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THE ε'/ε-STORY: 1976–2021*

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This story is dedicated to my great ε'/ε-collaborator Jean-Marc Gérard on the occasion of the 35th anniversary of our collaboration and his 64th birthday.

The ratio ε'/ε measures the size of the direct CP violation in $K_{\mathrm{L}} \to \pi\pi$ decays (ε') relative to the indirect one described by ε, and is very sensitive to new sources of CP violation. As such, it played a prominent role in particle physics already for 45 years. Due to the smallness of ε'/ε, its measurement required heroic efforts in the 1980s and the 1990s on both sides of the Atlantic with final results presented by NA48 and KTeV collaborations at the beginning of this millennium. On the other hand, even 45 years after the first calculation of ε'/ε, we do not know to which degree the Standard Model agrees with this data and how large is the room left for New Physics (NP) contributions to this ratio. This is due to significant non-perturbative (hadronic) uncertainties accompanied by partial cancellation between the QCD penguin contributions and electroweak penguin contributions. In addition to the calculation of hadronic matrix elements of the relevant operators including isospin breaking effects and QED corrections, it is crucial to accurately evaluate the Wilson coefficients of the relevant operators. While the significant control over the latter short-distance effects has been achieved already in the early 1990s, with several improvements since then, different views on the non-perturbative contributions to ε'/ε have been expressed by different authors over last thirty years. In fact, even at the dawn of the 2020s, the uncertainty in the room left for NP contributions to ε'/ε is still very significant, which I find to be very exciting. My own work on ε'/ε started in 1983 and involved both perturbative and non-perturbative calculations. This writing is a non-technical recollection of the steps which led to the present status of ε'/ε including several historical remarks not known to everybody. The present status of the $\Delta I = 1/2$ rule is also summarized.

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1. Overture

Let me open this writing with the 2020 formula for ε'/ε within the SM presented in [1]. It reads

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} = {\rm Im}\lambda_t \left[\left(1-\hat\Omega_{\rm eff}\right)\left(-2.9+15.4\, B_6^{(1/2)}(\mu^*)\right)+2.0-8.0\, B_8^{(3/2)}(\mu^*)\right]. \tag{1}$$

It includes NLO QCD corrections to the QCD penguin (QCDP) contributions and NNLO contributions to electroweak penguins (EWP). The coefficients in this formula and the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are scale-dependent. Their values for different scales are collected in Table 1 of [1]. Here, we will set $\mu^* = 1$ GeV because at this scale, it is most convenient to compare the values for $B_6^{(1/2)}$ and $B_8^{(3/2)}$ obtained in various non-perturbative approaches. The four contributions in (1) are dominated by the following operators:

— The terms involving the non-perturbative parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ contain only the contributions from the dominant QCDP operator Q_6 and the dominant EWP operator Q_8, respectively. There are two main reasons why Q_8 can here compete with Q_6 despite the smallness of the electroweak couplings relative to the QCD one. In the basic formula for ε'/ε, its contribution is enhanced relative to the Q_6's one by the factor ${\rm Re}A_0/{\rm Re}A_2 = 22.4$ with $A_{0,2}$ being isospin amplitudes. In addition, its Wilson coefficient is enhanced for the large top-quark mass which is not the case of the Q_6's one. The expressions for these two operators and the remaining operators mentioned below are given in Appendix A.

— The term -2.9 is fully dominated by the QCDP operator Q_4.

— The term $+2.0$ is fully dominated by EWP operators Q_9 and Q_{10}.

— The quantity $\hat\Omega_{\rm eff}$ represents the isospin breaking corrections and QED corrections beyond EWP contributions. It is not in the ballpark of a few percent as one would naively expect, because in ε'/ε it is enhanced by the factor ${\rm Re}A_0/{\rm Re}A_2 = 22.4$.

— ${\rm Im}\lambda_t$ is the CKM factor that within a few percent is in the ballpark of 1.45×10^{-4}.

On the other hand, the experimental world average of ε'/ε from NA48 [2] and KTeV [3, 4] collaborations reads

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}\,. \tag{2}$$

In what follows, we will describe the theoretical developments which led over four decades to the present values of $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $\hat{\Omega}_{\rm eff}$ obtained in various non-perturbative approaches. Inserting them into (1) will allow us to get various expectations for ε'/ε in the SM and to compare them with the data in (2).

2. The first period: 1976–1989

The story of ε'/ε begins on February 1, 1976 when John Ellis, Mary K. Gaillard and Dimitri Nanopoulos submitted a paper to *Nuclear Physics B* in which the first calculation of ε'/ε has been presented [5]. This pioneering calculation does not resemble by any means the present calculations of ε'/ε but this was the first one. In particular, it does not include renormalization group effects and takes only QCDP contributions into account. Moreover, in 1976, one had no idea about the matrix elements of QCDP operators contributing to ε'/ε.

However, already in 1975, Shifman, Vainshtein and Zakharov [6] suggested that the QCDPs could be responsible for the $\Delta I = 1/2$ rule: a large enhancement of the $K \to \pi\pi$ isospin amplitude A_0 over A_2 one as seen already above. This was quite natural at that time because QCDPs contribute only to A_0 and the attempts in 1974 to explain this rule through current–current operators (Q_1 and Q_2) including leading order QCD corrections in [7, 8] turned out to be unsuccessful. As found twelve years later in the framework of the Dual QCD (DQCD) approach [9] and recently confirmed by the RBC-UKQCD lattice QCD Collaboration [10], this failure was caused by the poor knowledge of hadronic matrix elements of these operators at that time. We will be more explicit about it at the end of this writing.

In order to explain the $\Delta I = 1/2$ rule with the help of QCDPs, Shifman, Vainshtein and Zakharov have simply chosen the values of their hadronic matrix elements so that this rule could be reproduced. However, the same matrix elements enter ε'/ε and Gilman and Wise [11] using their values from the Russian trio and performing renormalization group analysis for m_t much smaller than M_W, as thought in 1979, found ε'/ε to be in the ballpark of 5×10^{-2}. Soon after other analyses appeared, in particular the one by Guberina and Peccei in [12], finding values of ε'/ε in the ballpark of 10^{-3}–10^{-2}.

My first real encounter with ε'/ε goes back to 1981 when Bruce Winstein, the spokesman of future E731 and KTeV collaborations, entered my office at Fermilab and asked me about the work of Gilman and Wise. I told him that I never studied this ratio, but that one should seriously consider their result after they wrote other important papers, in particular [13].

Bruce told me that if ε'/ε was as large as claimed by Fred and Mark, he can definitely measure it and I encouraged him to do it because of the importance of CP violation.

My first paper on ε'/ε, in collaboration with Słominski and Steger, appeared in 1984 [14]. It was just a phenomenology of this ratio as a function of the hadronic matrix element of Q_6 of the CKM parameters and of m_t in the ballpark of 30 GeV as expected at that time.

Over the 1980s, the calculations of ε'/ε were refined through the inclusion of isospin breaking in the quark masses [15, 16], the inclusion of QED penguin effects for $m_t \leq M_W$ [15–19] and, in particular, through the first calculation of hadronic matrix elements of QCDP and EWP operators in QCD by Bardeen, Gérard and myself in the framework of the DQCD [9, 20, 21]. The latter calculations were done in the strict large-N limit of colours, but this was sufficient to see that QCDPs are not responsible for the $\Delta I = 1/2$ rule[1] and that ε'/ε was rather $\mathcal{O}(10^{-3})$ than $\mathcal{O}(10^{-2})$ as claimed by Gilman and Wise in [11]. Thus already in 1986, the values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ were known

$$B_6^{(1/2)} = 1.0\,, \qquad B_8^{(3/2)} = 1.0\,, \qquad (N \to \infty)\,, \tag{3}$$

and the study of their renormalization scale dependence for $\mu \geq 1$ GeV in the leading logarithmic (LO) approximation indicated that it was small, although as we will see later, it is non-negligible when higher order QCD corrections are taken into account and scales below 1 GeV in the framework of DQCD are considered.

It is rather surprizing that we did not calculate these two parameters including $1/N$ corrections in the 1980s. Such a calculation has been performed by Jean-Marc Gérard and me 29 years later with interesting consequences. I will report on it in Section 5.

While the numerical coefficients in (1) include higher order QCD corrections that were unknown in the 1980s, it is tempting to use this formula for $B_6^{(1/2)} = 1.0$ and $\hat{\Omega}_{\rm eff} = 0.29$ setting the last two terms to zero because EWP are irrelevant for low values of m_t expected at that time. Setting $\mathrm{Im}\lambda_t = 1.45 \times 10^{-4}$, we find

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} = 12.9 \times 10^{-4}\,, \tag{4}$$

in a good agreement with (2) which was first known fifteen years later. Setting $\hat{\Omega}_{\rm eff} = 0$ would increase this value to 18.1×10^{-4}. This is also consistent with experiment but this change illustrates the importance of isospin breaking corrections in the evaluation of ε'/ε. We will return to it

[1] More about it in Section 7.

in Section 5, where we will explain the dynamics behind $\hat{\Omega}_{\rm eff} = 0.29$. There is no point in discussing the errors in these estimates already now. We will do it in Section 6.

One of the last analyses of ε'/ε in the SM for $m_t \le M_W$ that included all effects known at that time, is the one in [22] in which typical values for ε'/ε were found in the ballpark of a few 10^{-3}.

In December 1988, I attended the Kaon conference in Vancouver. Fred Gilman during a lunch told me about his paper with Claudio Dib and Isard Dunietz [23], in which they calculated electroweak contributions to $K_{\rm L} \to \pi^0 e^+ e^-$ for an arbitrary top-quark mass finding for $m_t > 150$ GeV a large contribution from the Z^0 penguin that increased with m_t roughly as m_t^2. A similar result has been obtained by Jonathan Flynn and Lisa Randall [24]. This gave me the idea to calculate Z^0-penguin contribution to ε'/ε for large m_t. The QCDP and photon penguin contributions were known already at that time to have a very weak m_t dependence.

I moved to TUM in November 1988, but already in January 1989, I got a number of very good diploma students. Two of them, Gerhard Buchalla and Michaela Harlander were supposed to perform the calculation of ε'/ε together with me for an arbitrary top-quark mass. This was their first calculation of that type and I was simultaneously busy preparing first lectures in my life as well as starting the project on NLO QCD corrections to all flavour-violating processes in collaboration with Peter Weisz [25]. Consequently, my project with Gerhard and Michaela took longer than I initially expected and in April 1989[2], we were surprised by a paper by Flynn and Randall [26] in which ε'/ε including QCDP, EWP (γ and Z^0 penguins) and the relevant box diagrams were calculated for an arbitrary top-quark mass. Significant suppression of ε'/ε for $m_t > 150$ GeV by EWPs has been found by them. While definitely Flynn and Randall should be given the credit for pointing out the importance of Z^0 penguins in ε'/ε in print, fortunately for us their calculation of the QCDP contribution for large m_t was incorrect so that in fact the first correct calculation of ε'/ε in the SM for arbitrary m_t including LO QCD corrections has been presented by us in [27]. In fact, the suppression of ε'/ε for large m_t turned out to be significantly stronger than reported by Jonathan and Lisa in [26]. The strong cancellation between QCDP and EWP for $m_t > 150$ GeV found by us was soon confirmed in the erratum to [26] and, subsequently, by other authors [28, 29]. In fact, at that time, due to the aforementioned cancellation between QCDP and EWP contributions, the vanishing of ε'/ε in the SM and negative values for it could not be excluded.

[2] The paper appeared already in March but in 1989 the `arXiv` did not exist and we learned about it several weeks later.

It is then tempting to include next the EWP terms in (1) valid for the presently known m_t and set $B_6^{(1/2)} = B_8^{(3/2)} = 1.0$ and $\hat{\Omega}_{\rm eff} = 0.29$ to find this time

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} = 4.2 \times 10^{-4}\,, \tag{5}$$

that is by a factor of 4 below the experimental value in (2). Setting $\hat{\Omega}_{\rm eff} = 0$ would increase this value to 9.4×10^{-4}, roughly by a factor of two below the data and showing the importance of isospin breaking effects dominating the parameter $\hat{\Omega}_{\rm eff}$.

As the upper bound on m_t from electroweak precision tests was still unknown in 1989, one can find in our paper plots of ε'/ε as a function of m_t with ε'/ε vanishing for $m_t \approx 200$ GeV. I was visiting CERN in November 1989, one month after our paper appeared. In the CERN cafeteria, I met Jack Steinberger and asked him the standard question how he was doing. He told me that the NA31 Collaboration was shocked by the theory. I asked him which theory. Your paper with Buchalla and Harlander, he replied. In fact, at that time, only the first result from the NA31 Collaboration was known [30] implying $\varepsilon'/\varepsilon = (33 \pm 11) \times 10^{-4}$. The second NA31 analysis in 1992 gave $\varepsilon'/\varepsilon = (23 \pm 7) \times 10^{-4}$ [31] and the one from the E731 experiment at Fermilab — $\varepsilon'/\varepsilon = (7.4 \pm 5.9) \times 10^{-4}$ [32]. Therefore, the experimental situation of ε'/ε was still unclear in the first half of the 1990s.

It should be emphasized that the calculations described above were done in the leading logarithmic approximation (*e.g.* one-loop anomalous dimensions of the relevant operators) with the exception of the m_t dependence which in the analyses [26–28] has been already included at the NLO level. While such a procedure is not fully consistent, m_t dependence enters ε'/ε first at NLO in the renormalization group (RG) improved perturbation theory, it allowed for the first time to exhibit the strong m_t dependence of the EWP contributions and to identify the partial cancellation between QCDP and EWP contributions which is not seen in the strict leading logarithmic approximation.

3. The second period: 1990–2000

During the early 1990s, considerable progress has been made by calculating complete NLO corrections to ε'/ε. This means both QCD and electroweak corrections at NLO in the RG improved perturbation theory. This rather heroic effort has been accomplished by the Munich and Rome teams led by me [33–36] and Guido Martinelli [37, 38], respectively. I stress *heroic* because in that days the technology for two-loop calculations in weak decays, involving issues of γ_5 and evanescent operators, was at an early stage and, moreover, some of my colleagues told me that I was crazy doing such calculations in view of poorly known hadronic matrix elements. While the main goal in these papers was to calculate Wilson coefficients of QCDP and EWP operators at the NLO level in the context of ε'/ε, as a byproduct, many of our results could also be soon used for non-leptonic B and D decays. Reviews of this period of NLO calculations can be found in [39, 40] and in my recent book on weak decays [41].

Thus by 1993 ε'/ε was known at the NLO level as far as the Wilson coefficients of the contributing operators are concerned. On the other hand, the matrix elements of the contributing operators were either calculated in the large-N limit, represented for Q_6 and Q_8 operators by (3), or vacuum insertion method which for these two operators gives basically the same result as the large-N limit[3]. However, as other operators also play some role, significant uncertainties in the estimate ε'/ε have been found. In order to reduce them, it has been suggested in [36] to assume that the $\Delta I = 1/2$ rule can be completely explained by the SM dynamics, and to determine the matrix elements of the current–current operators Q_1 and Q_2 by using the experimental values of the $K \to \pi\pi$ isospin amplitudes A_0 and A_2. This in turn allowed, with the help of isospin symmetry, to determine the matrix elements of $(V-A)\times(V-A)$ operators such as Q_4, Q_9, Q_{10}. This method has been used in all analyses performed in Munich since then with some refinements made recently in [44, 45]. In this manner, the terms such as -2.9 and $+2.0$ in (1) could be more accurately determined than the remaining dominant terms.

However, this procedure has one weak point. If NP is responsible for a fraction of the $\Delta I = 1/2$ rule, this trick will not provide fully correct results for hadronic matrix elements and it is safer to calculate them using non-perturbative methods such as LQCD or DQCD which avoids automatically any NP contributions to hadronic operator matrix elements. NP can only affect the Wilson coefficients of these operators. Therefore, in fact, formula (1) does not assume that the SM explains fully the $\Delta I = 1/2$ rule, and to find the terms -2.9 and $+2.0$, only hadronic matrix elements from the RBC-UKQCD Collaboration [10] have been used, properly evolved to $\mu = 1$ GeV. Formula (1) exhibits only central values of various contributions. We will return to uncertainties in the final sections of this writeup.

Unfortunately in the 1990s, there were no results from LQCD for any of the terms in (1) and the relevant hadronic matrix elements of Q_6 and Q_8 operators, known only from DQCD at that time, and given by [16, 42]

[3] It should be stressed that the vacuum insertion method has no QCD basis and gives even wrong signs for the $1/N$ corrections to hadronic matrix elements as pointed out already in [9, 20, 21, 42] and confirmed numerically by the RBC-UKQCD Collaboration in [43].

$$\langle Q_6(\mu)\rangle_0 = -4\left[\frac{m_K^2}{m_s(\mu)+m_d(\mu)}\right]^2 (F_K - F_\pi)\, B_6^{(1/2)}(\mu)\,, \tag{6}$$

$$\langle Q_8(\mu)\rangle_2 = \sqrt{2}\left[\frac{m_K^2}{m_s(\mu)+m_d(\mu)}\right]^2 F_\pi\, B_8^{(3/2)}(\mu)\,, \tag{7}$$

with $B_6^{(1/2)} = B_8^{(3/2)} = 1.0$ in the large-N limit, were subject to the large uncertainty in the strange quark mass. We did not expose this dependence in (1) because in 2021 this uncertainty is very small. It is exhibited in the older Munich papers such as [46, 47]. Moreover, in the 1990s, it was not clear what was the error on $B_6^{(1/2)}$ and $B_8^{(3/2)}$, so that often in the phenomenological analyses of the Munich and also Rome group [48] their values were varied within the range of 0.8–1.2. We will see later that while presently the $B_8^{(3/2)}$ is already accurately known, this is not the case for $B_6^{(1/2)}$.

In any case, it turned out that NLO corrections led to additional suppression of ε'/ε beyond the one by EWP, so that typical values of ε'/ε in Munich papers were in the ballpark of 7×10^{-4} and the ones from Rome more like 3×10^{-4}, both values being significantly below the experimental result in (2).

The origin of the difference in ε'/ε resulting from the phenomenological analyses performed in Munich and Rome has been clarified only in 1998 in a paper written in collaboration with Paolo Gambino and Ulrich Haisch [49]. We have pointed out that without NNLO QCD corrections to EWP contribution, the results for ε'/ε are renormalization-scheme-dependent and exhibit significant non-physical dependences on the scale μ_t at which the top-quark mass $m_t(\mu_t)$ is evaluated as well as on the scale μ_W, at which the full SM is matched onto the low-energy effective theory without top, $W^\pm$, Z and the Higgs. It turned out then that the dominant difference between Munich and Rome NLO results originated in the use of different schemes for γ_5, NDR scheme in Munich and t'Hooft–Veltman scheme in Rome. This difference, as well as scale uncertainties mentioned above, have been significantly reduced in our paper, the first NNLO analysis of ε'/ε that concentrated on the EWP contributions. The corresponding NNLO analysis for QCDPs should hopefully be completed in 2021 [50, 51], although the first steps have already been made in 2004 by Martin Gorbahn and Ulrich Haisch, who calculated the three-loop anomalous dimension matrix of the relevant operators [52]. However, we already know today from preliminary results in [50, 51] that while the NNLO QCD corrections to EWPs increase their importance in ε'/ε [49], the ones to QCDPs suppress the latter. Consequently, there is a visible suppression of ε'/ε by the NNLO QCD corrections but the percentual effect of this suppression strongly depends on the values of the relevant hadronic matrix elements which in the case of QCDPs are still poorly known.

One of the last papers of this period was the one in [53] in which we addressed the first messages from the new round of ε'/ε-experiments by NA48 and KTeV collaborations that led eventually to the result in (2). We found that it was not possible to explain the data within the SM which motivated a detailed analysis in the MSSM [54].

4. The third period: 2000–2015

Already in the second period, but with even increased energy in the third period, the authors in [55–61] using the ideas from Chiral Perturbation Theory (ChPT) made a strong claim that final-state interactions (FSI) enhance $B_6^{(1/2)}$ above unity and suppress $B_8^{(3/2)}$ below it, so that the SM value for ε'/ε according to them is fully consistent with experiment. Albeit only a very inaccurate value $(17 \pm 9) \times 10^{-4}$ [61] could be obtained at that time. More about it later. Critical remarks about some of these papers appeared in [62] but they did not slow down the ChPT experts.

The main theoretical activity on ε'/ε within the SM in this period was another look at the parameter $\hat{\Omega}_{\rm eff}$ which includes both isospin breaking corrections and QED corrections beyond EWP contributions. Intensive analyses of these corrections can be found in [63–65], where references to earlier analyses can be found. The renaissance of such analyses with ChPT updates and also including the update of my 1987 analysis with Jean-Marc Gérard in [16] will be described in the fourth period. It modified significantly ChPT results, basically confirming with smaller uncertainties our 1987 value for $\hat{\Omega}_{\rm eff}$ that was only slightly lower than $\hat{\Omega}_{\rm eff} \approx 0.29$ used in our examples above.

In the absence of LQCD results and the claim of ChPT experts that the SM can reproduce the experimental value in (2), the interest in ε'/ε among theorists decreased significantly in this period. This was also due to the fact that the new experimental results in other sectors of particle physics, in particular in B physics and neutrino physics, could be easier addressed theoretically than ε'/ε. Selected reviews of ε'/ε in this period can be found in [47, 66–68]. On my part, I have performed with my Ph.D. students, postdocs and other collaborators a number of analyses of ε'/ε in various extensions of the SM that can easily be found in the `hep-arXiv`. I will not review them here because the input parameters and often the scale of NP changed in the last decade and it is better to summarize the status of NP models in the last period presented here. Nevertheless, the formulae presented in these papers could still turn out to be useful one day when the hadronic uncertainties will be reduced by much.

5. The fourth period: 2015–2019

The new era for ε'/ε began in 2015, when the RBC-UKQCD Collaboration [69, 70] presented their first results for $K \to \pi\pi$ hadronic matrix elements. From these results, one could extract values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ for $\mu = 1.30$ GeV. They were [44]

$$B_6^{(1/2)} = 0.57 \pm 0.19\,, \qquad B_8^{(3/2)} = 0.76 \pm 0.05\,, \qquad (\text{RBC-UKQCD, } 2015)\,. \tag{8}$$

It is evident that with such values, there is a strong cancellation between QCDP and EWP contributions. With $\hat{\Omega}_{\rm eff} \approx 0.15$, as used in 2015, and NLO QCD corrections, one finds values of ε'/ε in the ballpark of $(1\text{–}2) \times 10^{-4}$. This means one order of magnitude below the experimental value. However, with an error in the ballpark of 5×10^{-4}, one could talk of an ε'/ε anomaly of at most 3σ. The relevant analyses that extracted some matrix elements from data by assuming the $\Delta I = 1/2$ rule in the SM can be found in [44, 71]. The RBC-UKQCD lattice Collaboration [69, 70], calculating directly hadronic matrix elements of all operators, but not including isospin breaking (IB) effects, found similar result but with an error of 7×10^{-4}.

Motivated by the RBC-UKQCD results in (8), Jean-Marc Gérard and me calculated already in July 2015 $1/N$ corrections to the large-N limit in (3) [72]. These corrections, loop corrections in the meson theory with a *physical cut-off* $\Lambda \approx 0.7$ GeV, are the leading non-factorizable corrections to hadronic matrix elements of Q_6 and Q_8. Two main results in this paper are:

— Realization that the large-N result in (3) is not valid at scales $\mathcal{O}(1\,\text{GeV})$, as assumed in all papers before, but at much lower scales $\mathcal{O}(m_\pi^2)$. In order to find it out one has to calculate the non-factorizable $1/N$ contributions represented in DQCD by meson loops with a physical cut-off Λ, which separates the long-distance and short-distance contributions[4]. In the large-N limit, one cannot determine the scale in $B_6^{(1/2)}$ and $B_8^{(3/2)}$ and as for $\mu \geq 1$ GeV, the μ dependence of these parameters is weak [36], without knowing $1/N$ corrections it was useful to neglect this dependence.

— Calculation of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ at scales $\mathcal{O}(1$ GeV) by performing the meson evolution from the low factorization scale $\mathcal{O}(m_\pi^2)$ to the physical cutoff Λ of DQCD with the result

$$B_6^{(1/2)} \leq 0.6\,, \qquad B_8^{(3/2)} = 0.80 \pm 0.10\,, \qquad (\text{DQCD} - 2015)\,, \tag{9}$$

in a very good agreement with the RBC-UKQCD results in (8).

[4] For a detailed exposition of this point and comparison with ChPT which uses dimensional regularization, see Section 3 in [73].

As already mentioned, for scales above 1 GeV, both parameters decrease very slowly. This was already known from the 1993 analysis in [36], but as seen in Figs. 11 and 12 of that paper, $B_6^{(1/2)}$ decreases faster with increasing scale than $B_8^{(3/2)}$ in accordance with the pattern at low scales found in 2015 by Jean-Marc and myself. This can also be shown analytically [72]. Unfortunately, not knowing $1/N$ corrections to $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in 1993, both parameters have been set at $\mu = m_c$ in [36] to unity, which is clearly wrong.

While we did not exclude the possibility that our bound on $B_6^{(1/2)}$ could be violated by $1/N^2$ corrections, vector meson contributions and other effects like final-state interactions (FSI) not taken by us into account, one should notice that with only pseudoscalars included in the loops, the cut-off Λ has to be chosen below 1 GeV so that these omitted effects, even if they would increase $B_6^{(1/2)}$, could still be at least partially compensated by the running to higher scales that are explored by lattice QCD. In any case, we expected $B_6^{(1/2)}$ at scales $\mathcal{O}(1\ \mathrm{GeV})$ to be below unity.

Therefore, it appeared to us in 2015 that we could understand the QCD dynamics behind the LQCD values which was important for the following reason. There is no other lattice collaboration beyond RBC-UKQCD one calculating $B_6^{(1/2)}$ and $B_8^{(3/2)}$ at present, so that in the lattice world the results of the RBC-UKQCD Collaboration for ε'/ε could not be tested in 2015 and this is also the case now. As we will emphasize below, ChPT by itself has no means to verify or disprove the RBC-UKQCD results for $B_6^{(1/2)}$ and $B_8^{(3/2)}$. As already stated above, according to our analysis in [72], the main QCD dynamics behind the lattice values in (8) was the meson evolution at long distances, analogous to the well-known quark–gluon evolution at short-distance scales.

At a flavour workshop in Mainz in January 2016, two important ChPT experts, Gilberto Colangelo and Antonio Pich, expressed serious doubts about the RBC-UKQCD result in (8), because the $(\pi\pi)_{I=0}$ phase shift $\delta_0 \approx (24 \pm 5)^\circ$ obtained by RBC-UKQCD disagreed with $\delta_0 \approx 34^\circ$ obtained by combining dispersion theory with experimental input [74].

This criticism appeared in print in 2017 [75] and in two subsequent conference proceedings [76, 77]. It is in line with the one expressed many years ago in [55–61], but one should realize that with $\delta_0 \approx 24^\circ$, a big portion of FSI has been already taken into account in (8). Therefore, from Jean-Marc's and my point of view it appeared rather unlikely that increasing δ_0 up to its dispersive value $\delta_0 \approx 34^\circ$ would shift ε'/ε upwards by one order of magnitude. In fact, soon after the Mainz workshop, we have expressed this view in [78] demonstrating that possible enhancement of ε'/ε by FSI cannot be as large as its suppression through meson evolution absent in the calculations in [75]. Subsequently, in *A Christmas Story* [79], I have illustrated possible impact of the meson evolution on the result of the authors of [76, 77], finding much lower values of ε'/ε than claimed by them. Moreover, based on the insight from DQCD and NNLO QCD corrections as well as isospin breaking effects, I summarized my expectations for 2026, the 50$^{\rm th}$ anniversary of the first ε'/ε calculation and also my 80$^{\rm th}$ birthday, by

$$\boxed{(\varepsilon'/\varepsilon)_{\rm SM} = (5 \pm 2) \times 10^{-4}\,, \qquad (2026)\,.} \tag{10}$$

This expectation is in accord with our analysis [78] from which values of $B_6^{(1/2)}$ above unity at $\mu = 1$ GeV are rather unlikely even after the inclusion of FSI. However, for $B_6^{(1/2)} = 1$ we found already (5) and the slight decrease of $B_8^{(3/2)}$ could increase the value of ε'/ε by a bit. The error is just a guess estimate but even if it is large, confirmation of this result by LQCD would imply a significant anomaly and NP at work.

While waiting for the new RBC-UKQCD result, the calculation of isospin breaking affects and QED corrections, represented by $\hat{\Omega}_{\rm eff}$, has been updated within ChPT in [80] with the result

$$\hat{\Omega}_{\rm eff}^{(8)} = (17 \pm 9) \times 10^{-2}\,, \qquad ({\rm ChPT} - 2019)\,, \tag{11}$$

where the index "(8)" indicates that only contributions from the octet of pseudoscalars have been taken explicitly into account. The large error of 50% in this estimate originates from the difficulties in the matching of the long-distance (LD) and short-distance (SD) contributions in this framework, so that the effects of the flavour singlet η_0 cannot be explicitly included in this approach. They are buried in a poorly known low-enery constant L_7.

Using (11), the most recent estimate from ChPT [81, 82] reads

$$(\varepsilon'/\varepsilon)_{\rm SM} = (14 \pm 5) \times 10^{-4}\,, \qquad ({\rm ChPT} - 2019)\,. \tag{12}$$

It is in contrast to (10) fully consistent with experiment but due to the large error related to the problematic matching of LD and SD contributions in this approach, it still allows for significant NP contributions. We will return to this result in the next section.

Parallel to these analytic developments, RBC-UKQCD presented already in 2018 a new result for δ_0 that with $\delta_0 = (32.3 \pm 2.1)^\circ$ is within 1σ from its dispersive value[5]. The most important recent result from this collaboration is the new result for ε'/ε to be presented soon.

[5] See talks by Ch. Kelly and T. Wang at Lattice 2018 Conference.

The hints for the possible ε'/ε anomaly motivated several authors to perform BSM analyses of this ratio. We collected a selection of these papers in Table I in Appendix A. Here, we just mention that if this anomaly is confirmed one day by more precise calculations, the leptoquark models, with the possible exception of the vector U_1 model, will not be able to explain it because of the constraints from rare kaon decays [83]. This shows how crucial correlations of ε'/ε with other observables in a given NP scenario are. As indicated in Table I, such correlations have been analysed in other NP scenarios.

Moreover, the lessons gained from the SMEFT analysis in [84] should be very helpful in identifying NP behind possible ε'/ε anomaly. Such a general analysis allows to take the constraints from other processes, in particular from electroweak precision tests and collider processes, into account. To this end, the master formula for ε'/ε in [85], valid in any extension of the SM, should facilitate the search for the dynamics behind the possible anomaly in question. This formula is based on hadronic matrix elements of SM operators from LQCD and the BSM four-quark operators from DQCD [86]. The ones of chromomagnetic operators are known from LQCD [87] and DQCD [88]. They turned out to be less important than expected in the past. This master formula has been updated in [1] and will be soon generalized to include NLO QCD corrections to the BSM contributions.

6. The fifth period: 2020

The fifth period begins with a surprising new result from the RBC-UKQCD Collaboration [10]

$$(\varepsilon'/\varepsilon)_{\rm SM} = (21.7 \pm 8.4) \times 10^{-4}\,, \tag{13}$$

where statistical, parametric and systematic uncertainties have been added in quadrature. The central value is by an order of magnitude larger than the central 2015 value presented by this collaboration but is subject to large systematic uncertainties which dominate the quoted error. It is based on the improved values of the hadronic matrix elements of QCDP, includes the Wilson coefficients at the NLO level but does not include isospin breaking effects and NNLO QCD effects.

However, as already demonstrated in [45], the inclusion of the effects in question that are absent in (13) is important. Indeed, after including isospin-breaking effects from [80] in (11) and NNLO QCD corrections to EWP contributions [49], one finds using the hadronic matrix elements of RBC-UKQCD [1]

$$(\varepsilon'/\varepsilon)^{(8)}_{\rm SM} = (17.4 \pm 6.1) \times 10^{-4} \tag{14}$$

instead of (13). The index "(8)" indicates that only the octet of pseudoscalars has been included in the evaluation of isospin breaking effects summarized by $\hat{\Omega}_{\rm eff}^{(8)}$ in (11).

Yet, already in 1987, Jean-Marc and me [16][6] pointed out that the contribution of η_0 and of the resulting η–η' mixing cannot be neglected in the evaluation of ε'/ε. Updating and significantly improving our 1987 analysis we presented last spring the improved estimate of $\hat{\Omega}_{\rm eff}$ [89]

$$\boxed{\hat{\Omega}_{\rm eff}^{(9)} = (29 \pm 7) \times 10^{-2}\,, \qquad ({\rm NIB} - 2020)\,,} \tag{15}$$

where the index "(9)" indicates that the full nonet of pseudoscalars has been taken into account. This is the value we have used in our examples. Note that the percentual error, even if sizable, amounts to 24%, a factor of two smaller than in (11). Most importantly, the central value is by more than 50% higher than in (11).

At the FCPC 2020 Conference, Toni Pich referred to our result in (15) as the *naive* IB. As a coauthor of the naive dimensional regularization scheme (NDR) [25], I have no problem with this terminology and accepted this name in (15). It is well-known that the NDR scheme for γ_5 is used these days by almost everybody even if it is not as sophisticated as the 't Hooft–Veltman scheme. I expect that the future of NIB will be similar. The point is that our analysis in [89] is really not as naive as one would conclude from its name. In the decoupling limit for η_0, our approach reproduces IB from ChPT within 10%, but in contrast to ChPT, we are able to include the effect of η_0 that is very important. We should stress again that in our approach, this effect is included explicitly, while in the octet scheme, necessarily used in ChPT, it is buried in a poorly-known low-energy constant L_7. However, L_7 can only be extracted from the data in the large-N limit which in decays like $K \to \pi\pi$ is a bad approximation. Therefore, I expect that including η_0 effect in ε'/ε will remain a big challenge for ChPT for some time and whether ChPT will ever be able to match the present NIB result is unclear to me at present.

Using the value of $\hat{\Omega}_{\rm eff}^{(9)}$ in (15) together with hadronic matrix elements of RBC-UKQCD, one finds [1, 89]

$$\boxed{(\varepsilon'/\varepsilon)_{\rm SM}^{(9)} = (13.9 \pm 5.2) \times 10^{-4}\,.} \tag{16}$$

This is in my view the present best estimate of this ratio in the SM, if one accepts the results from RBC-UKQCD on hadronic matrix elements. However, I will stress below that I am not ready to do it at present and the

[6] See also [15].

main reason for the analyses in [1, 89] was the service to the community by improving the RBC-UKQCD analysis through the addition of isospin breaking effects and NNLO QCD corrections to EWPs.

The result in (16) agrees well with experiment and with the ChPT expectations (12) but in view of our comments on the ChPT analysis, it is on a more solid footing. Moreover, as we will demonstrate soon, its agreement with the ChPT value in (12) is a pure numerical coincidence. We expect further reduction of ε'/ε by roughly (5–10)% when NNLO QCD corrections to QCDP contributions will be taken into account [50, 51]. We look forward to the final results of these authors.

It is a good place to list the values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ at $\mu = 1$ GeV that can be extracted from the most recent RBC-UKQCD results for hadronic matrix elements for QCDPs [10] and from [69] for EWPs. This collaboration calculated them respectively for $\mu = 4$ GeV and $\mu = 3$ GeV, and one extracts at these scales [1]

$$B_6^{(1/2)}(4\ \text{GeV}) = 1.11 \pm 0.20\,, \qquad B_8^{(3/2)}(3\ \text{GeV}) = 0.70 \pm 0.04\,. \tag{17}$$

Performing the RG evolution down to 1 GeV, one finds [1] instead

$$B_6^{(1/2)}(1\ \text{GeV}) = 1.49 \pm 0.25\,, \qquad B_8^{(3/2)}(1\ \text{GeV}) = 0.85 \pm 0.05\,, \tag{18}$$

demonstrating the decrease of both parameters with increasing μ in accordance with DQCD [72].

Let us begin with the good news. Comparing the LQCD value for $B_8^{(3/2)}$ with DQCD one in (9), we find a very good agreement between LQCD and DQCD as far as EWP contribution to ε'/ε is concerned. This implies that this contribution to ε'/ε, that is unaffected by leading IB corrections, is already known within the SM with acceptable accuracy

$$\boxed{(\varepsilon'/\varepsilon)^{\text{EWP}}_{\text{SM}} = -(7 \pm 1) \times 10^{-4}\,, \qquad (\text{LQCD and DQCD})\,.} \tag{19}$$

Since both LQCD and DQCD can perform much better in the case of EWPs than in the case of QCDPs, I expect that this result will remain with us for coming years.

On the other hand, ChPT expected $B_8^{(3/2)} \approx 0.55$ [81] with the suppression below unity caused by FSI. Evidently this large suppression has not been confirmed by the RBC-UKQCD Collaboration. Including only the last two terms in (1), we find the EWP contribution estimated by ChPT to be roughly by a factor of 2 below the result in (19). This already signals that the agreement of (16) with (12) is an accidental numerical coincidence. This is undermined by the fact that the ChPT result was obtained with $\hat{\Omega}_{\text{eff}}^{(8)}$ in place of $\hat{\Omega}_{\text{eff}}^{(9)}$ and $\text{Im}\lambda_t = (1.35) \times 10^{-4}$ instead of our $\text{Im}\lambda_t = (1.45) \times 10^{-4}$.

The case of QCDPs is a different story. Here, the LQCD value overshoots the DQCD one by more than a factor of two and, consequently, despite the agreement on EWP contribution, the result in (16) based on RBC-UKQCD hadronic matrix elements differs by roughly a factor of three from my expectations for 2026 in (10). The difference from the RBC-UKQCD result that does not include IB, QED corrections and NNLO QCD effects in (13) differs even by a factor of four. On the other hand, the ChPT estimate of $B_6^{(1/2)}$ being in the ballpark of 1.35 is at first sight in the ballpark of the LQCD value. However, such a direct comparison is incorrect because the ChPT value of $B_6^{(1/2)}$ corresponds to much lower scales and, as demonstrated in Fig. 2 of [79], the inclusion of the meson evolution would make it significantly smaller in the ballpark of unity.

Let me next make a few additional critical remarks about ChPT estimate of ε'/ε:

— First of all, one should realize that strictly speaking, ε'/ε cannot be calculated in ChPT by itself because several important contributions in this framework depend on low-energy constants which have to be taken from LQCD calculations or low-energy data. While in the case of semi-leptonic decays this procedure is rather successful, in the case of $K \to \pi\pi$, it encounters serious problems which I doubt will be solved in the coming years.

— In particular, the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are evaluated in the strict large-N limit which, as we stressed, corresponds to the scales much lower than the scales at which Wilson coefficients can be calculated. However, ChPT by itself does not have meson evolution and, consequently, the matching with Wilson coefficients is practically impossible. The authors of [81] admit this stating by themselves that the dominant error in their estimate of ε'/ε originates from their ignorance about this matching. The low-energy constant which they have to know to overcome this difficulty is L_5. Its value obtained from LQCD is still rather uncertain implying large error in (12). However, this error could be an underestimate for the following reasons.

— The expression for ε'/ε in terms of low-energy constants presented by these authors is obtained in the large-N limit. While the numerical value for L_5 from LQCD certainly includes some $1/N$ corrections, there could still be some missing $1/N$ contributions both in the formula for ε'/ε used by them and also in the extraction of L_5 in case they would try to find its value from some low-energy data.

— Equally problematic is the inclusion of the singlet η_0 and of the η–η' mixing which has been known for more than 30 years to suppress ε'/ε significantly [15, 16, 89]. This effect is buried again in a poorly-known low-energy constant, this time L_7. If the authors of [81] would use the NIB of (15), they would end up, even without the meson evolution, with the value of ε'/ε in the ballpark of $(9–11) \times 10^{-4}$.

— There is one point where ChPT could be superior to DQCD. These are FSI. However, at least in the case of EWP contribution to ε'/ε, its strong suppression through FSI predicted by ChPT experts has not been confirmed by RBC-UKQCD which obtains the result for $B_8^{(3/2)}$ in agreement with DQCD. Here presently these effects are not included and the modest suppression of $B_8^{(3/2)}$ below unity in DQCD is due to the meson evolution.

Next, let me move to make several comments on the most recent analysis of the RBC-UKQCD Collaboration. I am doing it not only because all my recent analyses of ε'/ε [1, 45, 79, 89, 90] have been simply ignored by this collaboration. In particular, I want to list the arguments while I still expect the final result for ε'/ε in the SM to be close to my 2026 expectations and certainly by a factor of at least two below the present RBC-UKQCD value in (13).

While one should admire the RBC-UKQCD Collaboration for their heroic efforts over at least one decade to calculate ε'/ε, as they emphasize from first principles, here are my main problems with accepting their result in (13) despite the large error they admit.

— *The inclusion of isospin breaking corrections as a symmetric error to their value without these corrections.* This is like stating that these corrections could, in principle, enhance ε'/ε implying thereby an anomaly in this ratio that would require NP to *suppress* this ratio to agree with data. Equivalently, it amounts to question all the work done over more than thirty years by different authors, with different methods that imply significant suppression of ε'/ε by these corrections, in particular by the presence of the η_0 and related η–η' mixing. This is evident by comparing the value in (13) with (16) which use the same hadronic matrix elements from RBC-UKQCD but in (16) also isospin breaking corrections are included. While it could be legitimate to use LQCD for the calculation of all effects, talking to various colleagues who are closer to LQCD than me, it is likely that we will not see a value for ε'/ε from LQCD including isospin corrections, in particular those from η–η' mixing, before 2026. I do hope very much that these expectations are wrong and the RBC-UKQCD Collaboration or other LQCD groups will surprise us again by calculating this time these corrections with respectable precision.

— *The absence of GIM mechanism, the crucial property of the SM and of a great relevance for kaon physics.* RBC-UKQCD works at 4 GeV without the inclusion of charm. While this omission is more important for the QCDP contributions to the $\Delta I = 1/2$ rule (see below), because in the case of ε'/ε GIM mechanism is broken already at higher scales by the disparity of top-quark and charm-quark masses, it is to be expected that the inclusion of charm will play a role also for ε'/ε. I expect that this issue will be solved before a satisfactory inclusion of isospin breaking effects by LQCD in general.

— *Matching of the lattice renormalization scheme to the $\overline{MS}$ scheme used for the calculation of Wilson coefficients.* In order to reduce the errors in the matching, that is known presently at the one-loop level, RBC-UKQCD works at 4 GeV, which without charm is problematic but indeed could help in improving the accuracy of the matching because of the smaller value of α_s at this scale than at $\mu = 1.5$ GeV used by them in 2015. On the other hand, the problem with the omission of charm at this lower scale is smaller.

Finally, I am looking forward to more accurate LQCD results on ε'/ε from Japan [91]. This could help in resolving the controversy described above.

7. The present status of the $\Delta I = 1/2$ rule

I cannot resist to add a few lines about the $\Delta I = 1/2$ rule [92, 93] after RBC-UKQCD indicated in [10] that they should be credited for the identification of the dynamics behind this rule. This is disappointing, because in their first paper [43] they were more careful about the history of this rule as are the authors of [94, 95]. This history is summarized in our 2014 paper [73] and, in particular, in Section 7.2.3 of my recent book [41]. From this, it is evident that the credit for the identification of the basic dynamics behind this rule should go to the authors of [9] who demonstrated that the current–current operators and not QCDP operators[7], as claimed by the Russian masters [6], are dominantly responsible for this rule. While the short-distance contributions analysed in [7, 8] provided only a small enhancement of the $K \to \pi\pi$ isospin amplitude A_0 over A_2 one, the continuation

[7] The fact that QCDPs cannot be important for the $\Delta I = 1/2$ rule has also been noticed by the authors of [96].

of this *quark evolution* by *meson evolution* in the non-perturbative region down to very low energy scales within DQCD approach allowed to obtain the enhancement of the ratio $\text{Re}A_0/\text{Re}A_2$ up to 16 ± 2 from $\sqrt{2}$, in the absence of QCD dynamics, compared with the experimental value of 22.4. While a number of authors suggested different solutions that were published in the 1990s [55, 66, 97–103], the recent result from the RBC-UKQCD Collaboration seems to confirm our findings of 1986, although I have the impression that they cannot see it doing purely numerical work. In itself, it is a very important result and the RBC-UKQCD Collaboration should be congratulated for it. Besides, recent dissection of the $\Delta I = 1/2$ rule at large-N by other LQCD experts [94, 95] shows that in this decade, we should know whether NP plays any role in this rule. Here, also the studies from [91] will be important.

As stated above, it is dominantly the meson evolution responsible for the enhancement of $\text{Re}A_0$ and the suppression of $\text{Re}A_2$ with respect to the values in which QCD interactions are switched off by going to the large-N limit[8]

$$\text{Re}A_0 = 3.59 \times 10^{-8}\ \text{GeV}\,, \qquad \text{Re}A_2 = 2.54 \times 10^{-8}\ \text{GeV}\,, \qquad \frac{\text{Re}A_0}{\text{Re}A_2} = \sqrt{2}\,, \tag{20}$$

in plain disagreement with the experimental value of 22.4. It should be emphasized that the explanation of the missing enhancement factor of 15.8 through some dynamics must simultaneously give the correct values for $\text{Re}A_0$ and $\text{Re}A_2$ [104]

$$\text{Re}A_0 = 27.04(1) \times 10^{-8}\ \ \text{GeV}\,, \qquad \text{Re}A_2 = 1.210(2) \times 10^{-8}\ \ \text{GeV}\,. \tag{21}$$

This means that these dynamics should suppress $\text{Re}A_2$ by a factor of 2.1, not more, and enhance $\text{Re}A_0$ by a factor of 7.5. This tells us that while the suppression of $\text{Re}A_2$ is an important ingredient in the $\Delta I = 1/2$ rule, it is not the main origin of this rule. It is the enhancement of $\text{Re}A_0$, as already emphasized in [6], even if, in contrast to this paper, as demonstrated in [9], the current–current operators are responsible dominantly for this rule and not QCD penguins. More details can be found in our papers and in my book.

I am making this point because the RBC-UKQCD Collaboration in their papers and talks stressed more the suppression of $\text{Re}A_2$ and not the enhancement of $\text{Re}A_0$ as the *major* dynamics behind this rule. The simple discussion above shows that this is simply not true. Working numerically at 4 GeV and being not able to switch-off QCD interactions in LQCD, one simply cannot

[8] Only operator Q_2 contributes in this limit and the mesons are non-interacting in this limit. See [41, 73] for a detailed presentation.

see properly what is really going on. Yet, as demonstrated by Jean-Marc and myself in [105], in the context of BSM K^0–$\bar{K}^0$ matrix elements, meson evolution allows to explain analytically the values obtained by various lattice collaborations [106–110]. Without the meson evolution, one would fail to explain some of these matrix elements by factors of three. Therefore, we are convinced that this is also the case of $K \to \pi\pi$, that is of the $\Delta I = 1/2$ rule and of ε'/ε. In other words, meson evolution is bound to be present in RBC-UKQCD calculations and its effects will hopefully be more visible in ε'/ε in the next round of calculations by this collaboration.

This is also supported by the comparison of LQCD with DQCD in this context in Section 9 of [73], where we use the language of contractions used by LQCD experts. As already noticed in [43], the dominant two contractions work constructively to enhance $\mathrm{Re}A_0$ and destructively to suppress $\mathrm{Re}A_2$, but at 4 GeV, this only describes in the lattice language the property of asymptotic freedom that has been found in corresponding Wilson coefficients by Altarelli and Maiani [7], and Gaillard and Lee [8] by now 45 years ago. This in itself is an important result because it indicates that a proper matching of hadronic matrix elements from LQCD with the Wilson coefficients is possible.

However, in order to understand why the contribution of the sum of these contractions $2C_1 + C_2$ to $\mathrm{Re}A_0$ [9] is by a factor of 22 larger than their difference $(C_1 - C_2)$ entering $\mathrm{Re}A_2$, one has to understand physically the dynamics behind the values of C_1 and C_2. However, these dynamics must be below 1 GeV, not at 4 GeV. Otherwise the authors of [7, 8] working above 1 GeV would identify it 45 years ago, but they did not. Thus, while DQCD working below 1 GeV can identify these dynamics as the meson evolution, LQCD, working at 4 GeV can only see its implications in their numerical values which summarize all the contributions below 4 GeV.

Yet, we agree with CHPT experts that a part of the $\mathrm{Re}A_0$ enhancement over $\mathrm{Re}A_2$ comes from FSI, although they were not the first to point it out. To my knowledge, these were nuclear physicists [111] motivated by our first results in the 1980s.

Finally, let me emphasize the following problem in the present calculation of the $\Delta I = 1/2$ rule by the RBC-UKQCD Collaboration despite the impressive result they obtained for the ratio in question:

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 19.9(2.3)(4.4)\,, \qquad \text{RBC-UKQCD (2020)} \tag{22}$$

that is consistent with the DQCD value of 16 ± 2 and is in agreement with the experimental value 22.4.

[9] As seen in equation (122) of [73] and also in [43], there is a second constructive contribution $C_1 + 2C_2$ but with a much smaller coefficient multiplying it, it is subleading.

The inspection of various contributions to ReA_0 in the RBC-UKQCD result reveals the surprising fact that at the scale of 4 GeV, the QCDP contribution, known to be governed due to GIM mechanism by the $m_c - m_u$ difference, not only amounts to 10% of ReA_0 at this scale but even *suppresses* this amplitude thereby working against the $\Delta I = 1/2$ rule. This contribution would be absent in the presence of charm contributions due to GIM mechanism shifting the result in (22) towards the experimental value. However, in the presence of charm, other contributions could modify this result and we have to wait until next round of LQCD calculations which hopefully will include GIM mechanism. I am well aware of the fact that QCDP contribution to ReA_0 is scale-dependent and increases with lowering the scale but at 4 GeV it should be negligible. From DQCD estimates at 1 GeV, it could amount to a 10–15% effect but then it would slightly enhance and not suppress ReA_0.

I do hope very much that the RBC-UKQCD Collaboration appreciates my detailed analysis of their results[10] despite my reservations and that my critical remarks will motivate them to provide next time much improved results for both ε'/ε and the $\Delta I = 1/2$ rule with all important effects taken into account and with much smaller errors. There is no doubt that from present perspective LQCD will one day give us most precise values for the $\Delta I = 1/2$ rule and ε'/ε within the SM, hopefully revealing some NP contributions to both. Yet, this could still take a decade of tedious calculations. I expect that other LQCD collaborations like the ones in [91, 94] will make similar efforts.

In summary, in my view, the status of the $\Delta I = 1/2$ rule as of 2020 is as follows:

— The dominant dynamics behind this rule is our beloved QCD. It is simply the *quark evolution* from M_W down to scale $\mathcal{O}(1\text{ GeV})$ as analysed first by Altarelli and Maiani [7], and Gaillard and Lee [8], followed by the *meson evolution* down to very low scales at which QCD becomes a theory of weakly interacting mesons and free theory of mesons in the strict large-N limit [112–115]. This non-perturbative evolution within the Dual QCD approach dominates by far the enhancement of ReA_0 over ReA_2 as demonstrated by Bardeen, Gérard and myself in [9, 73].

— This picture appears to be confirmed in particular by the RBC-UKQCD Collaboration [10, 43] when one takes the insight from [105] into account, but also other LQCD collaborations [91, 94] made significant progress here. Very importantly, from present perspective, only LQCD can provide satisfactory estimate of the room left for New Physics in

[10] Partly presented already in [1, 89].

this rule. Most likely, it is at most at the level of 20%. A detailed analysis in [116] shows that heavy neutral coloured gauge bosons G' but not Z' could provide such contributions while satisfying all existing constraints.

8. A strategy for the coming years

Evidently, there is no question about that the situation with ε'/ε is very unclear at present. Personally, I am truly delighted that my expectations for ε'/ε in the SM, based on the collaboration with Jean-Marc, are very different from the ones of RBC-UKQCD and of the ChPT experts. There are two reasons for this. First of all, if we all agreed that the SM agrees with data on ε'/ε, the future of ε'/ε would be rather boring. Equally important, if one day my expectations in (10) will be confirmed by several LQCD collaborations, it will be evident who should be credited for the identification of the ε'/ε anomaly.

However, even ChPT practitioners and the RBC-UKQCD Collaboration, who strongly disagree with Jean-Marc's and my claims about ε'/ε, cannot exclude that at a certain level, NP will be required to fit its experimental value. Yet, analysing NP models containing new parameters for various values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ complicates the search for NP by much. Here comes one idea which in my view could give us a clue which NP models could have a chance to explain possible anomaly dependent on its size [117].

Instead, of varying $B_6^{(1/2)}$ and $B_8^{(3/2)}$, we can just write

$$\frac{\varepsilon'}{\varepsilon} = \left(\frac{\varepsilon'}{\varepsilon}\right)^{\rm SM} + \left(\frac{\varepsilon'}{\varepsilon}\right)^{\rm BSM} \tag{23}$$

and assume that NP provides a shift in ε'/ε

$$\left(\frac{\varepsilon'}{\varepsilon}\right)^{\rm BSM} = \kappa_{\varepsilon'} \times 10^{-3}\,, \qquad -0.5 \le \kappa_{\varepsilon'} \le 1.0\,, \tag{24}$$

with the range for $\kappa_{\varepsilon'}$ indicating conservatively the room left for BSM contributions. This range is dictated by the recent analyses in [1, 89]. Personally, I would vary $\kappa_{\varepsilon'}$ only in the range of $0.5 \le \kappa_{\varepsilon'} \le 1.5$ but this would mean ignoring the results from ChPT and RBC-UKQCD which I do not want to do today. We are fortunate that there is no interference between these two contributions although the Wilson coefficients of SM operators can be affected by NP. The corresponding modifications are included in the BSM term.

Now, in a given NP model, ε'/ε is correlated with other observables, in particular those in the K-meson decays like $K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu}$, $K^+ \to \pi^+ \nu\bar{\nu}$, $K_{\mathrm{L}} \to \pi^0 \mu^+ \mu^-$ and $K_{\mathrm{S}} \to \mu^+ \mu^-$. As a consequence, one can study the dependence of the corresponding branching ratios as functions of $\kappa_{\varepsilon'}$ which depends on the model considered. We refer to numerous plots of such dependences in [117, 118].

One can now ask what is the uncertainty in BSM contributions due to hadronic matrix elements. Here comes a piece of good news. It turns out that in most NP models considered until now, the dominant shift in ε'/ε comes from the modifications of the EWP contributions because similar to isospin breaking effects, they are enhanced by a factor $\mathrm{Re}A_0/\mathrm{Re}A_2$ relative to those of QCDPs. However, $B_8^{(3/2)}$ is already well-known so that the main uncertainty in NP contributions comes in this case from new parameters in BSM models which can be constrained by other processes.

Yet, there are also contributions from BSM operators, in particular scalar and tensor operators. Their matrix elements have only been calculated within DQCD [86]. However, the master formula for all BSM scenarios presented in [84, 85] demonstrates very clearly the dominance of the $\Delta I = 3/2$ contributions over the $\Delta I = 1/2$ ones also in this case. While the computation of the matrix elements of these new operators within LQCD has still to be done, it is expected that the uncertainties in the dominant $\Delta I = 3/2$ contributions will be smaller than the present uncertainty in the matrix element of the QCDP operator Q_6.

9. Final remarks

Our ε'/ε story approaches the end. I have concentrated here on the non-perturbative calculations because no consensus has been reached among theorists until now. However, being privileged to be one of two[11] theorists who calculated both short-distance and long-distance contributions to ε'/ε, I want to emphasize that without NLO calculations of Wilson coefficients performed in the early 1990s in Munich [33–36] and Rome [37, 38], the uncertainties in the prediction for ε'/ε would be even larger. In particular, without these corrections the matching of Wilson coefficients to hadronic matrix elements performed by LQCD would not be possible. The result would be simply renormalization-scheme-dependent. The story of these calculations is described in [40] and in this context, I want to thank Guido Martinelli and his strong team for a very friendly competition we had.

[11] The second is Guido Martinelli in the context of the contributions of chromomagnetic penguins to ε'/ε [87].

The uncertainties in various steps leading to (16) should still be significantly decreased in the coming years and I do hope very much that by 2026 the picture of ε'/ε with respect to possible NP contributions will be much clearer than it is today. In particular, the identification of new sources of CP violation in the data of NA48 and KTeV collaborations would be very important because they could play in principle a role in the explanation of our existence. I really have no idea whether NP in ε'/ε, if found, would be responsible for our existence. Not only because this is presently beyond my skills but also because we did not yet identify what this NP could be, although several ideas have been put forward. They are listed in Table I. My bet would be a heavy Z' and/or vector-like quarks but not leptoquarks.

My recollection of the ε'/ε efforts dealt dominantly with theory. Yet, without the measurements of ε'/ε by NA48 and KTeV collaborations, all these discussions between RBC-UKQCD, ChPT and DQCD experts would be much less exciting and we should thank these two important experimental groups for the result in (2).

Over 37 years I had 39 collaborators with whom I have written papers on ε'/ε but Jean-Marc Gérard is the one with whom I have written the highest number of ε'/ε papers and, in fact, among the *male* physicists, the one with whom I have written the highest number of papers to date. In particular this includes papers on the $\Delta I = 1/2$ rule, and large-N calculations in the context of the Dual QCD approach, 20 journal papers in total. Only Monika Blanke and Jennifer Girrbach-Noe can compete with him in this respect. The important virtue of this collaboration was that we had to struggle for 35 years against the spanish matadors, of three physics generations by now, Eduardo de Rafael, Antonio Pich and Hector Gisbert. In this context, we were declared to be *naive*, both as far as final-state interactions and isospin breaking corrections to ε'/ε are concerned, which clearly united us. Fortunately, the collected joint number of citations (2656) demonstrates that our work has not been ignored by the community and there is no doubt that this number will increase in the future. I want to thank Jean-Marc for this great and most pleasant collaboration and, in particular, for his deep insight into low-energy QCD from which I benefited in many ways. Thanks go also to those with whom I performed many analyses of ε'/ε within the SM and in various NP scenarios. Several of their names appeared in the reference list to this writing. The remaining ones can be found in INSPIRE.

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Appendix A

Operators and New Physics analyses

We list the operators mentioned in the text:

Current–Current:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}\,(\bar{u}_\beta d_\alpha)_{V-A}\,, \qquad Q_2 = (\bar{s}u)_{V-A}\,(\bar{u}d)_{V-A}\,, \tag{A.1}$$

QCD Penguins:

$$Q_3 = (\bar{s}d)_{V-A}\sum_{q=u,d,s,c,b}(\bar{q}q)_{V-A}\,, \qquad Q_4 = (\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s,c,b}(\bar{q}_\beta q_\alpha)_{V-A}\,, \tag{A.2}$$

$$Q_5 = (\bar{s}d)_{V-A}\sum_{q=u,d,s,c,b}(\bar{q}q)_{V+A}\,, \qquad Q_6 = (\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s,c,b}(\bar{q}_\beta q_\alpha)_{V+A}\,, \tag{A.3}$$

Electroweak Penguins:

$$\begin{aligned} Q_7 &= \tfrac{3}{2}\,(\bar{s}d)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V+A}\,, \\ Q_8 &= \tfrac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V+A}\,, \qquad \text{(A.4)} \\ Q_9 &= \tfrac{3}{2}\,(\bar{s}d)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V-A}\,, \\ Q_{10} &= \tfrac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V-A}\,. \qquad \text{(A.5)} \end{aligned}$$

Here, α,β denote colour indices and e_q denotes the electric quark charges reflecting the electroweak origin of $Q_7,\ldots,Q_{10}$. Finally, $(\bar{s}d)_{V-A} \equiv \bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha$.

TABLE I

Papers studying implications of a possible ε'/ε anomaly.

NP Scenario	References	Correlations with
LHT	[119]	$K_{\rm L}\to\pi^0\nu\bar\nu$
Z-FCNC	[117, 120, 121]	$K^+\to\pi^+\nu\bar\nu$ and $K_{\rm L}\to\pi^0\nu\bar\nu$
Z'	[117]	$K^+\to\pi^+\nu\bar\nu$, $K_{\rm L}\to\pi^0\nu\bar\nu$ and ΔM_K
Simplified models	[122]	$K_{\rm L}\to\pi^0\nu\bar\nu$
331 models	[123, 124]	$b\to s\ell^+\ell^-$
Vector-like quarks	[125]	$K^+\to\pi^+\nu\bar\nu$, $K_{\rm L}\to\pi^0\nu\bar\nu$ and ΔM_K
Supersymmetry	[126–130]	$K^+\to\pi^+\nu\bar\nu$ and $K_{\rm L}\to\pi^0\nu\bar\nu$
2HDM	[131, 132]	$K^+\to\pi^+\nu\bar\nu$ and $K_{\rm L}\to\pi^0\nu\bar\nu$
Right-handed currents	[133, 134]	EDMs
Left–right symmetry	[135, 136]	EDMs
Leptoquarks	[83]	all rare kaon decays
SMEFT	[84]	several processes
SU(8)	[137]	$b\to s\ell^+\ell^-$, $K^+\to\pi^+\nu\bar\nu$ $K_{\rm L}\to\pi^0\nu\bar\nu$
Diquarks	[138, 139]	ε_K, $K^+\to\pi^+\nu\bar\nu$ $K_{\rm L}\to\pi^0\nu\bar\nu$
3HDM + ν_R	[140]	$R(K^{(*)})$, $R(D^{(*)})$
Vector-like compositeness	[141]	$R(K^{(*)})$, $R(D^{(*)})$, ε_K $K^+\to\pi^+\nu\bar\nu$, $K_{\rm L}\to\pi^0\nu\bar\nu$
U(2)^3 flavour symmetry	[142]	hadronic $B\to K\pi$, $B_{s,d}\to(KK,\pi\pi)$ $B_s\to\phi(\rho^0,\pi^0)$

REFERENCES

[1] J. Aebischer, C. Bobeth, A.J. Buras, «ε'/ε in the Standard Model at the Dawn of the 2020s», *Eur. Phys. J. C* **80**, 705 (2020), arXiv:2005.05978 [hep-ph].

[2] NA48 Collaboration (J. Batley *et al.*), «A precision measurement of direct CP violation in the decay of neutral kaons into two pions», *Phys. Lett. B* **544**, 97 (2002), arXiv:hep-ex/0208009.

[3] KTeV Collaboration (A. Alavi-Harati *et al.*), «Measurements of direct CP violation, CPT symmetry, other parameters in the neutral kaon system», *Phys. Rev. D* **67**, 012005 (2003), arXiv:hep-ex/0208007.

[4] KTeV Collaboration (E. Abouzaid *et al.*), «Precise measurements of direct CP violation, CPT symmetry, other parameters in the neutral kaon system», *Phys. Rev. D* **83**, 092001 (2011), arXiv:1011.0127 [hep-ex].

[5] J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, «Left-handed currents and CP violation», *Nucl. Phys. B* **109**, 213 (1976).

[6] M.A. Shifman, A. Vainshtein, V.I. Zakharov, «Light quarks and the origin of the $\Delta I = 1/2$ rule in the non-leptonic decays of strange particles», *Nucl. Phys. B* **120**, 316 (1977).

[7] G. Altarelli, L. Maiani, «Octet enhancement of non-leptonic weak interactions in asymptotically free gauge theories», *Phys. Lett. B* **52**, 351 (1974).

[8] M.K. Gaillard, B.W. Lee, «$\Delta I = 1/2$ Rule for Nonleptonic Decays in Asymptotically Free Field Theories», *Phys. Rev. Lett.* **33**, 108 (1974).

[9] W.A. Bardeen, A.J. Buras, J.-M. Gérard, «A consistent analysis of the $\Delta I = 1/2$ rule for K decays», *Phys. Lett. B* **192**, 138 (1987).

[10] RBC and UKQCD collaborations (R. Abbott *et al.*), «Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the Standard Model», *Phys. Rev. D* **102**, 054509 (2020), arXiv:2004.09440 [hep-lat].

[11] F.J. Gilman, M.B. Wise, «The $\Delta I = 1/2$ rule and violation of CP in the six-quark model», *Phys. Lett. B* **83**, 83 (1979).

[12] B. Guberina, R.D. Peccei, «Quantum chromodynamic effects and CP violation in the Kobayashi–Maskawa model», *Nucl. Phys. B* **163**, 289 (1980).

[13] F.J. Gilman, M.B. Wise, «Effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays in the six-quark model», *Phys. Rev. D* **20**, 2392 (1979).

[14] A.J. Buras, W. Słominski, H. Steger, «B-meson decay, CP violation, mixing angles and the top quark mass», *Nucl. Phys. B* **238**, 529 (1984).

[15] J.F. Donoghue, E. Golowich, B.R. Holstein, J. Trampetic, «Electromagnetic and isospin breaking effects decrease ε'/ε», *Phys. Lett. B* **179**, 361 (1986); *Erratum ibid.* **188**, 511 (1987).

[16] A.J. Buras, J.-M. Gérard, «Isospin breaking contributions to ε'/ε», *Phys. Lett. B* **192**, 156 (1987).

[17] J. Bijnens, M.B. Wise, «Electromagnetic contribution to ε'/ε», *Phys. Lett. B* **137**, 245 (1984).

[18] S.R. Sharpe, «On the contribution of electromagnetic penguins to ϵ'», *Phys. Lett. B* **194**, 551 (1987).

[19] M. Lusignoli, «Electromagnetic corrections to the effective Hamiltonian for strangeness changing decays and ϵ'/ϵ», *Nucl. Phys. B* **325**, 33 (1989).

[20] W.A. Bardeen, A.J. Buras, J.-M. Gérard, «The $\Delta I = 1/2$ rule in the large-N limit», *Phys. Lett. B* **180**, 133 (1986).

[21] W.A. Bardeen, A.J. Buras, J.-M. Gérard, «The $K \to \pi\pi$ decays in the large-N limit: quark evolution», *Nucl. Phys. B* **293**, 787 (1987).

[22] A.J. Buras, J.-M. Gérard, «ε'/ε in the Standard Model», *Phys. Lett. B* **203**, 272 (1988).

[23] C.O. Dib, I. Dunietz, F.J. Gilman, «CP violation in the $K_{\mathrm{L}} \to \pi^0\ell^+\ell^-$ decay amplitude for large m_t», *Phys. Lett. B* **218**, 487 (1989).

[24] J.M. Flynn, L. Randall, «The CP violating contribution to the decay $K_{\mathrm{L}} \to \pi^0e^+e^-$», *Nucl. Phys. B* **326**, 31 (1989); *Erratum ibid.* **334**, 580 (1990).

[25] A.J. Buras, P.H. Weisz, «QCD nonleading corrections to weak decays in dimensional regularization and 't Hooft–Veltman schemes», *Nucl. Phys. B* **333**, 66 (1990).

[26] J.M. Flynn, L. Randall, «The electromagnetic penguin contribution to ε'/ε for large top quark mass», *Phys. Lett. B* **224**, 221 (1989).

[27] G. Buchalla, A.J. Buras, M.K. Harlander, «The anatomy of ε'/ε in the Standard Model», *Nucl. Phys. B* **337**, 313 (1990).

[28] E.A. Paschos, Y.L. Wu, «Correlations between ε'/ε and heavy top», *Mod. Phys. Lett. A* **6**, 93 (1991).

[29] M. Lusignoli, L. Maiani, G. Martinelli, L. Reina, «Mixing and CP violation in K- and B-mesons: a lattice QCD point of view», *Nucl. Phys. B* **369**, 139 (1992).

[30] NA31 Collaboration (H. Burkhardt *et al.*), «First evidence for direct CP violation», *Phys. Lett. B* **206**, 169 (1988).

[31] NA31 Collaboration (G.D. Barr *et al.*), «A new measurement of direct CP violation in the neutral kaon system», *Phys. Lett. B* **317**, 233 (1993).

[32] L.K. Gibbons *et al.*, «Measurement of the CP-violation parameter $\mathrm{Re}(\epsilon'/\epsilon)$», *Phys. Rev. Lett.* **70**, 1203 (1993).

[33] A.J. Buras, M. Jamin, M.E. Lautenbacher, P.H. Weisz, «Effective Hamiltonians for $\Delta S = 1$ and $\Delta B = 1$ non-leptonic decays beyond the leading logarithmic approximation», *Nucl. Phys. B* **370**, 69 (1992); *Addendum*, *ibid.* **375**, 501 (1992).

[34] A.J. Buras, M. Jamin, M.E. Lautenbacher, P.H. Weisz, «Two-loop anomalous dimension matrix for $\Delta S = 1$ weak non-leptonic decays. (I) $\mathcal{O}(\alpha_{\mathrm{s}}^2)$», *Nucl. Phys. B* **400**, 37 (1993), `arXiv:hep-ph/9211304`.

[35] A.J. Buras, M. Jamin, M.E. Lautenbacher, «Two-loop anomalous dimension matrix for $\Delta S = 1$ weak nonleptonic decays. (II) $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$», *Nucl. Phys. B* **400**, 75 (1993), `arXiv:hep-ph/9211321`.

[36] A.J. Buras, M. Jamin, M.E. Lautenbacher, «The anatomy of ε'/ε beyond leading logarithms with improved hadronic matrix elements», *Nucl. Phys. B* **408**, 209 (1993), `arXiv:hep-ph/9303284`.

[37] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, «ε'/ε at the next-to-leading order in QCD and QED», *Phys. Lett. B* **301**, 263 (1993), `arXiv:hep-ph/9212203`.

[38] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, «The $\Delta S = 1$ effective Hamiltonian including next-to-leading order QCD and QED corrections», *Nucl. Phys. B* **415**, 403 (1994), arXiv:hep-ph/9304257.

[39] G. Buchalla, A.J. Buras, M.E. Lautenbacher, «Weak decays beyond leading logarithms», *Rev. Mod. Phys.* **68**, 1125 (1996), arXiv:hep-ph/9512380.

[40] A.J. Buras, «Climbing NLO and NNLO Summits of Weak Decays», arXiv:1102.5650 [hep-ph].

[41] A.J. Buras, «Gauge Theory of Weak Decays», *Cambridge University Press*, 2020.

[42] A.J. Buras, J.-M. Gérard, «$1/N$ expansion for kaons», *Nucl. Phys. B* **264**, 371 (1986).

[43] RBC and UKQCD collaborations (P.A. Boyle *et al.*), «Emerging Understanding of the $\Delta I = 1/2$ Rule from Lattice QCD», *Phys. Rev. Lett.* **110**, 152001 (2013), arXiv:1212.1474 [hep-lat].

[44] A.J. Buras, M. Gorbahn, S. Jäger, M. Jamin, «Improved anatomy of ε'/ε in the Standard Model», *J. High Energy Phys.* **1511**, 202 (2015), arXiv:1507.06345 [hep-ph].

[45] J. Aebischer, C. Bobeth, A.J. Buras, «On the importance of NNLO QCD and isospin-breaking corrections in ε'/ε», *Eur. Phys. J. C* **80**, 1 (2020), arXiv:1909.05610 [hep-ph].

[46] A.J. Buras, M. Jamin, M.E. Lautenbacher, «A 1996 analysis of the CP violating ratio ε'/ε», *Phys. Lett. B* **389**, 749 (1996), arXiv:hep-ph/9608365.

[47] A.J. Buras, M. Jamin, «ε'/ε at the NLO: 10 years later», *J. High Energy Phys.* **0401**, 048 (2004), arXiv:hep-ph/0306217.

[48] M. Ciuchini *et al.*, «An Upgraded analysis of ε'/ε at the next-to-leading order», *Z. Phys. C* **68**, 239 (1995), arXiv:hep-ph/9501265.

[49] A.J. Buras, P. Gambino, U.A. Haisch, «Electroweak penguin contributions to non-leptonic $\Delta F = 1$ decays at NNLO», *Nucl. Phys. B* **570**, 117 (2000), arXiv:hep-ph/9911250.

[50] M. Cerdá-Sevilla, M. Gorbahn, S. Jäger, A. Kokulu, «Towards NNLO accuracy for ε'/ε», *J. Phys.: Conf. Ser.* **800**, 012008 (2017), arXiv:1611.08276 [hep-ph].

[51] M. Cerdá-Sevilla, «NNLO QCD Contributions to ε'/ε», *Acta Phys. Pol. B* **49**, 1087 (2018).

[52] M. Gorbahn, U. Haisch, «Effective Hamiltonian for non-leptonic $|\Delta F| = 1$ decays at NNLO in QCD», *Nucl. Phys. B* **713**, 291 (2005), arXiv:hep-ph/0411071.

[53] S. Bosch *et al.*, «Standard Model confronting new results for ε'/ε», *Nucl. Phys. B* **565**, 3 (2000), arXiv:hep-ph/9904408.

[54] A.J. Buras *et al.*, «ε'/ε and rare K and B decays in the MSSM», *Nucl. Phys. B* **592**, 55 (2001), arXiv:hep-ph/0007313.

[55] V. Antonelli, S. Bertolini, M. Fabbrichesi, E.I. Lashin, «The $\Delta I = 1/2$ selection rule», *Nucl. Phys. B* **469**, 181 (1996), arXiv:hep-ph/9511341.

[56] S. Bertolini, J.O. Eeg, M. Fabbrichesi, «A new estimate of ε'/ε», *Nucl. Phys. B* **476**, 225 (1996), arXiv:hep-ph/9512356.

[57] E. Pallante, A. Pich, «Strong Enhancement of ε'/ε through Final State Interactions», *Phys. Rev. Lett.* **84**, 2568 (2000), arXiv:hep-ph/9911233.

[58] E. Pallante, A. Pich, «Final state interactions in kaon decays», *Nucl. Phys. B* **592**, 294 (2001), arXiv:hep-ph/0007208.

[59] M. Büchler, G. Colangelo, J. Kambor, F. Orellana, «A note on the dispersive treatment of $K \to \pi\pi$ with the kaon off-shell», *Phys. Lett. B* **521**, 29 (2001), arXiv:hep-ph/0102289.

[60] M. Büchler, G. Colangelo, J. Kambor, F. Orellana, «Dispersion relations and soft pion theorems for $K \to \pi\pi$», *Phys. Lett. B* **521**, 22 (2001), arXiv:hep-ph/0102287.

[61] E. Pallante, A. Pich, I. Scimemi, «The Standard Model prediction for ε'/ε», *Nucl. Phys. B* **617**, 441 (2001), arXiv:hep-ph/0105011.

[62] A.J. Buras *et al.*, «Final state interactions and ε'/ε: a critical look», *Phys. Lett. B* **480**, 80 (2000), arXiv:hep-ph/0002116.

[63] V. Cirigliano, A. Pich, G. Ecker, H. Neufeld, «Isospin Violation in ϵ'», *Phys. Rev. Lett.* **91**, 162001 (2003), arXiv:hep-ph/0307030.

[64] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, «Isospin breaking in $K \to \pi\pi$ decays», *Eur. Phys. J. C* **33**, 369 (2004), arXiv:hep-ph/0310351.

[65] J. Bijnens, F. Borg, «Isospin breaking in $K \to 3\pi$ decays III: Bremsstrahlung and fit to experiment», *Eur. Phys. J. C* **40**, 383 (2005), arXiv:hep-ph/0501163.

[66] S. Bertolini, M. Fabbrichesi, J.O. Eeg, «Theory of the CP-violating parameter ϵ'/ϵ», *Rev. Mod. Phys.* **72**, 65 (2000), arXiv:hep-ph/9802405.

[67] V. Cirigliano *et al.*, «Kaon decays in the Standard Model», *Rev. Mod. Phys.* **84**, 399 (2012), arXiv:1107.6001 [hep-ph].

[68] S. Bertolini, J.O. Eeg, A. Maiezza, F. Nesti, «New physics in ϵ' from gluomagnetic contributions and limits on left–right symmetry», *Phys. Rev. D* **86**, 095013 (2012), arXiv:1206.0668 [hep-ph].

[69] RBC, UKQCD Collaboration (Z. Bai *et al.*), «Standard Model Prediction for Direct CP Violation in $K \to \pi\pi$ Decay», *Phys. Rev. Lett.* **115**, 212001 (2015), arXiv:1505.07863 [hep-lat].

[70] T. Blum *et al.*, «$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit», *Phys. Rev. D* **91**, 074502 (2015), arXiv:1502.00263 [hep-lat].

[71] T. Kitahara, U. Nierste, P. Tremper, «Singularity-free next-to-leading order $\Delta S = 1$ renormalization group evolution and ϵ'_K/ϵ_K in the Standard Model and beyond», *J. High Energy Phys.* **1612**, 078 (2016), arXiv:1607.06727 [hep-ph].

[72] A.J. Buras, J.-M. Gérard, «Upper bounds on ε'/ε parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from large N QCD and other news», *J. High Energy Phys.* **1512**, 008 (2015), arXiv:1507.06326 [hep-ph].

[73] A.J. Buras, J.-M. Gérard, W.A. Bardeen, «Large N approach to kaon decays and mixing 28 years later: $\Delta I = 1/2$ rule, $\hat{B}_K$ and ΔM_K», *Eur. Phys. J. C* **74**, 2871 (2014), arXiv:1401.1385 [hep-ph].

[74] G. Colangelo, J. Gasser, H. Leutwyler, «$\pi\pi$ scattering», *Nucl. Phys. B* **603**, 125 (2001), arXiv:hep-ph/0103088.

[75] H. Gisbert, A. Pich, «Direct CP violation in $K^0 \to \pi\pi$: Standard Model Status», *Rep. Prog. Phys.* **81**, 076201 (2018), arXiv:1712.06147 [hep-ph].

[76] H. Gisbert, A. Pich, «Updated Standard Model Prediction for ε'/ε», *Nucl. Part. Phys. Proc.* **300–302**, 137 (2018), arXiv:1810.04904 [hep-ph].

[77] H. Gisbert, «Current status of ε'/ε in the Standard Model», *PoS* **Confinement2018**, 178 (2018), arXiv:1811.12206 [hep-ph].

[78] A.J. Buras, J.-M. Gérard, «Final state interactions in $K \to \pi\pi$ decays: $\Delta I = 1/2$ rule *vs.* ε'/ε», *Eur. Phys. J. C* **77**, 10 (2017), arXiv:1603.05686 [hep-ph].

[79] A.J. Buras, «ϵ'/ϵ-2018: A Christmas Story», arXiv:1812.06102 [hep-ph].

[80] V. Cirigliano, H. Gisbert, A. Pich, A. Rodríguez-Sánchez, «Isospin-violating contributions to ϵ'/ϵ», *J. High Energy Phys.* **2002**, 032 (2020), arXiv:1911.01359 [hep-ph].

[81] V. Cirigliano, H. Gisbert, A. Pich, A. Rodríguez-Sánchez, «Theoretical status of ε'/ε», *J. Phys.: Conf. Ser.* **1526**, 012011 (2020), arXiv:1912.04736 [hep-ph].

[82] H. Gisbert, «Nonleptonic $K \to 2\,\pi$ decay dynamics», arXiv:2012.09483 [hep-ph].

[83] C. Bobeth, A.J. Buras, «Leptoquarks meet ε'/ε and rare kaon processes», *J. High Energy Phys.* **1802**, 101 (2018), arXiv:1712.01295 [hep-ph].

[84] J. Aebischer, C. Bobeth, A.J. Buras, D.M. Straub, «Anatomy of ε'/ε beyond the Standard Model», *Eur. Phys. J. C* **79**, 219 (2019), arXiv:1808.00466 [hep-ph].

[85] J. Aebischer *et al.*, «Master formula for ε'/ε beyond the Standard Model», *Phys. Lett. B* **792**, 465 (2019), arXiv:1807.02520 [hep-ph].

[86] J. Aebischer, A.J. Buras, J.-M. Gérard, «BSM hadronic matrix elements for ε'/ε and $K \to \pi\pi$ decays in the Dual QCD approach», *J. High Energy Phys.* **1902**, 021 (2019), arXiv:1807.01709 [hep-ph].

[87] ETM Collaboration (M. Constantinou *et al.*), «$K \to \pi$ matrix elements of the chromomagnetic operator on the lattice», *Phys. Rev. D* **97**, 074501 (2018), arXiv:1712.09824 [hep-lat].

[88] A.J. Buras, J.-M. Gérard, «$K \to \pi\pi$ and $K - \pi$ matrix elements of the chromomagnetic operators from Dual QCD», *J. High Energy Phys.* **1807**, 126 (2018), arXiv:1803.08052 [hep-ph].

[89] A.J. Buras, J.-M. Gérard, «Isospin-breaking in ε'/ε: impact of η_0 at the dawn of the 2020s», *Eur. Phys. J. C* **80**, 701 (2020), arXiv:2005.08976 [hep-ph].

[90] A.J. Buras, «The Optimal Strategy for ε'/ε in the SM: 2019», *J. Phys.: Conf. Ser.* **1526**, 012019 (2020), arXiv:1912.12306 [hep-ph].

[91] N. Ishizuka, K.-I. Ishikawa, A. Ukawa, T. Yoshié, «Calculation of $K \to \pi\pi$ decay amplitudes with improved Wilson fermion action in nonzero momentum frame in lattice QCD», *Phys. Rev. D* **98**, 114512 (2018), arXiv:1809.03893 [hep-lat].

[92] M. Gell-Mann, A. Pais, «Behavior of Neutral Particles under Charge Conjugation», *Phys. Rev.* **97**, 1387 (1955).

[93] M. Gell-Mann, A.H. Rosenfeld, «Hyperons and Heavy Mesons (Systematics and Decay)», *Annu. Rev. Nucl. Part. Sci.* **7**, 407 (1957).

[94] A. Donini, P.Hernández, C. Pena, F. Romero-López, «Dissecting the $\Delta I = 1/2$ rule at large $N_{\rm c}$», *Eur. Phys. J. C* **80**, 638 (2020), arXiv:2003.10293 [hep-lat].

[95] P.Hernández, F. Romero-López, «The large $N_{\rm c}$ limit of QCD on the lattice», arXiv:2012.03331 [hep-lat].

[96] R.S. Chivukula, J.M. Flynn, H. Georgi, «Polychromatic penguins don't fly», *Phys. Lett. B* **171**, 453 (1986).

[97] A. Pich, E. de Rafael, «Four-quark operators and non-leptonic weak transitions», *Nucl. Phys. B* **358**, 311 (1991).

[98] M. Jamin, A. Pich, «QCD corrections to inclusive $\Delta S = 1, 2$ transitions at the next-to-leading order», *Nucl. Phys. B* **425**, 15 (1994), arXiv:hep-ph/9402363.

[99] A. Pich, E. de Rafael, «Weak K-amplitudes in the chiral and $1/N_{\rm c}$ expansions», *Phys. Lett. B* **374**, 186 (1996), arXiv:hep-ph/9511465.

[100] S. Bertolini, J.O. Eeg, M. Fabbrichesi, E.I. Lashin, «The $\Delta I = 1/2$ rule and B_K at $\mathcal{O}(p^4)$ in the chiral expansion», *Nucl. Phys. B* **514**, 63 (1998), arXiv:hep-ph/9705244.

[101] J. Bijnens, J. Prades, «The $\Delta I = 1/2$ rule in the chiral limit», *J. High Energy Phys.* **9901**, 023 (1999), arXiv:hep-ph/9811472.

[102] T. Hambye *et al.*, «$1/N_{\rm c}$ corrections to the hadronic matrix elements of Q_6 and Q_8 in $K \to \pi\pi$ decays», *Phys. Rev. D* **58**, 014017 (1998), arXiv:hep-ph/9802300.

[103] R.J. Crewther, L.C. Tunstall, «$\Delta I = 1/2$ rule for kaon decays derived from QCD infrared fixed point», *Phys. Rev. D* **91**, 034016 (2015), arXiv:1312.3319 [hep-ph].

[104] Particle Data Group Collaboration (M. Tanabashi *et al.*), «Review of Particle Physics», *Phys. Rev. D* **98**, 030001 (2018).

[105] A.J. Buras, J.-M. Gérard, «Dual QCD Insight into BSM Hadronic Matrix Elements for K^0–$\bar{K}^0$ Mixing from Lattice QCD», *Acta Phys. Pol. B* **50**, 121 (2019), arXiv:1804.02401 [hep-ph].

[106] ETM Collaboration (N. Carrasco *et al.*), «$\Delta S = 2$ and $\Delta C = 2$ bag parameters in the Standard Model and beyond from $N_\mathrm{f} = 2+1+1$ twisted-mass lattice QCD», *Phys. Rev. D* **92**, 034516 (2015), arXiv:1505.06639 [hep-lat].

[107] SWME Collaboration (B.J. Choi *et al.*), «Kaon BSM B-parameters using improved staggered fermions from $N_\mathrm{f} = 2+1$ unquenched QCD», *Phys. Rev. D* **93**, 014511 (2016), arXiv:1509.00592 [hep-lat].

[108] RBC/UKQCD Collaboration (N. Garron, R.J. Hudspith, A.T. Lytle), «Neutral kaon mixing beyond the Standard Model with $n_\mathrm{f} = 2+1$ chiral fermions. Part 1: bare matrix elements and physical results», *J. High Energy Phys.* **1611**, 001 (2016), arXiv:1609.03334 [hep-lat].

[109] RBC/UKQCD Collaboration (P.A. Boyle *et al.*), «Neutral kaon mixing beyond the Standard Model with $n_\mathrm{f} = 2+1$ chiral fermions. Part 2: non perturbative renormalisation of the $\Delta F = 2$ four-quark operators», *J. High Energy Phys.* **1710**, 054 (2017), arXiv:1708.03552 [hep-lat].

[110] P. Boyle *et al.*, «BSM Kaon Mixing at the Physical Point», *EPJ Web Conf.* **175**, 13010 (2018), arXiv:1710.09176 [hep-lat].

[111] G.E. Brown, J.W. Durso, M.B. Johnson, J. Speth, «Final state interactions in K meson decays», *Phys. Lett. B* **238**, 20 (1990).

[112] G. 't Hooft, «A planar diagram theory for strong interactions», *Nucl. Phys. B* **72**, 461 (1974).

[113] G. 't Hooft, «A two-dimensional model for mesons», *Nucl. Phys. B* **75**, 461 (1974).

[114] E. Witten, «Baryons in the $1/N$ expansion», *Nucl. Phys. b* **160**, 57 (1979).

[115] S. Treiman, R. Jackiw, «Current Algebra and Anomalies», *Princeton University Press*, 1986.

[116] A.J. Buras, F. De Fazio, J. Girrbach, «$\Delta I = 1/2$ rule, ε'/ε and $K \to \pi\nu\bar{\nu}$ in $Z'(Z)$ and G' models with FCNC quark couplings», *Eur. Phys. J. C* **74**, 2950 (2014), arXiv:1404.3824 [hep-ph].

[117] A.J. Buras, «New physics patterns in ε'/ε and ε_K with implications for rare kaon decays and ΔM_K», *J. High Energy Phys.* **1604**, 071 (2016), arXiv:1601.00005 [hep-ph].

[118] J. Aebischer, A.J. Buras, J. Kumar, «Another SMEFT story: Z' facing new results on ε'/ε, ΔM_K and $K \to \pi\nu\bar{\nu}$», *J. High Energy Phys.* **2012**, 097 (2020), arXiv:2006.01138 [hep-ph].

[119] M. Blanke, A.J. Buras, S. Recksiegel, «Quark flavour observables in the Littlest Higgs model with T-parity after LHC Run 1», *Eur. Phys. J. C* **76**, 182 (2016), arXiv:1507.06316 [hep-ph].

[120] C. Bobeth, A.J. Buras, A. Celis, M. Jung, «Yukawa enhancement of Z-mediated new physics in $\Delta S = 2$ and $\Delta B = 2$ processes», *J. High Energy Phys.* **1707**, 124 (2017), arXiv:1703.04753 [hep-ph].

[121] M. Endo, T. Kitahara, S. Mishima, K. Yamamoto, «Revisiting kaon physics in general Z scenario», *Phys. Lett. B* **771**, 37 (2017), arXiv:1612.08839 [hep-ph].

[122] A.J. Buras, D. Buttazzo, R. Knegjens, «$K \to \pi\nu\bar{\nu}$ and ϵ'/ϵ in simplified new physics models», *J. High Energy Phys.* **1511**, 166 (2015), arXiv:1507.08672 [hep-ph].

[123] A.J. Buras, F. De Fazio, «ε'/ε in 331 models», *J. High Energy Phys.* **1603**, 010 (2016), arXiv:1512.02869 [hep-ph].

[124] A.J. Buras, F. De Fazio, «331 Models Facing the Tensions in $\Delta F = 2$ Processes with the Impact on ε'/ε, $B_s \to \mu^+\mu^-$ and $B \to K^*\mu^+\mu^-$», *J. High Energy Phys.* **1608**, 115 (2016), arXiv:1604.02344 [hep-ph].

[125] C. Bobeth, A.J. Buras, A. Celis, M. Jung, «Patterns of flavour violation in models with vector-like quarks», *J. High Energy Phys.* **1704**, 079 (2017), arXiv:1609.04783 [hep-ph].

[126] M. Tanimoto, K. Yamamoto, «Probing SUSY with 10 TeV stop mass in rare decays and CP violation of kaon», *Prog. Theor. Exp. Phys.* **2016**, 123B02 (2016), arXiv:1603.07960 [hep-ph].

[127] T. Kitahara, U. Nierste, P. Tremper, «Supersymmetric Explanation of CP Violation in $K \to \pi\pi$ Decays», *Phys. Rev. Lett.* **117**, 091802 (2016), arXiv:1604.07400 [hep-ph].

[128] M. Endo, S. Mishima, D. Ueda, K. Yamamoto, «Chargino contributions in light of recent ϵ'/ϵ», *Phys. Lett. B* **762**, 493 (2016), arXiv:1608.01444 [hep-ph].

[129] A. Crivellin, G. D'Ambrosio, T. Kitahara, U. Nierste, «$K \to \pi\nu\bar{\nu}$ in the MSSM in light of the ϵ'_K/ϵ_K anomaly», *Phys. Rev. D* **96**, 015023 (2017), arXiv:1703.05786 [hep-ph].

[130] M. Endo *et al.*, «Gluino-mediated electroweak penguin with flavor-violating trilinear couplings», *J. High Energy Phys.* **1804**, 019 (2018), arXiv:1712.04959 [hep-ph].

[131] C.-H. Chen, T. Nomura, «Re(ϵ'_K/ϵ_K) and $K \to \pi\nu\bar{\nu}$ in a two-Higgs doublet model», *J. High Energy Phys.* **1808**, 145 (2018), arXiv:1804.06017 [hep-ph].

[132] C.-H. Chen, T. Nomura, «ϵ'/ϵ from charged-Higgs-induced gluonic dipole operators», *Phys. Lett. B* **787**, 182 (2018), arXiv:1805.07522 [hep-ph].

[133] V. Cirigliano, W. Dekens, J. de Vries, E. Mereghetti, «An ε' improvement from right-handed currents», *Phys. Lett. B* **767**, 1 (2017), arXiv:1612.03914 [hep-ph].

[134] S. Alioli *et al.*, «Right-handed charged currents in the era of the Large Hadron Collider», *J. High Energy Phys.* **1705**, 086 (2017), arXiv:1703.04751 [hep-ph].

[135] N. Haba, H. Umeeda, T. Yamada, «ϵ'/ϵ anomaly and neutron EDM in $\mathrm{SU(2)_L \times SU(2)_R \times U(1)}_{B-L}$ model with charge symmetry», *J. High Energy Phys.* **1805**, 052 (2018), arXiv:1802.09903 [hep-ph].

[136] N. Haba, H. Umeeda, T. Yamada, «Direct CP violation in Cabibbo-favored charmed meson decays and ϵ'/ϵ in $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{B-L}$ model», *J. High Energy Phys.* **10**, 006 (2018), arXiv:1806.03424 [hep-ph].

[137] S. Matsuzaki, K. Nishiwaki, K. Yamamoto, «Simultaneous interpretation of K and B anomalies in terms of chiral-flavorful vectors», *J. High Energy Phys.* **1811**, 164 (2018), arXiv:1806.02312 [hep-ph].

[138] C.-H. Chen, T. Nomura, «ϵ_K and ϵ'/ϵ in a diquark model», *J. High Energy Phys.* **1903**, 009 (2019), arXiv:1808.04097 [hep-ph].

[139] C.-H. Chen, T. Nomura, «Left-handed color-sextet diquark in the kaon system», *Phys. Rev. D* **99**, 115006 (2019), arXiv:1811.02315 [hep-ph].

[140] C. Marzo, L. Marzola, M. Raidal, «Common explanation to the $R_{K^{(*)}}$, $R_{D^{(*)}}$ and ϵ'/ϵ anomalies in a 3HDM with right-handed neutrinos and connections to neutrino physics», *Phys. Rev. D* **100**, 055031 (2019), arXiv:1901.08290 [hep-ph].

[141] S. Matsuzaki, K. Nishiwaki, K. Yamamoto, «Simultaneous explanation of K and B anomalies in vectorlike compositeness», 18$^{\mathrm{th}}$ Hellenic School and Workshops on Elementary Particle Physics and Gravity (CORFU2018), Corfu, Greece, August 31–September 28, 2018, arXiv:1903.10823 [hep-ph].

[142] A. Crivellin, C. Gross, S. Pokorski, L. Vernazza, «Correlating ϵ'/ϵ with hadronic B decays via $U(2)^3$ flavour symmetry», *Phys. Rev. D* **101**, 015022 (2020), arXiv:1909.02101 [hep-ph].