CONSTRAINTS ON THE HIGH-DENSITY EQUATION OF STATE FROM THE GRAVITATIONAL-WAVE SIGNAL OF NEUTRON STAR MERGERS*

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We present a constraint on neutron star radii, which is derived from a multi-messenger interpretation of GW170817, the first unambiguously detected neutron star merger event. The optical and infrared emission of the electromagnetic counterpart was relatively bright compared to simulation results. We argue that the remnant of the merger did not undergo a prompt collapse to a black hole because this results in relatively dim electromagnetic transients. If this interpretation is correct, neutron star radii cannot be too small to prevent direct collapse. We find that the radius of a nonrotating neutron star with a mass of 1.6 $M_{\odot}$ should be larger than about 10.7 km excluding very soft nuclear matter. We emphasize the potential of future multi-messenger observations to which the same arguments and procedures can be applied, and which can then yield more stringent radius limits. Furthermore, a prompt collapse event can place an upper bound on the maximum mass of nonrotating neutron stars.

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1. Introduction

In August 2017, the network of Advanced LIGO and Advanced Virgo measured for the first time the gravitational wave signal from a neutron star merger [1]. The event is called GW170817 referring to the detection date. The total binary mass was found to be $M_{\rm tot} = M_1 + M_2 = 2.73\ M_\odot$ and the binary mass ratio $q = M_1/M_2$ was contrained to be between 0.7 and 1. The event took place at a distance of about 40 Mpc. This was sufficiently close to extract finite-size effects during the late inspiral stage, the phase just prior to merging. Finite-size effects are described by the combined tidal deformability $\tilde{\Lambda} = \frac{16}{13}\left(\frac{(M_1+12M_2)M_1^4}{(M_1+M_2)^5}\Lambda_1 + \frac{(M_2+12M_1)M_2^4}{(M_1+M_2)^5}\Lambda_2\right)$, with Λ_1 and Λ_2 being the tidal deformabilties of the individual stars [1–3]. The tidal deformability quantifies the response, *i.e.*, the induced quadrupole moment of a star to a tidal field, which is generated by the companion star [4, 5]. As other stellar parameters such as the density profile and the stellar radius, the tidal deformability depends on the mass and the equation of state of high-density matter. Specifically, the tidal deformability is defined as $\Lambda = \frac{2}{3}k_2\left(\frac{c^2R}{GM}\right)^5$ with the tidal Love number k_2 and the stellar radius R, which both can be computed from the stellar structure equations for a given equation of state. The tidal deformability strongly correlates with the stellar radius.

The equation of state of high-density matter is not precisely known because of the challenges to solve the nuclear many-body problems such as specifying the nuclear interactions and the fundamental constituents of high-density matter, *e.g.*, [6]. Moreover, a phase transition to deconfined quark matter might take place at densities of a few times nuclear saturation density occurring in neutron stars. Because stellar properties of neutron stars such as the tidal deformability or the mass–radius relation are uniquely linked to the equation of state, measuring or at least constraining such stellar parameters represents an important effort to understand the properties of high-density matter and fundamental interactions.

The analysis of GW170718 revealed that the combined tidal deformability was smaller than about 800 [1]. See Ref. [3] for a reanalysis which yielded a limit of $\tilde{\Lambda} < 650$ (see also Refs. [2, 7, 8]). The upper limit on $\tilde{\Lambda}$ implies that neutron star radii are smaller than about 13 to 14 km. This excludes very stiff nuclear matter.

In this contribution, we present a multi-messenger interpretation of GW170817, which yields a lower limit on neutron star radii and thus excludes very soft nuclear matter [9, 10].

2. Constraints from a multi-messenger interpretation

The derivation of the lower limit on neutron star radii is based on the observation of a relatively bright electromagnetic counterpart associated with GW170817, *e.g.*, [11, 12]. The properties of the observed electromagnetic emission in the optical and infrared wavebands are in excellent agreement with matter which is ejected from a neutron star merger and heated by the radioactive decays during the rapid neutron-capture process [13–15]. The electromagnetic transient evolved on a time scale of hours to days. Although the exact modeling of such type of transients is challenging, the data allows to estimate the amount of ejected matter and its outflow velocity. Different groups inferred ejecta masses of typically a few 0.01 $M_\odot$ somewhat depending on the underlying model (see Ref. [16] for a compilation of the different estimates).

The deduced ejecta masses are at the high end of what is expected from numerical simulations, but overall the observations are compatible with theory (see *e.g.*, Ref. [17] for a compilation of simulation results). This represents a remarkable achievement considering the challenges to model the small amounts of ejecta, which comprise at most a few per cent of the total stellar matter, which originate from different ejection mechanisms and which are highly dynamical [18].

In Ref. [9], we have argued that the very bright electromagnetic counterpart and the correspondingly high ejecta mass suggest that there was no direct black-hole formation in GW170817. Simulations show that binary systems which undergo a prompt gravitational collapse lead to small ejecta masses (*e.g.*, [18, 19]) disfavoring this scenario for GW170817. The interpretation of the data as pointing to no direct collapse in GW170817 is a simple and relatively robust argument. It is the only information in the derivation of the following equation-of-state constraint which is inferred from the properties of the electromagnetic counterpart.

2.1. Method and application to GW170817

The following argumentation uses the fact that the threshold binary mass $M_{\rm thres}$ for prompt black-hole formation depends sensitively on the equation of state. Introducing $M_{\rm thres}$ is motivated by the fact that binaries with a total mass $M_{\rm tot}$ exceeding $M_{\rm thres}$ undergo a direct gravitational collapse, whereas systems with $M_{\rm tot} < M_{\rm thres}$ lead to the formation of an at least temporarily stable neutron star merger remnant [20, 21].

Simulations of binary systems with many different binary masses and equations of state have revealed that to good approximation the threshold binary mass can be estimated by

$$M_{\rm thres} = \left(-3.38 \frac{G\, M_{\rm max}}{c^2\, R_{\rm max}} + 2.43\right) M_{\rm max}\,. \qquad (1)$$

Here, $M_{\rm max}$ is the maximum mass of nonrotating neutron star and $R_{\rm max}$ is the radius of this maxium-mass neutron star [21]. Both quantities are uniquely determined by the equation of state and are used here to characterize the equation-of-state dependence of $M_{\rm thres}$.

Employing the above argumentation that GW170817 did not undergo a direct collapse implies that the measured total binary mass of GW170817 is smaller than the threshold binary mass, *i.e.*,

$$2.73\ M_\odot < \left(-3.38\frac{G\,M_{\rm max}}{c^2\,R_{\rm max}} + 2.43\right) M_{\rm max}\,. \tag{2}$$

Both, $M_{\rm max}$ and $R_{\rm max}$ are unknown.

Causality restricts the speed of sound being smaller than the speed of light and thus constrains the stiffness of the equation of state [22, 23]. This results in the strict limit

$$M_{\rm max} < \frac{1}{2.82}\,\frac{c^2\,R_{\rm max}}{G}\,. \tag{3}$$

Inserting Eq. (3) into Eq. (2) finally leads to

$$2.73\ M_\odot < \left(\frac{-3.38}{2.82} + 2.43\right)\frac{1}{2.82}\,\frac{c^2\,R_{\rm max}}{G} = 0.437\,\frac{c^2\,R_{\rm max}}{G}\,. \tag{4}$$

This means that $R_{\rm max} < \frac{G\,2.73\ M_\odot}{0.437\,c^2} = 9.26$ km. Refining the arguments and taking into account error bars (see Refs. [9, 10, 24] for more details) finally yields

$$R_{\rm max} > 9.60^{+0.14}_{-0.03}\ {\rm km}\,. \tag{5}$$

We remark that the mass ratio has only a small impact on $M_{\rm thres}$, which is why it can be neglected in this derivation (see Ref. [9] for more details).

The equation-of-state dependence of the threshold binary mass can be equally well described by a relation $M_{\rm thres}(M_{\rm max}, R_{1.6})$ with $R_{1.6}$ being the radius of a nonrotating neutron star with a mass of 1.6 $M_\odot$ (see Ref. [21]). Following the same line of arguments as above and employing a similar causality limit constraining $M_{\rm max}$ by $R_{1.6}$ (see Refs. [10, 24] for details) leads to a lower bound on $R_{1.6}$, which is given by

$$R_{1.6} > 10.68^{+0.15}_{-0.04}\ {\rm km}\,. \tag{6}$$

The resulting constraints on neutron star radii are visualized in Fig. 1. The limits are overplotted on a number of mass–radius relations for different equation-of-state models available in the literature. The figure illustrates that the softest models are ruled out by our constraint.

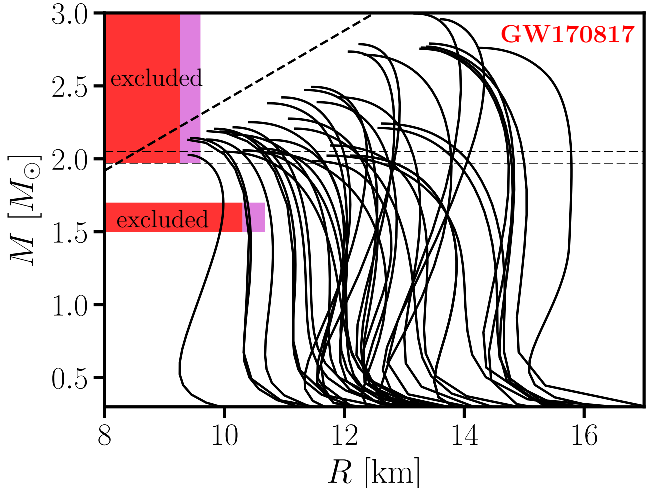

Fig. 1. (Color online) Lower bounds on radii of nonrotating neutron stars with a mass of 1.6 $M_\odot$ and with a mass of $M_{\max}$ in dark gray/red and gray/purple based on a multi-messenger interpretation of GW170817. The dark gray/red areas indicate a very conservative limit, whereas the gray/purple areas show a more realistic lower limit, see Ref. [9] for more details. The black curves display different equation-of-state models available in the literature. The softest of those models are excluded. Figure taken from Ref. [9].

See also Refs. [25–28] for equation-of-state constraints that employ very similar arguments. Note that our constraint implies a weaker limit on the tidal deformability than in Refs. [25, 26]. The reason is that the study of Refs. [25, 26] considers only four different equation-of-state models. Therefore, it does not allow to determine precisely the threshold between models that lead to dim transients and those that lead to bright electromagnetic counterparts. Since the tidal deformability strongly correlates with the neutron star radii, it is straightforward to convert our constraint to a lower limit on Λ. We find that our analysis implies that the tidal deformability has to be larger than $\Lambda > 210$ [9, 10, 24], whereas equations of state with larger tidal deformability are compatible with the current observations.

2.2. Future application

While the idea sketched above excludes only some relatively extreme models, the method generally bears a lot of potential for future applications to upcoming new measurements. Essentially, it can be immediately applied to any new detection which allows the distinction between a direct and no prompt gravitational collapse of the merger remnant. Additionally, with new multi-messenger observations, the identification of these events as being a prompt collapse or not may become more reliable as the interpretation and the theoretical understanding of the emission features and the underlying parameters will increase with more data.

The lower limit on neutron star radii will become stronger if an event with a higher total binary mass is measured, which indicates no direct gravitational collapse. This may also complement future constraints on the tidal deformability which mostly provide upper limits on neutron star radii.

We emphasize that the detection of a prompt collapse event (either through a very dim electromagnetic counterpart or the absence of strong postmerger gravitational wave emission) will be particularly interesting. Using very similar arguments as above, such a measurement will lead to an upper limit on neutron star radii and, importantly, on M_{max}. We refer to Refs. [9, 24] for more details.

This prospect is very promising because many other methods to constrain M_{max} from above may be rather model-dependent, *e.g.*, [29–32], whereas the use of the threshold binary mass appears to yield relatively robust limits [21]. Moreover, any new or improved pulsar mass measurement can only yield a lower limit on M_{max} [33–35].

We illustrate the potential of our method by assuming two hypothetical future events: one detection with $M_{\mathrm{tot}} = 2.9\ M_{\odot}$ with evidence for no direct black-hole formation and one detection with $M_{\mathrm{tot}} = 3.1\ M_{\odot}$ with indications for a prompt collapse of the merger remnant. The resulting constraints on neutron star radii and M_{max} are displayed by the shaded/purple areas in Fig. 2. It is obvious that such two measurements would strongly constrain the high-density equation of state and would provide very valuable information about the underlying physics. In particular, an upper limit on M_{max} has the potential to rule out a large number of equation-of-state models.

Considering in particular the impact of a prompt collapse event, we point out that we sketch an observing strategy in Ref. [10] to identify the most rewarding events for follow-up observations of gravitational wave triggers. Searching electromagnetic counterparts can be challenging and expensive for events at larger distances and for potential prompt-collapse events, which are expected to be dimmer. However, scientifically the most rewarding events will be those that further improve constraints on the threshold binary mass. Ideally, this will result in a relatively precise determination of M_{thres}. In combination with a future radius measurement, it may allow a direct inversion of $M_{\mathrm{thres}}(M_{\mathrm{max}}, R_{1.6})$ and thus a direct determination of M_{max}.

Moreover, knowing M_{thres} may be relevant for the interpretation of future gravitational wave measurements at larger distances. For instance, in the case of a coincident measurement with a gamma-ray burst, M_{thres} and the measured binary mass can inform about the precise conditions, which are required for launching a relativistic outflow.

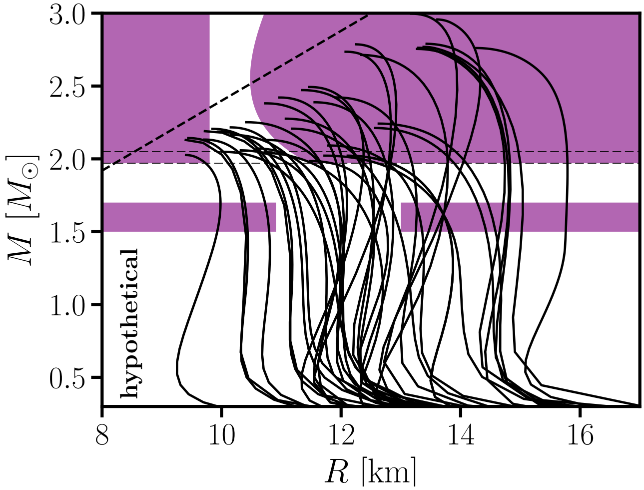

Fig. 2. (Color online) Shaded/purple areas display excluded combinations of mass and radius in the case of hypothetical detections of neutron star mergers and their counterparts with different total binary masses. Black curves show mass–radius relations for different equation-of-state models available in the literature. Figure taken from Ref. [9].

3. Summary

In this contribution, we demonstrate how a multi-messenger interpretation of GW170817 can place a robust lower limit on neutron star radii and thus rule out very soft nuclear matter. The constraint is based on a minimum set of assumptions and relies on the argument that the brightness of the electromagnetic counterpart of GW170817 points to no direct black-hole formation of the remnant. We emphasize that the method introduced in Ref. [9] has a lot of potential for stronger constraints on neutron star properties and the high-density equation of state. In particular, the observation of a prompt-collapse event can yield an upper limit on the maximum mass of neutron stars. Our discussion stresses the importance of future follow-up observations of gravitational-wave detections.

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