

VELTMAN, RENORMALIZABILITY, CALCULABILITY*

GIAMPIERO PASSARINO

Dipartimento di Fisica Teorica, Università di Torino, Italy
and
INFN, Sezione di Torino, Italy

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Dedicated to the memory of Professor Martinus Veltman, one of the founding fathers of our discipline: his legacy lives on. Many times we have to turn back and follow his footprints to find the right path. After reviewing general aspects of high-energy physics where he gave a seminal contribution we will introduce recent developments in the Standard Model Effective Field Theory, showing how the whole movement from renormalization to predictions plays from Veltman to SMEFT.

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1. Introduction

In his talk at Higgs Hunting 2015¹, Martinus Veltman said:
Higgs particle ... What does it mean? What does it do? It is claimed to give mass to all other particles. What does that mean? Can we now predict the masses of all particles? In this talk it will be attempted to explain why the Higgs particle is important to the theory of the Standard Model. The importance of the Higgs construction is that it made the theory of Yang–Mills fields renormalizable. Observable results can be calculated and compared with experiment, and that has happened in a multitude of ways in the last 40 years, up to and including the recent discovery of the Higgs particle.

It is clear that he meant “strictly renormalizable” but it is evident that the main emphasis is on: *Observable results can be calculated and compared with experiment.* In order to understand the situation before 1971, it is interesting to observe [1] that most of the papers on the subject came to the same conclusion “... it is concluded that all theories based on simple Lie groups are unrenormalizable”. All difficulties in the Yang–Mills theories disappeared

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¹ <https://indico.ijclab.in2p3.fr/event/2722/contributions/5996/>

in 1971 and the theories became fully renormalizable, that is all occurring infinities could be absorbed in the available free parameters. Theories with a Yang–Mills structure were now renormalizable theories and a precise model for the weak interactions existed already (although it had received virtually no attention). The phase of “precise calculations” started, extending the pioneering work of Berends, Gastmans and collaborators [2–4] to the full Standard Model [5–9], and continued till the most recent successes [10–13]. For a complete list of references, see Ref. [14].

To summarize a long journey, we can say that the transition was from the Fermi theory to the Standard Model (SM); for that, Veltman [15, 16] had to convince the community that the weak interactions were some form of a Yang–Mills theory. As he wrote: *I could not have done that without the knowledge of experimental physics that I had acquired at CERN*. It is worth noting that the journey never contemplated the extension of the Fermi theory with the inclusion of even higher operators; therefore, the SM represented the beyond-Fermi physics. Comparing with the present: we are now in a beyond-SM desert looking for alternative paths, although the regulative ideal of an ultimate theory remains a powerful aesthetic ingredient.

Almost equally important, and a landmark through the whole region of “strict renormalizability”, is the work on quantum gravity: “In case of gravitation interacting with scalar particles, divergencies in physical quantities remain” [1, 17]. It is possible that at some very large energy scale, all nonrenormalizable interactions disappear. This seems unlikely, given the difficulty with gravity. It is possible that the rules change drastically. It may even be possible that there is no end, simply more and more scales [18].

2. The Standard Model before LEP

The last step in the renormalization procedure is the connection between renormalized quantities and physical observables. Since all quantities at this stage are UV-free, we term it *finite renormalization*. Note that the absorption of UV divergencies into local counterterms is, to some extent, a trivial step (except for the problem of overlapping divergencies [19]); finite renormalization, instead, requires more attention. For example, beyond one loop, one cannot use on-shell masses but only complex poles for all unstable particles [20–23]. The complete formulation of finite renormalization is beyond the goal of this work. However, let us show some examples where the concept of an on-shell mass can be employed. Suppose that we renormalize a physical observable F

$$F = F_{\text{B}} + g^2 F_{1\text{L}}(M^2) + g^4 F_{2\text{L}}(M^2) , \quad (1)$$

where M is some renormalized mass which appears at one and two loops in F_{1L} and F_{2L} but does not show up in the Born term F_B . In this case, we can use the concept of an on-shell mass identifying $M = M_{\text{OS}}$ for the two-loop term and performing a finite mass renormalization at one loop

$$M^2 = M_{\text{OS}}^2 \left\{ 1 + \frac{g^2}{16\pi^2} \left[\text{Re} \Sigma_M^{(1)} \Big|_{p^2=-M_{\text{OS}}^2} - \delta Z_M^{(1)} \right] \right\} = M_{\text{OS}}^2 + g^2 \Delta M^2, \quad (2)$$

where M_{OS} is the on-shell mass and Σ is extracted from the required one-particle irreducible Green function. It is worth noting that Eq. (2) is still meaningful (no dependence on gauge parameters) and will be used inside the one-loop result

$$F = F_B + g^2 F_{1L}(M_{\text{OS}}^2) + g^4 [F_{2L}(M_{\text{OS}}^2) + F'_{1L}(M_{\text{OS}}^2) \Delta M^2], \quad (3)$$

where

$$F'_{1L}(M_{\text{OS}}^2) = \frac{\partial F_{1L}(M^2)}{\partial M^2} \Big|_{M^2=M_{\text{OS}}^2}. \quad (4)$$

If we focus on renormalization, we can safely state that all the necessary ingredients are available. Here, the crucial point is to connect a set of input experimental data (an input-parameter set, hereafter IPS) to the free parameters of the theory:

- mass renormalization involves the calculation of self-energies;
- renormalization of coupling constants requires additional elements, which depend on the choice of the observables in the IPS.

The most obvious selection of an IPS is based on the choice of those data which are known with the best experimental precision, *e.g.* the electromagnetic coupling constant, the Fermi coupling constant and the Z -boson mass.

Before the advent of the LEP operations, we had very few options for the IPS [24, 25]. To give an example, we consider a first approximation where the lowest-order expressions are compared with a set of low-energy data points, electric charge, neutral currents and μ -decay

$$g^2 = 4\pi\alpha(0), \quad M^2 = \frac{g^2\sqrt{2}}{8G_F}, \quad s_\theta^2 \quad \text{from} \quad R = \frac{\sigma_{\bar{\nu}e}}{\sigma_{\nu e}}. \quad (5)$$

The values for g^2 , M^2 and s_θ^2 , correct in the lowest order, will subsequently be used in the expressions for the radiative corrections. In the next order, we replace

$$g^2 \rightarrow g^2(1 + \delta g^2), \quad M^2 \rightarrow M^2(1 + \delta M^2), \quad s_\theta^2 \rightarrow s_\theta^2(1 + \delta s_\theta^2). \quad (6)$$

The counterterms are chosen to compensate precisely for the radiative corrections for $e\text{-}\mu$ scattering, μ -decay and the ratio R . Having determined these quantities, we may proceed to making predictions. The ratio R is $R(s_\theta)$. We use the fact that it does not depend on g^2 or M^2 at the lowest order and at zero energy and momentum transfer. Thus,

$$R_0 (s_\theta^2 + s_\theta^2 \delta s_\theta^2) = R_0 (s_\theta^2) + R'_0 s_\theta^2 \delta s_\theta^2, \quad (7)$$

with $R' = dR/ds_\theta^2$. The one-loop radiative corrections to R will be some R_1 , then δs_θ^2 is fixed by

$$\delta s_\theta^2 = -\frac{R_1}{R'_0 s_\theta^2}. \quad (8)$$

The rest is standard and gives $e^4 \rightarrow e^4(1 + \delta e^4)$ and

$$\delta g^2 = \frac{1}{2} \delta e^4 - \delta s_\theta^2, \quad \delta M^2 = \frac{1}{2} (\delta_\mu - \delta_\mu^{\text{em}}) + \delta g^2. \quad (9)$$

Let us denote by $s_{\nu e}^2$ the low-energy weak mixing angle defined through some R_{exp} . Then we can derive masses for the vector bosons in the low-energy convention. They are given by

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2} G_F s_{\nu e}^2}, \quad M_Z^2 = \frac{\pi\alpha}{\sqrt{2} G_F s_{\nu e}^2 c_{\nu e}^2}. \quad (10)$$

Let us consider the amplitude for $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$. In real life, many different contributions should be considered, but to illustrate some of the relevant points in the procedure, it is enough to limit the calculation to contributions from the heavy-quark doublet, (t - b), to the ZZ and $Z\gamma$ transitions.

In this case, we obtain

$$A(\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-) = \left(\frac{ig}{4c_\theta} \right)^2 \gamma^\mu \gamma_+ \otimes \gamma_\mu (a + b\gamma^5), \quad (11)$$

where $\gamma^\alpha \otimes \gamma^\alpha = \bar{\nu}_\mu \gamma^\alpha \bar{\nu}_\mu \bar{e} \gamma^\alpha e$, *etc.* and

$$a = \left[4s_\theta^2 - 1 - \frac{g^2 s_\theta^2}{4\pi^2} \frac{\Sigma_{Z\gamma}(p^2)}{p^2} \right] \Delta_Z(p^2), \quad b = -\Delta_Z(p^2), \quad (12)$$

with a propagator Δ_Z given by

$$\Delta_Z^{-1}(p^2) = p^2 + M_0^2 - \frac{g^2}{16\pi^2 c_\theta^2} \Sigma_{ZZ}(p^2), \quad (13)$$

where Σ_{ZZ} is the Z -boson self-energy and $\Sigma_{Z\gamma}$ is the corresponding transition. Then the total cross section $\sigma_{\nu e}$ can be computed and the data point $R = \sigma_{\bar{\nu}e}/\sigma_{\nu e}$ used

$$R = \frac{\xi_{\nu e}^2 - \xi_{\nu e} + 1}{\xi_{\nu e}^2 + \xi_{\nu e} + 1}, \quad \xi_{\nu e} = \frac{a}{b}, \quad (14)$$

where we assume the approximation of zero momentum transfer. Subtracting the terms involving UV poles and introducing a mass scale μ , we define the counterterm for s_θ and fix $s_\theta^{\overline{\text{MS}}}$ to first order in α . The whole renormalization procedure amounts to throwing away infinities. If we subtract the terms involving UV poles then the $\overline{\text{MS}}$ redefinition of the parameters is obtained (including s_θ), but we could as well assign any finite value to $1/\bar{\varepsilon}$ (to be defined in Eq. (20)) and check for the independence of the physical quantities of $1/\bar{\varepsilon}$. From the t - b quark doublet, we obtain

$$\begin{aligned} s_\theta^{\overline{\text{MS}}} &= s_{\nu e}^2 + \frac{\alpha}{12\pi} \left[2 \left(1 - \frac{8}{3} s_{\nu e}^2 \right) \ln \frac{m_t^2}{\mu^2} + \left(1 - \frac{4}{3} s_{\nu e}^2 \right) \ln \frac{m_b^2}{\mu^2} \right], \\ s_{\nu e}^2 &= \frac{1 - \xi_{\nu e}}{4}. \end{aligned} \quad (15)$$

As expected, there are no terms quadratic in the quark masses.

At this point, we are ready to make predictions. Starting from $p^2 = 0$, we can introduce an effective p^2 -dependent weak mixing angle, *etc.*

There is an important lesson to learn: there are Lagrangian parameters (*e.g.* s_θ), input parameters (*e.g.* $\xi_{\nu e}$) and predictions (or “pseudo-observables”). In the SM, there is no one-to-one correspondence between Lagrangian parameters and input parameters. It was different in the older days of QED, where only two Lagrangian parameters are present, e and m_e . In this case, one uses $m_{\text{exp}}(1 + \Delta m_e)$ reflecting some vague intuition about the physical meaning of the bare mass. The strategy to prescribe precisely what a Lagrangian parameter offers is a problem when there is no unique experimental quantity that can play the role of defining the parameter.

At LEP, $s_{\nu e}^2$ will be replaced by a $\sin \theta_{\text{eff}}^f$, related to the vector and axial couplings of the Z -boson.

3. LEP: the ρ -parameter and pseudo-observables

One way to explain renormalization is to say that infinities are unobservable and can thus be absorbed into the parameters of the Lagrangian. What about potentially large effects in the renormalized theory? When are they observable? For instance, the m_t^2 -terms at one loop, which are

there and show up in physical observables. Conversely, the M_H dependence of one-loop radiative corrections was another seminal contribution of Veltman and it was described by the screening theorem [26, 27]: the one-loop M_H -dependence in physical observables is only logarithmic. Terms proportional to M_H^2 are unobservable at the one-loop level (they start at two loops) and can be absorbed into the parameters of the SM Lagrangian, apart from the case of one-loop diagrams with external (on-shell) Higgs-lines. The ρ -parameter [28–30] was born.

A comment is needed for the “original” ρ -parameter, defined as the ratio of the W and Z masses squared divided by c_θ^2 . We start by introducing a bare quantity: $\rho_0 = M_W^2/(c_\theta^2 M_Z^2)$. The quantities appearing in this relation must be related to experimental data. There is no ambiguity in what is meant by an experimental mass. There remains the experimental c_θ^2 which should be extracted from data (at the time, low-energy data). In the original formulation, where everything was extracted from low-energy data ($p^2 = 0$), a ρ was introduced as

$$\rho = 1 + \frac{g^2}{16\pi^2 M_W^2} [\Sigma_{WW}(0) - \Sigma_{ZZ}(0)] = 1 + \frac{g^2}{16\pi^2} \Delta\rho, \quad (16)$$

where the 1 in the r.h.s. of the equation is actually ρ_{bare} in the SM. The ρ -parameter of Eq. (16) is finite and numerically very close to the experimental one, in any scheme where the counterterms are prescribed. The main correction is due to the top-quark mass. Usually, we do not attach any particular relevance to bare parameters, only the renormalized Lagrangian predicts meaningful — measurable — quantities. However, the ρ -parameter of the SM plays a special role. It is finite because of a residual symmetry which is nothing but the usual isospin invariance. Individual components in Eq. (16) are by themselves infinite, but the combination occurring in this equation for the ρ -parameter is finite, as it should be.

The Nobel Prize in Physics 1999 was awarded “for elucidating the quantum structure of electroweak interactions in physics” which was crucial for LEP. At LEP, we had the SM with one missing ingredient, therefore the strategy was: test the SM hypothesis *versus* M_H , introduce Pseudo-observables (generalizations of the ρ -parameter [26]), fit them and derive limits on the Higgs boson mass [31].

Ideally, the strategy should have been to combine the results of the LEP experiments at the level of the measured cross sections and asymmetries — a goal that has never been achieved because of the intrinsic complexity, given the large number of measurements with different cuts and the complicated structure of the experimental covariance matrices relating their errors. This reflects computing limitations at the time.

What the experimenters did [31] was just collapsing (and/or transforming) some “primordial quantities” (say a number of observed events in some pre-defined set-up) into some “secondary quantities” (the POs) which are closer to the theoretical description of the phenomena. The practical attitude of the experiments was to stay with a fit from “primordial quantities” to POs (with an SM remnant) for each experiment, and these sets of POs were averaged. The result of this procedure are the best values for POs. The extraction of Lagrangian parameters was based on the LEP-averaged POs. The PO-strategy was made possible thanks to high-precision QED calculations and tools, from the pioneering work of Ref. [32] (for an update, see Ref. [33]) up to the Monte Carlos that were instrumental at LEP in extracting the realistic observables; for instance, KKMC, the most advanced MC for 2-fermion production [34] (including second-order QED with resummation, initial-final state interference and spin polarizations, for an update, see Ref. [35]).

4. LHC: after the discovery of the Higgs boson

After the LHC Run 1, the SM has been completed, raising its status to that of a full theory [36]. Despite its successes, this SM has shortcomings *vis-à-vis* cosmological observations. At the same time, there is presently a lack of direct evidence for new physics phenomena at the accelerator energy frontier. From this state of affairs arises the need for a consistent theoretical framework in which deviations from the SM predictions can be calculated.

Theoretical physics suffers from some inherent difficulties: great successes during the 20th century, increasing difficulties to do better, as the easier problems get solved. The lesson of experiments from 1973 to today is that it is extremely difficult to find a flaw in the SM²: maybe the SM includes elements of a truly fundamental theory. The conventional vision is: some very different physics occurs at the Planck scale, the SM is just an effective field theory (EFT). What about the next SM? A new weakly-coupled renormalizable model? A tower of EFTs? Of course, there is a different vision: is the SM close to a fundamental theory?

We need a consistent theoretical framework in which deviations from the SM (or next-SM) predictions can be calculated. Such a framework should be applicable to comprehensively describe measurements in all sectors of particle physics: LHC Higgs measurements, past electroweak precision data (EWPD), *etc.* Here, we outline the strategy:

² Note however the long-standing tension between experiment and SM prediction in the anomalous magnetic moment of the muon, recently reaffirmed by the Fermilab experiment, Ref. [37].

- Consider the SM augmented with the inclusion of higher dimensional operators (say theory T_1); not strictly renormalizable. Although workable to all orders, T_1 fails above a certain scale, Λ_1 .
- Consider any Beyond-Standard-Model (BSM) model that is strictly renormalizable and respects unitarity (T_2); its parameters can be fixed by comparison with data, while masses of heavy states are presently unknown. $T_1 \neq T_2$ in the UV but they must have the same IR behavior.
- Consider now the whole set of data below Λ_1 . T_1 should be able to explain them by fitting Wilson coefficients, T_2 adjusting the masses of heavy states (as SM did with the Higgs mass at LEP) should be able to explain the data. Goodness of both explanations is crucial in understanding how well they match and how reasonable is to use T_1 instead of the full T_2 .
- Does T_2 explain everything? Certainly not, but it should be able to explain something more than T_1 .
- We could now define T_3 as T_2 augmented with (its own) higher dimensional operators; it is valid up to a scale Λ_2 .
- Continue.

This prompts the important question whether there is a last fundamental theory in this tower of EFTs that supersede each other with rising energies [38]. Some people conjecture that this deeper theory could be a string theory, *i.e.* a theory which is not a field theory any more. Or should one ultimately expect from physics theories that they are only valid as approximations and in a limited domain? To summarize: experiments occur at finite energy and measure $S_{\text{eff}}(\Lambda)$ (an effective S -matrix); whatever QFT should give low energy $S_{\text{eff}}(\Lambda)$, $\forall \Lambda < \infty$, *i.e.* there is no fundamental scale above which $S_{\text{eff}}(\Lambda)$ is not defined. However, $S_{\text{eff}}(\Lambda)$ loses its predictive power if a process at $E = \Lambda$ requires an infinite number of renormalized parameters [39–41].

To summarize: before LHC, we had the SM, a weakly coupled, strictly renormalizable (a theory with n Lagrangian parameters, requiring n data points, requiring n calculations; the $(n + 1)^{\text{th}}$ calculation is a prediction) theory with one unknown, M_H . The strategy was: test data against predictions *versus* M_H . At the LHC, after the discovery, with a lack of direct evidence for new physics phenomena, we have all the ingredients required to assess (in)consistency of the SM against data.

We briefly review the SMEFT Lagrangian [42–44]: consider the Standard Model, described by a Lagrangian $\mathcal{L}_{\text{SM}}^{(4)}$ with a symmetry group $G = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$. The SMEFT extension is described by a Lagrangian

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \sum_{d>4} \sum_i \frac{a_i^d}{\Lambda^{4-d}} \mathcal{Q}_i^{(d)}, \quad (17)$$

where Λ is the cutoff of the effective theory, a_i^d are Wilson coefficients and $\mathcal{Q}_i^{(d)}$ are G -invariant operators of mass-dimension d involving the $\mathcal{L}_{\text{SM}}^{(4)}$ fields. In this work, we will use the so-called “Warsaw basis” [45].

An unconventional approach to EFT: derivative-coupled field theories are known to develop ghosts [46]. The EFT option [47] replaces the original \mathcal{L} with some \mathcal{L}_{eff} truncated at some order in the Λ -expansion; the “dangerous” terms are substituted by using the equations of motion where, for instance, we neglect terms of $\mathcal{O}(\Lambda^{-2})$. We assume that \mathcal{L}_{eff} will be replaced by a well-behaved \mathcal{L}' at some larger scale, therefore justifying a truncated perturbative expansion; the EFT does not have ghosts while remaining within its regime of validity.

4.1. SMEFT and renormalization

At this point, we have lost strict renormalizability but this should not come at the price of losing computability [48, 49]; whether the predictions of a theory are matched by Nature is a completely different matter and can be decided only by comparing the predictions with experiment (calculability).

A renormalizable theory is determined by a fixed number of parameters; once these are determined (after finite renormalization) we can make definite predictions at a fixed accuracy. An EFT theory requires at higher and higher energies more and more counterterms; the asymptotic expansion in E/Λ may break down completely above some scale. Given a truncated expansion, we still have a large family of UV-complete theories with these low-order terms, which have different behavior at higher energies. Note that the notion of UV completion adopted here [50] is the claim that a theory is “formally” predictive up to all (possible) high energies, but we do not include the additional criterion that the theory be a final, unified “theory of everything”.

Therefore, it is crucial to prove that our EFT is closed under renormalization, order-by-order in the asymptotic expansion, although the number of counterterms will grow with the order (as mentioned above, the predictive power is lost at scales approaching the cutoff). For any given process, the amplitude can be written as follows:

$$A = \sum_{n=N}^{\infty} g^n A_n^{(4)} + \sum_{n=N_6}^{\infty} \sum_{l=1}^n \sum_{k=1}^{\infty} g^n g_{4+2k}^l A_{nlk}^{(4+2k)}, \quad (18)$$

where g is the $SU(2)$ coupling constant and $g_{4+2k} = 1/(\sqrt{2} G_F \Lambda^2)^k = g_6^k$, where G_F is the Fermi coupling constant and Λ is the scale around which new physics (NP) must be resolved. For each process, N defines the $\text{dim} = 4$ LO (*e.g.* $N = 1$ for $H \rightarrow VV$ *etc.* but $N = 3$ for $H \rightarrow \gamma\gamma$). $N_6 = N$ for tree initiated processes and $N - 2$ for loop-initiated ones. Here, we consider single insertions of $\text{dim} = 6$ operators, which defines the so-called NLO SMEFT. To be more precise, we define an NLO SMEFT amplitude as the one containing SMEFT vertices inserted in tree-level SM diagrams, tree-level (SMEFT-induced) diagrams with a non-SM topology, SMEFT vertices inserted in one-loop SM diagrams, and one-loop (SMEFT-induced) non-SM diagrams.

The amplitude can be rewritten as

$$A = g^N A_{\text{LO}}^{(4)}(\{p\}) + g^N g_6 A_{\text{LO}}^{(6)}(\{p\}, \{a\}) + \frac{g^{N+2}}{16\pi^2} A_{\text{NLO}}^{(4)}(\{p\}) + \frac{g^{N+2} g_6}{16\pi^2} A_{\text{NLO}}^{(6)}(\{p\}, \{a\}), \quad (19)$$

where $\{p\}$ is the set of SM parameters and $\{a\}$ the set of Wilson coefficients. Counterterms are introduced using

$$\Delta_{\text{UV}} = \frac{2}{4-d} - \gamma - \ln \pi - \ln \frac{\mu_R^2}{\mu^2} = \frac{1}{\bar{\epsilon}} - \ln \frac{\mu_R^2}{\mu^2}, \quad (20)$$

where d is the space-time dimension, γ is the Euler–Mascheroni constant, the loop measure is $\mu^{4-d} d^d q$ and μ_R is the renormalization scale. The counterterms are defined by

$$Z_i = 1 + \frac{g^2}{16\pi^2} \left(dZ_i^{(4)} + g_6 dZ_i^{(6)} \right) \Delta_{\text{UV}}. \quad (21)$$

With field/parameter counterterms, we can make UV finite (at $\mathcal{O}(g^2 g_6)$) all self-energies and transitions and the corresponding Dyson resummed propagators. Of course, we have to prove cancellation of UV poles for all Green's functions. This means that we need to make the SMEFT S -matrix UV (and IR) finite, including $\text{dim} = 6$ operators and, at least, $\text{dim} = 8$ operators (truncation uncertainty). The verification of any claim with explicit computations is of importance. The role of symmetry is crucial. The best way to understand the connection between UV poles and symmetry is given by the background-field method [17, 51]. Let us give an example of the complexity

of proving cancellation of UV poles in any EFT. Consider a scalar theory

$$\begin{aligned} \mathcal{L}(\phi_c + \phi) = & \mathcal{L}(\phi_c) + \phi_i \mathcal{L}'_i(\phi_c) + \frac{1}{2} \partial_\mu \phi_i \partial_\mu \phi_i \\ & + \phi_i N_{ij}^\mu \partial_\mu \phi_j + \frac{1}{2} \phi_i M_{ij}(\phi_c) \phi_j + \mathcal{O}(\phi^3) + \text{tot. der.} \end{aligned} \quad (22)$$

($\mathcal{L}'_i(\phi_c) = 0$). All one-loop diagrams are generated by $\mathcal{L}_2(\phi)$, the part quadratic in ϕ

$$\mathcal{L}_2(\phi) \rightarrow -\frac{1}{2} (\partial_\mu \phi)^2 + \phi N^\mu \partial_\mu \phi + \frac{1}{2} \phi M \phi. \quad (23)$$

The counter-Lagrangian is given by

$$\begin{aligned} \Delta \mathcal{L} = & \frac{1}{8\pi^2(d-4)} \left[a_0 M^2 + a_1 (\partial_\mu N_\nu)^2 + a_2 (\partial_\mu N_\mu)^2 + a_3 M N^2 \right. \\ & \left. + a_4 N_\mu N_\nu \partial_\mu N_\nu + a_5 (N^2)^2 + a_6 (N_\mu N_\nu)^2 \right]. \end{aligned} \quad (24)$$

However, define $X = M - N^\mu N_\mu$ and see that \mathcal{L} is invariant under 't Hooft transformation (H), Λ antisymmetric

$$\phi' = \phi + \Lambda \phi, \quad N'_\mu = N_\mu - \partial_\mu \Lambda + [\Lambda, N_\mu], \quad X' = X + [\Lambda, X]. \quad (25)$$

Therefore, $\Delta \mathcal{L}$ also will be invariant ($\text{Tr } X$ is invariant)

$$\begin{aligned} \Delta \mathcal{L} = & \frac{1}{\varepsilon} \text{Tr} (a X^2 + b Y^{\mu\nu} Y_{\mu\nu}), \\ Y_{\mu\nu} = & \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu], \end{aligned} \quad (26)$$

and Y transforms as X . The counter-Lagrangian is made of products of objects transforming as X and of $\text{dim} = 4$. As a consequence of the H -invariance, the number of counterterms goes from 7 to 2. Any approach to SMEFT-violating invariance is doomed to failure [52]. An EFT (*e.g.* SMEFT) including $\text{dim} = 6, 8$ operators will contain a term (for $\text{dim} = 6$, see Ref. [53])

$$\begin{aligned} & \frac{1}{2} \partial_\mu \phi_i g_{ij}^{\mu\nu}(\phi_c) \partial_\nu \phi_j \\ & \text{matrix-valued metric tensor.} \end{aligned} \quad (27)$$

We should pay attention to the fact that in the SM, $g_{ij}^{\mu\nu} \propto \delta^{\mu\nu} \delta_{ij}$, while in quantum gravity (QGR), it remains diagonal only in the ij indices. Therefore, in SMEFT we will have matrix-valued Riemann tensors, *i.e.* more invariants for the counter-Lagrangian $\Delta \mathcal{L}$, *i.e.* $\text{Tr}(X R)$, $\text{Tr}(R^2) \dots$; as

a consequence, EFT is computationally more complex than QGR [54]. We should remember that the name of the game is to have the full $\Delta\mathcal{L}$, not the counterterms for one or two processes. If \mathcal{L} is invariant under a group G , then the relation between the G transformation and the H one is crucial in proving closure under renormalization (not the same as strict renormalizability). In other words, the H -invariant counterterms of Eq. (26) must also be G -invariant.

To give an example of Eq. (27), we consider a Lagrangian

$$\mathcal{L} = -\partial_\mu \Phi^* \partial_\mu \Phi - m^2 \Phi^* \Phi - \frac{1}{2} g^2 (\Phi^* \Phi) , \quad (28)$$

where $\Phi = (\phi_1 + v + i\phi_2)/\sqrt{2}$. We introduce $v = M/g$ and $m^2 = \beta - M^2/2$, and add

$$\mathcal{Q}^{(8)} = \left(\frac{gg_6}{M^2} \right)^2 \partial_\mu \Phi^* \partial_\mu \Phi \square (\Phi^* \Phi) . \quad (29)$$

After splitting $\phi_i = \phi_{ic} + \phi_i$, we obtain that the contribution of the operator of dim = 8 to the metric tensor is $\delta g = \delta_1 g + \delta_2 g + \mathcal{O}(g^3)$, where

$$\begin{aligned} \delta_1 g^{\mu\nu} &= \frac{gg_6^2}{M} \phi_{1c} \delta^{\mu\nu} I , \\ \delta_2 g^{\mu\nu} &= \frac{g^2 g_6^2}{M^4} \\ &\times \begin{pmatrix} (g_{11} + g_{22}) \delta^{\mu\nu} + 4 \partial^\mu \phi_{1c} \partial^\nu \phi_{1c} & 4 \partial^\mu \phi_{1c} \partial^\nu \phi_{2c} \\ 4 \partial^\mu \phi_{12c} \partial^\nu \phi_{1c} & (g_{11} + g_{22}) \delta^{\mu\nu} + 4 \partial^\mu \phi_{2c} \partial^\nu \phi_{2c} \end{pmatrix} , \end{aligned} \quad (30)$$

where $g_{ij} = \partial^\alpha \phi_{ic} \partial^\alpha \phi_{jc}$.

Here, we will summarize the main steps in the renormalization procedure for the SMEFT. Field/parameter counterterms are not enough to make UV finite the Green's functions with more than two legs. A mixing matrix among Wilson coefficients is needed

$$a_i = \sum_j Z_{ij}^W a_j^{\text{ren}} , \quad Z_{ij}^W = \delta_{ij} + \frac{g^2}{16\pi^2} dZ_{ij}^W \Delta_{UV} . \quad (31)$$

We can start by computing the amplitude for the (on-shell) decay $H \rightarrow \gamma\gamma$ and fix a few entries in the mixing matrix; we can continue with the $HZ\gamma$ and HZZ amplitudes. When we arrive at HWW , we find that the dim = 4 part can be made UV finite but for the dim = 6 part, there are no Wilson coefficients left free so that the UV finiteness follows from gauge cancellations. We then continue with the decay of the Higgs boson (and of the Z boson) into fermion pairs and make all the corresponding amplitudes UV finite.

At this point, we are left with the universality of the electric charge. In QED, there is a Ward identity telling us that e is renormalized in terms of vacuum polarization, Ward–Slavnov–Taylor identities allow us to generalize the argument to the full SM [12, 55].

We can give a quantitative meaning to the previous statement by saying that the contribution from vertices (at zero momentum transfer) exactly cancels those from (fermion) wave-function renormalization factors. Therefore, we need to compute the vertex $\bar{f}f\gamma$ (at $q^2 = 0$) and the f wave function factor in SMEFT, proving that the WST identity can be extended to $\dim = 6$; this is non-trivial since there are no free Wilson coefficients in these terms (after the previous steps); (non-trivial) finiteness of LEP processes follows.

Finite renormalization in SMEFT; let us recall that the renormalization procedure comprises the specification of the gauge-fixing term including, together with the corresponding FP Lagrangian, the choice of the regularization scheme — nowadays dimensional regularization — the prescription for the renormalization scheme and the choice of an input parameter set. For the SM parameters, we use on-shell renormalization which requires the choice of some input parameter set (IPS); for the Wilson coefficients, we use the $\overline{\text{MS}}$ scheme. Therefore, the final answer will contain “universal” logarithms (renormalization group) but also “non-universal” logarithms which depend on the choice of the IPS. The choice of the IPS should be such that the effect of “non-universal” logarithms is minimal, *e.g.* $\alpha(0)$ should be avoided. Once we work with NLO SMEFT, non-local effects (*e.g.* normal-threshold singularities or even anomalous thresholds [56]) will also show up [47, 57, 58] for all those observables where the light masses are small compared to the scale at which we test the process and much smaller than the cutoff Λ .

Finally, the infrared/collinear part of the one-loop virtual and of the real corrections shows double factorization and the total is finite [59] at $\mathcal{O}(g^4 g_6)$.

4.2. Asymptotics of BSM models

The background-field method is also used in combination with the heat kernel [60], a very convenient tool for studying various asymptotics of the effective action; for instance, in deriving the low-energy limit of some underlying BSM theory described by $\mathcal{L}(\{\Phi\})$, where $\{\Phi\}$ includes both heavy and light fields. We expand $\Phi = \Phi_c + \phi$ and derive

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_c + \langle \phi, D\phi \rangle = \mathcal{L}_c + \phi^\dagger Q \phi, \\ Z[\Phi_c] &= \int [\mathcal{D}\phi] \exp\{iS\} = \exp\{iS_c\} \det^{-1/2}(D),\end{aligned}\tag{32}$$

where D must be self-adjoint. Using $Q = \square - M + \hat{Q}(\Phi_c)$, the heat kernel expansion requires computing

$$\text{Tr} \ln Q(x) \delta^4(x-y) = \int d^4x \frac{d^4q}{(2\pi)^4} \text{tr} \ln \left[-q^2 - M + \square + 2i q \cdot \partial + \hat{Q}(x, \partial_x) \right]. \quad (33)$$

When there is one field or $M_{ij} = M^2 \delta_{ij}$, we write $\ln[-(q^2 + M^2)(I + K)]$, expand $\ln(I + K)$ in powers of K obtaining the large M -expansion of $S_{\text{eff}} = 1/2 \ln \det(D)$ in terms of tadpole integrals. Otherwise, with more heavy scales or mixed heavy-light scales, the correct Taylor expansion [61] is

$$\ln(A + B) - \ln A = \int_0^\infty d\mu^2 \left[A_+^{-1} B A_+^{-1} - A_+^{-1} B A_+^{-1} B A_+^{-1} + \dots \right], \quad (34)$$

where $A_+ = A + \mu^2 I$.

5. Advanced SMEFT

In this section, we will use the so-called “Warsaw basis” [45]; note however that we will rescale the Wilson coefficients: in front of an operator $\mathcal{Q}_i^{(d)}$ of dimension d containing n fields, we will write $g^{n-2} a_i^d / \Lambda^{d-4}$, where g is the SU(2) coupling constant. Our goal is to consider the SMEFT with its anomalies [39, 62–64]; in these calculations, a certain amount of γ -matrix manipulation is unavoidable and we must specify the regularization scheme to be used, in particular, we must specify how to treat γ^5 . We will use the scheme developed by Veltman in Ref. [65], which is based on the work of Refs. [66, 67]. Therefore, γ^μ, γ^5 , and $\varepsilon^{\mu\nu\alpha\beta}$ are formal objects, where

$$\begin{aligned} \{\gamma^\mu, \gamma^\nu\} &= 2\delta^{\mu\nu} I, & \text{Tr } I &= 4, \\ \delta^{\mu\nu} &= \delta^{\bar{\mu}\bar{\nu}} + \delta^{\hat{\mu}\hat{\nu}}, & \delta^{\bar{\mu}\bar{\mu}} &= 4, & \delta^{\hat{\mu}\hat{\mu}} &= d-4, \\ \delta^{\mu\alpha} \delta^{\alpha\nu} &= \delta^{\bar{\mu}\bar{\alpha}} \delta^{\bar{\alpha}\bar{\nu}} + \delta^{\hat{\mu}\hat{\alpha}} \delta^{\hat{\alpha}\hat{\nu}}, \end{aligned} \quad (35)$$

where d is the space-time dimension. As described in Ref. [65], we have the following relation:

$$\text{Tr} \left(\dots \gamma^\alpha \dots \gamma^\beta \dots \right) = \text{Tr} \left(\dots \gamma^{\bar{\alpha}} \dots \gamma^{\bar{\beta}} \dots \right) + \text{Tr} \left(\dots \gamma^{\hat{\alpha}} \dots \gamma^{\hat{\beta}} \dots \right), \quad (36)$$

where the dots indicate strings of four-dimensional gamma matrices and also γ^5 . The second trace in the r.h.s. of Eq. (36) is computed according to the following rules:

1. move all the $\gamma^{\hat{\mu}}$ matrices to the right using $\gamma^{\hat{\mu}} \gamma^{\bar{\nu}} = -\gamma^{\bar{\nu}} \gamma^{\hat{\mu}}$,
2. for a trace containing an odd number of γ^5 matrices, use $\gamma^{\hat{\mu}} \gamma^5 = \gamma^5 \gamma^{\hat{\mu}}$,
3. for a trace containing an even number of γ^5 matrices, use $\gamma^{\hat{\mu}} \gamma^5 = -\gamma^5 \gamma^{\hat{\mu}}$.

As a consequence, we obtain

$$\text{Tr} \left(\cdots \gamma^{\hat{\alpha}} \gamma^{\hat{\beta}} \cdots \right) = \text{Tr} (\cdots) \text{Tr} \left(\gamma^{\hat{\alpha}} \gamma^{\hat{\beta}} \right) , \quad (37)$$

where the first trace in the r.h.s. only contains four-dimensional quantities, while $\text{Tr} (\gamma^{\hat{\alpha}} \gamma^{\hat{\beta}}) = d - 4$ etc.

Using a γ^5 which does not anticommute with the other Dirac matrices leads to “spurious anomalies” which violate gauge invariance (spoiling renormalizability) and we must impose the relevant Ward–Takahashi (Refs. [68, 69]) and Slavnov–Taylor (Refs. [70–72]) identities (for a review, see Ref. [73]). The problems, of course, are related to the existence of the ABJ anomaly (Refs. [74–76]), which cancels in the SM. The goal of this section is to study the ABJ anomaly in the SMEFT extension of the SM.

5.1. SMEFT and WTST identities

Consider the following amplitudes:

$$\begin{aligned} A_{\mu; \nu_1, \dots, \nu_n}^Z(p; q_1 \dots, q_n; k_1, \dots, k_m) , \\ A_{\nu_1, \dots, \nu_n}^\phi(p; q_1 \dots, q_n; k_1, \dots, k_m) , \end{aligned} \quad (38)$$

involving a Z -boson (or a ϕ^0 Higgs–Kibble ghost [77]) of momentum p , n gauge bosons (A, Z, W) and m Higgs bosons (all momenta are flowing inwards). The corresponding WTST identity is

$$\Gamma_{\nu_1, \dots, \nu_n} = i p^\mu A_{\mu; \nu_1, \dots, \nu_n}^Z + M_Z A_{\nu_1, \dots, \nu_n}^\phi = 0 . \quad (39)$$

We will study 2 different schemes:

- (S1) Z and ϕ^0 off-shell, remaining gauge bosons coupled to physical sources, *i.e.* $\partial_\mu J_\mu = 0$, anti-commuting γ^5 ;
- (S2) Z and ϕ^0 off-shell, remaining gauge bosons coupled to physical sources, *i.e.* $\partial_\mu J_\mu = 0$, the Veltman prescription for γ^5 , four-dimensional, on-shell, external momenta.

It is worth noting that there is a third scheme (which will not be considered here): Z and ϕ^0 off-shell, remaining gauge bosons coupled to arbitrary sources, *i.e.* $\partial_\mu J_\mu \neq 0$, the Veltman prescription for γ^5 , d -dimensional, off-shell external momenta, *i.e.* $p_\mu = p_{\bar{\mu}} + p_{\hat{\mu}}$.

Consider the SMEFT, a theory which is not strictly renormalizable; if there were no anomalies, all the UV divergences could be cancelled in $S^{\text{eff}}(\Lambda)$, order-by-order in $1/\Lambda$. SMEFT loses its predictive power if a process at

$E = \Lambda$ requires an infinite number of renormalized parameters. Due to the anomaly at the one-loop level, the symmetry will be broken and the mechanism of cancelling divergences is disturbed, *i.e.* WTST identities break down. For example, one relevant identity concerns the amplitude for a Z to decay into two photons: if this identity is violated, we can still restore it by introducing a UV-finite counterterm as it would be the case in the SM with only an electron and a neutrino. However, this new term is of non-renormalizable type giving rise to infinities at higher orders; see Ref. [78] for a complete discussion. The “complete” SM is an anomaly-free theory and we want to investigate the SMEFT from the following point of view: are there anomalies in the SMEFT? If they show up, could we find relations among the Wilson coefficients that cancel the anomalies? In the following sections, we will consider the relevant WTST identities for SMEFT amplitudes

$$A = A^{(4)} + g_6 A^{(6)} , \quad g_6 = \frac{1}{\sqrt{2} G_F \Lambda^2} , \quad (40)$$

where G_F is the Fermi coupling constant and the first term in Eq. (40) is the SM contribution. In computing the WTST identity of Eq. (39) (schemes 1 and 2), we assume that all the sources for the n gauge bosons and the m Higgs bosons are physical and on-shell, *i.e.* $q_i \cdot J_i(q_i) = 0$; this means that external sources cannot absorb/emit Fadeev–Popov ghosts. Internal lines in one-loop diagrams represent fermions. Each one-loop amplitude can be decomposed as follows:

$$\begin{aligned} A = & \frac{S}{d-4} + R + \sum_a f_a A_0^{\text{fin}}(a) + \sum_b f_b B_0^{\text{fin}}(b) \\ & + \sum_c f_c C_0(c) + \sum_d f_d D_0(d) , \end{aligned} \quad (41)$$

where A_0, \dots, D_0 are scalar one, \dots four-point functions [5, 6], d is the space-time dimension and “fin” denotes the UV finite part. It is worth noting that anomalies can be cancelled by adding counterterms if and only if the anomaly is of R type, *i.e.* UV finite and local. Locality of the UV-finite counterterms is related to the unitarity of the theory.

5.2. Anomalies and anomalous terms

Anomalies and anomalous terms, although correlated, are not the same thing. Anomalies have to do with WTST identities, perhaps the best example is given by

$$\Gamma_{\alpha\beta} = ip^\lambda A_{\lambda;\alpha,\beta}^Z + M_Z A_{\alpha,\beta}^\phi , \quad (42)$$

the ZAA WTST identity. If this identity is violated, say

$$\Gamma_{\alpha\beta} = X \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu, \quad (43)$$

with X UV-finite and local, we can restore it by adding to the Lagrangian a term $X \varepsilon_{\mu\nu\alpha\beta} \phi^0 (\partial_\mu A_\alpha) (\partial_\nu A_\beta)$. One could think of introducing two counterterms, ZAA and $\phi^0 AA$, in order to restore the identity and to cancel, at the same time, the anomalous ZAA and $\phi^0 AA$ couplings. However, these counterterms are not local. Having loop-induced anomalous couplings is not a surprise, even in the SM. The important thing is the cancellation of anomalies, not the presence of anomalous couplings. The point was explained long ago in Ref. [21]; consider any VVV one loop, SM, vertex, it will contain ε -terms that contribute to the triangle anomaly. The cancellation of the anomaly requires the contribution of the massive top quark. Writing

$$V = \sum_f V_f(m_f = 0) + [V_t(m_t \neq 0) - V_t(m_t = 0)], \quad (44)$$

it follows from the anomaly-cancellation conditions that all ε -terms disappear in the (massless) sum on the right-hand side. The remainder contains ε -terms and is m_t -dependent. This dependence is known to produce effects of delayed unitarity cancellation, which may become relevant at high energies. Everything becomes clear when we consider the explicit expressions in the SM for $Z_\mu(P) \rightarrow A_\alpha(p_1) + A_\beta(p_2)$ and $\phi^0(P) \rightarrow A_\alpha(p_1) + A_\beta(p_2)$:

$$A_{\alpha\beta}^\phi = \frac{g^3}{12\pi^2} \frac{s_\theta^2}{c_\theta} \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \left[3 \sum_l \frac{m_l^2}{M_Z^2} C_0(m_l) + \sum_d \frac{m_d^2}{M_Z^2} C_0(m_d) - 4 \sum_u \frac{m_u^2}{M_Z^2} C_0(m_u) \right], \quad (45)$$

$$A_{\mu;\alpha\beta}^Z = -i \frac{g^3}{12\pi^2} \frac{s_\theta^2}{c_\theta} \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \left[3 \sum_l \frac{m_l^2}{s} C_0(m_l) + \sum_d \frac{m_d^2}{s} C_0(m_d) - 4 \sum_u \frac{m_u^2}{s} C_0(m_u) \right] P_\mu, \quad (46)$$

where $P = -p_1 - p_2$, $P^2 = -s$ and $C_0(m_f)$ is the scalar three-point function with internal f lines. The sum is over leptons, up and down quarks. The WTST identity is obviously satisfied but both amplitudes correspond to anomalous, non-local, couplings. As far as the $\phi^0 AA$ amplitude is concerned, this fact is not so relevant since ϕ^0 is not an asymptotic state. Furthermore, the ZAA amplitude gives zero when the Z boson is coupled to a

conserved current. To summarize, the no-anomaly scheme does not imply the absence of anomalous couplings. Cancellation of anomalous couplings is not the question, cancellation of anomalies is. As we will discuss in the next paragraph, this WTST identity is violated in the SMEFT.

5.3. Step 0: WTST identities for two-point functions

For two-point functions, scheme 1 and scheme 2 give the same result, *i.e.* all WTST identities are satisfied, both in the SM and in the SMEFT [42, 79]. It is worth noting that the identities are violated if we use a scheme where $\{\gamma^{\hat{\mu}}, \gamma^5\} = 0$ and $[\gamma^{\hat{\mu}}, \gamma^5] = 0$, *i.e.* without using the anomalous trace introduced in Ref. [65]. Of course, they can always be restored by introducing local and UV-finite counterterms even in the presence of evanescent terms [80]. These terms are formally zero in the limit $d \rightarrow 4$ but their effect must be carefully analyzed due to the presence of UV poles.

5.4. Step 1: WTST identities for three-point functions

In our notation, g is the SU(2) coupling constant, $c_\theta = M_W/M_Z$ is the cosine of the weak-mixing angle. Furthermore, we introduce the following combinations,

$$\begin{aligned} a_{lW} &= s_\theta a_{lWB} + c_\theta a_{lBW} , & a_{lB} &= -c_\theta a_{lWB} + s_\theta a_{lBW} , \\ a_{dW} &= s_\theta a_{dWB} + c_\theta a_{dBW} , & a_{dB} &= -c_\theta a_{dWB} + s_\theta a_{dBW} , \\ a_{uW} &= s_\theta a_{uWB} + c_\theta a_{uBW} , & a_{uB} &= c_\theta a_{uWB} - s_\theta a_{uBW} , \end{aligned} \quad (47)$$

where a_{lW} , *etc.* are Wilson coefficients in the Warsaw basis [45]. It is worth noting that in SMEFT we have both triangles and bubbles due to four-point vertices like $A\phi^0\bar{f}f$ *etc.* If they are not included, the anomaly contains an UV-divergent term. We introduce

$$C^A = -\partial_\mu A_\mu , \quad C^Z = -\partial_\mu Z_\mu + M_Z \phi^0 , \quad C^\pm = -\partial_\mu W_\mu^\pm + M_W \phi^\pm . \quad (48)$$

In the “diagrammatic” language of Veltman, the validity of the WTST identities is equivalent to the statement that the C are free fields and any Green’s function with one or more external C -sources is zero. Considering the effective action S , we observe that diagrams determine S only up to an arbitrary choice of local counterterms and we are free to redefine S by adding to it S_{ct} with an arbitrary coefficient [39].

5.4.1. WTST identity for $C^Z AA$

In this case, we have $C^Z(P) \rightarrow A_\alpha(p_1) + A_\beta(p_2)$. Summing over fermion generations, we obtain

$$\Gamma_{\alpha\beta} = \frac{g^3}{8\pi^2} \frac{s_\theta}{c_\theta^3} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left(\frac{m_l^2}{M_Z^2} a_{lWB} + \frac{m_d^2}{M_Z^2} a_{dWB} + 2 \frac{m_u^2}{M_Z^2} a_{uWB} \right), \quad (49)$$

for schemes 1 and 2. As expected, there is no anomaly in $\dim = 4$ but there is one in $\dim = 6$ which is mass dependent. The standard treatment is that the anomaly can be removed by adding to the Lagrangian a term proportional to $\varepsilon_{\mu\nu\alpha\beta} \phi^0 \partial^\mu A^\alpha \partial^\nu A^\beta$.

5.4.2. WTST identity for $C^Z ZA$

In this case, we have $C^Z(P) \rightarrow Z_\alpha(p_1) + A_\beta(p_2)$, and obtain

$$\begin{aligned} \Gamma_{\alpha\beta}^{S1} = & \frac{g^3}{32\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[2 \frac{s_\theta}{c_\theta^3} \left(\frac{m_l^2}{M_Z^2} a_{lBW} + \frac{m_d^2}{M_Z^2} a_{dBW} + 2 \frac{m_u^2}{M_Z^2} a_{uBW} \right) \right. \\ & + \frac{1}{c_\theta^4} \left(\frac{m_l^2}{M_Z^2} v_l a_{lWB} + 3 \frac{m_d^2}{M_Z^2} v_d a_{dWB} + 3 \frac{m_u^2}{M_Z^2} v_u a_{uWB} \right) \\ & + \frac{4}{3} \frac{s_\theta}{c_\theta^2} \left(3 a_{\phi q}^{(3)} + a_{\phi q}^{(1)} - 8 a_{\phi u} - 2 a_{\phi d} - 3 a_{\phi l}^{(3)} + 3 a_{\phi l}^{(1)} - 6 a_{\phi l} \right) \\ & \left. - \frac{8}{3} s_\theta \left(3 a_{\phi q}^{(3)} + 5 a_{\phi q}^{(1)} - 4 a_{\phi u} - a_{\phi d} - 3 a_{\phi l}^{(3)} + 3 a_{\phi l}^{(1)} - 3 a_{\phi l} \right) \right], \quad (50) \end{aligned}$$

$$\begin{aligned} \Gamma_{\alpha\beta}^{S2} = & \frac{g^3}{64\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[4 \frac{s_\theta}{c_\theta^3} \left(\frac{m_l^2}{M_Z^2} a_{lBW} + \frac{m_d^2}{M_Z^2} a_{dBW} + 2 \frac{m_u^2}{M_Z^2} a_{uBW} \right) \right. \\ & + \frac{1}{c_\theta^4} \left(\frac{m_l^2}{M_Z^2} v_l a_{lWB} + 3 \frac{m_d^2}{M_Z^2} v_d a_{dWB} + 3 \frac{m_u^2}{M_Z^2} v_u a_{uWB} \right) \\ & \left. - 4 \frac{s_\theta}{c_\theta^2} \left(a_{\phi q}^{(3)} + 3 a_{\phi q}^{(1)} + 2 a_{\phi u} + a_{\phi d} - a_{\phi l}^{(3)} + a_{\phi l}^{(1)} + a_{\phi l} \right) \right], \quad (51) \end{aligned}$$

where $v_f = 1 - 8 Q_f I_f^3 s_\theta^2$. There is an anomaly in $\dim = 6$ which is mass dependent but UV finite and local; however, the anomaly is scheme dependent.

5.4.3. WTST identity for $C^Z ZZ$

In this case, we have $C^Z(P) \rightarrow Z_\alpha(p_1) + Z_\beta(p_2)$, and obtain

$$\begin{aligned} \Gamma_{\alpha\beta}^{S1} = & \frac{g^3}{16\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \\ & \times \sum_{\{g\}} \left[\frac{1}{c_\theta^4} \left(\frac{m_l^2}{M_Z^2} v_l a_{lWB} + 3 \frac{m_d^2}{M_Z^2} v_d a_{dWB} + 3 \frac{m_u^2}{M_Z^2} v_u a_{uWB} \right) \right. \\ & + \frac{8}{3} \frac{1}{c_\theta} \left(3 a_{\phi q}^{(3)} + a_{\phi q}^{(1)} - 8 a_{\phi u} - 2 a_{\phi d} - 3 a_{\phi l}^{(3)} + 3 a_{\phi l}^{(1)} - 6 a_{\phi l} \right) \\ & - \frac{8}{3} c_\theta \left(3 a_{\phi q}^{(3)} + 5 a_{\phi q}^{(1)} - 4 a_{\phi u} - a_{\phi d} - 3 a_{\phi l}^{(3)} + 3 a_{\phi l}^{(1)} - 3 a_{\phi l} \right) \\ & \left. + \frac{4}{3} \frac{1}{c_\theta^3} \left(-a_{\phi q}^{(1)} + 8 a_{\phi u} + 2 a_{\phi d} - 3 a_{\phi l}^{(1)} + 6 a_{\phi l} \right) \right] , \end{aligned} \quad (52)$$

$$\begin{aligned} \Gamma_{\alpha\beta}^{S2} = & \frac{g^3}{32\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \\ & \times \sum_{\{g\}} \left[\frac{1}{c_\theta^4} \left(\frac{m_l^2}{M_Z^2} v_l a_{lBW} + 3 \frac{m_d^2}{M_Z^2} v_d a_{dBW} + 3 \frac{m_u^2}{M_Z^2} v_u a_{uBW} \right) \right. \\ & - \frac{4}{c_\theta} \left(a_{\phi q}^{(3)} + 3 a_{\phi q}^{(1)} + 2 a_{\phi u} + a_{\phi d} - a_{\phi l}^{(3)} + a_{\phi l}^{(1)} + a_{\phi l} \right) \\ & \left. + \frac{2}{3} \frac{1}{c_\theta^3} \left(6 a_{\phi q}^{(3)} + 6 a_{\phi q}^{(1)} + 9 a_{\phi u} + 3 a_{\phi d} - 6 a_{\phi l}^{(3)} + 2 a_{\phi l}^{(1)} + 5 a_{\phi l} \right) \right] , \end{aligned} \quad (53)$$

where, once again, there is an anomaly in $\dim = 6$ which is mass dependent, scheme dependent but UV finite and local.

5.4.4. WTST identity for $C^Z W^+ W^-$

In this case, we have $C^Z(P) \rightarrow W_\alpha^-(p_1) + W_\beta^+(p_2)$, and obtain

$$\begin{aligned} \Gamma_{\alpha\beta}^{S1} = & -\frac{g^3}{16\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[\left(\frac{m_l^2}{M_Z^2} a_{lWB} - \frac{m_d^2}{M_Z^2} a_{dWB} + \frac{m_u^2}{M_Z^2} a_{uWB} \right) \frac{s_\theta^3}{c_\theta^3} \right. \\ & \left. + \left(\frac{m_l^2}{M_Z^2} a_{lBW} - \frac{m_d^2}{M_Z^2} a_{dBW} + \frac{m_u^2}{M_Z^2} a_{uBW} \right) \frac{s_\theta^2}{c_\theta^2} - 4 \left(a_{\phi q}^{(3)} - a_{\phi l}^{(3)} \right) \frac{s_\theta^2}{c_\theta} \right] , \end{aligned} \quad (54)$$

$$\begin{aligned}
\Gamma_{\alpha\beta}^{S2} = & -\frac{g^3}{24\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[\left(\frac{m_l^2}{M_Z^2} a_{lWB} - 3 \frac{m_d^2}{M_Z^2} a_{dWB} \right) \frac{s_\theta}{c_\theta^3} \right. \\
& + \left(\frac{m_l^2}{M_Z^2} a_{lBW} - 3 \frac{m_d^2}{M_Z^2} a_{dBW} \right) \frac{1}{c_\theta^2} + 3 \left(2 \frac{m_d^2}{M_Z^2} a_{lWB} + \frac{m_u^2}{M_Z^2} a_{uWB} \right) \frac{s_\theta}{c_\theta} \\
& \left. + 3 \left(2 \frac{m_d^2}{M_Z^2} a_{lBW} + \frac{m_u^2}{M_Z^2} a_{uBW} \right) - 8 \left(a_{\phi q}^{(3)} - a_{\phi l}^{(3)} \right) \frac{s_\theta^2}{c_\theta} \right]. \quad (55)
\end{aligned}$$

5.4.5. WTST identity for $C^A W^+ W^-$

In this case, we have $C^A(P) \rightarrow W_\alpha^-(p_1) + W_\beta^+(p_2)$, and obtain

$$\begin{aligned}
\Gamma_{\alpha\beta}^{S1} = & \frac{g^3}{16\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[\left(\frac{m_l^2}{M_Z^2} a_{lWB} - \frac{m_d^2}{M_Z^2} a_{dWB} + \frac{m_u^2}{M_Z^2} a_{uWB} \right) \frac{s_\theta^2}{c_\theta^2} \right. \\
& \left. + \left(\frac{m_l^2}{M_Z^2} a_{lBW} - \frac{m_d^2}{M_Z^2} a_{dBW} + \frac{m_u^2}{M_Z^2} a_{uBW} \right) \frac{s_\theta}{c_\theta} - 4 \left(a_{\phi q}^{(3)} - a_{\phi l}^{(3)} \right) s_\theta \right], \quad (56)
\end{aligned}$$

$$\begin{aligned}
\Gamma_{\alpha\beta}^{S2} = & -\frac{g^3}{8\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[\left(2 \frac{m_d^2}{M_Z^2} a_{dWB} + \frac{m_u^2}{M_Z^2} a_{uWB} \right) \frac{s_\theta^2}{c_\theta^2} \right. \\
& \left. + \left(2 \frac{m_d^2}{M_Z^2} a_{dBW} + \frac{m_u^2}{M_Z^2} a_{uBW} \right) \frac{s_\theta}{c_\theta} + \frac{8}{3} \left(a_{\phi q}^{(3)} - a_{\phi l}^{(3)} \right) s_\theta \right]. \quad (57)
\end{aligned}$$

In this case, the anomaly can be removed by adding to the Lagrangian a term proportional to $\varepsilon^{\mu\alpha\beta\nu} A_\nu (W_\beta^+ \partial_\mu W_\alpha^- - W_\alpha^- \partial_\mu W_\beta^+)$.

5.4.6. WTST identity for $C^A ZZ$ and $C^A ZA$

We only present the results for scheme 1. They are

$$\begin{aligned}
\Gamma_{\alpha\beta}^{S1} = & \frac{g^3}{12\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[\frac{3}{2} \left(\frac{m_l^2}{M_Z^2} a_{lBW} + \frac{m_d^2}{M_Z^2} a_{dBW} + 2 \frac{m_u^2}{M_Z^2} a_{uBW} \right) \frac{s_\theta}{c_\theta^3} \right. \\
& + \left(3 a_{\phi q}^{(3)} - 3 a_{\phi l}^{(3)} - 8 a_{\phi u} - 2 a_{\phi d} + a_{\phi q}^{(1)} + 3 a_{\phi l}^{(1)} - 6 a_{\phi l} \right) \frac{s_\theta}{c_\theta^2} \\
& \left. - 2 \left(3 a_{\phi q}^{(3)} - 3 a_{\phi l}^{(3)} - 4 a_{\phi u} - a_{\phi d} + 5 a_{\phi q}^{(1)} + 3 a_{\phi l}^{(1)} - 3 a_{\phi l} \right) s_\theta \right] \quad (58)
\end{aligned}$$

for $C^A ZZ$ and

$$\Gamma_{\alpha\beta}^{S1} = \frac{g^3}{16\pi^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \sum_{\{g\}} \left[\left(\frac{m_t^2}{M_Z^2} a_{lBW} + \frac{m_d^2}{M_Z^2} a_{dBW} + 2 \frac{m_u^2}{M_Z^2} a_{uBW} \right) \frac{s_\theta}{c_\theta^3} \right. \\ \left. - \frac{4}{3} \left(3 a_{\phi q}^{(3)} - 3 a_{\phi l}^{(3)} - 4 a_{\phi u} - a_{\phi d} + 5 a_{\phi q}^{(1)} + 3 a_{\phi l}^{(1)} - 3 a_{\phi l} \right) \frac{s_\theta^2}{c_\theta} \right] \quad (59)$$

for $C^A Z A$.

5.4.7. WTST identity for $C^Z G G$

In this case (where G_μ^a is the gluon field), we have $C^Z(P) \rightarrow G_\alpha^a(p_1) + G_\beta^b(p_2)$, and obtain

$$\Gamma_{\alpha\beta}^{ab} = \frac{g g_S^2}{16\pi^2 c_\theta^2} g_6 \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \delta^{ab} \sum_{\{g\}} \left(\frac{m_d^2}{M_Z^2} a_{dG} + \frac{m_u^2}{M_Z^2} a_{uG} \right), \quad (60)$$

with a sum over the quark generations and where g_S is the SU(3) coupling constant. There is an anomaly in $\dim = 6$ which is mass dependent but UV finite and local.

5.4.8. WTST identity for $C^- W^+ Z$ and $C^- W^+ A$

In these two cases, we have found no anomaly in scheme 1. However, a technical remark is needed: for amplitudes having a Born term, we must take into account the relation between bare and renormalized parameters and also include Dyson-resummation of the propagators (when needed). The identity reads as follows:

$$i p^\lambda A_{\lambda\alpha\beta}^W + M A_{\alpha\beta}^\phi = 0, \quad (61)$$

where M is the bare W mass and we use $M_0 = M/c_\theta$ for the bare Z mass. Inside and in front of loops, we will use the on-shell masses, M_W and M_Z . However, in this case, there is a lowest order where the relation between bare and on-shell masses must be corrected at $\mathcal{O}(g^2)$, involving the corresponding self-energies.

There is an important remark to be made here: consider the SM ($\dim = 4$), where there is no anomaly; consider now the SM with one electron and one neutrino, scheme 1 gives zero anomaly which is in conflict with the master formula for the one-loop anomalies given in Ref. [78]. The anomalies require a counterterm proportional to

$$\varepsilon_{\mu\nu\alpha\beta} (F_{\mu\nu} G_{\alpha\beta}^a \phi^a + F_{\mu\nu} F_{\alpha\beta} \phi^0 \tan \theta), \quad (62)$$

where $F_{\mu\nu}$ is the field strength of the original $U(1)$ vector boson, $G_{\mu\nu}^a$ is the field strength of the original SU(2) vector boson and ϕ^a is the Higgs–Kibble

ghost (with $\phi^3 \equiv \phi^0$). The correct result is reproduced with scheme 2; here the $C^-(p_1)Z_\mu(P)W_\beta^+(p_2)$ identity in SMEFT is given by

$$\begin{aligned}
 \Gamma_{\beta\mu}^{S2} = & \frac{g^3}{192\pi^2 M_W^2} g_6 \varepsilon_{\alpha\nu\beta\mu} p_1^\alpha p_2^\nu \\
 & \times \sum_{\{g\}} \left[(5m_l^2 a_{lWB} + 15m_d^2 a_{dWB} - 3m_u m_d (a_{uWB} + a_{dWB}) + 15m_u^2 a_{uWB}) \frac{s_\theta}{c_\theta} \right. \\
 & + 12(m_l^2 a_{lBW} + 3m_d^2 a_{dBW} + 3m_u^2 a_{uBW}) \\
 & - 2(7m_l^2 a_{lWB} + 5m_d^2 a_{dWB} + m_u m_d (a_{uWB} - a_{dWB}) + 13m_u^2 a_{uWB}) \frac{s_\theta^3}{c_\theta} \\
 & - 2(7m_l^2 a_{lBW} + 5m_d^2 a_{dBW} + m_u m_d (a_{uBW} - a_{dBW}) + 13m_u^2 a_{uBW}) \frac{s_\theta^2}{c_\theta} \\
 & \left. + 32M_W^2 \left(a_{\phi q}^{(3)} - a_{\phi l}^{(3)} \right) \frac{s_\theta^2}{c_\theta} - 32M_W^2 \left(3a_{\phi q}^{(1)} + a_{\phi l}^{(1)} \right) \frac{1}{c_\theta} \right] . \quad (63)
 \end{aligned}$$

5.5. SMEFT and anomalies: conclusions

There are SMEFT anomalies, UV-finite, local, mass-dependent and scheme-dependent. There are different options, for instance use consistently a scheme, *e.g.* the (naive) anti-commuting γ^5 scheme or the Veltman scheme and introduce counterterms. This procedure is the one which we could use working with the SM with one electron and one neutrino: due to the anomaly, the WTST identities break down at the one-loop level. We then introduce counterterms and the identities are restored but the terms are of a non-renormalizable nature and they give rise to infinities, at the earliest at the two-loop level, as described in Ref. [78].

Alternatively, we could cancel the anomalies. For instance, within the anti-commuting γ^5 scheme we obtain the following relations:

$$\begin{aligned}
 a_{fB} &= a_{fW} = 0 \quad \forall f, & a_{uG} &= a_{dG} = 0, \\
 a_{\phi l}^{(3)} &= a_{\phi q}^{(3)}, & a_{\phi l}^{(1)} &= -3a_{\phi q}^{(1)}, & a_{\phi l} &= -\frac{1}{3} \left(4a_{\phi q}^{(1)} + 4a_{\phi u} + a_{\phi d} \right). \quad (64)
 \end{aligned}$$

Similar relations, obtained within the Veltman scheme, are less “transparent”. It is worth noting that in the SMEFT there is a breakdown of the WTST identities, no matter which scheme is used. As long as the anomalies are UV-finite and local (which is the “tested” case up to three-point functions) they can be removed, within a given scheme, by counterterms.

One possible question is whether to include Eq. (64) in fits [64, 81]. We have shown that the cancellation of SMEFT anomalies is scheme dependent (extending to the full SMEFT the work of Ref. [64]) but there is also another point [63]: what happens if we have a UV complete and anomaly-free underlying theory whose low-energy behavior violates the identities? There does not seem to be any obvious solution if we insist on this strategy; note that only a_{fB} and a_{fW} are loop-generated [82] $\dim = 6$ operators that we have inserted in loops and mass-dependent anomalies are due to loop-generated operators. Having said that, the fact that using the relations in Eq. (64) is enough to cancel the C^Z and C^A , anomalies remains an intriguing consideration involving the fermions.

A final comment follows from the work of Ref. [78]: one should investigate the divergences arising in higher orders from these counterterms, something happening if the counterterms are part of a loop (these diagrams must be seen as two-loop diagrams). The result is that there are several logarithmically divergent amplitudes, *e.g.* an anomalous magnetic moment for the fermions.

WTST identities for four-point functions show new interesting aspect to be presented elsewhere.

6. Conclusions

It is straightforward to see that when it comes to confronting theoretical predictions with experimental results, there will always be a reference to a work of Martinus Veltman.

REFERENCES

- [1] M.J.G. Veltman, «Nobel lecture: From weak interactions to gravitation», *Rev. Mod. Phys.* **72**, 341 (2000).
- [2] F.A. Berends, K.J.F. Gaemers, R. Gastmans, « α^3 -contribution to the angular asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ », *Nucl. Phys. B* **63**, 381 (1973).
- [3] F.A. Berends *et al.*, «Multiple bremsstrahlung in gauge theories at high-energies (II). Single bremsstrahlung», *Nucl. Phys. B* **206**, 61 (1982).
- [4] F.A. Berends *et al.*, «Single bremsstrahlung processes in gauge theories», *Phys. Lett. B* **103**, 124 (1981).
- [5] G. 't Hooft, M.J.G. Veltman, «Scalar one-loop integrals», *Nucl. Phys. B* **153**, 365 (1979).
- [6] G. Passarino, M.J.G. Veltman, «One-loop corrections for e^+e^- annihilation into $\mu^+\mu^-$ in the Weinberg model», *Nucl. Phys. B* **160**, 151 (1979).
- [7] M. Consoli, «One-loop corrections to $e^+e^- \rightarrow e^+e^-$ in the Weinberg model», *Nucl. Phys. B* **160**, 208 (1979).

- [8] M. Lemoine, M.J.G. Veltman, «Radiative corrections to $e^+e^- \rightarrow W^+W^-$ in the Weinberg model», *Nucl. Phys. B* **164**, 445 (1980).
- [9] F. Antonelli, M. Consoli, G. Corbo, «One-loop correction to vector boson masses in the Glashow–Weinberg–Salam model of electromagnetic and weak interactions», *Phys. Lett. B* **91**, 90 (1980).
- [10] S. Actis, A. Ferroglia, M. Passera, G. Passarino, «Two-loop renormalization in the Standard Model. Part I: Prolegomena», *Nucl. Phys. B* **777**, 1 (2007), [arXiv:hep-ph/0612122](#).
- [11] S. Actis, G. Passarino, «Two-loop renormalization in the Standard Model. Part II: Renormalization procedures and computational techniques», *Nucl. Phys. B* **777**, 35 (2007), [arXiv:hep-ph/0612123](#).
- [12] S. Actis, G. Passarino, «Two-loop renormalization in the Standard Model. Part III: Renormalization equations and their solutions», *Nucl. Phys. B* **777**, 100 (2007), [arXiv:hep-ph/0612124](#).
- [13] S. Actis, G. Passarino, C. Sturm, S. Uccirati, «NLO electroweak corrections to Higgs boson production at hadron colliders», *Phys. Lett. B* **670**, 12 (2008), [arXiv:0809.1301 \[hep-ph\]](#).
- [14] A. Denner, S. Dittmaier, «Electroweak radiative corrections for collider physics», *Phys. Rep.* **864**, 1 (2020), [arXiv:1912.06823 \[hep-ph\]](#).
- [15] M.J.G. Veltman, «Perturbation theory of massive Yang–Mills fields», *Nucl. Phys. B* **7**, 637 (1968).
- [16] J. Reiff, M.J.G. Veltman, «Massive Yang–Mills fields», *Nucl. Phys. B* **13**, 545 (1969).
- [17] G. 't Hooft, M.J.G. Veltman, «One loop divergencies in the theory of gravitation», *Ann. Inst. H. Poincaré Phys. Theor. A* **20**, 69 (1974).
- [18] H. Georgi, «Effective field theory», *Annu. Rev. Nucl. Part. Sci.* **43**, 209 (1993).
- [19] D. Kreimer, «On overlapping divergences», *Commun. Math. Phys.* **204**, 669 (1999), [arXiv:hep-th/9810022](#).
- [20] M.J.G. Veltman, «Unitarity and causality in a renormalizable field theory with unstable particles», *Physica* **29**, 186 (1963).
- [21] W. Beenakker *et al.*, «The Fermion loop scheme for finite width effects in e^+e^- annihilation into four fermions», *Nucl. Phys. B* **500**, 255 (1997), [arXiv:hep-ph/9612260](#).
- [22] A. Denner, S. Dittmaier, «The complex-mass scheme for perturbative calculations with unstable particles», *Nucl. Phys. B Proc. Suppl.* **160**, 22 (2006), [arXiv:hep-ph/0605312](#).
- [23] S. Actis, G. Passarino, C. Sturm, S. Uccirati, «Two-loop threshold singularities, unstable particles and complex masses», *Phys. Lett. B* **669**, 62 (2008), [arXiv:0809.1302 \[hep-ph\]](#).
- [24] M. Green, M.J.G. Veltman, «Weak and electromagnetic radiative corrections to low-energy processes», *Nucl. Phys. B* **169**, 137 (1980).
- [25] G. Passarino, M.J.G. Veltman, «On the definition of the weak mixing angle», *Phys. Lett. B* **237**, 537 (1990).

- [26] M.J.G. Veltman, «Limit on mass differences in the Weinberg model», *Nucl. Phys. B* **123**, 89 (1977).
- [27] M.J.G. Veltman, «The Screening Theorem and the Higgs System», *Acta Phys. Pol. B* **25**, 1627 (1994).
- [28] D.A. Ross, M.J.G. Veltman, «Neutral currents and the Higgs mechanism», *Nucl. Phys. B* **95**, 135 (1975).
- [29] M.B. Einhorn, D.R.T. Jones, M.J.G. Veltman, «Heavy particles and the ρ parameter in the Standard Model», *Nucl. Phys. B* **191**, 146 (1981).
- [30] J. van der Bij, M.J.G. Veltman, «Two-loop large Higgs mass correction to the ρ -parameter», *Nucl. Phys. B* **231**, 205 (1984).
- [31] D.Y. Bardin, M. Grunewald, G. Passarino, «Precision calculation project report», [arXiv:hep-ph/9902452](https://arxiv.org/abs/hep-ph/9902452).
- [32] F.A. Berends, W.L. van Neerven, G.J.H. Burgers, «Higher order radiative corrections at LEP energies», *Nucl. Phys. B* **297**, 429 (1988); *Erratum ibid.* **304**, 921 (1988).
- [33] J. Blümlein, A. De Freitas, C. Raab, K. Schönwald, «The $O(\alpha^2)$ initial state QED corrections to $e^+e^- \rightarrow \gamma^*/Z_0^*$ », *Nucl. Phys. B* **956**, 115055 (2020), [arXiv:2003.14289](https://arxiv.org/abs/2003.14289) [hep-ph].
- [34] S. Jadach, B.F.L. Ward, Z. Was, «The precision Monte Carlo event generator KK for two fermion final states in e^+e^- collisions», *Comput. Phys. Commun.* **130**, 260 (2000), [arXiv:hep-ph/9912214](https://arxiv.org/abs/hep-ph/9912214).
- [35] A. Arbuzov *et al.*, «The Monte Carlo Program KKMC, for the Lepton or Quark Pair Production at LEP/SLC Energies — Updates of electroweak calculations», *Comput. Phys. Commun.* **260**, 107734 (2021), [arXiv:2007.07964](https://arxiv.org/abs/2007.07964) [hep-ph].
- [36] C. Mariotti, G. Passarino, «Higgs boson couplings: measurements and theoretical interpretation», *Int. J. Mod. Phys. A* **32**, 1730003 (2017), [arXiv:1612.00269](https://arxiv.org/abs/1612.00269) [hep-ph].
- [37] Muon $g - 2$ Collaboration (T. Alababha *et al.*), «Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab», *Phys. Rev. Accel. Beams* **24**, 044002 (2021).
- [38] M. Kuhlmann, «Quantum Field Theory», in: E.N. Zalta (Ed.) «The Stanford Encyclopedia of Philosophy (Winter 2018 Edition)», *Stanford, CA: The Metaphysics Research Lab*, 2004.
- [39] J. Preskill, «Gauge anomalies in an effective field theory», *Ann. Phys.* **210**, 323 (1991).
- [40] K. Costello, «Renormalization and Effective Field Theory», in: «Mathematical Surveys and Monographs. Vol. 170», *American Mathematical Society*, 2011.
- [41] S. Hartmann, «Effective field theories, reductionism and scientific explanation», *Stud. Hist. Philos. Mod. Phys.* **32**, 267 (2001).
- [42] M. Ghezzi, R. Gomez-Ambrosio, G. Passarino, S. Uccirati, «NLO Higgs effective field theory and κ -framework», *J. High Energy Phys.* **1507**, 175 (2015), [arXiv:1505.03706](https://arxiv.org/abs/1505.03706) [hep-ph].

- [43] G. Passarino, M. Trott, «The Standard Model Effective Field Theory and Next to Leading Order», [arXiv:1610.08356 \[hep-ph\]](#).
- [44] I. Brivio, M. Trott, «The Standard Model as an Effective Field Theory», *Phys. Rep.* **793**, 1 (2019), [arXiv:1706.08945 \[hep-ph\]](#).
- [45] B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, «Dimension-six terms in the Standard Model Lagrangian», *J. High Energy Phys.* **1010**, 085 (2010), [arXiv:1008.4884 \[hep-ph\]](#).
- [46] M. Ostrogradski, «Mémoires sur les equations differentielles relatives au probleme des isoperimetres», *Mem. Ac. St. Petersbourg* **VI**, 385 (1850).
- [47] G. Passarino, «XEFT, the challenging path up the hill: $\text{dim} = 6$ and $\text{dim} = 8$ », [arXiv:1901.04177 \[hep-ph\]](#).
- [48] A. David, G. Passarino, «Through precision straits to next Standard Model heights», *Rev. Phys.* **1**, 13 (2016), [arXiv:1510.00414 \[hep-ph\]](#).
- [49] A. David, G. Passarino, «Use and reuse of SMEFT», [arXiv:2009.00127 \[hep-ph\]](#).
- [50] K. Crowther, N. Linnemann, «Renormalizability, fundamentality and a final theory: The role of UV-completion in the search for quantum gravity», *Brit. J. Phil. Sci.* **70**, 377 (2019), [arXiv:1705.06777 \[physics.hist-ph\]](#).
- [51] G. 't Hooft, «An algorithm for the poles at dimension four in the dimensional regularization procedure», *Nucl. Phys. B* **62**, 444 (1973).
- [52] G. Passarino, «Field reparametrization in effective field theories», *Eur. Phys. J. Plus* **132**, 16 (2017), [arXiv:1610.09618 \[hep-ph\]](#).
- [53] G. Buchalla, A. Celis, C. Krause, J.-N. Toelstede, «Master Formula for One-Loop Renormalization of Bosonic SMEFT Operators», [arXiv:1904.07840 \[hep-ph\]](#).
- [54] M.J.G. Veltman, «Quantum Theory of Gravitation», *Conf. Proc. C* **7507281**, 265 (1975).
- [55] S. Dittmaier, «Electric charge renormalization to all orders», *Phys. Rev. D* **103**, 053006 (2021), [arXiv:2101.05154 \[hep-ph\]](#).
- [56] G. Passarino, «Peaks and cusps: anomalous thresholds and LHC physics», [arXiv:1807.00503 \[hep-ph\]](#).
- [57] F. del Aguila, Z. Kunszt, J. Santiago, «One-loop effective lagrangians after matching», *Eur. Phys. J. C* **76**, 244 (2016), [arXiv:1602.00126 \[hep-ph\]](#).
- [58] J.F. Donoghue, M.M. Ivanov, A. Shkerin, «EPFL Lectures on General Relativity as a Quantum Field Theory», [arXiv:1702.00319 \[hep-th\]](#).
- [59] M. Boggia *et al.*, «The HiggsTools handbook: a beginners guide to decoding the Higgs sector», *J. Phys. G* **45**, 065004 (2018) [arXiv:1711.09875 \[hep-ph\]](#).
- [60] D.V. Vassilevich, «Heat kernel expansion: User's manual», *Phys. Rep.* **388**, 279 (2003), [arXiv:hep-th/0306138](#).
- [61] N. Lashkari, H. Liu, S. Rajagopal, «Perturbation Theory for the Logarithm of a Positive Operator», [arXiv:1811.05619 \[hep-th\]](#).

- [62] O. Cata, W. Kilian, N. Kreher, «Gauge anomalies in the Standard-Model Effective Field Theory», [arXiv:2011.09976 \[hep-ph\]](#).
- [63] Q. Bonnefoy *et al.*, «Comments on gauge anomalies at dimension-six in the Standard Model Effective Field Theory», [arXiv:2012.07740 \[hep-ph\]](#).
- [64] F. Feruglio, «A note on gauge anomaly cancellation in effective field theories», *J. High Energy Phys.* **2103**, 128 (2021), [arXiv:2012.13989 \[hep-ph\]](#).
- [65] M.J.G. Veltman, «GAMMATRICA», *Nucl. Phys. B* **319**, 253 (1989).
- [66] G. 't Hooft, M.J.G. Veltman, «Regularization and renormalization of gauge fields», *Nucl. Phys. B* **44**, 189 (1972).
- [67] P. Breitenlohner, D. Maison, «Dimensionally Renormalized Green's Functions for Theories with Massless Particles. 1.», *Commun. Math. Phys.* **52**, 39 (1977).
- [68] J.C. Ward, «An Identity in Quantum Electrodynamics», *Phys. Rev.* **78**, 182 (1950).
- [69] Y. Takahashi, «On the generalized Ward identity», *Nuovo Cim.* **6**, 371 (1957).
- [70] M.J.G. Veltman, «Generalized Ward identities and Yang–Mills fields», *Nucl. Phys. B* **21**, 288 (1970).
- [71] A.A. Slavnov, «Ward identities in gauge theories», *Theor. Math. Phys.* **10**, 99 (1972).
- [72] J.C. Taylor, «Ward identities and charge renormalization of the Yang–Mills field», *Nucl. Phys. B* **33**, 436 (1971).
- [73] F. Jegerlehner, «Facts of life with γ_5 », *Eur. Phys. J. C* **18**, 673 (2001), [arXiv:hep-th/0005255](#).
- [74] S.L. Adler, «Axial vector vertex in spinor electrodynamics», *Phys. Rev.* **177**, 2426 (1969).
- [75] J.S. Bell, R. Jackiw, «A PCAC puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the σ model», *Nuovo Cim. A* **60**, 47 (1969).
- [76] W.A. Bardeen, «Anomalous Ward identities in spinor field theories», *Phys. Rev.* **184**, 1848 (1969).
- [77] T.W.B. Kibble, «Symmetry Breaking in non-Abelian Gauge Theories», *Phys. Rev.* **155**, 1554 (1967).
- [78] T. Sterling, M.J.G. Veltman, «Decoupling in theories with anomalies», *Nucl. Phys. B* **189**, 557 (1981).
- [79] T. Corbett, M. Trott, «One loop verification of SMEFT Ward Identities», [arXiv:2010.08451 \[hep-ph\]](#).
- [80] G. Bonneau, «Preserving canonical Ward identities in dimensional regularization with a non-anticommuting γ_5 », *Nucl. Phys. B* **177**, 523 (1981).
- [81] J. Minn, J. Kim, C.-k. Lee, «Spinor loop anomalies with very general local fermion Lagrangians», *Phys. Rev. D* **35**, 1872 (1987).
- [82] M.B. Einhorn, J. Wudka, «The bases of effective field theories», *Nucl. Phys. B* **876**, 556 (2013), [arXiv:1307.0478 \[hep-ph\]](#).