

RELATIVISTIC HYDRODYNAMICS FOR SPIN-POLARIZED MEDIA^{*}

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(Received April 22, 2020)

We summarize the key ingredients of the recently proposed formalism of relativistic perfect-fluid hydrodynamics with spin. Based on the underlying kinetic theory definitions for the equilibrium distribution functions, we obtain the evolution equations governing the system's expansion. Employing the Bjorken symmetry, we study the spin polarization dynamics of the system.

DOI:10.5506/APhysPolB.51.1537

1. Introduction

The first positive measurements of spin polarization of Λ hyperons made lately by the STAR Collaboration [1–4] have revived the interest in the studies of the relation between the vorticity of the matter produced in relativistic heavy-ion collisions and the average spin polarization of particles produced in these processes [5–36]; for a recent review, see [37]. Recently, it has been shown that the thermal-based models [38–41] which correctly describe the global polarization, unfortunately are not able to explain the differential observables [4]. These models are based on the assumption that the spin polarization of particles emitted at freeze-out is entirely determined by the quantity known as thermal vorticity [6, 42] and does not include the possibility of its independent dynamical evolution, which may take place during the fluid expansion. In this work, following ideas put forward in Refs. [43–48], we study such a possibility within the framework of relativistic perfect-fluid hydrodynamics with spin. Throughout the text, we use natural units with $c = k_B = \hbar = 1$.

2. Equilibrium distribution functions

Relativistic fluid dynamics may be derived from the underlying kinetic theory assuming that the distribution function describing the equilibrium

^{*} Presented by R. Ryblewski at XXVI Cracow Epiphany Conference on LHC Physics: Standard Model and Beyond, Kraków, Poland, January 7–10, 2020.

state of the system is known [49]. Herein, following works by Becattini *et al.* [6], we assume that the local equilibrium state of the relativistic system of particles (+) and antiparticles (−) with spin 1/2 and mass m is described by the following phase-space distribution functions (spin density matrices):

$$f_{rs}^+(x, p) = \bar{u}_r(p) X^+ u_s(p), \quad f_{rs}^-(x, p) = -\bar{v}_s(p) X^- v_r(p), \quad (1)$$

where x is the space-time position and p is the four-momentum, and $u_r(p)$ and $v_r(p)$ are Dirac bispinors ($r, s = 1, 2$) with the normalization $\bar{u}_r(p) u_s(p) = \delta_{rs}$ and $\bar{v}_r(p) v_s(p) = -\delta_{rs}$.

The matrices $X^\pm$ have the form of generalized relativistic Boltzmann distributions

$$X^\pm = \exp \left[\pm \xi(x) - \beta_\mu(x) p^\mu \pm \frac{1}{2} \omega_{\mu\nu}(x) \Sigma^{\mu\nu} \right],$$

where $\beta^\mu \equiv U^\mu/T$ and $\xi \equiv \mu/T$, with T , μ and U^μ denoting the temperature, baryon chemical potential and four-velocity, respectively. The quantity $\omega_{\mu\nu}$ is the spin polarization tensor satisfying $\omega^{\mu\nu} = -\omega^{\nu\mu}$ and $\Sigma^{\mu\nu} \equiv \frac{i}{4} [\gamma^\mu, \gamma^\nu]$ is the spin operator.

Employing expressions derived in Ref. [50] and using definitions (1), we can determine the corresponding equilibrium Wigner functions

$$\begin{aligned} \mathcal{W}_{\text{eq}}^\pm(x, k) &= \frac{e^{\pm\xi}}{4m} \int dP e^{-\beta \cdot p} \delta^{(4)}(k \mp p) \\ &\times \left[2m(m \pm \not{p}) \cosh(\zeta) \pm \frac{\sinh(\zeta)}{2\zeta} \omega_{\mu\nu} (\not{p} \pm m) \Sigma^{\mu\nu} (\not{p} \pm m) \right], \end{aligned} \quad (2)$$

where k is the off-mass-shell four-momentum of particles, $dP = d^3p / ((2\pi)^3 E_p)$ with $E_p = \sqrt{m^2 + \mathbf{p}^2}$ being the on-shell particle energy, and $\zeta = \frac{1}{2\sqrt{2}} \sqrt{\omega_{\mu\nu} \omega^{\mu\nu}}$.

It is convenient to consider the Clifford-algebra expansion of the Wigner function (2)

$$\begin{aligned} \mathcal{W}_{\text{eq}}^\pm(x, k) &= \frac{1}{4} \left[\mathcal{F}_{\text{eq}}^\pm(x, k) + i\gamma_5 \mathcal{P}_{\text{eq}}^\pm(x, k) + \gamma^\mu \mathcal{V}_{\text{eq},\mu}^\pm(x, k) \right. \\ &\quad \left. + \gamma_5 \gamma^\mu \mathcal{A}_{\text{eq},\mu}^\pm(x, k) + \Sigma^{\mu\nu} \mathcal{S}_{\text{eq},\mu\nu}^\pm(x, k) \right], \end{aligned}$$

where the coefficient functions $\mathcal{X} \in \{\mathcal{F}, \mathcal{P}, \mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}_{\mu\nu}\}$ can be extracted by calculating the trace of $\mathcal{W}_{\text{eq}}^\pm(x, k)$ multiplied first by: $\{\mathbf{1}, -i\gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, 2\Sigma_{\mu\nu}\}$.

3. Kinetic equations

The general Wigner function satisfies the kinetic equation

$$(\gamma_\mu K^\mu - m) \mathcal{W}(x, k) = C[\mathcal{W}(x, k)], \quad (3)$$

where the differential operator reads $K^\mu = k^\mu + \frac{i\hbar}{2} \partial^\mu$. In the case of global equilibrium, the Wigner function satisfies exactly Eq. (3) with $C[\mathcal{W}(x, k)] = 0$. The usual treatment of Eq. (3) is to consider the semi-classical expansion of the coefficient functions

$$\mathcal{X} = \mathcal{X}^{(0)} + \hbar \mathcal{X}^{(1)} + \hbar^2 \mathcal{X}^{(2)} + \dots$$

The analysis of Eq. (3) up to the next-to-leading order in $\hbar$ yields the following kinetic equations for the two independent coefficients $\mathcal{F}_{\text{eq}}$ and $\mathcal{A}_{\text{eq}}^\nu$:

$$k^\mu \partial_\mu \mathcal{F}_{\text{eq}}(x, k) = 0, \quad k^\mu \partial_\mu \mathcal{A}_{\text{eq}}^\nu(x, k) = 0, \quad k_\nu \mathcal{A}_{\text{eq}}^\nu(x, k) = 0. \quad (4)$$

In global equilibrium Eqs. (4) are exactly fulfilled which results in the conditions that β_μ is a Killing vector, and ξ and $\omega_{\mu\nu}$ are constant, however $\omega_{\mu\nu}$ does not have to be equal to thermal vorticity $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) = \text{const.}$

4. Hydrodynamic equations

In local equilibrium, Eqs. (4) are not satisfied exactly. In this case, we adopt the standard treatment [51], namely, by allowing for x dependence of the β , ξ and ω , we require that only certain moments in momentum space of the kinetic equations (4) are satisfied. This method leads to equations expressing conservation laws for charge, energy, linear momentum and spin [47]

$$\partial_\mu N^\mu = 0, \quad (5)$$

$$\partial_\mu T_{\text{GLW}}^{\mu\nu} = 0, \quad (6)$$

$$\partial_\lambda S_{\text{GLW}}^{\lambda, \alpha\beta} = 0, \quad (7)$$

where the baryon current, the energy-momentum tensor, and the spin tensor are given by the De Groot–Van Leeuwen–Van Weert (GLW) [50] expressions

$$N^\alpha = n U^\alpha, \quad (8)$$

$$T_{\text{GLW}}^{\alpha\beta} = (\varepsilon + P) U^\alpha U^\beta - P g^{\alpha\beta}, \quad (9)$$

$$S_{\text{GLW}}^{\alpha, \beta\gamma} = \cosh(\xi) \left[n_{(0)} U^\alpha \omega^{\beta\gamma} + \mathcal{A}_{(0)} U^\alpha U^\delta U^{[\beta} \omega^{\gamma]}_\delta + \mathcal{B}_{(0)} \left(U^{[\beta} \Delta^{\alpha\delta} \omega^{\gamma]}_\delta + U^\alpha \Delta^{\delta[\beta} \omega^{\gamma]}_\delta + U^\delta \Delta^{\alpha[\beta} \omega^{\gamma]}_\delta \right) \right], \quad (10)$$

where $\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$ is the projector on the space orthogonal to U .

In the leading order in the polarization tensor, the energy density ε , the pressure P , and the baryon density n are given by the formulas

$$n = 4 \sinh(\xi) n_{(0)}(T), \quad (11)$$

$$\varepsilon = 4 \cosh(\xi) \varepsilon_{(0)}(T), \quad (12)$$

$$P = 4 \cosh(\xi) P_{(0)}(T), \quad (13)$$

where we defined the auxiliary quantities describing thermodynamic properties of the system of spin-less and neutral massive Boltzmann particles [52]

$$n_{(0)}(T) = \frac{T^3}{2\pi^2} \hat{m}^2 K_2(\hat{m}) , \quad (14)$$

$$\varepsilon_{(0)}(T) = \frac{T^4}{2\pi^2} \hat{m}^2 \left[3K_2(\hat{m}) + \hat{m} K_1(\hat{m}) \right] , \quad (15)$$

$$P_{(0)}(T) = T n_{(0)}(T) . \quad (16)$$

The quantities $\mathcal{B}_{(0)}$ and $\mathcal{A}_{(0)}$ are defined as follows:

$$\mathcal{B}_{(0)} = -\frac{2}{\hat{m}^2} s_{(0)}(T) , \quad \mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2n_{(0)}(T) \quad (17)$$

with $s_{(0)} = (\varepsilon_{(0)} + P_{(0)})/T$ being the entropy density and $\hat{m} = m/T$.

5. Bjorken expansion

Similarly to the Faraday tensor, the polarization tensor $\omega_{\mu\nu}$ may be decomposed into electric-like (κ) and magnetic-like (ω) components

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta , \quad (18)$$

where the four-vectors κ and ω satisfy the conditions

$$\kappa \cdot U = 0 , \quad \omega \cdot U = 0 . \quad (19)$$

In the case of transversely homogeneous systems undergoing boost-invariant expansion in the longitudinal direction, also known as the Bjorken flow [53], it is convenient to introduce the following four-vector basis:

$$\begin{aligned} U^\alpha &= \frac{1}{\tau} (t, 0, 0, z) = (\cosh(\eta), 0, 0, \sinh(\eta)) , \\ X^\alpha &= (0, 1, 0, 0) , \\ Y^\alpha &= (0, 0, 1, 0) , \\ Z^\alpha &= \frac{1}{\tau} (z, 0, 0, t) = (\sinh(\eta), 0, 0, \cosh(\eta)) , \end{aligned} \quad (20)$$

where $\tau = \sqrt{t^2 - z^2}$ is the longitudinal proper time and $\eta = \frac{1}{2} \ln((t+z)/(t-z))$ is the space-time rapidity.

Basis (20) satisfies the conditions

$$\begin{aligned} U \cdot U &= 1 , \\ X \cdot X &= Y \cdot Y = Z \cdot Z = -1 , \\ X \cdot U &= Y \cdot U = Z \cdot U = 0 , \\ X \cdot Y &= Y \cdot Z = Z \cdot X = 0 . \end{aligned} \quad (21)$$

Using Eqs. (19) and Eqs. (21), one can decompose the vectors κ^μ and ω^μ as follows:

$$\begin{aligned}\kappa^\alpha &= C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \\ \omega^\alpha &= C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha,\end{aligned}\quad (22)$$

where the coefficients $C_{\kappa X}$, $C_{\kappa Y}$, $C_{\kappa Z}$, $C_{\omega X}$, $C_{\omega Y}$, and $C_{\omega Z}$ are scalar functions of proper time solely.

Using Eqs. (22) in Eq. (7) and projecting the latter on $U_\mu X_\nu$, $U_\mu Y_\nu$, $U_\mu Z_\nu$, $Y_\mu Z_\nu$, $X_\mu Z_\nu$ and $X_\mu Y_\nu$, we obtain the following six evolution equations:

$$\text{diag}(\mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{P}, \mathcal{P}, \mathcal{P}) \cdot \dot{\mathbf{C}} = \text{diag}(\mathcal{Q}_1, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{R}_1, \mathcal{R}_1, \mathcal{R}_2) \mathbf{C}, \quad (23)$$

where $\mathbf{C} = (C_{\kappa X}, C_{\kappa Y}, C_{\kappa Z}, C_{\omega X}, C_{\omega Y}, C_{\omega Z})$, $(\dot{}) \equiv U \cdot \partial = \partial_\tau$ and

$$\begin{aligned}\mathcal{L}(\tau) &= \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3, \\ \mathcal{P}(\tau) &= \mathcal{A}_1, \\ \mathcal{Q}_1(\tau) &= -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right], \\ \mathcal{Q}_2(\tau) &= -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right), \\ \mathcal{R}_1(\tau) &= -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right], \\ \mathcal{R}_2(\tau) &= -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right),\end{aligned}$$

with

$$\begin{aligned}\mathcal{A}_1 &= \cosh(\xi) (n_{(0)} - \mathcal{B}_{(0)}), \\ \mathcal{A}_2 &= \cosh(\xi) (\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}), \\ \mathcal{A}_3 &= \cosh(\xi) \mathcal{B}_{(0)}.\end{aligned}$$

From Eqs. (23) we observe that in the case of Bjorken expansion, the $\mathbf{C}$ coefficients evolve independently. Due to the rotational symmetry in the transverse plane, the functions $C_{\kappa X}$ and $C_{\kappa Y}$ (as well as $C_{\omega X}$ and $C_{\omega Y}$) obey the same differential equations.

Employing the Bjorken symmetry, the conservation of the charge current (5) can be written as

$$\frac{dn}{d\tau} + \frac{n}{\tau} = 0, \quad (24)$$

while the conservation of energy and linear momentum (6) (projected on U) gives

$$\frac{d\varepsilon}{d\tau} + \frac{(\varepsilon + P)}{\tau} = 0. \quad (25)$$

6. Spin polarization of particles at freeze-out

The information about the space-time evolution of the spin polarization tensor may be used to determine the average spin polarization per particle which is defined as follows [47]:

$$\langle \pi_\mu \rangle = E_p \frac{d\Pi_\mu(p)}{d^3p} \bigg/ E_p \frac{d\mathcal{N}(p)}{d^3p}, \quad (26)$$

where $E_p \frac{d\Pi_\mu(p)}{d^3p}$ is the total value (integrated over freeze out hypersurface $\Delta\Sigma_\lambda$) of the Pauli–Lubański vector

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta \cdot p} \tilde{\omega}_{\mu\beta} p^\beta,$$

and

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4 \cosh(\xi)}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta \cdot p}$$

is the momentum density of particles and antiparticles.

The polarization vector $\langle \pi_\mu^* \rangle$ in the particle rest frame is obtained by performing the canonical boost [54] of (26)

$$\langle \pi_\mu^* \rangle = -\frac{1}{8m} \begin{bmatrix} 0 \\ \left(\frac{p_x \sinh y_p}{b} \right) a_1 + \left(\frac{\chi p_x \cosh y_p}{b} \right) a_2 + 2C_{\kappa Z} p_y - \chi C_{\omega X} m_T \\ \left(\frac{p_y \sinh y_p}{b} \right) a_1 + \left(\frac{\chi p_y \cosh y_p}{b} \right) a_2 - 2C_{\kappa Z} p_x - \chi C_{\omega Y} m_T \\ - \left(\frac{m \cosh y_p + m_T}{b} \right) a_1 - \left(\frac{\chi m \sinh y_p}{b} \right) a_2 \end{bmatrix}, \quad (27)$$

where $a_1 = \chi(C_{\kappa X} p_y - C_{\kappa Y} p_x) + 2C_{\omega Z} m_T$, $a_2 = C_{\omega X} p_x + C_{\omega Y} p_y$, $b = m_T \cosh y_p + m$, and $\chi = (K_0(\hat{m}_T) + K_2(\hat{m}_T)) / K_1(\hat{m}_T)$ with $\hat{m}_T = m_T/T$. Above, we used the following parametrization of the four-momentum $p^\lambda = (m_T \cosh y_p, p_x, p_y, m_T \sinh y_p)$.

7. Results

In this section, we present the results obtained by solving numerically the differential equations (23), (24), and (25). We initialize the system at the proper time $\tau_0 = 1$ fm with initial temperature $T_0 = T(\tau_0) = 155$ MeV and the baryon chemical potential $\mu_0 = \mu(\tau_0) = 800$ MeV. We assume that the system consists of Λ particles with mass $m = 1116$ MeV. In Fig. 1, we show the proper-time dependence of the (properly scaled) temperature and baryon chemical potential. We reproduce the well-known results that the temperature of such a system decreases with proper time, while the ratio of baryon chemical potential and temperature increases. In Fig. 2, we show the proper time dependence of the C coefficients that describe the evolution of the spin polarization.

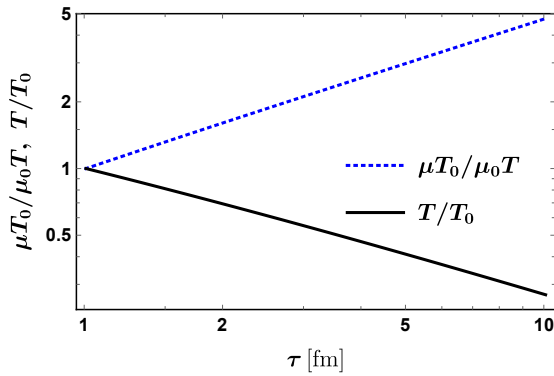


Fig. 1. (Color online) Proper-time dependence of the temperature scaled by its initial value (solid black line) and the ratio of baryon chemical potential over temperature rescaled by the initial ratio (dotted blue line).

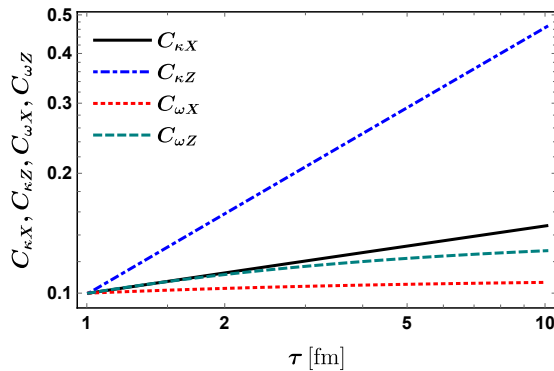


Fig. 2. (Color online) Proper-time dependence of the coefficients $C_{\kappa X}$ (solid black line), $C_{\kappa Z}$ (dash-dotted blue line), $C_{\omega X}$ (dotted red line) and $C_{\omega Z}$ (dashed green line).

The knowledge of the evolution of thermodynamic parameters and C coefficients allows us to calculate the components of the particle-rest-frame mean polarization vector $\langle \pi_\mu^\star \rangle$ at freeze-out as functions of particle three-momentum, see Fig. 3. We observe that the component $\langle \pi_y^\star \rangle$ is negative, which reflects the initial spin polarization of the system. Due to the Bjorken symmetry, the longitudinal component ($\langle \pi_z^\star \rangle$) is vanishing which does not agree with the characteristic quadrupole structure of the longitudinal polarization observed in the experiment. On the other hand, we observe that $\langle \pi_x^\star \rangle$ exhibits quadrupole structure. Clearly, we observe that the Bjorken symmetry is too restrictive to address the experimental measurements correctly.

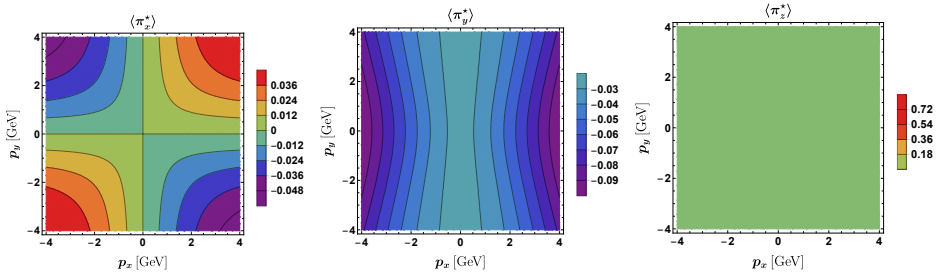


Fig. 3. Components of the particle-rest-frame mean polarization three-vector of Λ particles obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $C_{\kappa,0} = (0,0,0)$ and $C_{\omega,0} = (0,0.1,0)$ for $y_p = 0$.

8. Summary

In this work, we briefly reviewed the basic ingredients of the recently formulated approach of relativistic perfect-fluid hydrodynamics with spin. Using the kinetic theory definitions for the local equilibrium distribution functions, we derived the evolution equations governing the system's expansion. Assuming the Bjorken flow of the matter, we studied numerically the spin polarization dynamics of the system. We have shown that the coefficient functions characterizing the spin polarization evolve independently. We have used these results to determine the spin polarization of particles at the freeze-out. We have shown that within the simple Bjorken setup, the characteristic features observed in the experiment cannot be properly reproduced.

Supported in part by the National Science Center, Poland (NCN) grants No. 2016/23/B/ST2/00717 and No. 2018/30/E/ST2/00432.

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