

QZE AND IZE IN A SIMPLE APPROACH
AND THE NEUTRON DECAY*

FRANCESCO GIACOSA

Institute of Physics, Jan Kochanowski University
Uniwersytecka 7, 25-406 Kielce, Poland*(Received April 16, 2020)*

We discuss a simple and analytically solvable measurement model which describes the famous Quantum Zeno Effect (QZE) and Inverse Zeno Effect (IZE), that correspond to the slow down and to the increase of the decay rate caused by measurements (or, more general, by the interaction of an unstable state with the detector and the environment). Within this model, one can understand quite universal features of the QZE and IZE: by considering an unstable quantum state, such as an unstable particle, whose decay width as a function of energy is $\Gamma(\omega) = g^2 \omega^\alpha$, then — under quite general assumptions — the QZE occurs for $\alpha \in (0, 1)$, while the IZE for $\alpha \in (-\infty, 0) \cup (1, \infty)$. This result is also valid for more realistic measurement models than the one described in this work. We then apply these considerations to the decay of the neutron, for which $\alpha = 5$. Hence, the realization of the IZE for the neutron decay (and for the majority of weak decays) is in principle possible. Indeed, trap experiments find a lifetime that is 8.7 ± 2.1 s shorter than beam experiments, suggesting that the IZE could have taken place.

DOI:10.5506/APhysPolB.51.1345

1. Introduction

The Quantum Zeno Effect (QZE) and the Inverse Zeno Effect (IZE) are the slow down and the increase of the decay rate of an unstable state (or particle) when it is ‘measured’ often enough, see *e.g.* Refs. [1–7] for a review. Both the QZE and the IZE are a consequence of the fact that the decay law is not exactly exponential [8–13]. The experimental verification of nonexponential decays both at short and long times can be found in Refs. [14, 15], while the QZE and IZE on a genuine unstable quantum system (tunnelling through an optical potential) is described in Ref. [16].

* Presented at XXVI Cracow Epiphany Conference on LHC Physics: Standard Model and Beyond, Kraków, Poland, January 7–10, 2020.

Basically, each system can undergo the QZE if probed at short enough time intervals. Here, with ‘probed’ we do not necessarily mean a standard textbook measurement, but also a sufficiently strong interaction of the system with the environment that can lead to a decoherence which is for all practical purposes analogous to a measurement. As discussed in Refs. [3, 4, 6], the IZE can be even easier to realize than the QZE if some conditions are met, most importantly a strong — but not too strong — system–environment interaction.

In this work, we first briefly review in Sec. 2 the main features of the QZE and IZE: the key element is the so-called response function, which models the environment–system interaction. If the system is — in average — probed at a certain given rate, even if the underlying decay law is not an exponential, one still measures an exponential decay law, whose decay width (the inverse of the lifetime) is, however, different from the ‘on-shell’ or bare value obtained in the limit in which the unstable state weakly couples to the environment (for instance, by doing a single collapse measurement after a sufficiently long time). The effective or measured decay width emerges as the average of the decay width as a function of the energy convoluted with the previously-mentioned response function.

Next, in Sec. 3, we present a simple model for the response function which allows to present in a clear way under which conditions the QZE and IZE are realized. Even if this model may be regarded as *too simple* to be realistic, one can also understand results that go beyond the specific employed form. Namely, one can understand why the IZE is actually even more general than the QZE. By denoting the decay width function as $\Gamma(\omega)$ and assuming that in the energy range of interest $\Gamma(\omega) = g^2\omega^\alpha$, we show that — under quite broad assumptions — the QZE occurs for $\alpha \in (0, 1)$, while the IZE for the much broader range of $\alpha \in (-\infty, 0) \cup (1, \infty)$.

In particular, the IZE is expected to take place in the case of weak decays, since $\alpha > 1$. As a specific applications of our considerations, we present in Sec. 4 the example of the decay of the neutron, for which $\alpha = 5$. At present, an anomaly exists [17]: beam experiments which measure the emitted protons find the lifetime $\tau_n^{\text{beam}} = 888.1 \pm 2.0$ s, while trap or cavity experiments which monitor the surviving neutrons deliver the result $\tau_n^{\text{trap}} = 879.37 \pm 0.58$ s. As recently proposed in Ref. [18], the possibility that the mismatch is due to the IZE realized in trap experiments is discussed.

Finally, in Sec. 5, we present our conclusions.

2. QZE and IZE: general discussion

In this section, we summarize the results of Refs. [3–6], where a theoretical approach for the description of the measurement has been put forward. A decay process of an unstable state (or particle) called n is described by the decay function $\Gamma(\omega)$. The energy ω reads

$$\omega = m - \sum_{j=1}^N m_j, \quad (1)$$

where m_j are the ‘masses’ (or energies) of the N decay products of the state n , and m is the ‘running mass’ of the state. The quantity ω (and so m) can vary, since the mass of an unstable state is not fixed. Moreover, $\omega \geq 0$, since the running mass cannot be smaller than the sum of the masses in the final state. The on-shell value is obtained for

$$\omega_{\text{on-shell}} \equiv \omega_n = m_n - \sum_{j=1}^N m_j, \quad (2)$$

where

$$m_n = m_{\text{on-shell}}. \quad (3)$$

The ‘on-shell’ decay width is given by

$$\Gamma_n = \Gamma(\omega_n) = \Gamma_{\text{on-shell}} = \frac{1}{\langle t_n \rangle}, \quad (4)$$

where $\langle t_n \rangle$ is the mean lifetime of the unstable state n .

The form of the function $\Gamma(\omega)$ can be evaluated in the framework of the given model/approach. One possibility goes through the so-called Lee model [19] (see also Refs. [13, 20–23] and references therein) or within a certain given quantum field theoretical approach, *e.g.* Refs. [24, 25] (for the link of QFT to nonexponential decays, see also Refs. [12, 13, 26]).

A general result is that, in presence of a series of measurements and/or interactions of the system with the environment, the effective measured decay width may change according to the weighted average

$$\Gamma^{\text{measured}}(\tau) = \int_0^\infty f(\tau, \omega) \Gamma(\omega) d\omega, \quad (5)$$

where the parameter $\tau = \lambda^{-1}$ (with λ being the corresponding rate) describes how strong is the coupling of the environment with the system: large τ (small λ) means weak coupling (in which one should recover the on-shell decay width of Eq. (4)), while small τ (large λ) implies a strong coupling, in which deviations from the on-shell width are expected.

The function $f(\tau, \omega)$ can be regarded as the ‘response function’ of the environment/detector on the quantum system. Its form is generally peaked and symmetric w.r.t. ω_n , but the details depend on the system–environment–detector interaction(s). Nevertheless, three general constraints are

$$(i) \quad \int_0^{\infty} f(\tau, \omega) d\omega = 1, \quad (6)$$

$$(ii) \quad f(\tau \rightarrow \infty, \omega) = \delta(\omega - \omega_n), \quad (7)$$

$$(iii) \quad f(\tau \rightarrow 0, \omega) = \text{small constant}. \quad (8)$$

The first condition in Eq. (6) guarantees the normalization. As a consequence, in the Breit–Wigner limit, in which $\Gamma(\omega) = \Gamma_{\text{BW}}$ is a simple constant and no deviation from the exponential decay occurs [27], one has

$$\Gamma^{\text{measured}}(\tau) = \int_0^{\infty} f(\tau, \omega) \Gamma(\omega) d\omega = \Gamma_{\text{BW}} \int_0^{\infty} f(\tau, \omega) d\omega = \Gamma_{\text{BW}}, \quad (9)$$

for each measurement function $f(\tau, \omega)$. Then, as expected, neither QZE nor the IZE can take place. This case is however unphysical, since a constant decay width and the corresponding Breit–Wigner distribution are only an approximation (which in many cases is so good that it is hard to see any difference).

The second condition in Eq. (7) assures that, if the system is undisturbed, one obtains the ‘on-shell’ free decay width

$$\Gamma^{\text{measured}}(\tau \rightarrow \infty) = \Gamma_n = \Gamma_{\text{on-shell}}. \quad (10)$$

Finally, the third condition in Eq. (8) implies that, for τ very small, $f(\tau \rightarrow 0, \omega)$ is a (small) constant, hence

$$\Gamma^{\text{measured}}(\tau \rightarrow 0) = (\text{small constant}) \int_0^{\infty} \Gamma(\omega) d\omega \rightarrow 0, \quad (11)$$

assuming the convergence of $\int_0^{\infty} \Gamma(\omega) d\omega$: this is the famous QZE mentioned above.

The functional form of $f(\tau, \omega)$ depends on which type of measurement is performed. Two famous forms were considered in Refs. [3, 6]. For the case of instantaneous ideal Bang-Bang measurements at time intervals $0, \tau, 2\tau, \dots$, one gets

$$f(\tau, \omega) = \frac{\tau}{2\pi} \frac{\sin^2[\tau(\omega - \omega_n)/2]}{[\tau(\omega - \omega_n)/2]^2}. \quad (12)$$

In the case of a continuous measurement (in the form of *e.g.* a continuous detector–system interaction, see Refs. [3, 4] for details; for the general concept of a continuous monitoring, see also Refs. [28–30]), one gets

$$f(\tau, \omega) = \frac{1}{\pi\tau} \frac{1}{(\omega - \omega_n)^2 + \tau^{-2}}. \quad (13)$$

More general, the response function is not solely caused by measurements. The time-scale τ may be regarded as the dephasing/decoherence time for the whole environment–object–detector system. Actually, in various physical examples, the value of τ determined by the environment is more efficient than the actual measurements performed by a detector [31].

One may also note that the very convergence of Eq. (9) is not necessarily guaranteed. This is why in various applications one needs to further restrict the off-shellness of the unstable state to a certain range, upon replacing $\int_0^\infty d\omega[\dots] \rightarrow \int_{\omega_n - \Delta E}^{\omega_n + \Delta E} d\omega[\dots]$. Moreover, also the normalization (i) of Eq. (6) is not fulfilled for the functions in Eqs. (12) and (13) (even if numerically very well realized).

In the next section, we describe a simple model which fulfills all conditions (i), (ii), and (iii) exactly and — in addition — guarantees always the convergence of $\Gamma^{\text{measured}}(\tau)$.

3. QZE and IZE: a simple model

Here, as a concrete and simple model, we introduce the following rectangular response function:

$$f_{\text{rect}}(\tau = \lambda^{-1}, \omega) = N_\lambda \theta(\omega) \theta\left(\lambda^2 - (\omega - \omega_n)^2\right). \quad (14)$$

The constraint N_λ is necessary to guarantee condition (i) of Eq. (6)

$$N_\lambda = \begin{cases} \frac{1}{2\lambda} & \text{for } \omega_n - \lambda > 0 \\ \frac{1}{\omega_n + \lambda} = \frac{1}{\omega_C} & \text{for } \omega_n - \lambda > 0 \end{cases}, \quad (15)$$

where the upper limit $\omega_C = \omega_n + \lambda$ has been introduced and where the subscript C stands for (high-energy) cutoff. Note, for $\omega_n - \lambda > 0$, the function takes the form of

$$f_{\text{rect}}(\tau = \lambda^{-1}, \omega) = \begin{cases} 0 & \text{for } |\omega - \omega_n| > \lambda \\ \frac{1}{2\lambda} & \text{for } |\omega - \omega_n| \leq \lambda \end{cases}. \quad (16)$$

It then follows that for $\lambda \rightarrow 0$ (that is, $\tau \rightarrow \infty$), the function $f_{\text{rect}}(\tau = \lambda^{-1}, \omega)$ is a possible representation of the δ -function:

$$f_{\text{rect}}(\tau \rightarrow \infty, \omega) = \delta(\omega - \omega_n), \quad (17)$$

hence the condition (ii) of Eq. (7) is also guaranteed

$$\Gamma^{\text{measured}}(\tau \rightarrow \infty) = \int_0^\infty f_{\text{rect}}(\tau \rightarrow \infty, \omega) \Gamma(\omega) d\omega = \Gamma(\omega_n) = \Gamma_{\text{on-shell}}. \quad (18)$$

Next, let us consider the limit $\lambda \rightarrow \infty$. It is then clear that

$$f_{\text{rect}}(\tau = \lambda^{-1}, \omega) = \frac{1}{\omega_n + \lambda} \theta(\omega) \theta(\omega_n + \lambda - \omega) \stackrel{\lambda \gg \omega_n}{\simeq} \frac{1}{\lambda} \theta(\omega) \theta(\lambda - \omega). \quad (19)$$

Ergo, the QZE is easily realized (condition (iii) of Eq. (8))

$$\Gamma^{\text{measured}}(\tau) = \int_0^\infty f_{\text{rect}}(\tau \rightarrow 0, \omega) \Gamma(\omega) d\omega = \lim_{\lambda \rightarrow \infty} \frac{1}{\lambda} \int_0^\lambda \Gamma(\omega) d\omega = 0, \quad (20)$$

as long as $\int_0^\infty \Gamma(\omega) d\omega$ is finite (as it must be in each physical case).

Thus, all the conditions are fulfilled and — in addition — the response function cuts abruptly energies outside a certain range and is constant within a given range. This is different from Eqs. (12) and (13): the question if the response function in Eq. (14) can be — at least in some cases — partially realistic is hard to answer. Yet, as we shall see below, it is useful to show quite general properties.

Next, let us consider the case in which $\omega_n - \lambda > 0$ and assume that, within the range $(\omega_n - \lambda, \omega_n - \lambda)$, we can approximate the decay function as

$$\Gamma(\omega) = g^2 \omega^\alpha \quad \text{for } \omega \in (\omega_n - \lambda, \omega_n - \lambda). \quad (21)$$

This is only an approximation, but as long as λ is small enough, one may consider $g^2 \omega^\alpha$ as the dominant contribution. Yet, even in the case in which this approximation is not possible, one can always consider $\Gamma(\omega)$ as a polynomial function, hence one can easily generalize the argument that we are about to present. Indeed, if λ is very large, also such an approximation would break down. A typical expression that would include a form factor is given by

$$\Gamma(\omega) = g^2 \omega^\alpha e^{-(\omega - \omega_n)^2 / \Lambda^2}, \quad (22)$$

which guarantees the necessary convergence to guarantee the realization of QZE, see Eqs. (11) and (20), if λ is large enough.

We come back to the approximation of Eq. (21). The integral can be solved exactly

$$\begin{aligned} \Gamma^{\text{measured}}(\tau) &= \frac{1}{2\lambda} \int_{\omega_n - \lambda}^{\omega_n + \lambda} \Gamma(\omega) d\omega = \frac{g^2}{2\lambda} \frac{1}{\alpha + 1} (\omega^{\alpha+1})_{\omega_n - \lambda}^{\omega_n + \lambda} \\ &= \frac{g^2}{2\lambda} \frac{1}{\alpha + 1} \left((\omega_n + \lambda)^{\alpha+1} - (\omega_n - \lambda)^{\alpha+1} \right) \\ &= \frac{g^2}{2\lambda} \frac{\omega_n^{\alpha+1}}{\alpha + 1} \left((1 + x)^{\alpha+1} - (1 - x)^{\alpha+1} \right), \end{aligned} \quad (23)$$

where the ratio

$$x = \frac{\lambda}{\omega_n} \quad (24)$$

has been introduced. The number x is expected to be safely smaller than unity. Next, let us consider the following Taylor expansion up to third order:

$$(1+x)^{\alpha+1} = 1 + (\alpha+1)x + \frac{1}{2}(\alpha+1)\alpha x^2 + \frac{1}{3!}(\alpha+1)\alpha(\alpha-1)x^3 + \dots \quad (25)$$

Note, going up to x^3 is necessary for our purposes. By plugging in

$$\begin{aligned} \Gamma^{\text{measured}}(\tau) &= \frac{g^2}{2\lambda} \frac{\omega_n^{\alpha+1}}{\alpha+1} \\ &\times \left[1 + (\alpha+1)x + \frac{1}{2}(\alpha+1)\alpha x^2 + \frac{1}{3!}(\alpha+1)\alpha(\alpha-1)x^3 + \dots \right. \\ &\quad \left. - \left(1 - (\alpha+1)x + \frac{1}{2}(\alpha+1)\alpha x^2 - \frac{1}{3!}(\alpha+1)\alpha(\alpha-1)x^3 \right) \right], \quad (26) \end{aligned}$$

we obtain

$$\begin{aligned} \Gamma^{\text{measured}}(\tau) &= \frac{g^2}{2\lambda} \frac{\omega_n^{\alpha+1}}{\alpha+1} \left[2(\alpha+1)x + \frac{2}{3!}(\alpha+1)\alpha(\alpha-1)x^3 + \dots \right] \\ &= \frac{g^2}{2\lambda} \frac{\omega_n^{\alpha}\omega_n}{\alpha+1} \left[2(\alpha+1)\frac{\lambda}{\omega_n^{\alpha}} + \frac{2}{3!}(\alpha+1)\alpha(\alpha-1) \left(\frac{\lambda}{\omega_n} \right)^3 + \dots \right] \\ &= g^2 \omega_n^{\alpha} \left[1 + \frac{1}{3!}\alpha(\alpha-1) \left(\frac{\lambda}{\omega_n} \right)^2 + \dots \right]. \quad (27) \end{aligned}$$

We then find the following result, which can be regarded as the main achievement of the present work:

$$\Gamma^{\text{measured}}(\tau) = \Gamma_n \left[1 + \frac{\alpha(\alpha-1)}{6} \frac{\lambda^2}{\omega_n^2} + \mathcal{O}\left(\frac{\lambda^4}{\omega_n^4}\right) \right]. \quad (28)$$

One sees that the result depends on α and, in particular, on the sign of the quantity $\alpha(\alpha-1)$. We have

$$\text{QZE: } \Gamma^{\text{measured}}(\tau) < \Gamma_n \quad \text{if } 0 < \alpha < 1, \quad (29)$$

the well-known QZE is realized. We recall that the QZE is anyhow realized if τ is small enough (λ large enough, see Eqs. (11) and (20)), but it can be also realized for a relatively large value of τ if the condition $0 < \alpha < 1$ is met.

Next, the IZE takes place for

$$\text{IZE: } \Gamma^{\text{measured}}(\tau) > \Gamma_n \quad \text{if} \quad \alpha < 0 \text{ or } \alpha > 1. \quad (30)$$

Thus, one can see that the IZE is actually easier to obtain than the QZE (of course, for sufficiently small (but not too small) $\tau = \lambda^{-1}$). In most physical cases, indeed $\alpha > 1$ is realized.

In between, one has

$$\Gamma^{\text{measured}}(\tau) = \Gamma_n \quad \text{for} \quad \alpha = 0 \quad \text{and} \quad \alpha = 1. \quad (31)$$

This result is expected for $\alpha = 0$ (this is the Breit–Wigner limit) but, quite interestingly, holds also for $\alpha = 1$, when $\Gamma(\omega) = g^2\omega$ is linear.

This result can be actually extended to any symmetric response function

$$f(\tau, \omega) = \sum_k c_k f_{\text{rect}}(\tau_k = \lambda_k^{-1}, \omega), \quad (32)$$

where all c_k are positive functions of τ and are such that $\sum_k c_k = 1$. For instance, the functions in Eqs. (12) and (13) can be re-expressed in this way. It follows that

$$\Gamma^{\text{measured}} = \Gamma_n \left[1 + \sum_k c_k \frac{\alpha(\alpha-1)}{6} \frac{\lambda^2}{\omega_n^2} + \mathcal{O}\left(\frac{\lambda^4}{\omega_n^4}\right) \right], \quad (33)$$

hence the final results of Eqs. (29), (30) and (31) are still valid for a generic response function that fulfills Eq. (32).

Another interesting case is obtained when two (or more) terms are present (for instance, as in the case of different decay channels, an interesting topic in nonexponential decay [13, 32])

$$\Gamma(\omega) = g_1^2 \omega^\alpha + g_2^2 \omega^\beta, \quad (34)$$

out of which

$$\Gamma^{\text{measured}}(\tau) = \Gamma_n^{(1)} \left[1 + \frac{\alpha(\alpha-1)}{6} \frac{\lambda^2}{\omega_n^2} \right] + \Gamma_n^{(2)} \left[1 + \frac{\beta(\beta-1)}{6} \frac{\lambda^2}{\omega_n^2} \right]. \quad (35)$$

It is then clear that if both α and $\beta \in (0, 1)$, the QZE is realized, while otherwise the IZE takes place. Yet, if $\alpha \in (0, 1)$ and β does not (or *vice versa*), then there are two conflicting phenomena and no general statement can be made: the precise values of the coupling constants are necessary to assess if the decay width has decreased or increased.

Next, we consider the case in which ω_n is close enough to 0 (the lowest possible energy) such that $\lambda > \omega_n$. In this case, one has

$$\Gamma^{\text{measured}}(\tau) = \int_0^\infty f_{\text{rect}}(\tau, \omega) \Gamma(\omega) d\omega = \frac{1}{\omega_C} \int_0^{\omega_C} \Gamma(\omega) d\omega \quad (36)$$

$$= \frac{1}{\omega_C} g^2 \frac{\omega_C^{\alpha+1}}{\alpha+1} = \Gamma_n \frac{1}{\alpha+1} \frac{\omega_C^\alpha}{\omega_n^\alpha} \quad (37)$$

$$= \Gamma_n \left[1 + \frac{(\omega_C/\omega_n)^\alpha - (\alpha+1)}{\alpha+1} \right], \quad (38)$$

where $\omega_C = \omega_n + \lambda \geq 2\omega_n$. Then, one has

$$\text{IZE} \quad \text{for} \quad \alpha < 0 \quad \text{and for} \quad \alpha > \alpha_0 > 0 \quad (39)$$

with

$$(\omega_C/\omega_n)^{\alpha_0} = \alpha_0 + 1. \quad (40)$$

Since $\omega_C/\omega_n \geq 2$, it turns out that $\alpha_0 < 1$: *the range for the IZE increases even further.*

The QZE is confined to the interval

$$\text{QZE} \quad \text{for} \quad 0 < \alpha < \alpha_0 < 1, \quad (41)$$

thus the corresponding range decreased. Finally,

$$\Gamma^{\text{measured}}(\tau) = \Gamma_n \quad \text{for} \quad \alpha = 0 \quad \text{and} \quad \alpha = \alpha_0 < 1. \quad (42)$$

4. The neutron decay

Let us finally discuss two physical examples. First, we discuss the case of the neutron, since as mentioned in the introduction, it is particularly interesting in view of a persisting anomaly. The neutron decay is a weak decay whose decay width function has the form of

$$\Gamma(\omega) = g_n^2 \omega^5 \quad (43)$$

(this is actually the leading term, for the full formula see *e.g.* Ref. [25]), thus $\alpha = 5$: the IZE is possible. The on-shell values are $\omega_{\text{on-shell}} = m_n - m_p - m_e = 0.782333$ MeV, and $\Gamma_{\text{on-shell}} = g_n^2 \omega_{\text{on-shell}}^5 = \hbar/\tau_n^{\text{beam}} = \hbar/888.1 \text{ s}^{-1} = 7.41146 \times 10^{-25}$ MeV (implying $g_n = 1.59028 \times 10^{-12} \text{ MeV}^{-2}$). [Note, the anyhow small errors are neglected here.] Of course, the function $\Gamma(\omega)$ cannot rise indefinitely. An expression of the type as in Eq. (21) is expected to hold. To be more precise, the following behavior for the neutron is realistic:

$$\Gamma(\omega) \propto \begin{cases} \omega^5 & \text{for } \omega \lesssim M_W \\ \omega & \text{for } M_W \lesssim \omega \lesssim M_X \\ \omega e^{-\omega/\Lambda} & \text{for } \omega \gtrsim M_X \end{cases} . \quad (44)$$

where M_X is some large scale, $X = \text{GUT}$ or M_{Planck} . Also Λ is some large number of the order of M_X .

Ergo, for λ up to M_W , we are basically in the IZE regime: that means that in practice, only the IZE is realistic for the neutron. For even larger λ , the contribution $\propto \omega$ enters [26] and only for (an unrealistic) λ larger than M_X , the decreasing of $\Gamma^{\text{measured}}(\tau)$ would become visible.

Let us now discuss the IZE for ongoing experiments. In beam experiments, the value of τ was estimated in Ref. [18] to be quite large, thus λ turns out to be very small, sizably smaller than 10^{-6} MeV. Hence, the IZE is very small; to a very good extent

$$\Gamma^{\text{measured-beam}}(\tau_{\text{beam}}) \simeq \Gamma_{\text{on-shell}} . \quad (45)$$

On the other hand, for trap experiments, neutrons are kept in a very cold trap and they are constantly monitored by the environment. Together with the high degree of correlation in the wave function, it was proposed in Ref. [18] that τ can be sizably smaller, hence λ could be sufficiently large for a sizable IZE. Using the model explained in Sec. 3, for the value $\lambda = 0.0424$ MeV, one obtains $\Gamma^{\text{measured}}(\tau = \lambda^{-1})/\Gamma_{\text{on-shell}} = 1.0098$. Namely, in this way, one could understand why the trap experiment finds a larger decay. We also refer to [33] in which this topic is discussed by using both response functions presented in Sec. 2. The results were found to be very similar. More general, even if the present neutron anomaly is due to some systematic effects, one may still speculate that the IZE can be realized in future experimental setups.

As a second example, we also mention the decay of the muon, for which also $\alpha = 5$. Here, $\omega_{\text{on-shell}} \simeq m_\mu = 105.658$ MeV and $\Gamma_\mu = \hbar/(2.19698 \times 10^{-6} \text{ s})$ [34]. In order to get an increase of 1% on this value, one would need $\lambda = 5.787$ MeV, which is quite large. It seems, therefore, hard to measure the IZE in experiments involving muons.

5. Conclusions

In this work, we have introduced a simple measurement model, based on a rectangular response function, that allows to understand under which conditions the QZE and the IZE are realized. We have found that for realistic measurements, the IZE is actually favoured w.r.t. the QZE. Namely, when the decay widths scales as ω^α with $\alpha > 1$ or $\alpha < 0$, the IZE takes place, while the QZE is possible only for $0 < \alpha < 1$. The latter interval is further reduced if the on-shell energy $\omega_{\text{on-shell}}$ is close to zero (the left-energy threshold).

We have then applied these ideas to the decay of the neutron. In recent works, an experimental anomaly between trap and beam experiments has been found [17]: the lifetime measured in trap experiments — in which neutrons are monitored — is shorter than the one in beam experiments, where protons are counted. This mismatch has been interpreted in Refs. [35, 36] as the effect of a *beyond Standard Model* (BSM) invisible dark decay of the neutron that is undetected in the beam method. This idea has been criticized in Ref. [37], according to which a dark neutron would undermine the stability of neutron stars, as well as in Refs. [38, 39], where it is shown that the present Standard Model result coincides with the beam method. The conclusion would be that there is some unknown systematic error that affects the beam experimental setup.

In our approach, there is no need to use BSM physics: the IZE takes place for the case of neutron decays in traps, hence the shorter lifetime w.r.t. the beam result. Yet, if the Standard Model result is correct [38, 39] and the present anomaly is just an experimental artifact, there is also no need for an IZE in trap experiments. Nevertheless, our study shows that, for neutron decays, the IZE is not far from reach, hence it could be measured in future experiments dealing with cold neutrons.

The author thanks G. Pagliara for collaboration leading to Ref. [18]. F.G. acknowledges support from the National Science Centre, Poland (NCN) through the OPUS No. 2018/29/B/ST2/02576.

REFERENCES

- [1] A. Degasperis, L. Fonda, G.C. Ghirardi, *Nuovo Cim. A* **21**, 471 (1973).
- [2] B. Misra, E.C.G. Sudarshan, *J. Math. Phys.* **18**, 756 (1977).
- [3] A.G. Kofman, G. Kurizki, *Nature* **405**, 546 (2000); *Z. Naturforsch. A* **56**, (2001).
- [4] P. Facchi, S. Pascazio, *Fortschr. Phys.* **49**, 941 (2001).
- [5] P. Facchi, H. Nakazato, S. Pascazio, *Phys. Rev. Lett.* **86**, 2699 (2001);
P. Facchi, S. Pascazio, *Phys. Lett. A* **241**, 139 (1998).
- [6] P. Facchi, S. Pascazio, *J. Phys. A* **41**, 493001 (2008).
- [7] K. Koshino, A. Shimizu, *Phys. Rep.* **412**, 191 (2005).
- [8] L.A. Khalfin, *Zh. Eksp. Teor. Fiz.* **33**, 1371 (1957) [Engl. transl. *Sov. Phys. JETP* **6**, 1053 (1957)].
- [9] L. Fonda, G.C. Ghirardi, A. Rimini, *Rep. Prog. Phys.* **41**, 587 (1978).
- [10] C.A. Nicolaides, T. Mercouris, *J. Phys. B: At. Mol. Opt. Phys.* **29**, 1151 (1996); E. Torrontegui, J.G. Muga, J. Martorell, D.W.L. Sprung, *Phys. Rev. A* **80**, 012703 (2009).
- [11] K. Raczyńska, K. Urbanowski, *Acta Phys. Pol. B* **49**, 1683 (2018);
K. Urbanowski, K. Raczyńska, *Phys. Lett. B* **731**, 236 (2014),
[arXiv:1303.6975](https://arxiv.org/abs/1303.6975) [astro-ph.HE].

- [12] F. Giacosa, G. Pagliara, *Mod. Phys. Lett. A* **26**, 2247 (2011), [arXiv:1005.4817 \[hep-ph\]](#); G. Pagliara, F. Giacosa, *Acta Phys. Pol. B Proc. Suppl.* **4**, 753 (2011).
- [13] F. Giacosa, *Found. Phys.* **42**, 1262 (2012), [arXiv:1110.5923 \[nucl-th\]](#).
- [14] S.R. Wilkinson *et al.*, *Nature* **387**, 575 (1997).
- [15] C. Rothe, S.I. Hintschich, A.P. Monkman, *Phys. Rev. Lett.* **96**, 163601 (2006).
- [16] M.C. Fischer, B. Gutiérrez-Medina, M.G. Raizen, *Phys. Rev. Lett.* **87**, 040402 (2001).
- [17] F.E. Wietfeldt, *Atoms* **6**, 70 (2018); F.E. Wietfeldt, G.L. Greene, *Rev. Mod. Phys.* **83**, 1173 (2011).
- [18] F. Giacosa, G. Pagliara, *Phys. Rev. D* **101**, 056003 (2020).
- [19] T.D. Lee, *Phys. Rev.* **95**, 1329 (1954); C.B. Chiu, E.C.G. Sudarshan, G. Bhamathi, *Phys. Rev. D* **46**, 3508 (1992).
- [20] Z.W. Liu *et al.*, *Phys. Rev. Lett.* **116**, 082004 (2016), [arXiv:1512.00140 \[hep-lat\]](#).
- [21] Z. Xiao, Z.Y. Zhou, *J. Math. Phys.* **58**, 062110 (2017), [arXiv:1608.06833 \[hep-ph\]](#).
- [22] P. Facchi, M. Ligabò, D. Lonigro, [arXiv:1910.05957 \[math-ph\]](#).
- [23] F. Giacosa, [arXiv:2001.07781 \[hep-ph\]](#).
- [24] M.E. Peskin, D.V. Schroeder, «An Introduction to Quantum Field Theory», Addison-Wesley, Reading, USA 1995, p. 842.
- [25] W. Greiner, B. Mueller, «Gauge Theory of Weak Interactions», *Springer Berlin Heidelberg*, Berlin, Heidelberg 2000.
- [26] F. Giacosa, G. Pagliara, *Phys. Rev. D* **88**, 025010 (2013); *Phys. Rev. C* **76**, 065204 (2007).
- [27] V. Weisskopf, E.P. Wigner, *Z. Phys.* **63**, 54 (1930); *ibid.* **65**, 18 (1930); G. Breit, in: S. Flügge (Ed.) «Handbuch der Physik, Vol. 41/1», Springer, Berlin, Göttingen, Heidelberg 1959.
- [28] L.S. Schulman, *Phys. Rev. A* **57**, 1509 (1998).
- [29] E.W. Streed, *Phys. Rev. Lett.* **97**, 260402 (2006).
- [30] F. Giacosa, G. Pagliara, *Phys. Rev. A* **90**, 052107 (2014).
- [31] R. Omnes, «General theory of the decoherence effect in quantum mechanics», LPTHE-ORSAY-92-102.
- [32] F. Giacosa, P. Kościk, T. Sowiński, [arXiv:1912.06394 \[quant-ph\]](#).
- [33] F. Giacosa, *Acta Phys. Pol. B* **51**, 77 (2020).
- [34] M. Tanabashi *et al.*, *Phys. Rev. D* **98**, 030001 (2018).
- [35] B. Fornal, B. Grinstein, *Phys. Rev. Lett.* **120**, 191801 (2018).
- [36] Z. Berezhiani, *Lett. High Energy Phys.* **2**, 118 (2019); B. Belfatto, R. Beradze, Z. Berezhiani, *Eur. Phys. J. C* **80**, 149 (2020).
- [37] G. Baym, D.H. Beck, P. Geltenbort, J. Shelton, *Phys. Rev. Lett.* **121**, 061801 (2018).
- [38] D. Dubbers *et al.*, *Phys. Lett. B* **791**, 6 (2019).
- [39] A. Czarnecki, W.J. Marciano, A. Sirlin, *Phys. Rev. Lett.* **120**, 202002 (2018); *Phys. Rev. D* **100**, 073008 (2019).