ROTATIONAL BEHAVIOR OF TRIAXIALLY DEFORMED EVEN–EVEN TELLURIUM ISOTOPES

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The statistical theory of hot rotating nuclei (STHRN) method was implied to investigate the effect of shape transition in thermodynamical parameters for the isotopes of tellurium for (mass number) $A = 114$, 120, 122, 124 and 126. The single particle energy levels and intrinsic spin were obtained by diagonalizing the triaxial Nilsson Hamiltonian for deformation parameter $\epsilon = 0.0$ to 0.6 and shape parameter $\gamma = -120^\circ$ to -180°. The calculated statistical parameters such as rotational frequency, spin cutoff parameter, separation energy and moment of inertia indicate a sudden change around the angular momentum $M = 12\,\hbar$ at which the even–even isotopes of tellurium are found to change their shape from spherical to noncollective oblate. The results obtained show reasonable agreement with the experimental data and also with other theoretical models such as Interacting Boson Model-1 (IBM-1). The rotational frequency and moment of inertia values calculated from the STHRN method give reasonable agreement with the experimental data compared to IBM-1 model.

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1. Introduction

The structure of tellurium isotopes has drawn attention in both theoretical [1, 2] and experimental aspects [3–8] owing to its atomic number $Z = 52$, with two protons outside the major shell ($Z = 50$). Several experimental techniques were implied to explore the structure of Te isotopes. Generally, doubly even Te isotopes in the mass region of $114 \leq A \leq 122$ exhibit a vibrational-like structure [9]. However, Saxena *et al.* [1] found from the reduced transition probability $B(\mathrm{E}_2)$ values that tellurium isotopes with mass number $A = 120$, 122 and 124 exhibit rotational behavior in contrary to a vibrational-like structure. Meanwhile, Fotiades *et al.* [2] extended the yrast state of even–even isotopes of $^{120-124}$Te for a high spin up to $I = 16^+$. The results predict that the high-spin state up to 14^+ exhibits the characteristics of favored non-collective oblate structure, whereas at the highest spin *i.e.*, $I = 16^+$ state, it behaves as a weakly deformed collective structure. Similar phenomenon was later observed by Astier *et al.* [10] for ^{124}Te and ^{126}Te nucleus. Moreover, the discussion by Somnath Nag *et al.* [3] on the shape change of ^{120}Te with respect to spin explained that this observation may be due to opposite shape driving tendencies of protons and neutrons at high orbitals. They concluded that the shape of nucleus changes to oblate inspite of the continuous alignment of neutrons and protons and it was observed as $\epsilon \approx 0.15$ for $A = 120$ mass region.

Many theoretical approaches [4–8] have been employed to study the structure of Te isotopes. The structure of $^{124-127}$Te isotopes was also discussed via the shell model theory by incorporating SN100PN interaction [4]. This method was meant to analyze the structure of nucleus from its yrast states. The statistical theory of hot rotating nuclei (STHRN) method [11–13] was utilized in perceiving the shape of Te isotopes near the proton drip line by Mamta Aggarwal [8]. The discussions reveal that proton-rich Te nuclei are less deformed with prolate collective or nearly prolate shapes, while those near the stability valley are well-deformed with oblate non-collective shape. Based on this prediction, the shape transition phenomenon would be observed more prominently in Te isotopes near the stability valley.

The rotational behavior of even–even isotopes of Te was also reported by implying the Interacting Boson Model (IBM-1 and IBM-2) [6]. The obtained energy states from IBM-2 theory appear to be in good agreement with the experimental data. However, the moment of inertia calculated from IBM-1 model by Hossain *et al.* [7] for ^{120,122,124}Te nucleus shows a linear increase with angular momentum M, whereas the experimental data shows some sudden steep increase in the moment of inertia value at a certain M. Thus, an alternate theory is indeed necessary to overcome the linear increase of moment of inertia as a function of angular momentum in Te isotopes.

The statistical theory of hot rotating nuclei (STHRN) [14–16] is used to predict the shape transition behavior from thermodynamical properties. In STHRN method, the thermodynamical system of fermions is nothing but the compound nuclei formed through heavy-ion fusion reaction at high-excitation energy [17–20]. Statistical descriptions of finite nuclear systems are generally based on grand canonical ensemble averages with a condition that it conserves energy, particle number, and total angular momentum M of the system [21–24], and is described in the formalism section. Moreover, the statistical theory incorporates different degrees of freedom such as deformation parameter, angular momentum and temperature to investigate thermodynamical parameters which describe the structural properties of nuclei [21]. The following are some of the parameters that can be studied on the basis of statistical theory: nuclear level density parameter, entropy, rotational energy, excitation energy, moment of inertia, level density and single nucleon separation energy. Moreover, the observation of shape transition with respect to angular momentum and temperature in the STHRN method is a remarkable outcome, adopted by Moretto [22]. In the present work, temperature-dependent rotational properties for Te isotopes such as ^{114}Te, ^{120}Te, ^{122}Te, ^{124}Te and ^{126}Te were studied by the STHRN method. This method was able to reproduce moment of inertia at high-spin states which shows a good agreement with the experimental data for the even–even isotopes of tellurium.

2. Formalism

2.1. Statistical theory of hot rotating nuclei

The grand canonical partition function for the hot rotating nuclei is given by [21, 22]

$$\begin{aligned}\ln Q &= \sum_i \ln\left[1+\exp\left(\alpha_N+\lambda m_i^N-\beta\epsilon_i^N\right)\right]\\&\quad+\sum_i \ln\left[1+\exp\left(\alpha_Z+\lambda m_i^Z-\beta\epsilon_i^Z\right)\right]\,,\end{aligned}\tag{1}$$

where the Lagrangian multipliers, α_N, α_Z and λ conserve the number of neutrons, protons and total angular momentum of the system for a given temperature $T = 1/\beta$. The average number of particles, the average total energy, and the average total angular momentum are projected out from the partition function by the following equations:

$$\langle N\rangle = \frac{\partial \ln Q}{\partial \alpha_N} = \sum_i \left[1+\exp\left(\alpha_N+\lambda m_i^N-\beta\epsilon_i^N\right)\right]^{-1}\,,\tag{2}$$

$$\langle Z\rangle = \frac{\partial \ln Q}{\partial \alpha_Z} = \sum_i \left[1+\exp\left(\alpha_Z+\lambda m_i^Z-\beta\epsilon_i^Z\right)\right]^{-1}\,,\tag{3}$$

$$\langle E\rangle = -\frac{\partial \ln Q}{\partial \beta} = \sum_i \epsilon_i^N n_i^N + \sum_i \epsilon_i^Z n_i^Z\,,\tag{4}$$

$$\langle M\rangle = \frac{\partial \ln Q}{\partial \lambda} = \sum_i \epsilon_i^N m_i^N + \sum_i \epsilon_i^Z m_i^Z\,,\tag{5}$$

where ϵ_i^Z and ϵ_i^N are the single particle energy levels of protons and neutrons with spin projection m_i^Z and m_i^N, and n_i^N and n_i^Z are occupational probabilities of the i^{th} shell corresponding to neutron and proton, respectively.

Equations (2), (3) and (5) are solved to determine the Lagrangian multipliers for a given temperature T. The Lagrangian multipliers α_Z, α_N and λ conserve the number of protons, neutrons and total angular momentum of the system and their definitions given in Refs. [22–24] are

$$\alpha_N = \beta E_\mathrm{F}^N\,; \qquad \alpha_Z = \beta E_\mathrm{F}^Z\,; \qquad \lambda = \beta\eta\,,$$

where E_F^Z and E_F^N are the Fermi energy of proton and neutron system and the total angular momentum is generated from the Lagrangian multiplier η corresponding to the single particle spins m_i [17] at a given temperature $T = 1/\beta$. The entropy is given by

$$\begin{aligned} S(M,T) &= -\sum_i \left[n_i^N \ln n_i^N + \left(1-n_i^N\right)\ln\left(n_i^N\right)\right] \\ &\quad -\sum_i \left[n_i^Z \ln n_i^Z + \left(1-n_i^Z\right)\ln\left(n_i^Z\right)\right]\,. \end{aligned} \tag{6}$$

The excitation energy $E^*(M,T)$ is obtained using the following equation:

$$E^*(M,T) = E(M,T) - E(0,0)\,, \tag{7}$$

where $E(0,0)$ is the ground-state energy of the nucleus and is given by

$$E(0,0) = \sum_{i=1}^{N} \epsilon_i^N + \sum_{i=1}^{Z} \epsilon_i^Z\,. \tag{8}$$

The free energy of the system is given as

$$F(M,T) = E(M,T) - TS(M,T)\,. \tag{9}$$

The nucleon separation energy as a function of angular momentum M and temperature T is calculated using the following expressions:

$$S_n(M,T) = TN\left[\sum_{i=1}\left(1-n_i^N\right)n_i^N\right]^{-1}, \tag{10}$$

$$S_z(M,T) = TZ\left[\sum_{i=1}\left(1-n_i^Z\right)n_i^Z\right]^{-1}. \tag{11}$$

The above formulae for the nucleon separation energy have been used by us in the framework of the statistical theory and are reported in Ref. [25]. It is obvious from Eqs. (9) and (10) that nucleon separation energy depends on the single particle level density at the Fermi energy. In this method, only Z-component M of the total angular momentum I is considered. As mentioned by Moretto [22–24], the laboratory fixed z-axis can be made to coincide with the body-fixed z'-axis, and it is possible to identify and substitute M for the total angular momentum I within the limit of quantum mechanics, the Z-component M of the total angular momentum is $M \to \sqrt{I(I+1)} = I + \frac{1}{2}$, where I is the total angular momentum of the system.

Rotational energy $E_{\rm rot}$ and rotational frequency $\omega_{\rm rot}$ are expressed as

$$E_{\rm rot} = E(M,T) - E(0,T)\,, \tag{12}$$

$$\omega_{\rm rot} = \frac{\partial E_{\rm rot}}{\partial M}\,. \tag{13}$$

Kinematical moment of inertia j is calculated from rotational energy as

$$j = \hbar^2 M \left(\frac{\partial E_{\rm rot}}{\partial M}\right)^{-1}\,, \tag{14}$$

and spin cut-off parameter [26] is given as

$$\sigma^2(M,T) = \sum_i \left[n_i^N \left(1 - n_i^N\right) \left(m_i^N\right)^2\right] + \sum_i \left[n_i^Z \left(1 - n_i^Z\right) \left(m_i^Z\right)^2\right]\,. \tag{15}$$

The behavior of rotational states of the highly-excited compound nuclear system formed in fusion reactions can be studied from the rotational energy and kinematic moment of inertia. The statistical theory with single particle level structure as the input can be used to extract information about the complex phenomena such as phase transitions and shape transitions. The energy levels and the intrinsic spin for proton and neutron systems were generated by diagonalising the triaxial Nilsson Hamiltonian for the deformation parameter (ϵ) 0.0 to 0.6 insteps of 0.1 and shape parameter (γ) from $-120°$ to $-180°$ insteps of $-20°$.

2.2. Triaxially deformed Nilsson Hamiltonian

The triaxial Nilsson Hamiltonian [27, 28] for a single particle in the non-rotating system, is given by

$$H^{\rm o} = \frac{P^2}{2m} + \frac{1}{2}m\left\{\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right\} + C\,\vec{l}\cdot\vec{s} + D\left(\vec{l}^{\,2} - 2\left\langle \vec{l}^{\,2}\right\rangle\right)\,, \tag{16}$$

where $C = 2\kappa\hbar\omega_{\rm o}$ and $D = \kappa\mu\hbar\omega_{\rm o}$, the values of κ and μ are taken from [27] and $\omega_{\rm o}$ is the harmonic oscillator parameter that involves the principle of volume conservation for nuclei that are deformed from spherical shapes. The symbol $\vec{l}$ represents the orbital angular momentum and $\vec{s}$ represents the intrinsic nuclear spin. The three oscillator frequencies ω_x, ω_y and ω_z are expressed as

$$\omega_x = \omega_{\mathrm{o}}\left[1-\frac{2}{3}\epsilon\cos\left(\gamma-\frac{2\pi}{3}\right)\right], \tag{17}$$

$$\omega_y = \omega_{\mathrm{o}}\left[1-\frac{2}{3}\epsilon\cos\left(\gamma+\frac{2\pi}{3}\right)\right], \tag{18}$$

$$\omega_z = \omega_{\mathrm{o}}\left[1-\frac{2}{3}\epsilon\cos(\gamma)\right], \tag{19}$$

where ϵ is the deformation parameter and γ is the Euler angle which is together called as shape parameters. Under volume conservation condition,

$$\omega_x\,\omega_y\,\omega_z = \dot{\omega_{\mathrm{o}}}^3 = \text{constant}\,. \tag{20}$$

The value of undeformed oscillator spacing is given as

$$\hbar\omega_{\mathrm{o}} = \frac{41}{(A^{1/3}+0.77)}\ \mathrm{MeV}\,. \tag{21}$$

The microscopic single particle energies ϵ_i and their corresponding spin projection m_i are generated by diagonalizing the triaxial Nilsson Hamiltonian for principal quantum number N up to 11 as a function of ϵ and γ. The deformation parameter ϵ range from 0.0 to 0.6 in steps of 0.1 for the shape parameter $\gamma = -120^\circ$ (collective prolate) to -180° (non-collective oblate) in steps of -20°.

3. Results and discussion

The present work involves the calculations of excitation energy, neutron and proton separation energies, spin cut-off parameter, kinematic moment of inertia and rotational frequency based on the STHRN method for the tellurium isotopes for $A = 114$, 120, 122, 124 and 126. The calculations are done for a temperature range of 0.5 MeV to 3.0 MeV in steps of 0.25 MeV and for an angular momentum range of $0\,\hbar$ to $25\,\hbar$ in steps of $1\,\hbar$. Calculations have been carried out for the energy eigenvalues obtained from triaxial Nilsson Hamiltonian for the deformation parameters $\epsilon = 0.0$ to 0.6 in steps of 0.1 and for the shape parameters $\gamma = -120^\circ$ to -180° in steps of -20°. Though the investigation of all the parameters have been extended to the above-mentioned isotopes of Te, discussions on separation energies, rotational frequency, kinematic moment of inertia and spin cut-off parameter with respect to angular momentum and temperature has been extended to certain isotopes based on their availability of experimental data. The other markable results of the Te isotopes have been tabulated. The equilibrium shape of the system is attained for minimized free energy with respect to the deformation parameters ϵ, γ at finite angular momentum M and temperature T. Thus, the energy level diagram for the nucleus ^{124}Te and ^{126}Te has been plotted for the available experimental levels [29] in Fig. 1.

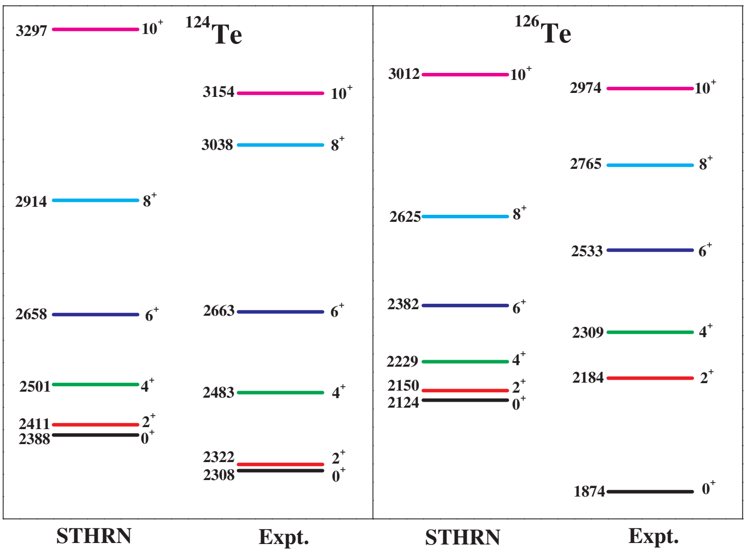

Fig. 1. Excitation energy (E^*) [MeV] [non-yrast states] for ^{124}Te and ^{126}Te as a function of the angular momentum M at the temperature $T = 0.75$ MeV. The results are compared with available experimental data from the ENSDF database [29].

Figure 1 gives a comparative analysis of excitation energy calculated by the STHRN method with the available experimental data [29] for ^{124}Te and ^{126}Te. It is quite obvious that the non-yrast states calculated from the STHRN method closely match with experimental data. From the excitation energy diagram, it is observed that the energy levels obtained by the STHRN method for a given angular momentum states give agreement with the experimental data. The small deviation in the energy levels may be due to the absence of neutron–neutron, proton–proton and neutron–proton interactions [4]. This may be rectified by the inclusion of pairing interaction in the STHRN method. However, the calculation carried out in this article does not include pairing correlations.

The single neutron (S_n) and proton (S_p) separation energy as functions of angular momentum for various temperatures for the isotopes ^{122}Te and ^{126}Te are shown in Fig. 2 and Fig. 3. From the figures, it is observed that

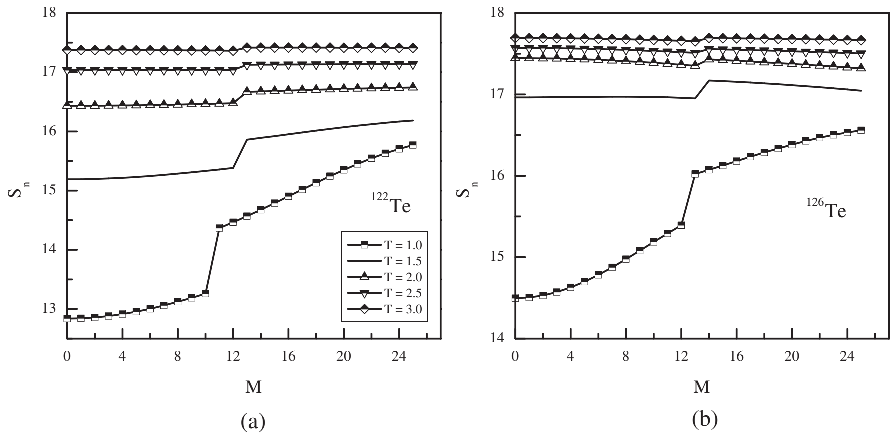

Fig. 2. Neutron separation energy S_n [MeV] as a function of angular momentum M for (a) ^{122}Te and (b) ^{126}Te for temperatures 1.0 to 3.0 MeV in steps of 0.5 MeV.

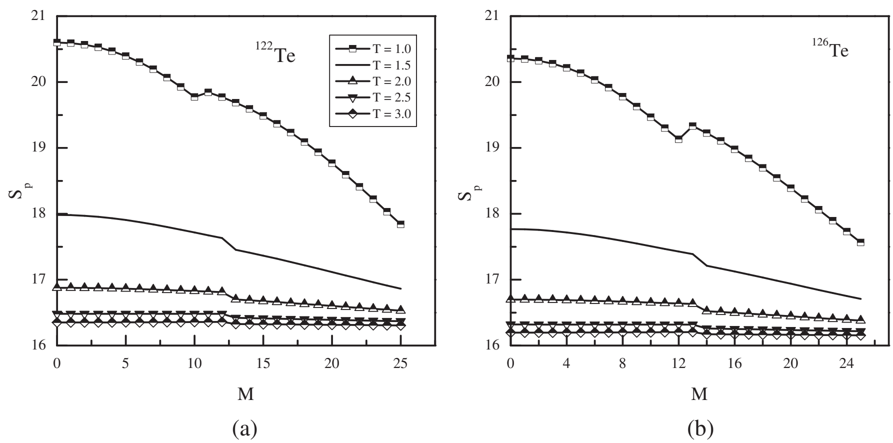

Fig. 3. Proton separation energy S_p [MeV] as a function of angular momentum M for (a) ^{122}Te and (b) ^{126}Te with the same description as in Fig. 2.

there occurs a sudden change in the separation energy at a certain angular momentum M. This sudden change was observed for ^{122}Te at $M = 10\,\hbar$ and ^{126}Te at $M = 12\,\hbar$, and it corresponds to a shape transition [30] from spherical ($\epsilon = 0.0$) to non-collective oblate shape ($\epsilon = 0.1$, $\gamma = -180°$). This shape transition phenomenon is observed in all the Te isotopes mentioned in this article. In the STHRN method, the deformation parameter ϵ at which the Te isotopes changes its shape to non-collective oblate is $\epsilon = 0.1$. This similar phenomenon was also observed experimentally for ^{120}Te nucleus and the quadrupole deformation ϵ at which the oblate shape was observed is $\epsilon = 0.15$. Thus, the experimentally obtained deformation value is in good agreement with the one obtained from the STHRN method. Moreover, the theoretical study carried out by Sabri *et al.* [9] by implying transitional interacting boson model including both IBM-1 and IBM-2 reported that Te isotopes change from spherical to deformed shapes which also serve as an additional reference for the observed shape transition phenomenon by the STHRN method. It is also reported [25, 26] that separation energy plays a major role in determining the structural change of a microscopic nuclear system. From Fig. 2 and Fig. 3, it can be seen that though the shape transition observed in S_n and S_p occurs at the same M, the S_n increases for increasing M, while S_p decreases as a function of M. The increase in S_n indicates a tight binding of neutrons [31], whereas the decrease in S_p indicates the loosely bound proton system with two protons outside the closed shell ($Z = 50$). Thus, from the separation energy plot, it is concluded that the microscopic system associated with the change in shape of the Te isotopes is the protons.

Figure 4 illustrates the spin cut-off parameter σ^2 as a function of temperature T and angular momentum M for the system ^{122}Te and ^{126}Te. The spin cut-off parameter plays a major role in determining the nuclear level density [32] and it can be derived by both semi-classical and quantum mechanical approach. In the present work, σ^2 is evaluated as a function of M and T by quantum mechanical approach *i.e.* $\sigma^2 = n\langle m^2\rangle$, where $\langle m^2\rangle$ is the average angular momentum projection squared on z-axis. The value of σ^2 tends to decrease for closed shell and then it increases [33]. Moreover, from Fig. 4, it is also observed that σ^2 increases with increasing temperature because the shell effect vanishes at higher temperature [34]. The expression of σ^2 given in Eq. (15) also known as microscopic expression for spin cut-off parameter, reflects the structural effect near the Fermi energy. This effect is not observed in either macroscopic approach of σ^2, probably known as rigid body value expression [$\sigma^2 = 0.0138A^{2/3}\sqrt{U/a}$] and in the Gilbert–Cameron expression [$\sigma^2 = 0.0138A^{2/3}\sqrt{a(U - E_{\mathrm{o}})}$], where a is the level density parameter, U is the excitation energy and E_{o} is the back shift energy [35]. It is also obvious that the value of σ^2 is constant for increasing angular momentum, but a sudden change is observed at a certain angular momentum. This sudden increase in the σ^2 implies a sharp spin distribution function and it is an indication of shape transition behavior. Moreover, all the parameters such as excitation energy, separation energy, spin cut-off parameter are plotted for free energy minimized equilibrium system, the value of σ^2 up to $M = 10\,\hbar$ in ^{122}Te corresponds to spherical shape with deformation $\epsilon = 0.0$, while above $M = 10\,\hbar$, the value of σ^2 corresponds to non-collective oblate shape ($\epsilon = 0.1, \gamma = -180°$). Thus, the shape of tellurium isotopes ^{114}Te, ^{120}Te, ^{122}Te, ^{124}Te and ^{126}Te is found to change its shape from spherical to oblate non-collective as a function of angular momentum. The separation energy and spin cut-off parameter for ^{114}Te and ^{124}Te isotopes for temperature 1.0 and 3.0 MeV have been tabulated in Table I.

TABLE I

Comparison of spin cut-off parameter σ^2, separation energy [MeV] for proton S_p and neutron S_n as a function of angular momentum M for the isotopes ^{114}Te and ^{124}Te for temperatures $T = 1$ and 3 MeV.

M [$\hbar$]	$T = 1$ MeV						$T = 3$ MeV					
	^{114}Te			^{124}Te			^{114}Te			^{124}Te		
	S_n [MeV]	S_p [MeV]	σ^2	S_n [MeV]	S_p [MeV]	σ^2	S_n [MeV]	S_p [MeV]	σ^2	S_n [MeV]	S_p [MeV]	σ^2
0	11.693	14.688	30.565	12.694	14.222	30.085	12.616	10.974	92.495	14.277	11.242	104.962
2	11.696	14.646	30.671	12.704	14.181	30.195	12.616	10.974	92.525	14.278	11.241	105.006
4	11.703	14.525	30.983	12.730	14.064	30.521	12.616	10.973	92.614	14.279	11.241	105.137
6	11.714	14.341	31.471	12.770	13.886	31.054	12.615	10.973	92.763	14.282	11.240	105.357
8	11.731	14.108	32.111	12.815	13.665	31.781	12.614	10.972	92.974	14.285	11.240	105.665
10	12.012	14.134	55.636	12.860	13.424	32.680	12.613	10.971	93.248	14.290	11.239	106.064
12	12.033	14.014	56.058	13.691	13.658	60.681	12.612	10.970	93.588	14.295	11.238	106.555
14	12.062	13.879	56.521	13.803	13.547	60.584	12.593	10.984	166.302	14.266	11.250	191.333
16	12.101	13.731	57.012	13.919	13.423	60.571	12.593	10.985	166.663	14.268	11.251	191.678
18	12.152	13.576	57.515	14.035	13.291	60.673	12.593	10.987	167.073	14.269	11.252	192.068
20	12.208	13.419	58.022	14.143	13.154	60.911	12.593	10.989	167.531	14.271	11.254	192.504
22	12.276	13.263	58.518	14.239	13.016	61.299	12.594	10.991	168.040	14.273	11.256	192.984
24	12.351	13.114	58.985	14.317	12.881	61.842	12.594	10.993	168.597	14.276	11.257	193.507

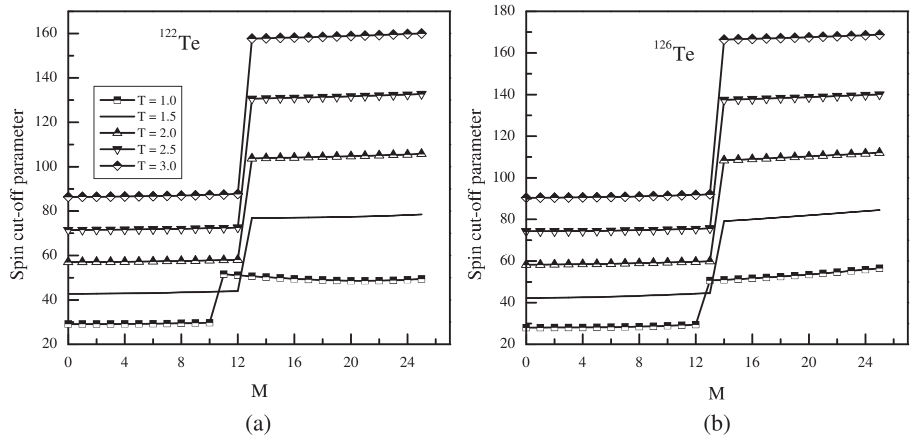

Fig. 4. Spin cut-off parameter σ^2 as a function of angular momentum M for (a) ^{122}Te and (b) ^{126}Te. The temperatures mentioned are in MeV.

In Fig. 5 (a)–(d), angular momentum M as a function of rotational frequency ω_{rot} also known as back bending phenomenon [36, 37] is plotted for ^{120}Te, ^{122}Te, ^{124}Te and ^{126}Te, and the results are compared with available experimental values [2, 10]. The rotational frequency calculated from the

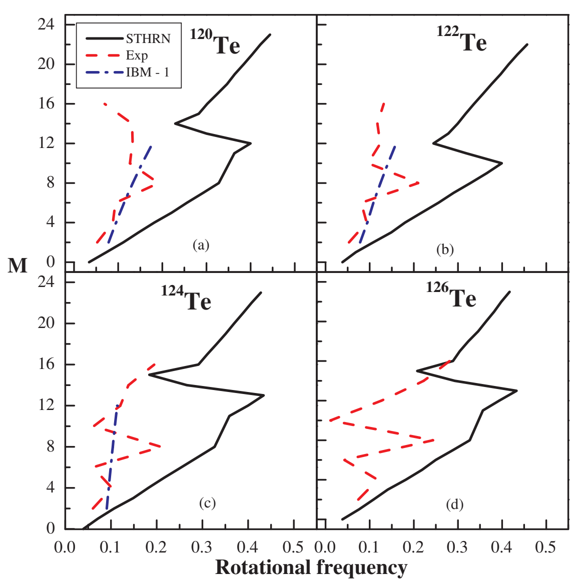

Fig. 5. (Color online) Angular momentum M as a function of rotational frequency $\omega\hbar$/MeV for (a) ^{120}Te (b) ^{122}Te, (c) ^{124}Te and, (d) ^{126}Te at $T = 3.0$ MeV. The solid black line represent the moment of inertia value calculated from the STHRN method, the dashed red line corresponds to experimental data [2, 10] and the dash-dotted blue line represents the IBM-1 data [7].

STHRN method gives comparable result with the experimental data. As discussed earlier in Introduction, the Interacting Boson Model (IBM-1) has been applied for the study of rotational behavior in Te isotopes [7], the moment of inertia plot increases linearly for increasing angular momentum, while a slight deviation is observed in the experimental value. This discrepancy is surpassed in the STHRN method because the results are reasonably comparable with the experimental data and follows almost the same pattern as that of the experimental data. This similar behavior has been observed in the moment of inertia plot which is shown in Fig. 6. The sudden deviation in the linear pattern of rotational frequency at certain angular momentum states corresponds to band crossing. For the even–even isotopes of Te, a single band crossing around $12\,\hbar$ is observed. Significant deviation of rotational levels at a particular angular momentum is due to both the single particle alignment and collective rotation contributing to the structural change.

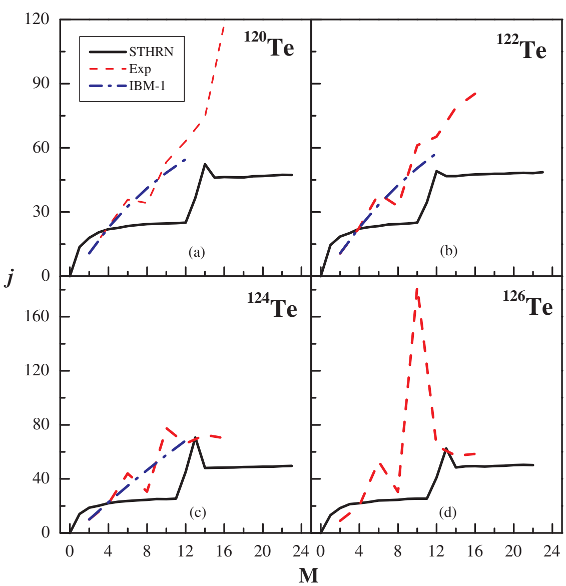

Fig. 6. (Color online) Moment of inertia j [$\hbar^2$/MeV] as a function of angular momentum $\hbar$ for (a) ^{120}Te, (b) ^{122}Te (c), ^{124}Te, and (d) ^{126}Te with the same description as in Fig. 5.

In Fig. 6, the kinematic moment of inertia as a function of angular momentum is shown for ^{120}Te, ^{122}Te, ^{124}Te and ^{126}Te. The dashed red lines correspond to the experimental values while the solid black lines correspond to the moment of inertia calculated from the STHRN method. At low- and high-angular momentum, the calculated moment of inertia is compared with the experimental values. The kinematical moment of inertia changes around

$M = 12\,\hbar$ for the even–even isotopes of Te. This change in the moment of inertia corresponds to the shape transition phenomenon due to band crossing and these are compared with experimental data [2, 10, 29].

TABLE II

Comparison of rotational frequency as a function of angular momentum calculated from STHRN method with the available IBM-1 model [7] and experimental data [2, 10] for the isotopes ^{120}Te, ^{122}Te and ^{124}Te for temperature 3.0 MeV respectively.

M $[\hbar]$	Rotational frequency $[\omega\hbar/\mathrm{MeV}]$								
	^{120}Te			^{122}Te			^{124}Te		
	STHRN	Exp	IBM-1	STHRN	Exp	IBM-1	STHRN	Exp	IBM-1
2	0.11157	0.0523	0.0784	0.10791	0.0530	0.0784	0.10693	0.0605	0.0908
4	0.18201	0.0882	0.0951	0.18018	0.0931	0.0946	0.18164	0.1020	0.0951
6	0.25586	0.0936	0.1132	0.25586	0.0805	0.1083	0.25391	0.0604	0.0995
8	0.32800	0.1913	0.1329	0.32837	0.2099	0.1243	0.32666	0.2095	0.1039
10	0.35079	0.1263	0.1542	0.39917	0.0962	0.1414	0.34862	0.0622	0.1086
12	0.39966	0.1325	0.1777	0.24438	0.1241	—	0.39966	0.1207	0.1133
14	0.22922	0.1315	—	0.29932	0.1174	—	0.26587	0.1382	—
16	0.30188	0.0692	—	0.33643	0.1317	—	0.29126	0.1947	—

TABLE III

Comparison of moment of inertia j as a function of angular momentum calculated from the STHRN method with the available IBM-1 model [7] and experimental data [2, 10] for the isotopes ^{120}Te, ^{122}Te and ^{124}Te for temperature 3.0 MeV respectively.

M $[\hbar]$	Moment of inertia $[\hbar^2\ \mathrm{MeV}]$								
	^{120}Te			^{122}Te			^{124}Te		
	STHRN	Exp	IBM-1	STHRN	Exp	IBM-1	STHRN	Exp	IBM-1
2	17.9256	10.7066	10.707	18.5339	10.6366	10.713	18.7032	9.5554	9.956
4	21.9772	23.3061	22.701	22.2005	22.6847	22.760	22.0215	21.6768	22.698
6	23.4504	35.7956	32.699	23.4504	38.5918	33.424	23.6307	44.1430	34.871
8	24.3900	34.2192	41.147	24.3628	32.6637	42.541	24.4903	30.4985	46.529
10	24.5510	53.3782	48.392	25.0519	61.1768	50.518	25.0214	77.5305	57.654
12	25.0214	63.1339	54.658	49.1029	65.2223	57.522	45.1350	66.1271	68.341
14	52.3450	74.3996	—	46.7732	78.8321	—	48.0671	72.5803	—
16	46.3760	117.781	—	47.5588	85.3642	—	48.3304	70.2154	—

The rotational frequency and kinematic moment of inertia as a function of angular momentum have been calculated by the STHRN method for ^{120}Te, ^{122}Te and ^{124}Te, and compared with the experimental [2, 10, 29] and other theoretical model such as IBM-1 [7], and are tabulated in Table II and Table III, respectively. The results show comparable agreement with the available experimental data, and the STHRN method follows the same pattern as that of experimental value instead of a linear pattern obtained in the IBM-1 model.

4. Summary and conclusion

Depending upon the thermodynamic parameters such as excitation energy, separation energy for protons and neutrons, rotational frequency, kinematic moment of inertia and spin cut-off parameter, shape transition behavior has been investigated for tellurium isotopes. The equilibrium shape for a non-zero temperature is found by minimizing free energy function. From the thermodynamical parameters, it is observed that the isotopes ^{114,120,122}Te are found to be spherical for $\epsilon = 0.0$ at the angular momentum range $M = 0$–$9\,\hbar$ and finally reaches the deformed non-collective oblate shape for $\epsilon = 0.1$ and $\gamma = -180^\circ$ at $M = 10$–$25\,\hbar$. The nucleus ^{124,126}Te remains of spherical shape with $\epsilon = 0.0$ for the angular momentum range $M = 0$–$12\,\hbar$ and becomes of oblate shape with $\epsilon = 0.1$ and $\gamma = -180^\circ$ for above $M \geq 13\,\hbar$. The observed oblate shape with deformation $\epsilon = 0.1$ is found to be in good agreement with the experimentally obtained deformation $\epsilon = 0.15$.

A sudden change arises in the spin cut-off parameter and separation energy of neutron and proton at the angular momentum $M = 12\,\hbar$ for the tellurium isotopes which confirms that the shape transition behavior has made an impact on all the thermodynamical parameters. The non-yrast states or the energy level diagram in Fig. 1 gives a comparable agreement with the experimental data. The moment of inertia and rotational frequency describes the spin distribution of nuclear levels. The calculated results are comparable with the experimental data and it overcomes the linear pattern obtained by IBM-1 model. Thus, using the triaxial Nilsson model, the shape transition phenomenon was discussed for the even–even isotopes of Te. From the results obtained, the STHRN method is found to be the most suitable method to study the even–even isotopes of tellurium nucleus at high-spin states compared to IBM-1 model. However, the small deviations can be overthrown by the inclusion of pairing interactions in the STHRN method.

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