CALCULATIONS OF THE ALPHA DECAY HALF-LIVES OF SOME POLONIUM ISOTOPES USING THE DOUBLE FOLDING MODEL

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The calculations of the α-decay half-lives of some polonium isotopes in the mass range of 186–218 have been carried out using the Wentzel–Kramers–Brillouin (WKB) semiclassical approximation. The α-nucleus effective potential used contains the Coulomb potential, centrifugal potential, and nuclear potential. The nuclear potential is obtained via the double folding model, with the microscopic NN effective interactions derived from the relativistic mean field theory Lagrangian (termed R3Y). Different parametrizations of the R3Y interactions have been employed in the computation of nuclear potentials. The results obtained using the R3Y NN interactions are compared with the ones obtained using the famous Michigan-3-Yukawa (M3Y) interactions. The use of density-dependent NN interaction is also considered. When compared to available experimental data, there are improvements in the results when density-dependent interaction potentials are used compared to when density-independent interactions are employed.

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1. Introduction

α-decay is an important decay mode that can give information about the structure of nuclei [1, 2]. The α-decay studies of nuclei have been investigated using various theoretical approaches such as the generalised liquid drop

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model [3–5], the effective liquid drop model [6], the modified generalised liquid drop model [7–9], the fission-like model [10], the preformed cluster model [11, 12], and the cluster formation model [13–16]. These models use various interaction potentials ranging from the phenomenological potential such as the proximity potentials [17], the Woods–Saxon, the squared Woods–Saxon, and the Cosh potentials to microscopic interactions such as the double folding model. The Geiger–Nuttall law was the first decay law to describe the α-decay half-life, and Gamow in 1928 gave a theoretical explanation of the Geiger–Nuttall law. Gamow explained that the α-decay was due to the quantum mechanical tunneling of a charged α particle through the nuclear Coulomb barrier [18]. Various empirical formulas have been introduced to compute the α-decay half-lives of many isotopes since the introduction of the Geiger–Nuttall law. Some of these formulas are: the Royer formula [19–21], the Viola–Seaborg formula [22], the universal decay law developed by Qi *et al.* [23, 24], the Akrawy formula [25], the Ren formula, [26, 27], the scaling law of Horoi [28], the scaling law of Brown, the AKRE formula developed by Akrawy and Poenaru [29], *etc.*

From a theoretical point of view, the α-decay half-lives can be studied using the semiclassical WKB framework. In this formalism, the effective interaction between the α-daughter system plays an important role in the calculations. The effective interaction consists of the nuclear potential, the Coulomb potential and the centrifugal potential. There have been various phenomenological [30, 31] and microscopic nuclear potentials [32–36] introduced to study the α-decay of various nuclei. In the microscopic approach, the nuclear potential is determined using the double folding model, where the nuclear densities are folded with the effective M3Y nucleon–nucleon interaction. The use of density-dependent double folding model has also been introduced [36–38] to study the α-decay half-lives of many nuclei. A microscopic NN interaction derived from the relativistic mean field theory Lagrangian (termed R3Y) was introduced in Ref. [12], where the authors used the derived NN interaction to compute the optical potential in the double folding model and studied cluster decays of some nuclei.

In this study, the α-decay half-lives of some polonium isotopes have been calculated using both density-independent and density-dependent double folding models. The nuclear potentials are calculated using the effective nucleon–nucleon interactions determined from the relativistic mean field theory (termed R3Y). The results of the calculations using the M3Y-Paris and M3Y-Reid effective nucleon–nucleon interactions have also been included for comparison. The article is organised as follows: the theoretical models employed to compute the α-decay half-lives of the polonium isotopes are described in Section 2. The results of the calculations are presented and discussed in Section 3, while the conclusion is given in Section 4.

2. Theoretical formalism

The effective α-nucleus potential $V(R)$ is given by

$$V_{\text{eff}}(R) = \lambda V_{\text{N}}(R) + V_{\text{C}}(R) + V_{\ell}\,, \tag{1}$$

where λ is the quantization factor, R is the relative distance between the α particle and daughter nucleus. The centrifugal term $V_{\ell} = \frac{\hbar^2 \ell(\ell+1)}{2\mu R^2}$, ℓ is the orbital angular momentum, $\mu = mA_1A_2/(A_1+A_2)$ is the reduced mass of the α particle and the daughter nucleus, and the nucleon mass $m = 931.494$ MeV. By using the Langer modification we have $\ell(\ell+1) \to \left(\ell+\frac{1}{2}\right)^2$. The values of ℓ are calculated by using the spin-parity selection rule [39]

$$|J_{\text{d}} - J_{\text{p}}| \;\leq\; \ell \leq J_{\text{d}} + J_{\text{p}}\,, \tag{2}$$

$$\pi_{\text{p}} \;=\; (-1)^{\ell}\,\pi_{\text{d}}\,. \tag{3}$$

The Coulomb potential $V_{\text{C}}(R)$ is given in the form of [36]

$$V_{\text{C}}(R) = Z_1 Z_2 e^2 \begin{cases} \frac{1}{R} & \text{for } R > R_{\text{C}} \\ \frac{1}{2R_{\text{C}}}\left[3 - \left(\frac{R}{R_{\text{C}}}\right)^2\right] & \text{for } R \leq R_{\text{C}} \end{cases}, \tag{4}$$

where Z_1 and Z_2 are the charge number of the α particle and daughter nucleus, respectively, and $R_{\text{C}} = 1.2\left(A_1^{1/3} + A_2^{1/3}\right)$.

The nuclear interaction potential $V_{\text{N}}(R)$ between the α and daughter nuclei in the double folding model is written as

$$V_{\text{N}}(R) = \int\int \rho_1(\boldsymbol{r}_1) F(\rho_1, \rho_2) \rho_2(\boldsymbol{r}_2) v(E_{\alpha}, s) \mathrm{d}\boldsymbol{r}_1 \mathrm{d}\boldsymbol{r}_2\,, \tag{5}$$

where $s = |R + \boldsymbol{r}_2 - \boldsymbol{r}_1|$ is the relative distance between interacting nucleon pair, $\rho_1(\boldsymbol{r}_1)$ and $\rho_2(\boldsymbol{r}_2)$ are the ground state matter density distributions of the α and daughter nuclei, respectively, and the kinetic energy of the α particle is denoted as E_{α}. The density distribution of the α particle is taken to be of the usual Gaussian form

$$\rho_1(r_1) = 0.4299\,\mathrm{e}^{-0.7024 r_1^2}\,, \tag{6}$$

and the density distribution of the daughter nucleus is taken to be of the Fermi form [37, 40]

$$\rho_2(r_2) = \frac{\rho_0}{1 + \exp\left(\frac{r_2 - R}{a}\right)}\,, \tag{7}$$

where the diffuseness parameter $a = 0.54$ fm, $R_{1(2)} = 1.07 A_{1(2)}^{1/3}$ fm, A_1 is the

mass number of the α particle and A_2 is the mass number of the daughter nucleus [37, 41]. The value of ρ_0 is obtained by integrating the matter density distribution equivalent to the mass number of the daughter nucleus.

In Eq. (5), the density-dependence factor $F(\rho, E_\alpha)$ is given as [42, 43]

$$F(\rho_1, \rho_2) = C\left[1 + \alpha\, \mathrm{e}^{-\beta(\rho_1+\rho_2)} - \gamma(\rho_1 + \rho_2)\right] . \tag{8}$$

The parameters of the interaction *viz.* C, α, β, γ were determined through reproducing the saturation properties of normal nuclear matter within Hartree–Fock calculations [44]. The density-dependent NN interactions used in this paper are the DDM3Y1 parametrizations. The parameters C, α, β, γ corresponding to the DDM3Y1 parametrizations are given in Table I.

TABLE I

The parameters of the various density-dependent NN interactions used in this work [43–45].

Interaction	Label	C	α	β	γ
D-independent	DD0	1	0	0	0
DDM3Y1 (Reid)	DD1	0.2843	3.6391	2.9605	0.0000
DDM3Y1 (Paris)	DD1	0.2963	3.7231	3.7384	0.0000

The popular choices for the nucleon–nucleon interactions in the double folding model have often been the M3Y interactions. The M3Y interactions were constructed to reproduce the G-matrix elements of both the Paris (M3Y-Paris) and Reid (M3Y-Reid) NN interactions in an oscillator basis [45]. They are given by

$$v^{\mathrm{M3Y-Paris}}(s, E_\alpha) = 11062\frac{\mathrm{e}^{-4s}}{4s} - 2537.5\frac{\mathrm{e}^{-2.5s}}{2.5s} + J_{00}^{\mathrm{P}}(E_\alpha)\delta(\boldsymbol{s}) \tag{9}$$

and

$$v^{\mathrm{M3Y-Reid}}(s, E_\alpha) = 7999\frac{\mathrm{e}^{-4s}}{4s} - 2134\frac{\mathrm{e}^{-2.5s}}{2.5s} + J_{00}^{\mathrm{R}}(E_\alpha)\delta(\boldsymbol{s}) , \tag{10}$$

respectively. In this study, the effective nucleon–nucleon interactions derived from the relativistic mean field (RMF) theory Lagrangian, with different parametrizations are also employed. Following Ref. [12], the effective nucleon–nucleon interaction, derived from the relativistic mean field Lagrangian is given by the sum of the scalar (σ) and vector (ω, ρ) parts of

the meson fields. That is,

$$\begin{aligned} v_{\text{eff}}(s) &= V_\omega + V_\sigma + V_\rho \\ &= \frac{g_\omega^2}{4\pi}\frac{\mathrm{e}^{-m_\omega s}}{s} - \frac{g_\sigma^2}{4\pi}\frac{\mathrm{e}^{-m_\sigma s}}{s} + \frac{g_\rho^2}{4\pi}\frac{\mathrm{e}^{-m_\rho s}}{s} + J_{00}(E)\delta(s)\,, \end{aligned} \tag{11}$$

where g_i and m_i $(i = \omega, \sigma, \rho)$ are the coupling constants and meson masses, respectively, and the last term is the exchange contribution. Different parameters of the RMF effective NN interaction have been employed in this work *viz.* R3Y-L1, R3Y-W, R3Y-Z, and R3Y-HS parametrizations. They are given, respectively, as [12]

$$\begin{aligned} v^{\text{R3Y-L1}}(s, E_\alpha) &= 9967.88\frac{\mathrm{e}^{-3.968s}}{4s} - 6660.95\frac{\mathrm{e}^{-2.787s}}{4s} \\ &\quad + J_{00}^{\text{R}}(E_\alpha)\delta(\boldsymbol{s})\,, \end{aligned} \tag{12}$$

$$\begin{aligned} v^{\text{R3Y-W}}(s, E_\alpha) &= 8550.74\frac{\mathrm{e}^{-3.968s}}{4s} - 5750.24\frac{\mathrm{e}^{-2.787s}}{4s} \\ &\quad + J_{00}^{\text{R}}(E_\alpha)\delta(\boldsymbol{s})\,, \end{aligned} \tag{13}$$

$$\begin{aligned} v^{\text{R3Y-Z}}(s, E_\alpha) &= 12008.98\frac{\mathrm{e}^{-3.9528s}}{4s} - 7861.80\frac{\mathrm{e}^{-2.7939s}}{4s} \\ &\quad + J_{00}^{\text{R}}(E_\alpha)\delta(\boldsymbol{s})\,, \end{aligned} \tag{14}$$

$$\begin{aligned} v^{\text{R3Y-HS}}(s, E_\alpha) &= 11956.94\frac{\mathrm{e}^{-3.968s}}{4s} - 6882.64\frac{\mathrm{e}^{-2.6352s}}{4s} \\ &\quad + 4099.06\frac{\mathrm{e}^{-3.902s}}{4s} + J_{00}^{\text{P}}(E_\alpha)\delta(\boldsymbol{s})\,. \end{aligned} \tag{15}$$

A complete description of the R3Y interactions is provided in Ref. [12]. The zero-range exchange terms are given by

$$J_{00}^{\text{R}}(E_\alpha) = -276(1 - 0.005E_\alpha/A_\alpha)\ \text{MeV fm}^3 \tag{16}$$

and

$$J_{00}^{\text{P}}(E_\alpha) = -590(1 - 0.002E_\alpha/A_\alpha)\ \text{MeV fm}^3\,. \tag{17}$$

Here, $E_\alpha = Q_\alpha A_1/A$, Q_α denotes the energy released in the α decay process, and A is the mass number of the parent nucleus. The quantization factor λ in equation (1) is determined through the Bohr–Sommerfeld quantization and Wildermuth rule [40, 41, 46]

$$\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar}\left[Q_\alpha - V_{\text{eff}}(R)\right]}\,\mathrm{d}R = (G - \ell + 1)\,\frac{\pi}{2}\,, \tag{18}$$

where the global quantum number, G, is given for α-decay process as

$$G_\alpha = \begin{cases} 18 & N \leq 82 \\ 20 & 82 < N \leq 126 \\ 22 & N > 126 \end{cases} . \tag{19}$$

The following formula is then used to calculate the α-decay half-life [41]:

$$T_{1/2} = \frac{\ln 2}{\nu P_\alpha P} , \tag{20}$$

where the assault frequency ν is determined using the WKB approximation [39]

$$\nu = \frac{\hbar}{2\mu} \left[\int_{r_1}^{r_2} \frac{\mathrm{d}R}{\sqrt{\frac{2\mu}{\hbar^2} |Q - V_{\text{eff}}(R)|}} \right]^{-1} , \tag{21}$$

and the tunneling probability P is calculated via

$$P = (1 + \mathrm{e}^q)^{-1} , \tag{22}$$

and

$$q = \frac{\sqrt{8\mu}}{\hbar} \int_{r_2}^{r_3} \sqrt{V_{\text{eff}}(R) - Q}\, \mathrm{d}R , \tag{23}$$

$r_i (i = 1, 2, 3)$ are the three turning points, and the pre-formation probability P_α is computed here using the empirical formula [39]

$$\log P_\alpha = s\sqrt{\mu Z_1 Z_2} + b , \tag{24}$$

where $a = -0.052$ and $b = 0.69$ for even–even nuclei. For odd-A nuclei, $b = 0.6$.

3. Results and discussions

Here, the results of the calculations using the theory described above are presented and discussed. In the calculations, both the density-independent (DD0) and density-dependent interactions (DDM3Y) were used. The experimental input data have been extracted from the NUBASE2020 database [47–49]. In the calculations of the double folding potentials, the R3Y interactions with the different parametrizations (R3Y-HS, R3Y-L1, R3Y-W, and R3Y-Z) have been used. The calculations using the M3Y interactions are included for comparison with the R3Y interactions. In figure 1, the plots of the effective α-nucleus interactions (equation (1)) using the density-independent (DD0) R3Y-W, R3Y-L1, R3Y-HS, R3Y-Z, M3Y-Paris, and M3Y-Reid interactions are shown. The quantization factor (λ) is not included in figure 1 (a),

whereas it is used in figure 1 (b). When the quantization factor is not included, the R3Y-Z can be seen to give the strongest potential, while the M3Y-Reid gives the weakest potential. However, when the quantization factor is used, only a slight difference is observed in the strengths of the potentials for the different models. The quantization factor has the most clear effect on the R3Y-Z potential, by drastically reducing the strength of the potential. The black dots in figures 1 (a) and 1 (b) indicate the Q_α values.

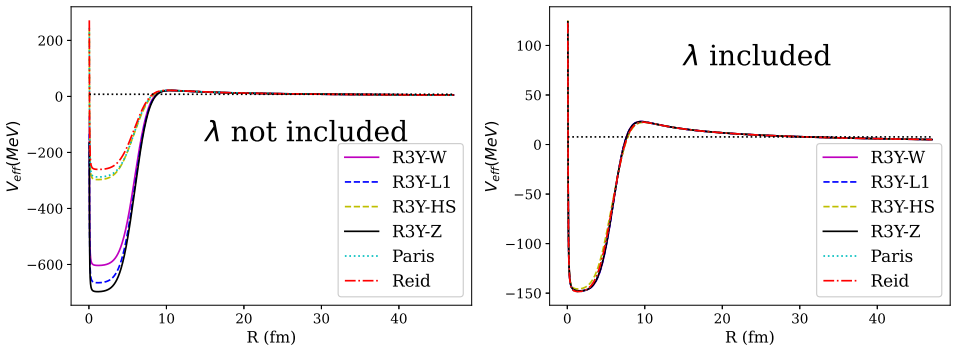

Fig. 1. Plot of the effective α-nucleus potential V_{eff} for ^{190}Po using the density-independent (DD0) R3Y-W, R3Y-L1, R3Y-HS, R3Y-Z, M3Y-Paris, and M3Y-Reid interactions. (a) quantization factor not applied and (b) quantization factor included.

In order to give a quantitative comparison between the theoretically calculated results and the experimental data, the root mean square standard deviation (σ) has been computed for the different models. The following formula was used to compute the standard deviation [17]:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(\log_{10} T^{\text{th}}_{1/2,i} - \log_{10} T^{\text{exp}}_{1/2,i}\right)^2\right]}. \tag{25}$$

Here, $T^{\text{exp}}_{1/2,i}$ are the experimental half-lives, while $T^{\text{th}}_{1/2,i}$ are the theoretical half-lives.

The calculated α-decay half-lives for the 33 polonium (Po) isotopes using the double folding model with density-independent interactions (*i.e.* DD0) are shown in Table II. Here, the preformation factor P_α is taken to be one. The first three columns show, respectively, the mass number (A), experimental Q_α values, and the logarithm of the experimental α-decay half-lives. The fourth to ninth columns show the results using the M3Y-Paris, M3Y-Reid, R3Y-HS, R3Y-L1, R3Y-W, and R3Y-Z parameters, respectively. The last

TABLE II

Calculated α-decay half-lives, $\log\left[T_{1/2}(s)\right]$, of Po isotopes ($Z = 84$) using density-independent (DD0) M3Y and R3Y interactions and setting $P_\alpha = 1$.

		$\log\left[T_{1/2}(s)\right]$						
A	Q_α	Expt.	M3Y-Paris	M3Y-Reid	R3Y-HS	R3Y-L1	R3Y-W	R3Y-Z
186	8.5012	−4.3980	−5.4067	−5.3986	−5.5411	−5.1470	−5.1298	−5.1767
187	7.9789	−2.8540	−3.6645	−3.6744	−3.8078	−3.4051	−3.3669	−3.4357
188	8.0823	−3.5610	−4.2642	−4.2736	−4.3805	−3.9779	−3.9813	−4.0288
189	7.6943	−2.4200	−2.8340	−2.8430	−2.9770	−2.5668	−2.5340	−2.5976
190	7.6933	−2.6090	−3.1242	−3.1327	−3.1493	−2.8523	−2.8201	−2.8689
191	7.8223	−1.6580	−3.5370	−3.5456	−3.5871	−3.2693	−3.2521	−3.2844
192	7.3196	−1.4920	−1.9469	−1.9498	−1.8905	−1.6678	−1.6500	−1.6987
193	7.0938	−0.4320	−1.1873	−1.1942	−1.0868	−0.9034	−0.8855	−0.9345
194	6.9871	−0.4070	−0.8235	−0.8301	−0.7033	−0.5378	−0.5157	−0.5689
195	6.7497	0.6670	0.0367	0.0306	0.1934	0.3272	0.3455	0.2959
196	6.6582	0.7450	0.3714	0.3654	0.5402	0.6635	0.6817	0.6319
197	6.4113	2.0800	1.3394	1.3337	1.5323	1.6357	1.6541	1.6038
198	6.3097	2.0260	1.7461	1.7403	1.9388	2.0435	2.0620	2.0113
199	6.0743	3.6400	2.7518	2.7463	2.9546	3.0513	3.0698	3.0192
200	5.9816	3.7900	3.1562	3.1504	3.3567	3.4558	3.4743	3.4237
201	5.7993	4.7600	3.9990	3.9934	4.1857	4.2972	4.3160	4.2657
202	5.7010	5.1500	4.4653	4.4593	4.6398	4.7626	4.7808	4.7307
203	5.4960	6.3000	5.7751	5.7683	5.9027	6.0713	6.0908	6.0396
204	5.4849	6.2800	5.5427	5.5365	5.6790	5.8348	5.8530	5.8032
205	5.3247	7.1800	6.3955	6.3892	6.4928	6.6820	6.7000	6.6509
206	5.3270	7.1500	6.3681	6.3615	6.4666	6.6538	6.6718	6.6228
207	5.2159	8.0000	6.9800	6.9725	7.0472	7.2603	7.2781	7.2297
208	5.2157	7.9610	6.9662	6.9592	7.0341	7.2459	7.2632	7.2174
209	4.9792	9.5070	8.6379	8.6304	8.6136	8.9069	8.9190	8.8773
210	5.4075	7.0780	5.8673	5.8609	5.9888	6.1545	6.1724	6.1233
211	7.5946	−0.2870	−2.0614	−2.0684	−2.2014	−1.7798	−1.7619	−1.8096
212	8.9542	−6.5240	−7.1707	−7.1783	−7.3548	−6.9657	−6.9520	−6.9799
213	8.5361	−5.4290	−6.1067	−6.1075	−6.2109	−5.8874	−5.8731	−5.9122
214	7.8335	−3.7840	−4.1262	−4.1323	−4.0893	−3.8779	−3.8620	−3.9052
215	7.5263	−2.7490	−3.1842	−3.1898	−3.0869	−2.9226	−2.9061	−2.9511
216	6.9063	−0.8390	−1.0826	−1.0937	−0.8790	−0.7942	−0.7763	−0.8251
217	6.6621	0.1800	−0.1825	−0.1871	0.0510	0.1151	0.1334	0.0834
218	6.1148	2.2690	2.0579	2.0532	2.3217	2.3686	2.3875	2.3357
	σ		0.8044	0.8099	0.7807	0.5729	0.5595	0.5950

row of Table II shows the calculated standard deviation values (σ) for the various models. The σ for the M3Y-Paris, M3Y-Reid, R3Y-HS, R3Y-L1, R3Y-W, and R3Y-Z models are 0.8044, 0.8099, 0.7807, 0.5729, and 0.5595, respectively. The R3Y models have lower σ than the M3Y models, which suggests that the R3Y models give better descriptions of the α-decay half-lives of the polonium isotopes than the M3Y models.

In Tables III and IV, the results of the calculated α-decay half-lives for the polonium isotopes are shown using density-independent and density-dependent interactions, respectively. In both tables, the pre-formation factor using equation (24) is included. The fourth to seventh columns show the results using the R3Y-HS, R3Y-L1, R3Y-W, and R3Y-Z models, respectively.

TABLE III

Calculated α-decay half-lives, $\log\left[T_{1/2}(s)\right]$, of Po isotopes ($Z = 84$) using density-independent (DD0) interactions and including the pre-formation factor P_α.

		$\log\left[T_{1/2}(s)\right]$					
A	Q_α	Expt.	R3Y-HS	R3Y-L1	R3Y-W	R3Y-Z	$\log P_\alpha$
186	8.5012	−4.3980	−4.9137	−4.5195	−4.5024	−4.5493	−0.6274
187	7.9789	−2.8540	−3.0902	−2.6876	−2.6494	−2.7182	−0.7175
188	8.0823	−3.5610	−3.7529	−3.3503	−3.3537	−3.4012	−0.6276
189	7.6943	−2.4200	−2.2593	−1.8492	−1.8163	−1.8799	−0.7177
190	7.6933	−2.6090	−2.5216	−2.2246	−2.1923	−2.2411	−0.6278
191	7.8223	−1.6580	−2.8693	−2.5515	−2.5343	−2.5666	−0.7178
192	7.3196	−1.4920	−1.2626	−1.0399	−1.0221	−1.0708	−0.6279
193	7.0938	−0.4320	−0.3689	−0.1855	−0.1675	−0.2165	−0.7180
194	6.9871	−0.4070	−0.0753	0.0903	0.1123	0.0592	−0.6280
195	6.7497	0.6670	0.9115	1.0453	1.0636	1.0140	−0.7181
196	6.6582	0.7450	1.1684	1.2916	1.3099	1.2601	−0.6282
197	6.4113	2.0800	2.2506	2.3539	2.3723	2.3220	−0.7183
198	6.3097	2.0260	2.5671	2.6719	2.6903	2.6396	−0.6283
199	6.0743	3.6400	3.6730	3.7697	3.7882	3.7376	−0.7184
200	5.9816	3.7900	3.9852	4.0843	4.1028	4.0522	−0.6285
201	5.7993	4.7600	4.9042	5.0157	5.0345	4.9842	−0.7185
202	5.7010	5.1500	5.2684	5.3913	5.4094	5.3593	−0.6286
203	5.4960	6.3000	6.6214	6.7900	6.8095	6.7583	−0.7187
204	5.4849	6.2800	6.3078	6.4635	6.4818	6.4320	−0.6287
205	5.3247	7.1800	7.2116	7.4008	7.4188	7.3697	−0.7188
206	5.3270	7.1500	7.0955	7.2827	7.3007	7.2517	−0.6289
207	5.2159	8.0000	7.7661	7.9793	7.9970	7.9486	−0.7189
208	5.2157	7.9610	7.6631	7.8749	7.8922	7.8464	−0.6290
209	4.9792	9.5070	9.3326	9.6260	9.6380	9.5963	−0.7190
210	5.4075	7.0780	6.6179	6.7836	6.8015	6.7524	−0.6291
211	7.5946	−0.2870	−1.4823	−1.0606	−1.0427	−1.0905	−0.7192
212	8.9542	−6.5240	−6.7256	−6.3365	−6.3228	−6.3506	−0.6292
213	8.5361	−5.4290	−5.4916	−5.1681	−5.1538	−5.1929	−0.7193
214	7.8335	−3.7840	−3.4600	−3.2485	−3.2327	−3.2758	−0.6293
215	7.5263	−2.7490	−2.3675	−2.2032	−2.1867	−2.2317	−0.7194
216	6.9063	−0.8390	−0.2495	−0.1647	−0.1468	−0.1957	−0.6295
217	6.6621	0.1800	0.7705	0.8346	0.8529	0.8029	−0.7195
218	6.1148	2.2690	2.9513	2.9981	3.0171	2.9652	−0.6296

TABLE IV

Calculated α-decay half-lives, $\log\left[T_{1/2}(s)\right]$, of Po isotopes ($Z = 84$) using density-dependent (DDM3Y) interactions and including the pre-formation factor P_α.

		$\log\left[T_{1/2}(s)\right]$					
A	Q_α	Expt.	R3Y-HS	R3Y-L1	R3Y-W	R3Y-Z	$\log P_\alpha$
186	8.5012	−4.3980	−4.5453	−4.7343	−4.7061	−4.7643	−0.6274
187	7.9789	−2.8540	−2.7375	−2.9087	−2.8805	−2.9267	−0.7175
188	8.0823	−3.5610	−3.4126	−3.5880	−3.5600	−3.6193	−0.6276
189	7.6943	−2.4200	−1.9107	−2.0720	−2.0431	−2.0955	−0.7177
190	7.6933	−2.6090	−2.3080	−2.4443	−2.4158	−2.4764	−0.6278
191	7.8223	−1.6580	−2.6032	−2.7693	−2.7412	−2.8009	−0.7178
192	7.3196	−1.4920	−1.1169	−1.2627	−1.2339	−1.2957	−0.6279
193	7.0938	−0.4320	−0.1526	−0.4102	−0.3812	−0.4440	−0.7180
194	6.9871	−0.4070	0.1346	−0.1342	−0.1058	−0.1684	−0.6280
195	6.7497	0.6670	0.9515	0.8178	0.8476	0.7814	−0.7181
196	6.6582	0.7450	1.3903	1.0634	1.0933	1.0261	−0.6282
197	6.4113	2.0800	2.2417	2.1237	2.1540	2.0844	−0.7183
198	6.3097	2.0260	2.5576	2.4411	2.4714	2.4055	−0.6283
199	6.0743	3.6400	3.6532	3.5376	3.5676	3.5017	−0.7184
200	5.9816	3.7900	3.9680	3.8520	3.8825	3.8161	−0.6285
201	5.7993	4.7600	4.9028	4.7805	4.8143	4.7439	−0.7185
202	5.7010	5.1500	5.2811	5.1589	5.1894	5.1233	−0.6286
203	5.4960	6.3000	6.6933	6.5548	6.5854	6.5192	−0.7187
204	5.4849	6.2800	6.3641	6.2324	6.2625	6.1974	−0.6287
205	5.3247	7.1800	7.3128	7.1707	7.2009	7.1369	−0.7188
206	5.3270	7.1500	7.1946	7.0534	7.0829	7.0192	−0.6289
207	5.2159	8.0000	7.9006	7.7514	7.7806	7.7179	−0.7189
208	5.2157	7.9610	7.7963	7.6475	7.6766	7.6140	−0.6290
209	4.9792	9.5070	9.5689	9.3973	9.4263	9.3653	−0.7190
210	5.4075	7.0780	6.6895	6.5548	6.5843	6.5203	−0.6291
211	7.5946	−0.2870	−1.1383	−1.2869	−1.2578	−1.3202	−0.7192
212	8.9542	−6.5240	−6.3515	−6.5192	−6.4972	−6.5430	−0.6292
213	8.5361	−5.4290	−5.2023	−5.3571	−5.3304	−5.3826	−0.7193
214	7.8335	−3.7840	−3.3201	−3.4498	−3.4241	−3.4791	−0.6293
215	7.5263	−2.7490	−2.2922	−2.4099	−2.3831	−2.4409	−0.7194
216	6.9063	−0.8390	−0.2873	−0.3826	−0.3536	−0.4172	−0.6295
217	6.6621	0.1800	0.6948	0.6129	0.6427	0.5772	−0.7195
218	6.1148	2.2690	2.8513	2.7702	2.8012	2.7327	−0.6296

The last column shows the calculated pre-formation factor ($\log P_\alpha$). A physical inspection of the tables indicates that the R3Y-models give very good descriptions of the α-decay half-lives of the polonium isotopes. Moreover, Table V shows the results of the standard deviation (σ) calculations using the data from Tables III and IV. When the density-independent model is

used, the four R3Y models *viz.* R3Y-HS, R3Y-L1, R3Y-W, and R3Y-Z have the respective standard deviation values of $0.4278, 0.4328, 0.4440$, and 0.4159. This confirms that all the R3Y models give very good descriptions of the α-decay half-lives of the 33 polonium isotopes. The R3Y-Z gives the lowest value of σ, while the R3Y-W gives the highest value. Furthermore, when the density-dependent interaction (DDM3Y) is used, the standard deviation values decrease for the four R3Y models. This shows the importance of using density-dependent interactions in the R3Y models.

TABLE V

The calculated root mean square standard deviations.

	R3Y-HS	R3Y-L1	R3Y-W	R3Y-Z
DD0	0.4278	0.4328	0.4440	0.4159
DDM3Y	0.3970	0.3627	0.3651	0.3626

The plots of the calculated α-decay half-lives $\log\left[T_{1/2}(s)\right]$ against the neutron number using the four R3Y-models with the experimental half-lives are shown in figure 2. The density-independent model (DD0) is shown in figure 2 (a), while the density-dependent DDM3Y model is shown in figure 2 (b). The maximum value of the α-decay half-lives is obtained at $N = 125$ which corresponds to the parent nucleus ^{209}Po. The minimum value of the α-decay half-lives is obtained at $N = 128$ which corresponds to the daughter nucleus ^{208}Pb with the neutron number $N = 126$. The maximum and minimum values are associated with the role of shell closure effects relative to the magicity (or near magicity) of the neutron number. A high

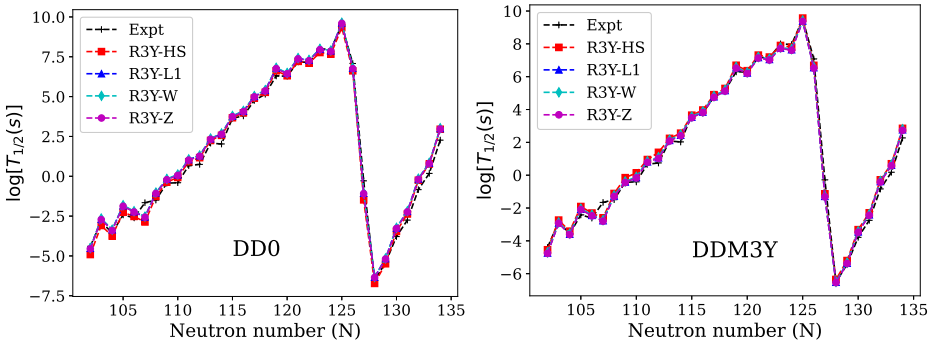

Fig. 2. Comparison of the calculated α-decay half-lives of the Po isotopes between the theoretical models and experiment. (a) using density-independent (DD0) interactions, (b) using density-dependent DDM3Y interactions.

half-life indicates the magicity of the parent nucleus, while a low half-life indicates the magicity of the daughter nucleus. Here, the daughter nucleus that corresponds to the lowest half-life (^{208}Pb) has a neutron magic number $N = 126$.

The difference between the experimental and theoretical α-decay half-lives has also been calculated using the following formula [17, 27]:

$$\Delta T_{1/2} = \left| \log_{10} \left[T_{1/2}^{\text{th}} \right] - \log_{10} \left[T_{1/2}^{\text{exp}} \right] \right| . \tag{26}$$

Figure 3 shows the plots of $\Delta T_{1/2}$ against the neutron number for the different models. In figure 3 (a), the computed $\Delta T_{1/2}$ using the density-independent models (DD0) are shown while figure 3 (b) shows the results using the density-dependent DDM3Y models. In the two plots (figure 3 (a) and figure 3 (b)), most of the points are below 0.6. This again confirms the accuracy of the use of the R3Y models to study the α-decay half-lives of the polonium isotopes.

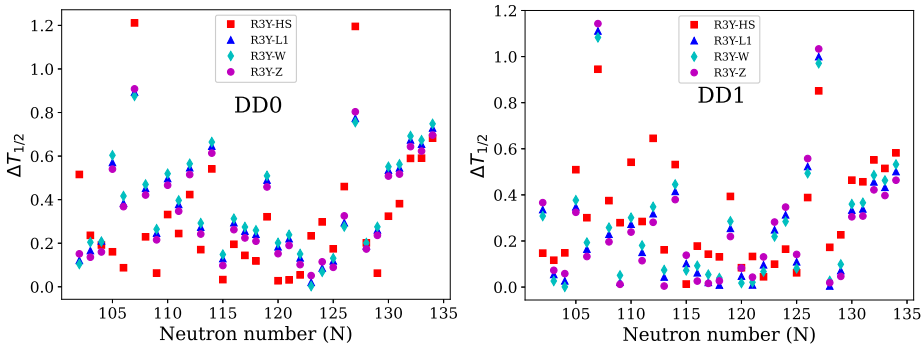

Fig. 3. Plot of the calculated $\Delta T_{1/2}$ against neutron the number (N) for the Po (a) using the density-independent (DD0) interactions, (b) using the density-dependent DDM3Y interactions.

4. Conclusion

The calculations of the α-decay half-lives of some polonium isotopes in the mass range of 186–218 have been carried out theoretically using the WKB semiclassical approximations and with the use of the Bohr–Sommerfeld quantization factor. The α-nucleus potential is obtained using the double folding model, with the R3Y nucleon–nucleon effective interactions. The R3Y effective nucleon–nucleon interactions are derived from the relativistic mean field theory Lagrangian. For comparison, the calculations using the M3Y interactions were also included. When compared with experimental

data, the results obtained using the R3Y models are found to be more accurate than the results obtained using the M3Y-Reid and M3Y-Paris NN interactions. When density-dependent DDM3Y interactions are used in the R3Y models, the results are found to be better than using the density-independent interactions, with the R3Y-Z giving the lowest deviation from experimental data. In general, when compared to experimental data, the R3Y models give maximum standard deviation value $\sigma = 0.4440$ when the density-independent interaction is used and maximum $\sigma = 0.3970$ when the density-dependent interaction is employed. This shows the importance of using the density-dependent interaction in the R3Y model. We conclude that the use of the R3Y effective NN interactions in the double folding model gives very good descriptions of the α-decay half-lives of the polonium isotopes.

REFERENCES

[1] E. Shin, Y. Lim, C.H. Hyun, Y. Oh, «Nuclear isospin asymmetry in α decay of heavy nuclei», *Phys. Rev. C* **94**, 024320 (2016).

[2] V. Zanganah *et al.*, «Calculation of α-decay and cluster half-lives for $^{197-226}$Fr using temperature-dependent proximity potential model», *Nucl. Phys. A* **997**, 121714 (2020).

[3] G. Royer, R. Moustabchir, «Light nucleus emission within a generalized liquid-drop model and quasimolecular shapes», *Nucl. Phys. A* **683**, 182 (2001).

[4] B. Xiaojun, H. Zhang, H. Zhang, G. Royer, J. Li, «Systematical calculation of α decay half-lives with a generalized liquid drop model», *Nucl. Phys. A* **921**, 85 (2014).

[5] G. Royer, H.F. Zhang, «Recent α decay half-lives and analytic expression predictions including superheavy nuclei», *Phys. Rev. C* **77**, 037602 (2008).

[6] J.P. Cui, Y.L. Zhang, S. Zhang, Y.Z. Wang, «α-decay half-lives of superheavy nuclei», *Phys. Rev. C* **97**, 014316 (2018).

[7] K.P. Santhosh, C. Nithya, H. Hassanabadi, D.T. Akrawy, «α-decay half-lives of superheavy nuclei from a modified generalized liquid-drop model», *Phys. Rev. C* **98**, 024625 (2018).

[8] K.P. Santhosh, T.A. Jose, «Alpha and cluster decay using modified generalized liquid drop model with iso-spin dependent pre-formation factor», *Nucl. Phys. A* **992**, 121626 (2019).

[9] K.P. Santhosh *et al.*, «α-decay half-lives of lead isotopes within a modified generalized liquid drop model», *Phys. Rev. C* **101**, 064610 (2020).

[10] Y.J. Wang, H.F. Zhang, W. Zuo, J.Q. Li, «Improvement of a Fission-Like Model for Nuclear α Decay», *Chinese Phys. Lett.* **27**, 062103 (2010).

[11] R.K. Gupta, W. Greiner, «Cluster radioactivity», *Int. J. Mod. Phys. E* **03**, 335 (1994).

[12] B.B. Singh, S.K. Patra, R.K. Gupta, «Cluster radioactive decay within the preformed cluster model using relativistic mean-field theory densities», *Phys. Rev. C* **82**, 014607 (2010).

[13] J.-G. Deng, J.-C. Zhao, P.-C. Chu, X.-H. Li, «Systematic study of α decay of nuclei around the $Z = 82, N = 126$ shell closures within the cluster-formation model and proximity potential 1977 formalism», *Phys. Rev. C* **97**, 044322 (2018).

[14] S.M.S. Ahmed, R. Yahaya, S. Radiman, M.S. Yasir, «Alpha-cluster preformation factors in alpha decay for even–even heavy nuclei using the cluster-formation model», *J. Phys. G: Nucl. Part. Phys.* **40**, 065105 (2013).

[15] D. Deng, Z. Ren, «α preformation factors of medium-mass nuclei and the structural effects in the region of crossing the $Z = 82$ shell», *Phys. Rev. C* **93**, 044326 (2016).

[16] S.M.S. Ahmed, «Alpha-cluster preformation factor within cluster-formation model for odd-A and odd–odd heavy nuclei», *Nucl. Phys. A* **962**, 103 (2017).

[17] W.A. Yahya, «Alpha decay half-lives of $^{171-189}$Hg isotopes using Modified Gamow-like model and temperature dependent proximity potential», *J. Niger. Soc. Phys. Sci.* **2**, 250 (2020).

[18] A. Zdeb, M. Warda, K. Pomorski, «Half-lives for α and cluster radioactivity within a Gamow-like model», *Phys. Rev. C* **87**, 024308 (2013).

[19] G. Royer, «Alpha emission and spontaneous fission through quasi-molecular shapes», *J. Phys. G: Nucl. Part. Phys.* **26**, 1149 (2000).

[20] G. Royer, «Analytic expressions for alpha-decay half-lives and potential barriers», *Nucl. Phys. A* **848**, 279 (2010).

[21] G. Royer, C. Schreiber, H. Saulnier, «Analytic relations for partial alpha decay half-lives and barrier heights and positions», *Int. J. Mod. Phys. E* **20**, 1030 (2011).

[22] V. Viola, G. Seaborg, «Nuclear systematics of the heavy elements — II Lifetimes for alpha, beta and spontaneous fission decay», *J. Inorg. Nucl. Chem.* **28**, 741 (1966).

[23] C. Qi *et al.*, «Microscopic mechanism of charged-particle radioactivity and generalization of the Geiger–Nuttall law», *Phys. Rev. C* **80**, 044326 (2009).

[24] C. Qi, F.R. Xu, R.J. Liotta, R. Wyss, «Universal decay law in charged-particle emission and exotic cluster radioactivity», *Phys. Rev. Lett.* **103**, 072501 (2009).

[25] D.T. Akrawy, A.H. Ahmed, «New empirical formula for α-decay calculations», *Int. J. Mod. Phys. E* **27**, 1850068 (2018).

[26] Z. Ren, C. Xu, Z. Wang, «New perspective on complex cluster radioactivity of heavy nuclei», *Phys. Rev. C* **70**, 034304 (2004).

[27] D.T. Akrawy, H. Hassanabadi, Y. Qian, K.P. Santhosh, «Influence of nuclear isospin and angular momentum on α-decay half-lives», *Nucl. Phys. A* **983**, 310 (2019).

[28] M. Horoi, «Scaling behaviour in cluster decay», *J. Phys. G: Nucl. Part. Phys.* **30**, 945 (2004).

[29] D.T. Akrawy, D.N. Poenaru, «Alpha decay calculations with a new formula», *J. Phys. G: Nucl. Part. Phys.* **44**, 105105 (2017).

[30] D. Naderi, «Study of Cluster Radioactivity: The Influence of Deformation of the Cluster and Daughter Nuclei on Cluster Decay Half-lives», *Int. J. Mod. Phys. E* **22**, 1350065 (2013).

[31] A. Adel, T. Alharbi, «Cluster decay half-lives of trans-lead nuclei based on a finite-range nucleon–nucleon interaction», *Nucl. Phys. A* **958**, 187 (2017).

[32] K.P. Santhosh, S. Sahadevan, B. Priyanka, M.S. Unnikrishnan, «Systematic study of heavy cluster emission from $^{210-226}$Ra isotopes», *Nucl. Phys. A* **882**, 49 (2012).

[33] T.T. Ibrahim, S.M. Wyngaardt, B.D.C. Kimene Kaya, «Analysis of the clustering in ^{212}Po, ^{218}Rn and ^{232}U», *Nucl. Phys. A* **966**, 73 (2017).

[34] A. Soylu, Y. Sert, O. Bayrak, I. Boztosun, «Role of the cluster deformations in explaining the exotic decay half-lives», *Eur. Phys. J. A* **48**, 128 (2012).

[35] A. Soylu, L.M. Robledo, M. Warda, «Calculations on the half-lives of Cluster decay in two-potential approach», *Int. J. Mod. Phys. E* **27**, 1850005 (2018).

[36] D. Ni, Z. Ren, «α-decay calculations of medium mass nuclei within generalized density-dependent cluster model», *Nucl. Phys. A* **828**, 348 (2009).

[37] C. Xu, Z. Ren, «Favored α-decays of medium mass nuclei in density-dependent cluster model», *Nucl. Phys. A* **760**, 303 (2005).

[38] C. Xu, Z. Ren, «Global calculation of α-decay half-lives with a deformed density-dependent cluster model», *Phys. Rev. C* **74**, 014304 (2006).

[39] N. Maroufi, V. Dehghani, S.A. Alavi, «Cluster Decay Half-life with Double-folding Potential: Uncertainty Analysis», *Acta Phys. Pol. B* **50**, 1349 (2019).

[40] B.D.C. Kimene Kaya, S.M. Wyngaardt, T.T. Ibrahim, W.A. Yahya, «Comparison of double-folding effective interactions within the cluster model», *Phys. Rev. C* **98**, 044308 (2018).

[41] F. Ghorbani, S.A. Alavi, V. Dehghani, «Temperature dependence of the alpha decay half-lives of even–even Th isotopes», *Nucl. Phys. A* **1002**, 121947 (2020).

[42] D. Deng, Z. Ren, «Improved double-folding α-nucleus potential by including nuclear medium effects», *Phys. Rev. C* **96**, 064306 (2017).

[43] I.I. Gontchar, M.V. Chushnyakova, «A C-code for the double folding interaction potential of two spherical nuclei», *Comput. Phys. Commun.* **181**, 168 (2010).

[44] G.R. Satchler, D.T. Khoa, W. von Oertzen, «Nuclear incompressibility and density dependent NN interactions in the folding model for nucleus–nucleus potentials», *Phys. Rev. C* **56**, 954 (1997).

[45] G.L. Zhang, H. Liu, X.Y. Le, «Nucleon–nucleon interactions in the double folding model for fusion reactions», *Chinese Phys. B* **18**, 136 (2009).

[46] N. Maroufi, V. Dehghani, S.A. Alavi, «Alpha and cluster decay of some deformed heavy and superheavy nuclei», *Nucl. Phys. A* **983**, 77 (2019).

[47] M. Wang *et al.*, «The AME 2020 atomic mass evaluation (II). Tables, graphs and references», *Chinese Phys. C* **45**, 030003 (2021).

[48] W.J. Huang *et al.*, «The AME 2020 atomic mass evaluation (I). Evaluation of input data, and adjustment procedures», *Chinese Phys. C* **45**, 030002 (2021).

[49] F.G. Kondev *et al.*, «The NUBASE2020 evaluation of nuclear physics properties», *Chinese Phys. C* **45**, 030001 (2021).