A PEDAGOGICAL DISCUSSION OF QUARKYONIC MATTER AND ITS IMPLICATION FOR NEUTRON STARS

Larry McLerran

Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA

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This paper is a brief pedagogical review of the ideas that motivate the concept of Quarkyonic Matter. If $N_{\rm c}$ is the number of quark colors, baryonic matter at very high density and low temperature remains confining to density scales of the order of $N_{\rm c}^{3/2}\Lambda_{\rm QCD}^3$ that is parametrically larger than that of the QCD scale $\Lambda_{\rm QCD}^3$. This implies that a description of nuclear matter will involve both quark and confined degrees of freedom. I argue that the equation of state of Quarkyonic Matter should be very hard, and the sound velocity should rise very rapidly from its small value at nuclear matter densities to a value of the order of 1 at a few times nuclear matter density.

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1. Introduction

In Fig. 1, a hypothetical phase diagram of QCD at finite temperature and density is shown [1]. The only trouble with this figure is that different speakers at different meetings will present different plots as *The Phase Diagram of QCD*. The reason for this is because we do not really know the phase diagram except at finite temperature when the quark chemical potential, μ_Q, is $\mu_Q \ll T$. In this region, there are now excellent experimental data and high quality lattice Monte Carlo computations which determine the thermodynamic properties of strongly interacting matter to good precision. This paper concerns a speculation about Quarkyonic Matter which may exist at very high density and low temperature [2].

At present, the only knowledge we have of the nature of matter at very high baryon density comes from information extracted from neutron stars [3–9], and from the properties of matter near at or below the density of nuclear matter. In these cases, $T \ll \mu_Q$. This information concerns the equation of state or sound velocity. It comes from parameterizing the equation of state of strongly interacting matter in the hydrostatic equations that allow the determination of the masses and radii of observed stars.

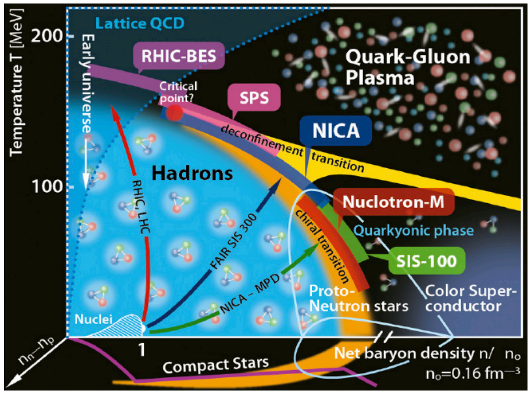

Fig. 1. A popular hypothetical diagram showing possible phases of QCD [1].

All the remaining information in Fig. 1 is based at best on educated conjecture and at worst on wild speculation. The content of this paper is, therefore, largely based on speculation. It is nevertheless true that, in spite of the fact that so little is really known, there are various diverse opinions that are very firmly held. I think the quotation from Mark Twain is very apt: *"It is not what you don't know that gets you in trouble. It is what you think you know but you don't."*

2. Phase structure at large N_c

To understand the phase diagram structure of QCD, it is useful to think in the limit of a large numbers of colors, N_c. In this limit, quark loops are suppressed by one power of N_c compared to gluon loops. The coupling is adjusted so that in the large-N_c limit, $g^2_{\text{'t Hooft}} = g^2 N_c$ is held fixed. In this limit, there is no quark pair production, so that the potential at long range between a static quark–antiquark pair is linear. Baryons are made of N_c quarks and are therefore heavy, with masses of the order of $M_B \sim N_c \Lambda_{\rm QCD}$. Mesons are weakly interacting, with interaction strength of $1/N_c$ [10, 11].

2.1. Finite temperature

At finite temperature, Debye screening of the quark–antiquark potential is induced by gluon loops. These loops are of the order of one in powers of N_c. The Debye screening length is

$$\frac{1}{r^2_{\rm Debye}} \sim g^2_{\text{'t Hooft}} T^2 \,. \tag{1}$$

The Debye screening length becomes of the order of the confinement scale, when $T \sim \Lambda_{\rm QCD}$. There is a deconfinement temperature at $T \sim \Lambda_{\rm QCD}$.

One can also understand the deconfinement transition from the properties of hadronic systems [12]. At low temperatures, there is a gas of weakly interacting mesons. There are no baryons present because they are so massive and their density is exponentially suppressed in $N_{\rm c}$: $\rho \sim {\rm e}^{-E/T} \sim {\rm e}^{-N_{\rm c}\Lambda_{\rm QCD}/T}$. In contrast, the density of mesons is of the order of one in powers of $N_{\rm c}$ because mesons are color singlets. On the other hand, at high temperatures, there is a gas of gluons with a density of the order of $N_{\rm c}^2$ because gluons are in the adjoint representation of the color group. The transition is the result of an exponentially growing density of states that leads to a Hagedorn limiting temperature where the partition function diverges. For the finite $N_{\rm c}$, this is cut off when the meson density becomes of the order of $N_{\rm c}^2$, and although the meson–meson cross section is of the order of $1/N_{\rm c}^2$, the interaction energy becomes of the order of $N_{\rm c}^2$.

With zero mass quarks present, there is in addition a chiral symmetry restoration temperature. For two or three flavors, chiral restoration seems to be at the temperature of the deconfinement transition. For the finite $N_{\rm c}$, the deconfinement transition disappears as does the chiral transition for realistic quark masses. The phase transitions are replaced by crossovers. One identifies the transition temperature by extrapolating results to the limit of very small quark masses where the chiral transition becomes a real phase transition.

2.2. Finite density

At finite density and zero temperature, Debye screening is generated only by quark loops. For a quark chemical potential μ_Q, the Debye screening length is

$$1/r^2_{\rm Debye} \sim g^2_{\rm 't\,Hooft}\,\mu_Q^2/N_{\rm c}\,. \tag{2}$$

In the infinite $N_{\rm c}$ limit, the quarks never can Debye screen away a linear potential. The deconfining temperature is independent of $N_{\rm c}$. For finite but large $N_{\rm c}$, the chemical potential at which there is deconfinement is very large

$$\mu_Q \sim \sqrt{N_{\rm c}}\,\Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}\,. \tag{3}$$

This condition follows from setting the Debye screening mass to be the confinement scale. This is shown in Fig. 2.

This result is absolutely amazing [2]. Confinement remains until quark energy scales are huge compared to the confinement scale. This is in spite of the fact that most interactions of the quarks are at energy scales gigantic compared to the confinement scale and where one would naively believe one can use weakly coupled perturbation theory.

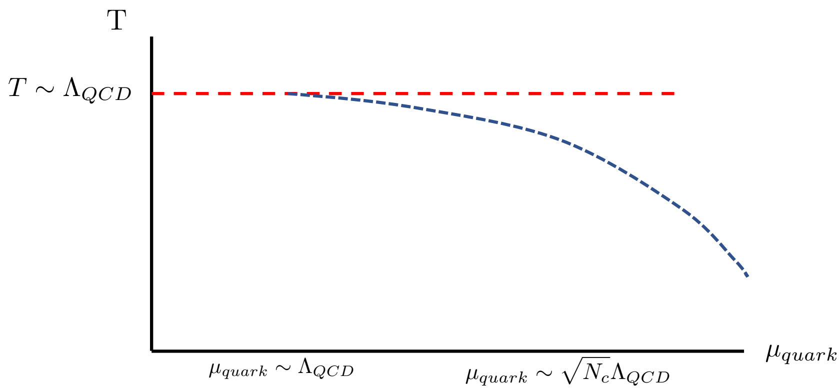

Fig. 2. (Color online) The deconfinement temperature in this plot is the long-dashed red line for infinite N_c and the dashed blue line for finite but large N_c.

High density baryonic matter, with $\mu_Q \geq \Lambda_\mathrm{QCD}$, is further differentiated from ordinary hadronic matter. First, we note that the baryon number chemical potential is $\mu_\mathrm{B} = N_\mathrm{c}\,\mu_Q$, so that for $\mu_\mathrm{B} > M_N$, where M_N is the nucleon mass, there must be baryons. On the other hand, we expect that the density of baryons is

$$n_\mathrm{baryon} \sim \mathrm{e}^{(\mu_\mathrm{B}-E_\mathrm{B})/T} \sim \mathrm{e}^{N_\mathrm{c}(\mu_Q-E_Q)/T}. \tag{4}$$

Therefore, hadronic matter will have no baryon number density for $\mu_Q < M_N/N_\mathrm{c} \sim \Lambda_\mathrm{QCD}$.

In addition, when the energy density of hadronic matter is of the order of one in the number of colors, but about the density at which baryons are present, the energy density will go like N_c, because of the baryons.

In the large-N_c limit, there is, therefore, a world of confined mesons with no baryons, or hadronic matter, a world of deconfined matter, or the quark–gluon plasma, and world where matter is confined into mesons, baryons and glueballs, even though the baryon density can be huge compared to the QCD scale. This last phase of matter we call Quarkyonic [2], because although it is confined, we expect that we can, in many cases, describe its constituents and their interactions as quarks, since most interactions will be at very high energy (for a review, see lecture III in Ref. [13]). The resulting phase diagram is shown in Fig. 3.

The picture of Quarkyonic Matter is that deep inside the baryonic Fermi sea, interactions are controlled by exchange interactions. These interactions are not so sensitive to the infrared, and we imagine that the degrees of freedom that allow us to compute these interactions are quarks. At the Fermi surface, interactions become sensitive to the infrared and one must account

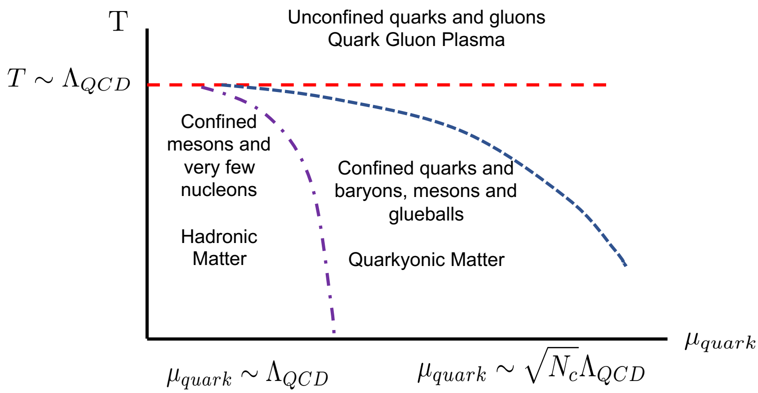

Fig. 3. A phase diagram of QCD appropriate for large N_c that includes Quarkyonic Matter.

for confinement. The degrees of freedom at the Fermi surface are confined, being therefore glueballs, mesons and nucleons. This is schematically illustrated in Fig. 4.

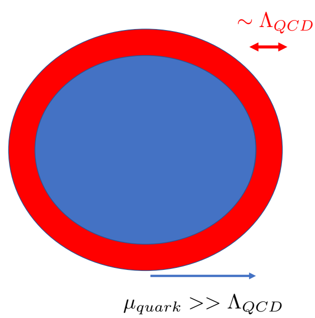

Fig. 4. (Color online) A Fermi sphere of quarks (inner/blue) surrounded by a shell of nucleonic matter (outer/red).

We need to understand how chiral symmetry breaking might occur. In the vacuum, chiral symmetry breaking is generated due to a sigma meson condensate forms. The sigma meson may be thought of as a bound state of a quark with positive binding energy with a hole, or antiparticle, in the negative energy Dirac sea. At finite baryon density, this can occur with the essential modification that the quark has an energy slightly above the Fermi energy, or chemical potential of the baryon Fermi sea, and the antiquark is a hole just below this Fermi energy. Due to this Fermi energy, the particle–hole pair has a net momentum of $2\mu_Q$ and the energy of the moving bound state must be compared to that of the $2\mu_Q$. This means that the sigma meson condensation will have net momentum, and the condensate of sigma mesons will break both rotational and translational invariance. This is similar to the problem of charge density wave in condensed matter physics. The condensates may end up making structure of crystals and quasi-crystals [14–16].

In Fig. 5, a summary of the structure we might expect for a phase diagram of QCD is shown.

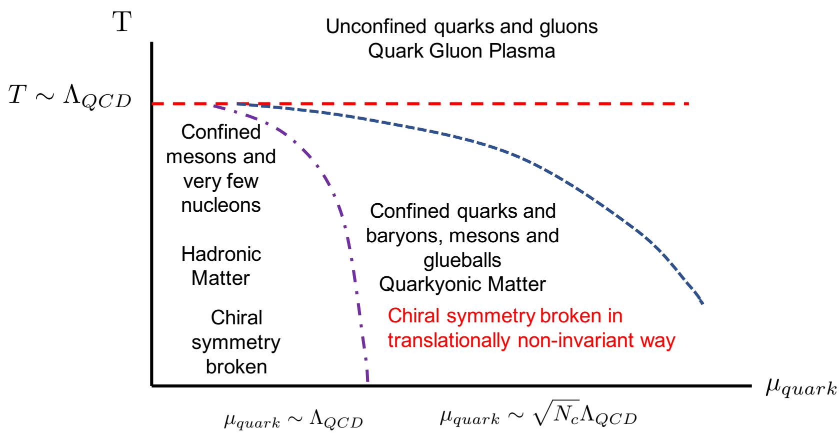

Fig. 5. A hypothetical phase diagram of QCD in the large-N_c limit.

3. Why quarkyonic matter can give hard equations of state

First, consider the properties of hadronic matter near and above the density of nuclear matter. In QCD for three colors, Fermi momentum of nuclear matter is of the order of Λ_{QCD} and Fermi momentum $k_{\mathrm{F}} \sim 200$–300 MeV. This means that the binding energy and kinetic energies of such a gas are small, $k_{\mathrm{F}}^2/2M_N \sim \Lambda_{\mathrm{QCD}}/N_{\mathrm{c}}$. By some miracle, nuclear matter arranges itself so that the typical energy scales are quite small, $\sim 1/N_{\mathrm{c}}$, even though the typical nuclear interactions are of the order of N_{c}. This can happen if the nuclear matter is a somewhat dilute gas. Then, as matter becomes squeezed, interactions become important, and the typical interaction energy becomes of the order of N_{c}. This suggests that hadronic matter can change its properties very rapidly, when it is squeezed to the density where nucleon hard cores overlap. This density is parametrically close to that of nuclear matter.

This seems to be what really occurs, based on an analysis of the masses and radii of neutron stars [3–9]. In Fig. 6, an analysis of neutron star masses and radii produces a sound velocity curve that rises very rapidly as a function of density. The sound velocity squared exceeds $1/3$ at a density of 3–4 times that of nuclear matter. This is a rapid increase by a factor of

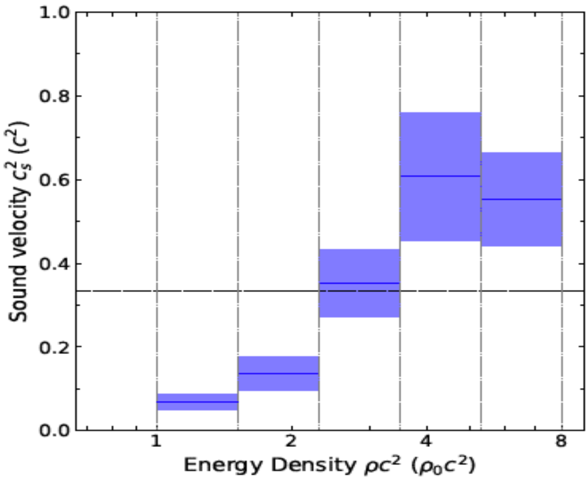

Fig. 6. The sound velocity as a function of density from the analysis of Fujimoto *et al.* [9].

1–2 orders of magnitude. On the other hand, the sound velocity must have a maximum, since at asymptotically high baryon number density, the sound velocity squared must approach 1/3 *from below.*

There are some generic conclusions one can draw from this phenomenological observation. The sound velocity of a zero temperature Fermi gas is

$$v_{\rm s}^2 = \frac{n_{\rm B}}{\mu_{\rm B}\,{\rm d}n_{\rm B}/{\rm d}\mu_{\rm B}}\,. \tag{5}$$

In this equation, $n_{\rm B}$ denotes the baryon number density, and the fully relativistic Fermi energy or chemical potential for baryon number that includes the nucleon mass is denoted as $\mu_{\rm B}$. Equation (5) implies parametrically that

$$\frac{\delta\mu_{\rm B}}{\mu_{\rm B}} \sim v_{\rm s}^2\frac{\delta n_{\rm B}}{n_{\rm B}}\,. \tag{6}$$

This relation implies that when the sound velocity becomes of the order of one, and when the baryon number density changes by the order of one, then the baryon chemical potential also changes by the order of one. The baryon chemical potential very near to nuclear matter density is very close to the nucleon mass. Therefore, the baryon number chemical potential minus the nucleon mass is of the order of the nucleon mass itself. The energy scales have become relativistic, even though the baryon number density has barely changed! If nuclear matter is described by nucleons as quasi-particles, then their phase space will increase like $k_{\rm F}^3$, and the rapid increase in phase space available without a corresponding increase in the baryon density suggests that the nucleons are only partially filling their available phase space. This observation suggests that the nucleons are occupying a shell of phase space at high momenta. This is precisely what happens in Quarkyonic Matter. In Quarkyonic Matter, the nucleons do not fill the Fermi sphere but a shell near the Fermi surface

To understand a little better what might occur, we need to remember that $\mu_Q \sim \mu_{\mathrm{B}}/N_{\mathrm{c}}$. Therefore, if the typical baryon momentum is of order the nucleon mass, then the quark chemical potential would be very low, and there would be very few quarks present. Only when the baryon chemical potential becomes large, can quarks be important. However, if the nucleons are in a Fermi sphere, the baryon density would be enormous. A viable possibility is that the nucleons sit in a shell at the Fermi surface of decreasing thickness as the density increases. The decreasing thickness of the shell compensates for the increasing Fermi momentum. The quarks sit under this shell, but they contribute very little to the overall density until the nucleon Fermi energy is of the order of $N_{\mathrm{c}}M_N$ that is, the nucleons become relativistic.

It is useful to see this more explicitly. Recall that an ideal gas of nucleons has

$$n_{\mathrm{B}}^n = \frac{2}{3\pi^2}\left(k_{\mathrm{F}}^n\right)^3 \tag{7}$$

and quarks have

$$n_{\mathrm{B}}^q = \frac{2}{3\pi^2}\left(k_{\mathrm{F}}^q\right)^3 . \tag{8}$$

In this equation, the Fermi momenta of nucleons and quarks are represented as k_{F}^q and k_{F}^n. The baryon number density of quarks and that of nucleon is identical when expressed in terms of their Fermi momenta. The problem is that of course the quark Fermi momentum is $1/N_{\mathrm{c}}$ smaller than that of the nucleons. So if we take relativistic gas of nucleons and compress it into a shell so that the density does not increase, until the typical momentum of the nucleons is very large, the quarks make little contribution to the density. One can construct a parallel argument to this for the energy density.

The argument is different for the pressure. The pressure for the nucleon for a non-relativistic gas is of the order of

$$P \sim \left(\frac{k_{\mathrm{F}}^B}{M}\right)^2 \epsilon_N , \tag{9}$$

where ϵ_N is the energy density. This gives a sound velocity that is of the order of $1/N_{\mathrm{c}}^2$ when the nucleons are relativistic but becomes of the order of one when the nucleons form a relativistic shell. The change in sound velocity occurs with an the order of one change in baryon number density. Note also that the quarks only begin to make a contribution to the pressure once the nucleon shell has formed.

The baryon chemical potential minus the nucleon mass also changes from the order of $\Lambda_{\mathrm{QCD}}/N_{\mathrm{c}}$ to the order of $N_{\mathrm{c}}\Lambda_{\mathrm{QCD}}$ in this narrow range of density.

It is important to note that the transition to Quarkyonic Matter is not a phase transition. In a phase transition, the pressure and chemical potential remain fixed, and in a first order phase transition, the number density and energy density vary. For the Quarkyonic Matter transition, completely the opposite occurs: The chemical potential and pressure vary rapidly, while the energy density and baryon number density do not. This is precisely what is needed to generate hard equations of state.

It is possible to generate a simple parametrization of the shell width that has sound velocities that correspond to what is phenomenologically observed. One has to match on to acceptable low-density equations of state at baryon densities of the order of twice that of nuclear matter. Generically, there is a maximum in the sound velocity. It is not surprising that this is the baryon density where nuclear matter is transforming into Quarkyonic Matter.

4. Dynamics of the formation of the nucleon shell

Perhaps the simplest way of getting a description of Quarkyonic Matter is to assume a quasi-particle description [17, 18]. We treat the quarks as existing in a Fermi sphere up to a quark Fermi energy. We treat the quarks as massive with a constituent mass

$$M_q = M_N/N_{\mathrm{c}}\,. \tag{10}$$

The top of the quark Fermi sea is an energy equal to μ_q. We will consider here isosinglet quark matter where there are equal numbers of up and down quarks. For the nucleons, we have equal numbers of protons and neutrons and, therefore, equal numbers of up and down quarks. Using the constituent quark model, the nucleon energy of quarks inside a nucleon is $1/3$ of the nucleon energy. This means the energy at the bottom of the nucleon Fermi sea is $E^{\mathrm{bottom}}_{\mathrm{nucleon}} = N_{\mathrm{c}}\,\mu_q$. Therefore, we take the nucleons to be in a shell between this lower energy and that of the top of the Fermi sea.

A simple way to describe the interactions of nucleons is to treat them as free particles in a volume that is reduced by a volume equal to that of their hard-core interactions [19–21]. If we let $n_0 = 1/(4/3\pi r_0^3)$ be the density scale associate with the hard core, then the baryon density is

$$\frac{n}{1-n/n_0} = 2\int^{k_{\mathrm{F}}} \frac{\mathrm{d}^3k}{(2\pi)^3}\,. \tag{11}$$

We see that the density of baryon in such a theory is limited by the hard-core density n_0. As this density is approached, the nucleon contributions must saturate. This occurs because the typical nucleon momentum grows. This means that the Fermi momentum at the top of the quark Fermi sea grows. The Fermi sea of quarks can accommodate more baryon density, while the momentum space thickness of the shell decreases, so that the total baryon density in the shell stops growing. This can be seen in an explicit model where the thickness of the sheet is determined by energy minimization.

We can understand what happens qualitatively: As the hard-core density is approached, the sound velocity rapidly rises. This is due to the rapid increase in the typical nucleon momenta, as argued above. At some point, either the sound velocity must saturate or must have a maximum, as the quarks begin to dominate the dynamics. Asymptotically, the sound velocity squared must approach 1/3, because this it the limit for a free quark gas.

This is shown to happen explicitly in Ref. [18]. Such a maximum or saturation is that occurs in determinations of equations of state for neutron start properties.

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