THE PROTOPHOBIC X-BOSON COUPLED TO QUANTUM ELECTRODYNAMICS[*]

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The possible origin of an X-boson having a mass value around 17 MeV had motivated us to investigate its interaction with leptons of QED. This new hypothetical particle can possibly be a candidate to describe the so-called fifth interaction in a new physics scenario beyond the Standard Model. The simplest X-boson model unified to the Standard Model is based on an $\mathrm{SU}_c(3) \times \mathrm{SU}_L(2) \times \mathrm{U}_Y(1) \times \mathrm{U}(1)_{B-L}$ symmetry, where the group $\mathrm{U}(1)_{B-L}$ is attached to the X-boson, with a kinetic mixing with the gauge field of $\mathrm{U}_Y(1)$. The Higgs sector was revisited to generate the mass for the new boson. Thus, the mass of 17 MeV fixes a vacuum expected value scale. Thereby, we could estimate the mass of the hidden Higgs field through both the VEV-scale and the Higgs' couplings. A model of QFT was constructed in a renormalizable R_ξ-gauge, and we analyze its perturbative structure. After that, the radiative correction of the X-boson propagator has been calculated at one-loop approximation to yield the Yukawa potential correction. The form factors associated with the QED-vertex correction were calculated to confirm the electron's anomalous magnetic moment together with the computation of the interaction magnitude. The muon case

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was discussed. Furthermore, we have introduced a renormalization group scheme to explore the running X-boson mass and its coupling constant with the leptons of the Standard Model.

1. Introduction

The anomalies of the excited state of 8-beryllium ($^8\mathrm{Be}^*$) relative to its ground state have revealed the existence of a new neutral X-boson through the nuclear decay $^8\mathrm{Be}^* \to {}^8\mathrm{Be}+X$ [1]. Immediately, the X-boson decays into the electron–positron pair $X \to e^+ + e^-$. It has a vector feature like the photon, but it must have a mass of approximately $m_X = 17$ MeV. Moreover, it must mediate a weak force with a range of 12 fm. In principle, its unification is associated with the introduction of an extra gauge symmetry $\mathrm{U}(1)_{B-L}$, where $B-L$ means the baryonic number (B) minus the leptonic number (L), besides the known gauge symmetry of the Standard Model (SM). The model is also a good candidate due to the absence of chiral anomaly.

The introduction of a light or heavy Z' particle via $\mathrm{U}(1)_{B-L}$ is an old idea in theoretical particle physics which includes several scenarios of Z' models beyond the SM [2–12]. Besides, many Z' models like $\mathrm{U}(1)_{B-L}$ also have motivated the theoretical research in dark matter scenarios [13–22]. Certainly, the existence of a new boson can lead to the emergence of a fifth fundamental interaction in Nature [23]. In this context, the extended SM is based on the $\mathrm{SU}_c(3) \times \mathrm{SU}_L(2) \times \mathrm{U}_Y(1) \times \mathrm{U}(1)_{B-L}$ gauge symmetry. For a complete review about the anomaly of beryllium decays, see [24]. Recently, a huge number of references show the alternative models to describe this extended SM [25–30]. The effective Lagrangian that could describe the Abelian sector of the X-boson model can be written as

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}X_{\mu\nu}^2 + \frac{\chi}{2}X_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu^2 + J_\mu X^\mu \,, \qquad (1)$$

where $X^{\mu\nu}$ is the field strength tensor of X^μ, $F^{\mu\nu}$ is the corresponding one for the photon, and J^μ is the fermionic current coupled to the X^μ-field

$$J^\mu = \sum_{f=e,u,d,\dots} e\chi_f \bar{f}\gamma^\mu f \,, \qquad (2)$$

where f represents any fermion of the SM. The X-boson can also interact chirally with the SM leptons via the axial current [24, 31, 32]. The current in Eq. (2) defines the so-called protophobic interaction where the nucleons' coupling constant (n,p) of the X-boson is multiplied by both the χ_n- and χ_p-parameters, whose magnitudes satisfy the inequality $\chi_n \gg \chi_p$. This is

also known as the millicharged interactions [5]. We list some values of χ_Ψ for the electron, neutron and neutrinos, following the X-boson phenomenology in the literature [8], namely,

$$2 \times 10^{-4} < |\chi_e| < 1.4 \times 10^{-3}\,, \qquad |\chi_n| < 2.5 \times 10^{-2}$$
$$\text{and} \quad \sqrt{|\chi_\nu \chi_e|} \lesssim 7 \times 10^{-5}\,. \tag{3}$$

By considering both the u- and d-quarks, the extreme protophobic limit $(\chi_p = 0)$ parameterizes χ_u and χ_d since [8]

$$\chi_u = -\frac{\chi_n}{3} \simeq \pm 3.7 \times 10^{-3} \quad \text{and} \quad \chi_d = +\frac{2\,\chi_n}{3} \simeq \mp 7.4 \times 10^{-3}\,. \tag{4}$$

In Eq. (1), the χ-parameter, estimated within the range of $10^{-6} < \chi < 10^{-3}$, mixes kineticaly the boson X^μ with the usual electromagnetic (EM) photon A^μ. It is clear that the massive term spoils the $U_{B-L}(1)$ symmetry. The Lagrangian in Eq. (1) has just the EM gauge symmetry U(1)$_{\rm em}$. Therefore, a Spontaneous Symmetry Breaking (SSB) mechanism spoils one of the Abelian symmetries to generate a mass value of $m_X = 17$ MeV in Eq. (1). Consequently, the experimental value of 17 MeV defines the scale of a VEV, consequently, we can estimate a mass range for the hidden Higgs.

In this paper, we will start with the SM extended by a U(1)$_{B-L}$ symmetry with a kinetic mixing term in the gauge sector. The hidden Higgs sector will be revisited in the R_ξ-gauge to spoil one of the U(1) gauge symmetries and, therefore, we will carry out the full diagonalization procedure to identify a massive gauge field from a VEV-scale as the X-boson. The remaining massless eigenstates will be identified as the EM-photon. Hence, the model is a candidate to describe the interaction between leptons and the X-boson as well as the usual QED interaction with photon. This approach is similar to the U(1)$_{B-L}$ gauge boson together with kinetic mixing discussed in [33].

Thus, we have a QED scenario coupled to the hypothetical fifth interaction associated with the X-boson mediator. Thereby, we can obtain a renormalizable and unitary model, where the interaction between both the X-boson and fermions of the SM satisfies the protophobic condition mentioned previously. We will discuss some aspects of the model from the point of view of the perturbative QFT with the motivation to confirm some constraints on the χ-parameters. For example, we can estimate the X-boson decay time by using the decay width of $X \to e^+ + e^-$. We obtain the differential cross section for electron–positron scattering $e^+e^- \to \mu^+\mu^-$ via X-boson at the tree level. Moreover, we will analyze some aspects of the perturbation theory at one-loop approximation: *(i)* The contribution of the X-boson to the electron physical mass; *(ii)* The X-boson full propagator, the m_X-renormalized mass and the correspondent Uehling potential;

(iii) The contribution of the X-boson into the QED-vertex calculation. Thus, the form factors yield a contribution of the χ-parameter to the electron's anomalous magnetic moment, which confirms the constraints in Eq. (3); *(iv)* Finally, we will obtain all the renormalization factors of the model by fixing the renormalization conditions. In this way, the renormalization group scheme will be introduced to investigate both the current X-boson mass and its respective coupling constant as functions of an arbitrary dimensionless scale.

The organization of this paper obeys the following schedule: in Section 2, we will discuss the model based on the $SU_c(3) \times SU_L(2) \times U_Y(1) \times U(1)_{B-L}$ symmetry with a kinetic mixing between the Abelian gauge fields. The hidden Higgs sector will also be analyzed with a detailed full diagonalization to give a mass for the X-boson, and the gauge sector in the R_ξ-gauge fixing. In Section 3, we will calculate the decay rate of the X-boson into the electron–positron pair. In Section 4, we will obtain the scattering $e^+ e^- \to \mu^+ \mu^-$ at tree level. In Section 5, the contribution of the X-boson to the electron self-energy will be calculated at one loop. In Section 6, the correction of the X-boson full propagator will be used to obtain the corresponding Uehling potential. In Section 7, the correction to the QED vertex will be computed concerning the interaction of the X-boson with the electron–positron pair. In Section 8, we will construct the model renormalization and discuss the functions that appear in the Callan–Symanzik equation. Finally, the conclusions and last remarks will be depicted in Section 9.

2. The Abelian model and the Higgs sector: New considerations

In this section, we will restate the sector of gauge fields and leptons/quarks of the model governed by a $SU_c(3) \times SU_L(2) \times U_Y(1) \times U(1)_{B-L}$ symmetry, with the kinetic mixing χ-parameter inside the Abelian gauge sector. This model is well-known in the literature to describe the hidden photon, or also, new gauge boson that could appear from the MeV-scale and below [34]. Here, we will study in detail the diagonalization of the Higgs sector to identify the corresponding massive and massless eigenstates.

Let us begin with the gauge fields sector composed by the massless vector fields A^μ, Y^μ and B^μ

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2}\text{tr}\left(F_{\mu\nu}^2\right) - \frac{1}{4}Y_{\mu\nu}^2 - \frac{1}{4}B_{\mu\nu}^2 + \frac{\chi}{2}Y_{\mu\nu}B^{\mu\nu}, \qquad (5)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g\,[A_\mu, A_\nu]$, $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$, and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

The fermions sector is defined by the usual Lagrangian

$$\mathcal{L}_{\text{lept/quarks}} = \sum_{f \in \text{SM}} \bar{f} i \slashed{D} f \,, \tag{6}$$

where the sum includes all the fermions (f) from Table I, and the covariant derivative operator is

$$D_\mu = \partial_\mu + i\, g_L\, A_{a\mu}\, \frac{\sigma_a}{2} + i Y\, g_Y\, Y_\mu + i Q_{BL}\, g_{BL}^f\, B_\mu \,, \tag{7}$$

where Y is the hypercharge generator, Q_{BL} is the generator corresponding to U(1)$_{B-L}$, and g_L, g_Y and g_{BL}^f are the coupling constants of SU$_L$(2), U$_Y$(1) and U(1)$_{B-L}$, respectively. The coupling constant g_{BL}^f was introduced to characterize the interaction of the hidden X-boson with the leptons/quarks of the SM. The particle content of the model is shown in Table I. The $B - L$ model is an anomaly free one by adding the three right-handed neutrinos (RHNs) with charge $Q_{BL} = -1$. In other words, the interactions from Eq. (6) between the fermions and gauge bosons are given by

$$\mathcal{L}^{\text{int}} = -\bar{f} \left(g_L\, I_{3L}\, \slashed{A}_3 + g_Y\, Y\, \slashed{Y} + Q_{BL}\, g_{BL}^f\, \slashed{B} \right) f \,. \tag{8}$$

In fact, we did not identify the physical EM-photon and the X-boson in the model yet. The physical fields are identified after a diagonalization procedure with the help of the SSB mechanism. Thereby, a hidden Higgs sector breaks one of the Abelian symmetries to give mass to the X-boson. After this SSB, the EM-symmetry can be written such that SU$_L$(2) $\times$ U(1)$_Y$ $\times$ U(1)$_{B-L}$ $\overset{v,v_{BL}}{\longmapsto}$

TABLE I

The anomaly-free particle content for the U(1)$_{B-L}$ model.

Fields	SU$_L$(2)	U(1)$_Y$	U(1)$_{B-L}$
L_i	$\underline{\mathbf{2}}$	$-1/2$	-1
ℓ_{iR}	$\underline{\mathbf{1}}$	-1	-1
N_{iR}	$\underline{\mathbf{1}}$	0	-1
Q_{iL}	$\underline{\mathbf{2}}$	$+1/6$	$+1/3$
Q_{iR}	$\underline{\mathbf{1}}$	$+2/3$	$+1/3$
q_{iR}	$\underline{\mathbf{1}}$	$-1/3$	$+1/3$
Φ	$\underline{\mathbf{2}}$	$+1/2$	0
ϕ_{BL}	$\underline{\mathbf{1}}$	0	$+2$

$U_{em}(1)$. To accomplish the task, the simplest scalar Lagrangian for the Higgs sector is

$$\mathcal{L}_{\text{Higgs}} = |D_\mu \Phi|^2 - \mu^2 |\Phi|^2 - \lambda |\Phi|^4 + |D_\mu \phi_{BL}|^2 - \mu_{BL}^2 |\phi_{BL}|^2$$
$$- \lambda_{BL} |\phi_{BL}|^4 - \lambda' |\Phi|^2 |\phi_{BL}|^2 , \qquad (9)$$

where μ, λ, μ_{BL}, λ_{BL} and λ' are real parameters. The complex scalar field Φ is the doublet one known from the SM, and ϕ_{BL} is a $SU_L(2)$ singlet scalar field associated with the symmetry breaking of $U(1)_{B-L}$. The covariant derivative operator in Eq. (7) acts on the scalars fields in accord with their charges in Table I.

The parametrization for the scalars fields is

$$\Phi(x) = \frac{v + H(x)}{\sqrt{2}} \exp \left[\frac{i}{v} \begin{pmatrix} \chi^3 & \sqrt{2}\,\chi^- \\ \sqrt{2}\,\chi^+ & -\chi^3 \end{pmatrix} \right]$$
$$\text{and} \quad \phi_{BL}(x) = \frac{v_{BL} + h_{BL}(x)}{\sqrt{2}}\, e^{i \frac{\eta(x)}{v_{BL}}} , \qquad (10)$$

where H and h_{BL} are real functions and η, χ^3, $\chi^\pm$ are the Goldstone bosons of the model. The VEV $v = 246$ GeV is the SSB scale for the SM and v_{BL} is the VEV scale that breaks $U(1)_{B-L}$ such that the SSB pattern satisfies the condition $v_{BL} \ll v = 246$ GeV to give mass for the light gauge boson X. Therefore, after the SSB, the Lagrangian of neutral gauge bosons is given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \left(A_{\mu\nu}^3\right)^2 - \frac{1}{4} Y_{\mu\nu}^2 - \frac{1}{4} B_{\mu\nu}^2 + \frac{\chi}{2} Y_{\mu\nu} B^{\mu\nu} + \frac{v_{BL}^2}{2} \left(4\,g_{BL\phi}^2\right) B_\mu^2$$
$$+ \frac{v^2}{8} \left(g_L A_\mu^3 - g_Y Y_\mu\right)^2 , \qquad (11)$$

where $A_{\mu\nu}^3 = \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3$. The sector in Eq. (11) indicates a diagonalization procedure to obtain the mass of X-boson and the physical gauge bosons. To do it, we will write the Lagrangian in a matrix form

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2} (V^\mu)^t \,\Box\, \theta_{\mu\nu} K\, V^\nu + \frac{1}{2} (V^\mu)^t \eta_{\mu\nu} M^2 V^\nu , \qquad (12)$$

where $(V^\mu)^t = \begin{pmatrix} Y^\mu & A^{\mu 3} & B^\mu \end{pmatrix}$, K is the kinetic matrix and $\tilde{M}^2$ is the mass matrix, respectively,

$$K := \begin{pmatrix} 1 & 0 & -\chi \\ 0 & 1 & 0 \\ -\chi & 0 & 1 \end{pmatrix} \quad \text{and} \quad \tilde{M}^2 = \frac{v^2}{4} \begin{pmatrix} g_Y^2 & -g_L\,g_Y & 0 \\ -g_L\,g_Y & g_L^2 & 0 \\ 0 & 0 & 16\,g_{BL\phi}^2 \frac{v_{BL}^2}{v^2} \end{pmatrix} . \qquad (13)$$

To diagonalize the Lagrangian in Eq. (12), we will carry out the non-orthogonal transformation

$$
\begin{pmatrix} Y^\mu \\ A^{\mu 3} \\ B^\mu \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{\chi}{\sqrt{1-\chi^2}} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-\chi^2}} \end{pmatrix} \begin{pmatrix} \tilde{Y}^\mu \\ \tilde{A}^{\mu 3} \\ \tilde{B}^\mu \end{pmatrix} . \tag{14}
$$

Thus, the mass matrix in the field basis $\begin{pmatrix} \tilde{Y}^\mu & \tilde{A}^{\mu 3} & \tilde{B}^\mu \end{pmatrix}$ can be written as

$$
\tilde{M}^2 = \frac{v^2}{4} \begin{pmatrix} g_Y^2 & -g_L\, g_Y & g_Y^2\, \chi \\ -g_L\, g_Y & g_L^2 & -g_L\, g_Y\, \chi \\ g_Y^2\, \chi & -g_L\, g_Y\, \chi & 16 g_{BL}^2 \frac{v_{BL}^2}{v^2}\left(1+\chi^2\right) + g_Y^2 \chi^2 \end{pmatrix} + \mathcal{O}\left(\chi^3\right) . \tag{15}
$$

The matrix $\tilde{M}^2$ can be diagonalized by the two rotation angles

$$
R(\xi, \theta_{\mathrm{W}}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\xi & \sin\xi \\ 0 & -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} \cos\theta_{\mathrm{W}} & \sin\theta_{\mathrm{W}} & 0 \\ -\sin\theta_{\mathrm{W}} & \cos\theta_{\mathrm{W}} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{16}
$$

where

$$
\tan(2\xi) = \frac{-2\chi\,\sin\theta_{\mathrm{W}}}{1-\delta} \quad \text{and} \quad \delta := \frac{16 g_{BL\phi}^2}{g_L^2 + g_Y^2}\frac{v_{BL}^2}{v^2} . \tag{17}
$$

The diagonal mass matrix is $M_{\mathrm{D}}^2 = R\,\tilde{M}^2\,R^T = \mathrm{diag}\left(0,\, m_Z^2,\, m_X^2\right)$ in which the null eigenvalue is identified as the photon mass, and the others eigenvalues are the Z mass and the X-boson mass, respectively,

$$
m_Z \simeq \frac{v}{2}\sqrt{g_L^2 + g_Y^2}\left(1 + \frac{\chi^2}{2}\sin^2\theta_{\mathrm{W}}\right),
$$

$$
m_X \simeq 2\, g_{BL}\, v_{BL}\left(1 + \frac{\chi^2}{2}\cos^2\theta_{\mathrm{W}}\right).
$$

The W mass is the same in the SM, *i.e.*, $m_W = g_L v/2$. The Weinberg angle is $\sin^2\theta_{\mathrm{W}} = 0.23$ in which the electric charge is parameterized as $e = g_L\sin\theta_{\mathrm{W}} = g_Y\cos\theta_{\mathrm{W}}$, and determines the constant couplings $g_L = 0.64$ and $g_Y = 0.34$. Using these values, the W and Z masses are like in the SM: $m_W = 80$ GeV and $m_Z = 91$ GeV.

The full diagonalization in the gauge sector yields the Lagrangian for the X-boson

$$
\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{4}X_{\mu\nu}^2 + \frac{1}{2}m_X^2 X_\mu^2 + \frac{1}{2}\left(\partial_\mu\eta\right)^2 + \frac{1}{2}m_X\,\partial_\mu\eta\,X^\mu , \tag{18}
$$

in which $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ is the X-boson strength field tensor. To eliminate the mixed last term $\eta - X^\mu$, we can add a gauge fixing Lagrangian

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\beta}\left(\partial_\mu X^\mu - \frac{\beta}{2}\, m_X\, \eta\right)^2 , \qquad (19)$$

where β is a real parameter. The surface terms can be eliminated since the Lagrangian is integrated throughout all the space-time. The inversion of the X-boson Lagrangian yields the renormalized X-propagator in the momentum space

$$\langle\, X_\mu\, X_\nu\,\rangle = \frac{-i}{k^2 - m_X^2}\left[\eta_{\mu\nu} + (\beta - 1)\frac{k_\mu\, k_\nu}{k^2 - \beta\, m_X^2}\right] . \qquad (20)$$

All the propagators are well-defined at the ultraviolet range, *i.e.*, the model is both renormalizable and unitary.

Therefore, the transformation $\left(Y^\mu,\, A^{\mu 3},\, B^\mu\right) \mapsto (A^\mu,\, Z^\mu,\, X^\mu)$ are given by

$$\begin{aligned}
Y_\mu &= -\sin\theta_{\rm W} Z_\mu + \cos\theta_{\rm W} A_\mu + \chi\cos^2\theta_{\rm W} X_\mu , \\
A_\mu^3 &= \cos\theta_{\rm W} Z_\mu + \sin\theta_{\rm W} A_\mu + \chi\sin\theta_{\rm W}\cos\theta_{\rm W} X_\mu , \\
B_\mu &= -\chi\sin\theta_{\rm W} Z_\mu + X_\mu .
\end{aligned} \qquad (21)$$

Substituting the diagonalizations into Eq. (8), the interaction between fermions (f) and gauge bosons in the basis of $\{A^\mu,\, Z^\mu,\, X^\mu\}$ is given by

$$-\mathcal{L}^{\rm int} = e\, Q_{\rm em}\, \bar{f}\, \slashed{A}\, f + e\, Q_Z\, \bar{f}\, \slashed{Z}\, f + e\, Q_X\, \bar{f}\, \slashed{X}\, f , \qquad (22)$$

where the electric charge is $Q_{\rm em}^f = I_{3L}^f + Y^f$, the Z generator is $Q_Z^f = (I_{3L}^f - \sin^2\theta_{\rm W}\, Q_{\rm em}^f)/(\sin\theta_{\rm W}\cos\theta_{\rm W})$, and the charge generator Q_X is defined by

$$Q_X^f = \chi\,\cos\theta_{\rm W}\, Q_{\rm em}^f + \frac{g_{BL}^f}{e}\, Q_{BL}^f . \qquad (23)$$

We have that the generator Q_X depends on the small parameters χ and g_{BL}^f that changes for each SM fermion.

The magnitude of these interactions is extremely small when compared to the couplings in the SM. Thereby, the generator Q_X is the origin of the millicharges in the model. As an example, using the charges from Table I, the generator Q_X for leptons (ℓ) is given by $Q_X^{(\ell)} = -\chi\cos\theta_{\rm W} - g_{BL}^\ell/e$. If we fix the mixing parameter at $\chi = 10^{-4}$ and constrain the coupling X from Eq. (22) with (3), *i.e.*, $|\chi^e| = |Q_X^{(\ell)}|$, we obtain an estimative of g_{BL}^ℓ for leptons, which is

$$2.01 \times 10^{-4} < |g_{BL}^\ell| < 1.59 \times 10^{-3} . \qquad (24)$$

Therefore, the constant coupling $e\,\chi_e$ yields the magnitude of the interaction of X-boson with left- and right-handed leptons similar to the interaction of leptons with the photon in quantum electrodynamics.

The couplings of X^μ with $W^\pm$ and Z are obtained by substituting $A_\mu \to A_\mu + \chi\,\cos\theta_{\mathrm{W}}\,X_\mu$ and $Z_\mu \to Z_\mu + \chi\,\delta\,\sin\theta_{\mathrm{W}} X_\mu$ in the SM vertex of A^μ with $W^+\,W^-$, and with Z. All these couplings are listed below, namely,

$$\mathcal{L}^{\mathrm{int}}_{X-W^\pm} = -i\,e\,\chi\,\cos\theta_{\mathrm{W}}\left(W^+_{\mu\nu}\,W^{\mu-} - W^-_{\mu\nu}\,W^{\mu+}\right)X^\nu$$
$$-i\,e\,\chi\,\cos\theta_{\mathrm{W}}\,X_{\mu\nu}\,W^{\mu+}\,W^{\nu-}\,, \tag{25}$$

$$\mathcal{L}^{\mathrm{int}}_{\gamma X-W^\pm} = -\frac{e^2}{2}\,\chi\,\cos\theta_{\mathrm{W}}$$
$$\times\left[2\,W^+_\mu\,W^{\mu-}\,A_\nu X^\nu - W^+_\mu\,W^-_\nu\,(A^\mu X^\nu + A^\nu X^\mu)\right]\,, \tag{26}$$

$$\mathcal{L}^{\mathrm{int}}_{XX-W^\pm} = -\frac{e^2}{2}\,\chi^2\cos^2\theta_{\mathrm{W}}\left(W^+_\mu\,W^{\mu-}\,X^\nu X^\nu - W^+_\mu\,W^-_\nu\,X^\mu X^\nu\right)\,, \tag{27}$$

$$\mathcal{L}_{XZ-W^\pm} = -\frac{1}{2}\,e\,g_L\,\chi\,\delta\,\cos^2\theta_{\mathrm{W}}$$
$$\times\left[2\,W^+_\mu\,W^{\mu-}\,Z_\nu\,X^\nu - W^+_\mu\,W^-_\nu\,(Z^\mu X^\nu + Z^\nu X^\mu)\right]\,, \tag{28}$$

where $W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu$ and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ are the field strength tensors of $W^\pm$ and Z, respectively. The interactions of scalar fields H and h_{BL} are represented by the Lagrangian

$$\mathcal{L}_{h_{BL}-X} = \frac{m_Z^2}{v}\,\chi\,\delta\,\sin\theta_{\mathrm{W}}\,H\,Z_\mu X^\mu + \frac{m_Z^2}{2v}\,\chi^2\,\delta^2\,\sin^2\theta_{\mathrm{W}}\,H\,X_\mu X^\mu$$
$$+g^2_{BL\phi}\,v_{BL}\,h_{BL}\,X^2_\mu + \frac{1}{2}\,g^2_{BL\phi}\,v^2_{BL}\,h^2_{BL}\,X^2_\mu\,, \tag{29}$$

and all the Feynman rules for the vertex of X-boson are organized in Table II. Using the value $m_X = 17$ MeV, the X^μ-boson mass fixes the VEV-scale v_{BL} at

$$v_{BL} \simeq \frac{8.5}{|g_{BL\phi}|}\,\mathrm{MeV}\,. \tag{30}$$

In this way, the hidden Higgs has a mass of $m_{h_{BL}} \simeq \sqrt{2\,\lambda_{BL}\,v^2_{BL}}$, with an upper bound at < 12 MeV. If we use the value of $|g_{BL\phi}|$ in the range of Eq. (24), the hidden Higgs estimate at few GeV-scale. It must be lighter than the SM Higgs which has mass equal to $m_H \simeq 125$ GeV. Therefore, we have gotten a similar QFT model consistent with both the requirements of renormalization and unitarity, and also with the X-boson phenomenology. Besides, the interaction sector of X-boson with the SMs leptons/quarks satisfies the experimental constraints through the generator Q_X. In the next sections of the paper, we will concentrate our attention on the interactions between leptons and X-boson and its respective phenomenology.

TABLE II

List of vertices of the X-boson present in the $U(1)_{B-L}$ model.

Interaction	Vertex factor
$X f \bar{f}$	$-i\,e\,Q_X^f\,\gamma^\mu$
$X W^+ W^-$	$-i\,e\,\chi\,\cos\theta_{\rm W}\,[(k_1-k_2)_\rho\eta_{\mu\nu}+(k_2-k_3)_\mu\eta_{\nu\rho}+(k_1-k_3)_\nu\eta_{\mu\rho}]$
$X X W^+ W^-$	$-i\,e^2\,\chi^2\,\cos^2\theta_{\rm W}\,(2\eta_{\mu\nu}\eta_{\lambda\rho}+\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\mu\rho}\eta_{\nu\lambda})$
$X A W^+ W^-$	$-i\,e^2\,\chi\,\cos\theta_{\rm W}\,(2\eta_{\mu\nu}\eta_{\lambda\rho}+\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\mu\rho}\eta_{\nu\lambda})$
$X Z W^+ W^-$	$-i\,e\,g_L\,\chi\,\delta\,\cos^2\theta_{\rm W}\,(2\eta_{\mu\nu}\eta_{\lambda\rho}+\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\mu\rho}\eta_{\nu\lambda})$
$H Z X$	$i\,\frac{m_Z^2}{v}\,\chi\,\delta\,\sin\theta_{\rm W}\,\eta_{\mu\nu}$
$H X X$	$i\,\frac{m_Z^2}{2v}\,\chi^2\,\delta^2\,\sin^2\theta_{\rm W}\,\eta_{\mu\nu}$
$h_{BL} X X$	$i\,g_{BL\phi}\,v_{BL}^2\,\eta_{\mu\nu}$
$h_{BL}^2 X X$	$\frac{i}{2}\,g_{BL\phi}^2\,v_{BL}^2\,\eta_{\mu\nu}$

3. The decay time of the X-boson

The X-boson phenomenology starts with the description of two decay processes. One of them is the π^0-decay into a massive dark photon with the coupling with SM particles that are proportional to their electric charge [23]. In an X-boson context, the neutral pion decay is $\pi^0 \to X\,\gamma$ with the bound for χ_p-proton parameter around the $|\chi_p| \lesssim 0.8$–1.2×10^{-3}. An SU(2)-model that includes the scalar pions can be the best description of this process. The second decay can be found in the ATOMKI pair spectrometer experiment that observes the excited 8-beryllium decay $^8{\rm Be}^\star \to {}^8{\rm Be} + X$ followed by $X \to e^+e^-$ [1, 39]. Thus, the model proposed here has the framework of interaction of the X-boson with $e^\pm$-pair proportional to the fundamental charge.

Using the usual rules of QFT, the decay rate is given by the expression

$$\Gamma = \frac{1}{2\pi^2}\frac{1}{2k^0}\int \frac{{\rm d}^3\boldsymbol{p}}{2p^0}\int \frac{{\rm d}^3\boldsymbol{p}'}{2p'^0}\,\delta^4\left(k-p-p'\right)\frac{1}{12}\sum_{\lambda,s,s'}|\mathcal{M}|^2\,, \qquad (31)$$

where k^μ is the X-boson four-momentum, and p^μ and p'^μ are the external momenta of the electron and positron, respectively. The electron–positron elastic scattering amplitude is

$$i\mathcal{M}\left(X \to e^+e^-\right) = \epsilon_\mu(k,\lambda)\,\bar{u}(p,s)\left(-ie\,\chi_e\,\gamma^\mu\right)v\left(p',s'\right)\,, \qquad (32)$$

where $\epsilon^\mu(k, \lambda)$ is the polarization vector, and $u(p, s)$ and $v(p', s')$ represent the wave plane amplitudes. Using the completeness relation

$$\sum_\lambda \epsilon_\mu(k, \lambda)\, \epsilon_\nu(k, \lambda) = -\eta_{\mu\nu} + \frac{k_\mu\, k_\nu}{m_X^2}\,,$$

$$\sum_s u_\alpha(p, s)\, \bar{u}_\beta(p, s) = (\not{p} + m)_{\alpha\beta}\,,$$

$$\sum_{s'} v_\alpha\left(p', s'\right)\, \bar{v}_\beta\left(p', s'\right) = (\not{p}' - m)_{\alpha\beta}\,, \tag{33}$$

the Γ-decay factor assumes the form of

$$\Gamma = \frac{e^2 \chi_e^2}{12\pi^2 m_X^3} \int \frac{\mathrm{d}^3 \boldsymbol{p}}{2p^0} \frac{\mathrm{d}^3 \boldsymbol{p}'}{2p'^0} \delta^4\left(k - p - p'\right) \left[(p \cdot k)\left(p' \cdot k\right) + \frac{1}{4} m_X^2 \left(m_X^2 + 4m^2\right)\right]\,, \tag{34}$$

where we have used the on-shell condition $k^2 = m_X^2$, and $m = 0.5$ MeV is the electron mass. Solving the above integral, we arrive at the decay rate

$$\Gamma\left(X \to e^+ e^-\right) = \frac{\alpha\, \chi_e^2}{3}\, m_X \sqrt{1 - 4\frac{m^2}{m_X^2}} \left(1 + 2\frac{m^2}{m_X^2}\right)\,, \tag{35}$$

where $\alpha = e^2/4\pi \simeq 1/137$ is the fine structure constant, and the X mass must satisfy the condition $m_X > 2\,m$. The decay $X \to e^+\, e^-$ can be approximated by $m_X \gg 2m = 1$ MeV, so the Γ-factor is given by

$$\Gamma\left(X \to e^+\, e^-\right) \approx \frac{\alpha\, \chi_e^2}{3}\, m_X\,. \tag{36}$$

Using $m_X = 17$ MeV and the constraints from Eq. (3), we obtain the range

$$1.6 \times 10^{-9}\ \text{MeV} < \Gamma\left(X \to e^+ e^-\right) < 8 \times 10^{-8}\ \text{MeV}\,. \tag{37}$$

Another possible process is the X-decay into light neutrinos, *i.e.*, $X \to \bar{\nu}\,\nu$. Following this framework, the decay width is

$$\Gamma\left(X \to \bar{\nu}\,\nu\right) \approx \frac{\alpha\, \chi_\nu^2}{3}\, m_X\,. \tag{38}$$

Thus, for the constraint in Eq. (3), we can obtain the range

$$5 \times 10^{-13}\ \text{MeV} < \Gamma\left(X \to \bar{\nu}\,\nu\right) < 2.3 \times 10^{-10}\ \text{MeV}\,. \tag{39}$$

Hence, the mode lifetime τ is given by

$$\tau = \frac{1}{\Gamma\left(X \to e^+\, e^-\right) + \Gamma\left(X \to \bar{\nu}\,\nu\right)}\,, \tag{40}$$

namely, it has the range[1]

$$8.3 \times 10^{-15}\,\mathrm{s} < \tau < 4.2 \times 10^{-12}\,\mathrm{s}\,. \tag{41}$$

However, the better way to investigate this process is through a model $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1) \times \mathrm{U}(1)_{B-L}$ which includes the left-handed neutrinos (and the right-components), where its interaction with the X-boson must depend on the mixing angles, like the θ_W — Weinberg angle, for example. This enlarged framework with the weak interaction sector is an ongoing research.

4. The muon–anti-muon scattering

The measurements of a neutron–nucleus scattering motivated us to explore the Yukawa potential acting on the X-boson, and the corresponding scattering process [40]. In the context of this model, we can use the e^2-leading order for the electron–positron scattering process $e^+e^- \to X \to \mu^+\mu^-$ to obtain its scattering cross section. Using the rules of QFT, the amplitude for this scattering at the tree level is[2]

$$\begin{aligned}
i\mathcal{M}\left(e^+e^- \to \mu^+\mu^-\right) &= \bar{v}\left(k,t\right)\left(-ie\,\chi_e\,\gamma^\mu\right)u\left(p,s\right)\frac{-i\eta_{\mu\nu}}{(p+k)^2 - m_X^2} \\
&\times \bar{u}\left(p',s'\right)\left(-ie\,\chi_e\,\gamma^\nu\right)v\left(k',t'\right)\,,
\end{aligned} \tag{42}$$

where the momentum are p and k for the electron–positron pair, and p' and k' for the muon pair μ^- and μ^+. Let us consider the collision in the center-of-mass (CM) frame of the e^+e^- pair. In this case, $\boldsymbol{p} + \boldsymbol{k} = \boldsymbol{p}' + \boldsymbol{k}' = \boldsymbol{0}$ and we denote the electron and muon energy as being $s = (p+k)^2 = (p'+k')^2$. Using the rules of QFT, the differential cross section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{\alpha^2\,\chi_e^4}{4\left(s - m_X^2\right)^2}\left[s\left(1 + \cos^2\beta\right) + 8\,m_\mu^2\,\sin^2\beta\right]\,, \tag{43}$$

where β is the scattering angle between the 3-momentum $\boldsymbol{p}$ and $\boldsymbol{p}'$, or between $\boldsymbol{k}$ and $\boldsymbol{k}'$ in the CM frame. We have also considered $m_e \approx 0$ when compared to the muon mass $m_\mu = 105.7$ MeV. The result in Eq. (43) is illustrated in Fig. 1 as a function of the β-angle. Using the value of $\sqrt{s} = 1$ MeV for the CM energy and the previous masses, the total cross section of this process is

$$\sigma\left(e^+e^- \to \mu^+\mu^-\right) = \pi\alpha^2\,\chi_e^4\,\frac{s + 8m_\mu^2}{\left(s - 4m_X^2\right)^2} = 6.92 \times 10^{10}\,\chi_e^4\,[\mathrm{pb}]\,. \tag{44}$$

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[1] Here, we have used the conversion formula 1 MeV $= 1.52 \times 10^{21}$ s^{-1} in the natural units $\hbar = c = 1$.

[2] We use the X-boson propagator in the Feynman gauge ($\beta = 1$).

Using the bounds in Eq. (3), we obtain the bounds of the cross section of this process at 0.11 fb $< \sigma \ (e^+ \, e^- \to \mu^+ \, \mu^-) < 0.26$ pb [3].
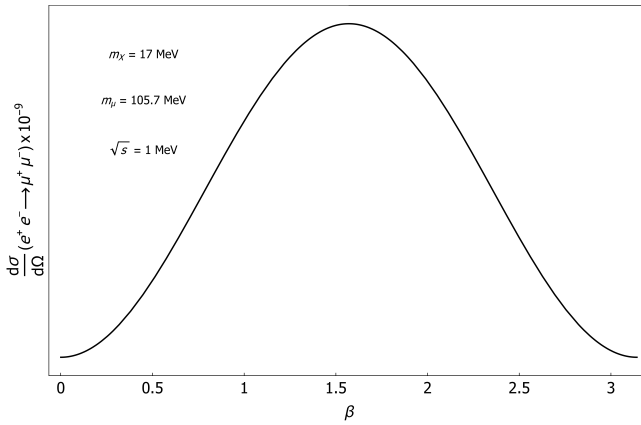

Fig. 1. The differential cross section of $e^+ e^- \to \mu^+ \mu^-$ as a function of the β-angle. We adopt the values of $m_X = 17$ MeV, $m_\mu = 105$ MeV and $\sqrt{s} = 1$ MeV for the CM frame energy.

5. The electron self-energy with the X-boson correction

Over the last sections, we have obtained some phenomenological results for the Abelian model of the X-boson at the leading order in a perturbative QFT. From now on, we will discuss the radiative corrections at one-loop approximation. The first one is due to the electron propagator, where we have investigated the influence of the X-boson mass in the physical electron mass. Using the previous rules, the electron propagator at one loop can be

$$\Sigma_1 \left(\not{p} \right) = \Sigma_1^{\mathrm{QED}} \left(\not{p} \right) + \Sigma_1^{(X)} \left(\not{p} \right) . \tag{45}$$

The expressions of $\Sigma_1^{\mathrm{QED}} \left(\not{p} \right)$ and $\Sigma_1^{(X)} \left(\not{p} \right)$ can be given by the integrals

$$\Sigma_1^{\mathrm{QED}}(\not{p}) = -i \, e^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\gamma^\mu \left(\not{k} + \not{p} + m \right) \gamma_\mu}{\left[(k+p)^2 - m^2 \right] \left(k^2 - m_\gamma^2 \right)} , \tag{46}$$

and

$$\Sigma_1^{(X)}(\not{p}) = -i \, e^2 \, \chi_e^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\gamma^\mu \left(\not{k} + \not{p} + m \right) \gamma_\mu}{\left[(k+p)^2 - m^2 \right] \left(k^2 - m_X^2 \right)} , \tag{47}$$

where m_γ is the photon mass introduced as a regulator to control the infrared divergence that emerges from Eq. (46) when $m_\gamma = 0$. It is clear that after

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[3] We obtain the cross section in the picobarn units.

the renormalization, we must withdraw the m_γ-parameter since we have a possible limit such as $m_\gamma \to 0$. Therefore, we have two similar integrals where the χ-factor is less than one. These integrals have linear divergences in the ultraviolet region, so we will use the dimensional regularization to control the divergences, *i.e.*, the integral dimension is altered by a ω-regularization parameter $4 \to 2\omega$, where the physical dimension is recovered, obviously, when $\omega \to 2$. In this way, the coupling constant is redefined to keep it dimensionless, *i.e.*, $e \to e \mu^{2-\omega}$, where μ is an arbitrary energy scale. In fact, when we withdraw the regularization parameter, the divergent term will be isolated from the physical term, which makes the expansion $\omega = 2 - \varepsilon$, for $\varepsilon \to 0$. Therefore, the QED-contribution in the Feynman gauge is known in the literature as the result below

$$\Sigma_1^{\text{QED}}(\not{p}, \varepsilon) = \frac{\alpha}{4\pi} \left(-\not{p} + 4m\right) \frac{\mu^{2\varepsilon}}{\varepsilon} - \frac{\gamma\, \alpha}{4\pi} \left(-\not{p} + 4m\right)$$

$$-\frac{\alpha}{2\pi} \int\limits_0^1 \mathrm{d}z \left[(1-z)\, \not{p} - 2\, m\right] \ln \left[\frac{4\pi\mu^2}{m^2\, z - p^2\, z\, (1-z)}\right] , \qquad (48)$$

where $p^2 < 2\, m^2$, and $\gamma \simeq 0.57$ is the Euler–Mascheroni constant. The regularized contribution of the X-boson is given by the integral

$$-i\, \Sigma_1^{(X)}(\not{p}, \omega) = -e^2\, \chi_e^2 \left(\mu^2\right)^{2-\omega} \int \frac{\mathrm{d}^{2\omega}k}{(2\pi)^{2\omega}} \frac{\gamma^\mu\, (\not{k} + \not{p} + m)\, \gamma_\mu}{\left[(k+p)^2 - m^2\right] \left(k^2 - m_X^2\right)} . \qquad (49)$$

Using the technique given in the literature concerning the Feynman integrals, the result of the one in Eq. (49), for $\omega = 2 - \varepsilon$, is

$$\Sigma_1^{(X)}(\not{p}, \varepsilon) \simeq \frac{\alpha}{4\pi} \left(-\not{p} + 4m\right) \frac{\mu^{2\varepsilon}}{\varepsilon} - \frac{\gamma\, \alpha}{4\pi} \left(-\not{p} + 4m\right)$$

$$-\frac{\alpha\, \chi_e^2}{2\pi} \int\limits_0^1 \mathrm{d}z \left[(1-z)\, \not{p} - 2\, m\right] \ln \left[\frac{4\pi\mu^2}{m_X^2(1-z) + m^2 z - p^2 z(1-z)}\right] , \quad (50)$$

where $p^2 < 2\, m^2 + 2\, m_X^2$. The renormalized full propagator is represented by the expression

$$s(\not{p}) = \frac{Z_2}{\not{p} - m - \Sigma(\not{p} = m)} , \qquad (51)$$

where the Z_2-renormalization factor connects the electron bare field Ψ_0 to the physical field Ψ via $\Psi_0 = \sqrt{Z_2}\, \Psi$. It is given by the on-shell condition

$$Z_2 = 1 + \left.\frac{\mathrm{d}\Sigma(\not{p})}{\mathrm{d}\not{p}}\right|_{\not{p}=m} . \qquad (52)$$

The electron physical mass is identified as being $m_e = m + \Sigma(\not{p} = m)$, and the leading order contribution to the electron's mass is

$$\Sigma_1\left(m,\varepsilon\right) \simeq \frac{3m\alpha}{4\pi}\frac{\mu^{2\varepsilon}}{\varepsilon} - \frac{3m\alpha\,\gamma}{4\pi} + \frac{3m\alpha}{4\pi}\ln\left(\frac{4\pi\mu^2}{m^2}\right) + \frac{5m\alpha}{4\pi}$$
$$+ \frac{7m\alpha\,\chi_e^2}{8\pi} - \frac{3m\alpha\,\chi_e^2}{2\pi}\ln\left(\frac{m_X}{m}\right), \tag{53}$$

where we have assumed $m^2/m_X^2 \ll 1$, and $1 + \chi_e^2 \simeq 1$. This result gives the contribution at the one-loop approximation for the electron's renormalization mass. Thereby, the electron's physical mass has the finite correction given by

$$\frac{m_e}{m} = 1 + \frac{5\alpha}{4\pi} + \frac{7\alpha\,\chi_e^2}{8\pi} - \frac{3\alpha\,\chi_e^2}{2\pi}\ln\left(\frac{m_X}{m}\right). \tag{54}$$

Finally, the Z_2-factor is

$$Z_2 = 1 - \frac{\alpha}{2\pi}\frac{\mu^{2\varepsilon}}{\varepsilon} - \frac{\alpha\,\gamma}{2\pi} - \frac{\alpha}{\pi} - \frac{\alpha}{4\pi}\ln\left(\frac{4\pi\mu^2}{m^2}\right) - \frac{\alpha\,\chi_e^2}{4\pi}\ln\left(\frac{4\pi\mu^2}{m_X^2}\right) - \frac{\alpha}{2\pi}\ln\left(\frac{m_\gamma}{m}\right). \tag{55}$$

Therefore, we have provided the renormalization result for both the propagator and the electron field with the contribution of the X-boson.

6. The full X-propagator and the Uehling potential

The analysis of the X-boson propagator is important in order to understand the X-boson physical mass. We will see that the m_X-physical mass leads us to the calculation of the on-shell complex renormalization. Furthermore, its radiative correction contributes to the Yukawa potential obtained in Section 4. Let us start with the renormalized field of the X-boson which is defined by the relation

$$X_0^\mu = \sqrt{Z_X}\,X^\mu, \tag{56}$$

where the full renormalized propagator of the X-boson depends on the Z_X-factor, and it is given by the expression

$$\Delta_{\mu\nu}\left(k^2\right) = \frac{-i\,\eta_{\mu\nu}\,Z_X^{-1}}{\left(1 - \Pi\left(k^2\right)\right)k^2 - m_{0X}^2}. \tag{57}$$

The $\Pi(k^2)$ is a scalar function that multiplies the transverse term in the vacuum polarization

$$\Pi_{\mu\nu}\left(k^2\right) = \Pi\left(k^2\right)\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right). \tag{58}$$

The conserved current guarantees that $k_\mu J^\mu = 0$. Hence, the terms like $k_\mu k_\nu / k^2$ are zero due to the term $J_\mu \, \Delta^{\mu\nu} \, J_\nu$ in the perturbation theory. The on-shell condition $k^2 = m_X^2$ fixes the propagator pole by using the Z_X-factor condition

$$Z_X = \frac{1}{1 - \Pi \left(m_X^2 \right)} \, , \tag{59}$$

thus, the X-propagator in Eq. (57) is finite and it is given by

$$\Delta_{\mu\nu} \left(k^2 \right) = \frac{-i \, \eta_{\mu\nu}}{\left[1 - \Pi_{\mathrm{R}} \left(k^2 \right) \right] k^2 - m_X^2} \, . \tag{60}$$

The scalar function $\Pi_{\mathrm{R}}(k^2) = \Pi \left(k^2 \right) - \Pi \left(m_X^2 \right)$ is finite, and it cancels out the divergence that appears from the vacuum polarization. Notice that for the physical mass m_X which is related to the unphysical mass m_{0X}, in accordance with the relation $m_{0X} = \sqrt{Z_X^{-1}} \, m_X$, the renormalization factor for the X-boson mass is $Z_{m_X} = Z_X^{-1}$.

Using the previous rules, the expression of the vacuum polarization at one-loop approximation is given by the integral

$$i \, \Pi_1^{\mu\nu}(m, k) = - \left(i \, e \, \chi_e \right)^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \, \mathrm{tr} \left(\frac{i\gamma^\mu}{\not{p} - m} \frac{i\gamma^\nu}{\not{p} - \not{k} - m} \right) , \tag{61}$$

which is the same expression given by the standard QED considering the factor χ. Consequently, the calculation of the vacuum polarization in this order is the same as the one considering the usual QED. Using the dimensional regularization, $i.e.$, $D = 4 \rightarrow D = 2\omega$, the previous integral can be calculated and the result is

$$\begin{aligned}
\Pi_1 \left(m^2, k^2, \omega \right) &= -\frac{\alpha \, \chi_e^2}{\pi} \left(\mu^2 \right)^{2-\omega} \omega \, \Gamma(2 - \omega) \\
&\quad \times \int_0^1 \mathrm{d}x \, x \left(1 - x \right) \left[\frac{4\pi\mu^2}{m^2 - k^2 x(1-x)} \right]^{2-\omega} ,
\end{aligned} \tag{62}$$

where $4m^2 > k^2$. We isolate the divergent part by writing $\omega = 2 - \varepsilon$, with $\varepsilon \rightarrow 0^+$ to obtain the result

$$\begin{aligned}
\Pi_1 \left(m^2, k^2, \varepsilon \right) &= -\frac{\alpha \, \chi_e^2}{3\pi} \frac{\mu^{2\varepsilon}}{\varepsilon} + \frac{\alpha \, \chi_e^2}{6\pi} (2\gamma + 1) - \frac{2\alpha \, \chi_e^2}{\pi} \\
&\quad \times \int_0^1 \mathrm{d}x \, x \left(1 - x \right) \ln \left[\frac{4\pi\mu^2}{m^2 - k^2 x(1-x)} \right] .
\end{aligned} \tag{63}$$

Using this result, the finite part that appears in Eq. (60) is given by the subtraction

$$\Pi_{\mathrm{R}}\left(k^2\right) = \frac{2\alpha\,\chi_e^2}{\pi}\int\limits_0^1 \mathrm{d}x\,x\left(1-x\right)\ln\left[\frac{m^2 - k^2\,x\left(1-x\right)}{m^2 - m_X^2\,x\left(1-x\right)}\right]. \qquad (64)$$

The integral in Eq. (64) can be calculated for all values of k^2. So, we obtain

$$\begin{aligned}\Pi_{\mathrm{R}}\left(k^2\right) &= -\frac{4\alpha\,\chi_e^2}{3\pi}\left(\frac{m^2}{k^2} - \frac{m^2}{m_X^2}\right) + \frac{\alpha\,\chi_e^2}{3\pi}\left(1 + \frac{2m^2}{k^2}\right)f\left(k^2\right)\\ &\quad -\frac{\alpha\,\chi_e^2}{3\pi}\left(1 + \frac{2m^2}{m_X^2}\right)f\left(m_X^2\right),\end{aligned} \qquad (65)$$

where the function $f(k^2)$ is defined by

$$f\left(k^2\right) = \begin{cases} 2\sqrt{1 - \frac{4m^2}{k^2}}\,\sinh^{-1}\!\left(\frac{\sqrt{-k^2}}{2m}\right) & \text{if } k^2 < 0\,,\\[2ex] \sqrt{\frac{4m^2}{k^2} - 1}\,\cot^{-1}\!\left(\sqrt{\frac{4m^2}{k^2} - 1}\right) & \text{if } 0 < k^2 \le 4m^2,\\[2ex] \sqrt{1 - \frac{4m^2}{k^2}}\left[2\cosh^{-1}\!\left(\frac{\sqrt{k^2}}{2m}\right) - i\pi\right] & \text{if } k^2 > 4m^2\,. \end{cases} \qquad (66)$$

In this expression, we can observe the appearance of an imaginary part, when $k^2 > 4m^2$. This is the X-boson case where the on-shell condition $k^2 = m_X^2$ fixes the inequality $m_X^2 > 4m^2$ for the masses $m_X = 17$ MeV and $m = 0.5$ MeV. The imaginary part means the instability of the X-boson and, as a consequence, it decays into the virtual electron–positron pair.

The result of the integral in Eq. (65) under the on-shell condition $k^2 = m_X^2$ yields the Z_X-factor

$$\begin{aligned}Z_X &\simeq 1 - \frac{\alpha\,\chi_e^2\,\mu^{2\varepsilon}}{3\pi}\,\frac{1}{\varepsilon} + \frac{\alpha\,\chi_e^2}{6\pi}\left(2\gamma + 1\right) - \frac{\alpha\,\chi_e^2}{3\pi}\ln\left(\frac{4\pi\mu^2}{m^2}\right) - i\,2\alpha\,\chi_e^2\\ &\quad + \frac{4\alpha\,\chi_e^2}{\pi}\ln\left(\frac{m_X}{m}\right). \end{aligned} \qquad (67)$$

Hence, the Z_{m_X}-factor can be obtained and the physical mass of the X-boson is

$$\begin{aligned}\frac{m_X^{\mathrm{R}}}{m_{0X}} &\simeq 1 - \frac{\alpha\,\chi_e^2\,\mu^{2\varepsilon}}{6\pi}\,\frac{1}{\varepsilon} + \frac{\alpha\,\chi_e^2}{12\pi}\left(2\gamma + 1\right) - \frac{\alpha\,\chi_e^2}{6\pi}\ln\left(\frac{4\pi\mu^2}{m^2}\right) - i\,\alpha\,\chi_e^2\\ &\quad + \frac{2\alpha\,\chi_e^2}{\pi}\ln\left(\frac{m_X}{m}\right). \end{aligned} \qquad (68)$$

The general expression for the energy potential is given by

$$U(\boldsymbol{r}) = -4\pi\alpha\,\chi_e^2 \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{r}}\, \Delta_{00}\left(-\boldsymbol{k}^2\right)\,. \tag{69}$$

Using the propagator in Eq. (57) into the energy potential in Eq. (69), where we have neglected the order α^3 terms, after some algebraic manipulation, by using the approximation $m^2/m_X^2 \ll 1$, the previous integral can be reduced to one quadrature in the following expression:

$$
\begin{aligned}
U(r) &= -\alpha\,\chi_e^2\, \frac{e^{-m_X r}}{r} - \frac{\alpha^2\,\chi_e^4}{3\pi r}\, e^{-m_X r} - i\,\frac{\alpha^2\,\chi_e^4}{3\,r}\left(1 - \frac{m_X r}{2}\right) e^{-m_X r}\\
&\quad - \frac{2\alpha^2\chi_e^4}{3\pi r} \int\limits_1^\infty d\xi \left(1 + \frac{1}{2\xi^2}\right) \frac{\left(\xi^2 - 1\right)^{1/2}}{\xi^2} \left(1 - \frac{m^2}{4m_X^2\xi^2}\right)^{-2} e^{-2mr\xi}\,.
\end{aligned}
\tag{70}
$$

This ξ-integral can be called as the integral representation of the *Uehling potential* with the correction of the X-boson mass. The ξ-integral is difficult to solve analytically, so we have to analyze it considering the asymptotic case $mr \gg 1$. For $mr \gg 1$, only the region $0 \le \xi - 1 \ll (mr)^{-1}$ contributes to the integral, so one can approximate $\xi \simeq 1$ to obtain the expression

$$
\begin{aligned}
U(r) &\simeq -\frac{\alpha\,\chi_e^2}{r}\, e^{-m_X r}\left(1 + \frac{\alpha\,\chi_e^2}{3\pi}\right) - \frac{\alpha^2\,\chi_e^4}{4\sqrt{\pi}\,r}\, \frac{e^{-2mr}}{(mr)^{3/2}}\\
&\quad -i\,\frac{\alpha^2\,\chi_e^4}{3\,r}\left(1 - \frac{m_X r}{2}\right) e^{-m_X r}\,,
\end{aligned}
\tag{71}
$$

whenever $m \neq 0$. Obviously, the imaginary part goes to zero when $m_X \to \infty$ (or $m_X \gg m$). In this calculation, we do not take into account the loop corrections of neutrinos and quarks (or pion and neutron). However, it would be convenient to do it in the entire model involving the $SU_L(2)$-group, concerning the unification of X-boson with the weak interaction.

7. The anomalous muon magnetic moment

The vertex is another diagram of the model useful for the renormalization of the constant coupling. Furthermore, it includes the electron's anomalous magnetic moment which is a famous QED calculation with an incredible agreement with the experimental result. At one loop, the QED vertex of leptons with the photon has a correction of the X propagator that can yield a significant contribution to the muon magnetic moment. We have two vertices in the X-boson model: the usual one of QED, and the interaction of X

with the SM f-fermions. As a consequence, we have two Z-renormalization factors for each vertex diagram. Thus, the computation of the anomalous moment motivates us to obtain a bound on the χ-parameter.

Let us start with the QED photon vertex where two diagrams contribute. So we will denote it by the sum

$$\Gamma^\mu\left(p,p'\right) = \Gamma^\mu_{1-\text{QED}}\left(p,p'\right) + \Gamma^\mu_{1-X}\left(p,p'\right) , \tag{72}$$

where the contribution of the X-boson propagator in this order is represented by the integral

$$\Gamma^\mu_{1-X}\left(p,p'\right) = -e^3\,\chi_e^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\gamma^\alpha\left(\slashed{k}+\slashed{p}'+m\right)\gamma^\mu\left(\slashed{k}+\slashed{p}+m\right)\gamma_\alpha}{\left[(k+p')^2-m^2\right]\left[(k+p)^2-m^2\right]\left(k^2-m_X^2\right)} . \tag{73}$$

The QED photon vertex has the similar expression exchanging $e^3\,\chi_e^2 \to e^3$ and $m_X \to m_\gamma$ for the photon mass, introduced to tame the infrared divergence. This integral has a ultraviolet divergence by a simple power counting. Therefore, we will use the previous technique of dimensional regularization to isolate the divergent term from the physical terms. The divergent part has the result

$$\Gamma^\mu\left(q^2\right) = -i\,e\,\mu^{2\epsilon}\gamma^\mu\left[1 + \frac{\alpha}{2\pi}\frac{\mu^{2\epsilon}}{\epsilon} + \text{finite part}\right] . \tag{74}$$

The vertex of X-boson with leptons has the correction at one loop given by

$$\Lambda^\mu\left(q^2\right) = -i\,e\,\chi_e\,\mu^{2\epsilon}\gamma^\mu\left[1 + \frac{\chi_e^2\,\alpha}{2\pi}\frac{\mu^{2\epsilon}}{\epsilon} + \text{finite part}\right] . \tag{75}$$

Thus, we have obtained the necessary terms for the renormalization vertex. The renormalization procedure will be performed in the next section. Now, we are interested in the finite part of Eq. (73).

The finite part of the vertex that contains the form factors is consequently defined by the following difference given by:

$$\Gamma^\mu_{\text{R}}\left(q^2\right) = \Gamma^\mu\left(q^2\right) - \Gamma^\mu\left(q^2 = 0\right) . \tag{76}$$

The Gordon identity of the Dirac current can be used such that the finite part of Eq. (73) yields the relation

$$\Gamma^\mu_{\text{R}}\left(q^2\right) = \gamma^\mu F_1\left(q^2\right) + i\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\,F_2\left(q^2\right) , \tag{77}$$

where F_1 and F_2 are the form factors, and q is defined as the photon's external momentum $q^\mu = p'^\mu - p^\mu$. We have also used the usual on-shell

conditions for the fermion external momentum, *i.e.*, $p^2 = p'^2 = m^2$. Thus, the form factors at one-loop approximation are given by

$$F_1\left(q^2\right) = \frac{\alpha}{2\pi} \int\limits_0^1 \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\delta(x{+}y{+}z{-}1)\,\frac{1{+}(2{-}z)^2{+}(1{-}x)(1{-}y)\,q^2/m^2}{(1{-}z)^2 + z\,m_\gamma^2/m^2 - x\,y\,q^2/m^2}$$

$$+\frac{\alpha\,\chi_e^2}{2\pi} \int\limits_0^1 \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\delta(x+y+z-1)\,\frac{1+(2-z)^2+(1-x)(1-y)\,q^2/m^2}{(1-z)^2 + z\,m_X^2/m^2 - x\,y\,q^2/m^2}$$

$$\tag{78}$$

and

$$F_2\left(q^2\right) = \frac{\alpha}{2\pi} \int\limits_0^1 \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\delta(x+y+z-1)\,\frac{2z(1-z)}{(1-z)^2 - x\,y\,q^2/m^2}$$

$$+\frac{\alpha\,\chi_e^2}{2\pi} \int\limits_0^1 \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\delta(x+y+z-1)\,\frac{2z(1-z)}{(1-z)^2 + z\,m_X^2/m^2 - x\,y\,q^2/m^2}\,. \tag{79}$$

The first factor F_1 is the origin of the infrared divergences in the model. When $q^2 = 0$, the photon mass m_γ is the parameter that regularizes the infrared divergence in the first integral of Eq. (78). The second factor F_2 gives an important contribution to the electron anomalous magnetic moment. This contribution appears when $q^2 = 0$, where the X-boson vertex carries the correction

$$F_2^{(e)}(0) = \frac{\alpha}{2\pi} + \frac{\alpha\,\chi_e^2}{2\pi}\,\frac{m_e^2}{m_X^2} \int\limits_0^1 \mathrm{d}z\,\frac{2\,z(1-z)^2}{z + (1-z)^2\,m_e^2/m_X^2}\,. \tag{80}$$

Here, the first term is just the contribution of the ordinary QED. The second integral is the contribution of the mass m_X. We have expanded the previous integral for $m^2/m_X^2 \ll 1$. Hence, we have obtained the result at the lower order

$$F_2^{(e)}(0) \simeq \frac{\alpha}{2\pi}\left(1 + \frac{2}{3}\,\chi_e^2\,\frac{m_e^2}{m_X^2}\right)\,. \tag{81}$$

If we use the well-known experimental uncertainty in the electron's anomalous magnetic moment, it leads us to the upper bound

$$|\chi_e|\,\frac{m_e}{m_X} \lesssim 4.2 \times 10^{-5}\,. \tag{82}$$

Using the values $m_e = 0.5$ MeV and $m_X = 17$ MeV, the upper bound for the χ_e-parameter is $|\chi_e| \lesssim 1.4 \times 10^{-3}$, which agrees with Eq. (3). This result just confirms the one present in the literature [38]. For the muon case, if we just consider its interaction with X-boson, the F_2-form factor when $q^2 = 0$ is given by

$$F_2^{(\mu)}(0) = \frac{\alpha}{2\pi} + \frac{\alpha \chi_e^2}{2\pi} \int\limits_0^1 \mathrm{d}z \, \frac{2z(1-z)^2}{(1-z)^2 + z \, m_X^2/m_\mu^2} \,, \tag{83}$$

and using $m_\mu = 105$ MeV, the z-integral has the result

$$F_2^{(\mu)}(0) \simeq \frac{\alpha}{2\pi} \left(1 + \frac{3}{2} \chi_e^2 \right) \,. \tag{84}$$

From the value reported by the E821 experiment $\Delta a_\mu(\mathrm{E821}) \simeq (116592080 \pm 63) \times 10^{-11}$ [41], we have the subtraction in relation to SM value

$$\Delta a_\mu(\mathrm{E821 - SM}) = (295 \pm 81) \times 10^{-11} \,, \tag{85}$$

which imposes the upper bound $|\chi_e| \lesssim 4.43 \times 10^{-5}$.

8. Renormalization

8.1. Renormalized perturbation theory

In this section, we will investigate the renormalized sector of the X-boson gauge and sector of leptons. To carry out the perturbative renormalization, both the renormalized fermion (leptons) and the X-boson gauge sectors are given by the Lagrangian

$$\mathcal{L}_{\mathrm{lept}-X} = \bar{f}\left(i\,\slashed{\partial} - m_f\right) f + \bar{f}\left(i\,\delta_2\,\slashed{\partial} - \delta_{m_f}\right) f - \frac{1}{4}\,F_{\mu\nu}^2 - \frac{1}{2\alpha}\left(\partial_\mu A^\mu\right)^2$$

$$- \frac{\delta_A}{4}\,F_{\mu\nu}^2 - \frac{\delta_A}{2\alpha}\left(\partial_\mu A^\mu\right)^2 - \frac{1}{4}\,X_{\mu\nu}^2 + \frac{1}{2}\,m_X^2\,X_\mu^2 - \frac{1}{2\beta}\left(\partial_\mu X^\mu\right)^2 - \frac{\delta_X}{4}\,X_{\mu\nu}^2$$

$$- \frac{\delta_X}{2\beta}\left(\partial_\mu X^\mu\right)^2 + \frac{1}{2}\,\delta_{m_X}\,X_\mu^2 - e\,\bar{f}\,\slashed{A}\,\Psi - \delta_e\,\bar{f}\,\slashed{A}\,f - \chi_e\,e\,\bar{f}\,\slashed{X}\,f - \delta_3\,\bar{f}\,\slashed{X}\,f \,, \tag{86}$$

where the relations between the bare and renormalized quantities are

$$A_\mu^{(0)} = \sqrt{Z_A}\,A_\mu \,, \tag{87}$$

and the counter-terms are

$$Z_A = 1 + \delta_A \,, \quad Z_X = 1 + \delta_X \,, \quad m + \delta_m = m_0\,Z_2 \,, \quad m_{0X}^2\,Z_X = m_X^2 + \delta_{m_X} \,,$$

$$e_0\,Z_2\,Z_A^{1/2} = e + \delta_e \,, \quad e_0\,\chi_e\,Z_2\,Z_X^{1/2} = \chi_e\,e + \delta_3 \,. \tag{88}$$

We have introduced six counter-terms $\{\delta_A, \delta_X, \delta_m, \delta_{m_X}, \delta_e, \delta_3\}$ to cancel out all the divergences that emerge from the Lagrangian in Eq. (86). Here, all the parameters in the renormalized Lagrangian are finite, and we need six conditions to fix all these counter-terms. They are

$$\Sigma(\not{p})\big|_{\not{p}=m} = 0\,, \qquad\qquad \frac{\mathrm{d}}{\mathrm{d}\not{p}}\,\Sigma(\not{p})\bigg|_{\not{p}=m} = 0\,,$$

$$\Pi\left(k^2\right)\big|_{k^2=0} = 0\,, \qquad \Pi_X\left(k^2\right)\big|_{k^2=m_X^2} = 0\,,$$

$$\Gamma^\mu\left(q^2\right)\big|_{q^2=0} = -i\,e\,\gamma^\mu\,, \qquad \Lambda^\mu\left(q^2\right)\big|_{q^2=m_X^2} = -i\,\chi_e\,e\,\gamma^\mu\,. \qquad (89)$$

The first constraint fixes the physical electron mass and the second one fixes the renormalization of the fermion field. The third one fixes the Z_A-factor and the fourth one fixes the Z_X-factor. The last two constraints fix both the vertex-photon, and X-boson vertex, respectively. In Sections 5 and 6, we have obtained the Z_2-, Z_X- and Z_{m_X}-factors. Using the conditions in Eq. (89), the counter-terms δ_e and δ_3 can be determined such that we can obtain the relations $e_0\,Z_2\,Z_A^{1/2} = e\,Z_1$ and $e_0\,Z_2\,Z_X^{1/2} = e\,Z_3$. The Ward identity guarantees that $Z_1 = Z_2$, and the Z_A- and Z_3-factors are given by

$$Z_A = 1 - \frac{\alpha}{3\pi}\frac{\mu^{2\epsilon}}{\epsilon} + \text{finite part} \quad \text{and} \quad Z_3 = 1 - \frac{\chi_e^2\,\alpha}{2\pi}\frac{\mu^{2\epsilon}}{\epsilon} + \text{finite part}\,. \quad (90)$$

Hence, we have determined all the Z-renormalization factors at one loop. This renormalization scheme allows us to investigate the physical parameters as a function of an arbitrary scale. In the next subsection, we will introduce the renormalization group through the Callan–Symanzik equation. Its solution yields the running physical mass and the constant coupling.

8.2. Renormalization group

The Callan–Symanzik equation concerning the renormalization group for the parameters of Eq. (86) is given by

$$\left[\mu\,\frac{\partial}{\partial\mu} + \beta(e)\,\frac{\partial}{\partial e} + \beta_\chi(e_\chi)\,\frac{\partial}{\partial e_\chi} + m\,\gamma_m(e)\,\frac{\partial}{\partial m} + m_X\,\gamma_{m_X}(e)\,\frac{\partial}{\partial m_X}\right.$$

$$\left. -n\,\gamma_A(e) - n\,\gamma_X(e)\right]\Gamma^{(n)}(e, e_\chi, m, m_X) = 0\,, \qquad (91)$$

where $\Gamma^{(n)}$ is the one-particle irreducible Green function of n-points, and μ is an arbitrary energy scale. We will use in this work the notation $e_\chi = \chi_e\,e$, where the beta function β_χ is associated with the X-boson vertex

renormalization. The functions β, β_χ, γ_m, γ_{m_X}, γ_A and γ_X are related to the Z-renormalization factors by

$$\beta(e) = \mu\, \frac{\partial e}{\partial \mu}\,, \qquad \beta_\chi(e_\chi) = \mu\, \frac{\partial e_\chi}{\partial \mu}\,, \qquad \gamma_m(e) = -\frac{\mu}{2}\, \frac{\partial}{\partial \mu} \ln Z_m\,,$$

$$\gamma_{m_X}(e) = -\frac{\mu}{2}\, \frac{\partial}{\partial \mu} \ln Z_{m_X}\,, \quad \gamma_A(e) = \frac{\mu}{2}\, \frac{\partial}{\partial \mu} \ln Z_A\,, \quad \gamma_X(e) = \frac{\mu}{2}\, \frac{\partial}{\partial \mu} \ln Z_X\,. \tag{92}$$

The function β is kept constant in this X-boson framework. This occurs because, at high momenta, the mass $m_X = 17$ MeV is negligible, where β is given explicitly by $\beta(e) = e^3/12\pi^2$, and the model is not asymptotically free. Combining the functions in Eq. (92) and the relations in Eq. (88), it is easy to see that $\beta(e) = e\,\gamma_A(e)$ and, consequently, that $\gamma_A(e) = \alpha/3\pi$. The new functions β_χ, γ_{m_X} and γ_X are

$$\beta_\chi(e) = \frac{e^3}{4\pi^2} \left(1 - \frac{2\,\chi_e^2}{3}\right) \quad \text{and} \quad \gamma_{m_X}(e) = \gamma_X(e) = -\frac{e^2\,\chi_e^2}{12\pi^2}\,. \tag{93}$$

The invariance of the Green function $\Gamma^{(n)}$ under scale transformation $\Gamma^{(n)}(e, e_\chi, m, m_X, \mu) = \Gamma^{(n)}\left(\bar{e}(t), \bar{e}_\chi(t), \bar{m}(t), \bar{m}_X(t), \bar{\mu}(t) = \mu\, e^t\right)$ leads us to the *effective coupling constants* $\bar{e}(t)$, $\bar{e}_\chi(t)$ and effective masses $\bar{m}(t)$ and $\bar{m}_X(t)$ as a function of the dimensionless scale t, all of them satisfy the equations

$$\frac{\partial \bar{e}}{\partial t} = \beta(\bar{e})\,, \qquad\qquad \frac{\partial \bar{e}_\chi}{\partial t} = \beta(\bar{e}_\chi)\,,$$

$$\frac{\partial \bar{m}}{\partial t} = \bar{m}(t)\,\gamma_m(\bar{e}(t))\,, \qquad \frac{\partial \bar{m}_X}{\partial t} = \bar{m}_X(t)\,\gamma_{m_X}(\bar{e}(t))\,, \tag{94}$$

where $\bar{e}(t=0) = e$, $\bar{e}_\chi(t=0) = \chi_e e$, $\bar{m}(t=0) = m$ and $\bar{m}_X(t=0) = m_X = 17$ MeV. The solution of Eqs. (94) provides the known results of QED for both running coupling constant and running electron mass. The solutions yield the running X-boson vertex with fermions

$$\bar{e}_\chi(t) = \chi_e\, e \left(1 - \frac{\chi_e^2\, e^2\, t}{2\pi^2}\right)^{-1/2}\,. \tag{95}$$

The previous function imposes the vertical asymptote at $t = 2\pi^2/\left(\chi_e^2\, e^2\right)$ and it corresponds to the so-called *Landau singularity*. The $\bar{e}_\chi$-running function is shown in Fig. 2.

Solving Eq. (94), we obtain that

$$\bar{m}_X(t) = 17\,\text{MeV} \left(1 - \frac{\chi_e^2\, e^2\, t}{2\pi^2}\right)\,, \tag{96}$$

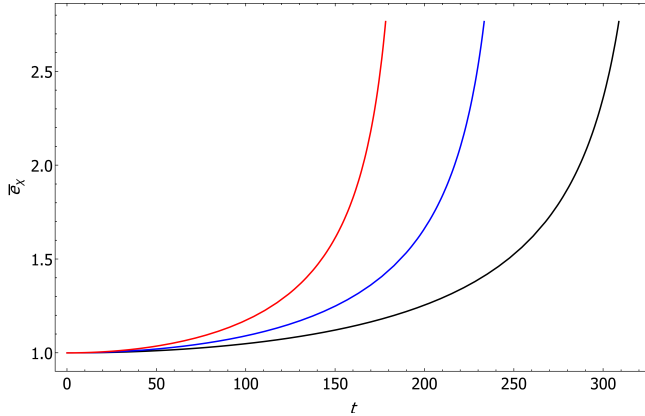which yields the X-boson running mass as a function of arbitrary t-scale.

Fig. 2. (Color online) The running $\bar{e}_\chi/(\chi_e\,e)$-coupling constant as a function of an arbitrary dimensionless t-scale, for $\chi = 6 \times 10^{-3}$ (left/red line), $\chi = 3.5 \times 10^{-3}$ (middle/blue line) and $\chi = 2 \times 10^{-3}$ (right/black line), respectively.

9. Conclusions and final remarks

Recently, we were aware of the introduction of a concept that declares that a new neutral boson explains the experimental anomalies that emerge from the 8-beryllium nuclear decay $^8\mathrm{Be}^* \to {}^8\mathrm{Be} + X$. The solution of this puzzle implies that the invariant mass of the X-boson must be around $m_X = 17$ MeV. This conjecture plays a fundamental role in a possible new physics at the MeV-scale, so that it could be the announcement of a fifth fundamental interaction. Besides, the X-boson couples kinetically through the χ-mixing kinetic with the usual massless photon. Other important property is the protophobic interaction of X-boson with the nucleons of the SM. Thereby, the X-boson introduces an extra Abelian group U(1) in the unification of the fundamental interactions. In this paper, we propose the SM with the extra U(1)$_{B-L}$.

With these ideas in mind, in this work we have investigated the SU$_L(2)\times$ U$_Y(1) \times$ U(1)$_{B-L}$ model with kinetic mixing in the gauge sector, which can describe the interaction between the new X-boson and the leptons of the SM. The Higgs model was introduced to give the mass $m_X = 17$ MeV, that consequently fixes the lower bound of $v_{BL} = 8.5$ MeV, by the recent experimental constraints. Thus, the hidden Higgs is estimated to have a mass within the range of GeV-scale, with a lightest mass relative to the usual Higgs of 125 GeV of the SM. After the spontaneous symmetry breaking mechanism, we have a renormalizable and unitary model in the R_ξ-gauge, with a finishing electromagnetic U(1)$_{\mathrm{em}}$ symmetry.

After that, we have discussed the interaction between the X-boson and the leptons through the elements of QFT, like the decay rate, where we have calculated a X-lifetime within the range of $8.3 \times 10^{-15}\,\text{s} < \tau < 4.2 \times 10^{-12}\,\text{s}$, just taking into account the decays: $X \to e^+\,e^-$, and in neutrino's case, $X \to \bar{\nu}\,\nu$. As a second application, the simplest case is the electron–positron scattering into muon–anti-muon pair, *i.e.*, $e^+\,e^- \to X \to \mu^+\,\mu^-$.

Following the usual QFT, we have computed the contributions of the X-boson mass to the electron physical mass. The perturbation theory for the X-boson full propagator was obtained. We have seen that the vacuum polarization at one loop gave a contribution to the Yukawa potential. The on-shell renormalization indicates the appearance of a complex contribution, as for example, in the case of the X-boson physical mass in Eq. (68), and in the Uehling potential calculation. In fact, this complex renormalization scenario is a consequence of the X-boson instability that decays into the $e^+\,e^-$-pair. The correction to the QED vertex was calculated, and the electron's anomalous magnetic moment $(g-2)_e$ estimates the χ_e-parameter around $|\chi_e| \lesssim 1.4 \times 10^{-3}$, which is in agreement with the literature results. The muon anomalous magnetic moment was also obtained with the X-boson correction, where the result reported from E821 experiment bounds the χ_e-parameter at $|\chi_e| \lesssim 4.43 \times 10^{-5}$.

We have introduced the renormalized model and the renormalization conditions to fix the physical parameters. Thereby, all the renormalization factors were obtained: for the physical fields, masses and the coupling constants. After that, we have applied these results into a renormalization group scheme to obtain the behavior of both, the current X-mass and the X-boson coupling constant with leptons.

It is clear that we have just considered a part of a bigger model in order to include neutrinos and quarks. The extended model $\text{SU}_L(2) \times \text{U}_Y(1) \times \text{U}(1)_{B-L}$ is the candidate to include the corrections to neutrinos/quarks to the X-propagator, together with the contribution of the X-propagator to the $(g-2)_\mu$ muon factor. Moreover, there is also a perspective to include a hidden fermion sector that could have a dark matter feature which interacts with the X-boson.

Other phenomenological approach would be to investigate the $\text{SU}_L(2) \times \text{U}_Y(1) \times \text{U}(1)_{B-L}$ symmetry group with a dark photon A' of mass bounded by $m_{A'} \lesssim 8$ GeV, and to analyze its interactions with the fermion part of the SM and a possible content of a dark matter fermion. It is an ongoing research that will be published elsewhere.

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