EFFECT OF Λ PARTICLE PHONON COUPLING ON THE ENERGY SPECTRA OF ${}^{5}_{\Lambda}$He AND ${}^{17}_{\Lambda}$O*

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The energy spectra of the hypernuclei ${}^{5}_{\Lambda}$He and ${}^{17}_{\Lambda}$O were studied within a multiphonon scheme, where the Λ particle is coupled to particle–hole Tamm–Dancoff phonons describing the excitations of the core. A chiral interaction was used. The calculations show that the core excitations push considerably, through their coupling, the Hartree–Fock energies down in energy and enrich the low-energy spectrum in ${}^{17}_{\Lambda}$O.

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1. Introduction

Studying hypernuclei starting from realistic interactions is very challenging. In fact, *ab initio* calculations were performed mostly for hypernu-

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clei up to the p-shell [1–4]. Only the auxiliary field diffusion Monte Carlo method [5, 6] and the hypernuclear mean-field model [7, 8] were adopted to investigate medium and heavy hypernuclei.

In a recent work [9], we adopted the Hartree–Fock (HF) method to study the structure of hypernuclei consisting of one Λ hyperon bound to even–even nuclear cores. We employed the chiral potential $\mathrm{N^2LO_{sat}}$ [10], which contains two-body (NN) and three-body (NNN) forces. The Λ–nucleon (ΛN) interaction was extracted from the $N\Lambda$–$N\Lambda$ channel of the chiral hyperon–nucleon (YN) potential [11].

The method was applied to ${}^{17}_{\Lambda}\mathrm{O}$ and ${}^{41}_{\Lambda}\mathrm{Ca}$. Although the relative distances between levels were in rough agreement with the empirical data, the overall theoretical spectra were shifted upward by few MeV.

On the other hand, as pointed out already in Ref. [9], there is room for improving the description of the Λ-hypernuclei spectra by including the excitations of the nuclear core. The equation-of-motion phonon method (EMPM) [12–14] is ideal for this purpose. This is a self-consistent method which, starting from a realistic potential, generates for even–even nuclei a multiphonon basis of n-phonon states ($n = s1,2,3,\ldots$) whose constituents are particle–hole (p–h) or quasiparticle (qp) Tamm–Dancoff (TD) phonons. The Pauli principle is completely fulfilled and no approximations are involved.

This method was then extended to odd nuclei. In this extension, it generates an orthonormal basis of states composed of the valence particle or hole coupled to n-phonon states describing the excitations of the nuclear core [15–17].

In the present work, we extend the latter scheme to odd hypernuclei by just replacing the odd valence nucleon with the Λ-hyperon. For illustrative purposes, we consider the simple case of Λ coupled to TDA phonons ($n = 1$) and perform numerical calculations for ${}^{5}_{\Lambda}\mathrm{He}$ and ${}^{17}_{\Lambda}\mathrm{O}$.

2. Theoretical framework

We adopted the intrinsic Hamiltonian

$$\widehat{H} = \widehat{T}_N + \widehat{T}_\Lambda + \widehat{V}_{NN} + \widehat{V}_{NNN} + \widehat{V}_{N\Lambda} - \widehat{T}_{\mathrm{CM}} \,. \tag{1}$$

The first step consists in generating an HF basis for the nucleons and Λ. We then derive and solve the TDA eigenvalue equation in the p–h nuclear subspace and generate the phonon states

$$|\alpha\rangle = Q^\dagger_\alpha |0\rangle = \sum_{ph} C^\alpha_{ph} a^\dagger_p a_{\bar{h}} |0\rangle \tag{2}$$

of energies E_α. Here $a^\dagger_p$ ($a_{\bar{h}}$), by acting on the HF vacuum $|0\rangle$, creates a nucleon particle (hole) of energy ϵ_p (ϵ_h).

We now proceed with deriving the eigenvalue equation in the subspace spanned by the Λ-phonon basis states

$$|(p_\Lambda \times \alpha)^v\rangle = \left(c_p^\dagger \times Q_\alpha^\dagger\right)^v |0\rangle\,, \tag{3}$$

where $c_p^\dagger$ is a Λ creation operator and v denotes the angular momentum of the system. To this purpose, we start with the equation of motion

$$\langle\nu_1|\left[\widehat{H}, c_p^\dagger\right]|\alpha\rangle = (E_{\nu_1} - E_\alpha) C^{\nu_1}_{p\alpha}\,, \tag{4}$$

where $C^{\nu_1}_{p\alpha} = \langle\nu_1|c_p^\dagger|\alpha\rangle$. After expanding the commutator, we obtain the eigenvalue equation

$$\sum_{p'\alpha'} \mathcal{A}^{\nu_1}_{p\alpha,p'\alpha'} C^{\nu_1}_{p'\alpha'} = E_{\nu_1} C^{\nu_1}_{p\alpha}\,. \tag{5}$$

The A-matrix has the form

$$\mathcal{A}^{\nu_1}_{p\alpha,p'\alpha'} = \left(E_\alpha + \varepsilon_p^\Lambda\right)\delta_{\alpha\alpha'}\delta_{pp'} + \sum_\lambda [\lambda]^{1/2} W\left(\nu_1\alpha p'\lambda; p\alpha'\right)\mathcal{F}^\lambda_{p\alpha p'\alpha'}\,, \tag{6}$$

where $[\lambda] = 2J_\lambda + 1$, W is a Racah coefficient, and

$$\mathcal{F}^\lambda_{p\alpha p'\alpha'} = \sum_{rs} F^\lambda_{rspp'}\langle\alpha||\left(a_r^\dagger \times a_s\right)^\lambda||\alpha'\rangle\,. \tag{7}$$

Here, the sum goes to particle $(rs) = (p_i p_k)$ and hole $(rs) = (h_i h_k)$ pairs, and F^λ is the Pandya transform of the two-body potential V^Ω

$$F^\lambda_{rspp'} = \sum_\Omega [\Omega](-)^{r+p'-\lambda-\Omega} W\left(rspp'; \lambda\Omega\right) V^\Omega_{rpsp'}\,. \tag{8}$$

The solution of Eq. (5) yields the eigenvalues E_{ν_1} and the eigenstates

$$|\nu_1\rangle = \sum_{p\alpha} C^{\nu_1}_{p\alpha}\,|(p \times \alpha)^v\rangle\,. \tag{9}$$

We can now solve the full eigenvalue problem in the space spanned by the single-particle Λ state $|\nu_0\rangle = c^\dagger_{\nu_0}|0\rangle$ of spin v plus the particle–phonon basis ν_1. To this purpose, we compute the off-diagonal term

$$\langle\nu_1|\widehat{H}|\nu_0\rangle = \frac{1}{[v]^{1/2}} \sum_{p\alpha} (-)^{v+p+\alpha}[\alpha]^{1/2}\mathcal{F}^v_{p\alpha} C^{\nu_1}_{p\alpha}\,, \tag{10}$$

where

$$\mathcal{F}^v_{p\alpha} = \sum_{p_i h_k} C^\alpha_{p_i h_k} F^\alpha_{pvp_ih_k}\,. \tag{11}$$

The diagonalization of the Hamiltonian in the full space gives the physical hypernuclear states $|\nu\rangle = \sum_{n=0,1} C^\nu_{\nu_n}|\nu_n\rangle$ of energies E_ν.

3. Calculations and results

We employed the SRG [18] transformed potential N^2LO_{sat} with a flow parameter $s = 2.0$ fm^{-1} to study the nuclear cores ^{4}He and ^{16}O. We first solved the HF equations to obtain the single particle basis. This was determined within a HO space encompassing up to the major shells $N_{\max} = 12$. The oscillator parameter was set to $\hbar\omega = 20$ MeV.

Using this basis, we have computed the unperturbed ground-state energy E_{HF} and, then, the correlation energy E_{corr} by resorting to the EMPM [19]. To this purpose, we solved the eigenvalue equation in a configuration space up to 2 phonons.

As shown in Table I, HF does not give sufficient binding in either nuclei. The inclusion of the correlation energies yields total energies close to the values obtained in the Λ-CCSD(T) calculation [10] and the experimental energies. The remaining gap in ^{16}O may be due to the truncation of the space which excludes three and four phonons.

TABLE I

Unperturbed ground-state energy E_{HF}, correlation energy E_{corr}, total energy E_{tot}, compared to the experimental values E_{exp}.

^{A}X	E_{HF} [MeV]	E_{corr} [MeV]	E_{tot} [MeV]	E_{exp} [MeV]
^{4}He	−22.217	−6.084	−28.301	−28.296
^{16}O	−95.429	−28.203	−123.632	−127.619

We studied the effect of the Λ particle–phonon coupling on the HF energies. Such a coupling pushes the levels of both hypernuclei downward by few MeV.

In $^5_\Lambda$He, we have computed the energy of the $1/2^+$ ground state using the $N\Lambda$–$N\Lambda$ channel of the bare YN potential [11] for two values of the regulator cutoff parameter, $\lambda = 550$ MeV and $\lambda = 600$ MeV. As shown in Fig. 1, both HF and total energies depend strongly on such a parameter.

In $^{17}_{\Lambda}$O, the lowest energy states $(1/2^+, 1/2^-, 3/2^-)$ were determined using the $N\Lambda$–$N\Lambda$ channel of the bare YN potential with regulator cutoff $\lambda = 550$ MeV. Coupling the Λ particle to the one-phonon excitations yields a downward energy shift. Moreover, the states $1/2^-$ and $3/2^-$ get split into doublets. Both doublet states are superpositions of the original single-particle Λ levels and the two states obtained by coupling the Λ $0s_{1/2}$ to the first 1^- TD phonon. Therefore, the theoretical energy spectrum seems to be richer than the empirical one deduced from experiments [21].

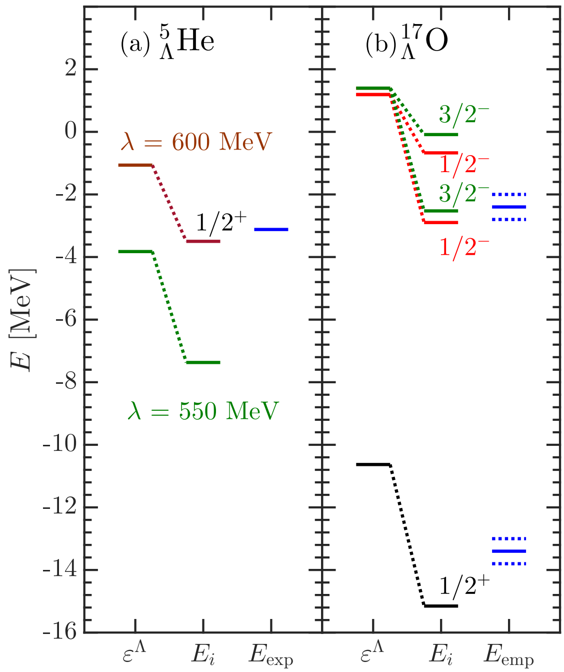

Fig. 1. Single-particle energies ε_Λ and hypernuclear energies E_i of ${}^{5}_{\Lambda}$He (a) and ${}^{17}_{\Lambda}$O (b) compared to the experimental values $E_{\rm exp}$ taken from [20] (a) and the empirical values $E_{\rm emp}$ extracted from experiments [21] (b). The dashed lines give the uncertainties of the empirical values.

4. Conclusions

According to our findings, the coupling of the Λ hyperon to the excitations of the nuclear core induces a systematic downward energy shift of the energy spectra of ${}^{5}_{\Lambda}$He and ${}^{17}_{\Lambda}$O.

In ${}^{17}_{\Lambda}$O we obtained an energy spectrum richer than the empirical one due to the splitting of the $1/2^-$ and $3/2^-$ levels into doublets.

The results calculated using the bare YN potential depend strongly on the regulator cutoff. This is consistent with our previous calculations which used the Hartree–Fock in the proton–neutron–Λ formalism and the $N\Lambda$ TDA [9].

We must point out that we did not include yet the Λ–Σ mixing in the YN interaction. Its inclusion is expected to shift further downward the hypernuclear ground-state energies [22]. In any case, we plan to take such a mixing into account in a forthcoming paper.

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