEC sample when satisfy the relation $g^{(2)}(Q)/g_{\max}^{(2)} > R$, where R is a random number uniformly generated in the range from 0 to 1, Q the two pions momentum difference. The probability of $g^{(2)}(Q)/g_{\max}^{(2)} > R$ is proportional to $g^{(2)}(Q)$. Thus, this method is capable of producing correct density distribution subject to Eq. (2). The BEC parameters for the BEC samples are typically set to be $r_0 = 0.8$ fm and $\lambda_2 = 1.0$.

With the event mixing method, the BEC effects can be obviously observed in the obtained correlation functions, as shown in Fig. 1 (b). For comparison, the ratio of Q spectrum of the BEC sample to that of the corresponding pure phase-space sample is also shown. It can be seen that the proposed mixing method can reproduce the correlation functions as the input ones. The BEC parameters r_0 and λ_2 are determined by fitting Eq. (2) to the correlation function. Table I compares the mixing-obtained BEC parameters with the input ones. It is found that the fit r_0 values are in good agreement with the input ones at all energy points, while the λ_2 values are a little bit underestimated.

TABLE I

Comparing input BEC parameters with those from the proposed mixing method.

E_γ [GeV]	Input			Fit		
	r_0 [fm]	λ_2	$\chi^2/\text{n.d.f.}$	r_0 [fm]	λ_2	$\chi^2/\text{n.d.f.}$
1.00	0.79 ± 0.02	0.97 ± 0.04	9.7/11	0.83 ± 0.04	0.79 ± 0.06	30.0/11
1.03	0.79 ± 0.03	0.94 ± 0.05	20.0/11	0.81 ± 0.04	0.86 ± 0.06	34.8/11
1.06	0.80 ± 0.03	0.95 ± 0.05	15.5/12	0.86 ± 0.05	0.77 ± 0.07	40.7/12
1.09	0.78 ± 0.02	0.98 ± 0.05	10.8/12	0.82 ± 0.04	0.78 ± 0.06	29.6/12
1.12	0.76 ± 0.02	0.92 ± 0.04	5.3/12	0.85 ± 0.04	0.79 ± 0.06	13.3/12
1.15	0.81 ± 0.03	0.95 ± 0.05	10.7/13	0.81 ± 0.04	0.74 ± 0.06	31.5/13
Ave.	0.79 ± 0.01	0.95 ± 0.02		0.83 ± 0.02	0.79 ± 0.02	

In order to study the systematic bias introduced by the proposed mixing method, the weighted mean fit values of both r_0 and λ_2 are compared to the weighted mean input values. It is found that the mean value r_0 (0.83 ± 0.02) over the six energies is about 5% overestimated compared to the input one, 0.79 ± 0.01 . The mean value of λ_2 is found to be 0.79 ± 0.02 , about 17% underestimated compared to the mean value of the input ones, 0.95 ± 0.02 .

Comparing with two previously proposed mixing methods [31, 32], this mixing method induces systematic bias of both r_0 and λ_2 to be smaller (Fig. 2), because it avoids the Q^2 -dependent fitting problem and hence improves the accuracy of the fitting and reduces the systematic bias of the fit BEC parameters.

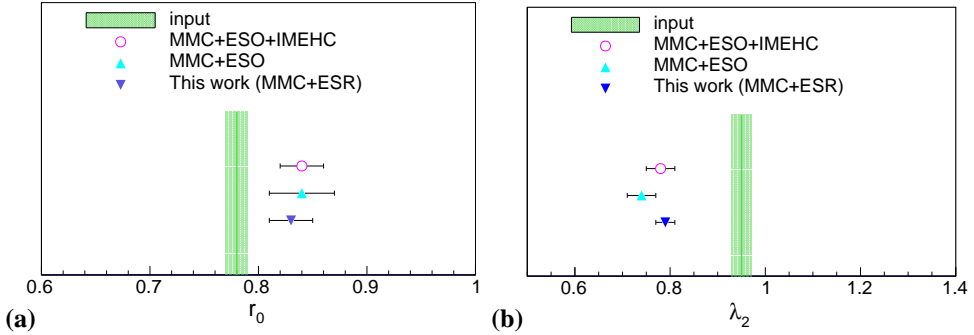


Fig. 2. Comparing input BEC parameters with those from the proposed mixing method. For comparison, the results from the other mixing methods are also presented [31, 32].

Although this new method improves the accuracy of fitting and reduces the systematic bias of fit BEC parameters, it still introduces systematic bias and in practical applications the BEC parameter obtained by this mixing method should be corrected. Future efforts may focus on improving the systematic bias.

On the experimental side, the two-particle BEC measurement is also affected by the detector acceptance. Generally, such measurements involve a 4π detection system. Compared to single particle's detecting efficiency, position resolution, energy resolution and geometric coverage, the joint two-particle detection acceptance may have a stronger impact on the event mixing. A single particle in a mixed event is from real experimental data and its existence is naturally reasonable, but the existence of the two particles in a mixed event may be problematic. This problem is common for GeV-energy particle detection with electro-magnetic calorimeter composed of many independent crystals (*e.g.* BGO). An incident energetic particle creates an electromagnetic shower, whose energy deposit commonly extends to several adjacent detector crystals. All corresponding detector crystals are grouped into a cluster. As a result, the detector cannot distinguish two particles if their clusters are overlapping. Although there exist algorithms to separate two overlapping clusters, it may give an ambiguous result. To cope with this situation for the real data BEC analysis, a cluster overlapping cut may be needed, which requires that any pair of clusters in an event must share no

overlapping crystals both for the real data and the mixed events. With this cut, events with two clusters having overlapping crystals are rejected from the event mixing and not counted as original events. Numerical simulations show that the correlation function has a steep drop at low Q s without the cluster overlapping cut. Generally speaking, for a given detector, a specific mixing cut should be taken to eliminate possible detector acceptance impacts on the event mixing. A detailed discussion of detection acceptance effects on the event mixing is, however, outside the scope of the present paper.

4. Summary

A new event mixing method is proposed for two-pion Bose–Einstein correlations (BEC) measurement in reactions with only two identical pions among three final-state particles. This mixing method with a new mixing constraint named energy sum range cut eliminates extra and unnecessary fitting parameter and hence improves the systematic bias for both BEC parameters r_0 and λ_2 . Numerical simulations with the $\gamma p \rightarrow \pi^0 \pi^0 p$ events at several incident photon energies around 1 GeV are performed to verify this mixing method. It is found that this new method has smaller systematic bias and better fitting uncertainties for both r_0 and λ_2 , compared to two prior proposed mixing methods. In future studies, improvements of the systematic bias are needed.

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