A HIDDEN, HEAVIER RESONANCE
OF THE HIGGS FIELD

Maurizio Consoli

INFN — Sezione di Catania, 95129 Catania, Italy
maurizio.consoli@ct.infn.it

In Veltman's original view, the Higgs particle mass M_h was naturally large thus marking a second threshold in weak interactions, as with the W mass and the non-renormalizable 4-fermion $V - A$ theory. Besides, with a large M_h, Spontaneous Symmetry Breaking (SSB) would essentially be determined by the pure scalar sector regardless of the other sectors of the theory. Surprisingly, this picture is not completely ruled out. In fact, if SSB in Φ^4 theories is a weak first-order phase transition, as indicated by most recent lattice simulations, in addition to the known resonance with mass $m_h \sim 125$ GeV, there might be a heavier excitation with mass $M_h^2 \sim m_h^2 \ln(\Lambda_s/M_h)$, where Λ_s is the ultraviolet cutoff of the scalar sector. The larger M_h controls vacuum stability and, differently from m_h, would remain finite in units of the weak scale $\langle \Phi \rangle \sim 246.2$ GeV when $\Lambda_s \to \infty$. Lattice simulations of the propagator performed in the 4D Ising limit of the theory are consistent with this two-mass structure and lead to the estimate $M_h \sim 700$ GeV. In spite of its large mass, however, the heavier state would couple to longitudinal vector bosons with the same typical strength of the low-mass state and would thus represent a relatively narrow resonance. In this way, such hypothetical resonance would naturally fit with some excess of 4-lepton events observed by ATLAS around 680 GeV. Analogous data from CMS are needed to confirm or disprove this interpretation. Finally, the effect of a two-mass structure on radiative corrections is discussed in connection with the value of $\alpha_s(M_Z)$ from $e^+e^- \to$ hadrons.

1. Introduction

In principle, there were many possible scenarios for the Higgs particle mass. At the extremes of the mass range, one could consider two basically different options. The (minimal) supersymmetry, where the mass of the lightest Higgs scalar is $m_h \sim M_W$, and Veltman's idea [1] of a large

$M_h \lesssim 1$ TeV. In his view, incorporating the idea of Spontaneous Symmetry Breaking (SSB) [2, 3] with a large M_h, the Standard Model was the natural completion of the non-renormalizable Glashow model [4]. In this sense, he was speaking of a second threshold in weak interactions, just like with the W mass and the non-renormalizable 4-fermion $V - A$ theory. Today, after the observation at the LHC [5, 6] of the narrow scalar resonance with mass $m_h \sim 125$ GeV, Veltman's large M_h seems to be definitely ruled out.

However, this is not necessarily true. So far, only the gauge and Yukawa couplings of the 125 GeV resonance have been tested. The effects of a genuine scalar self-coupling $\lambda = 3m_h^2/\langle \Phi \rangle^2$ are still below the accuracy of the measurements so that some uncertainty about the origin of SSB still persists.

At the beginning, the driving mechanism was just a classical potential with double-well shape. But then, after Coleman and Weinberg [7], the classical potential was replaced by the effective potential $V_{\text{eff}}(\varphi)$ which, in principle, includes the zero-point energy of all particles in the spectrum. Yet, SSB could essentially be determined by the pure scalar sector if the other contributions to the vacuum energy were negligible.

As argued in Refs. [8, 9], through some logical steps, this may become a natural perspective. One should first follow those lattice simulations of Φ^4 in 4D [10–12] indicating that SSB is a (weak) first-order phase transition. While in the presence of gauge bosons SSB is often described as a first-order transition, in pure Φ^4 this requires to replace standard perturbation theory with some alternative scheme. A scheme where massless Φ^4 (*i.e.* classically scale invariant) exhibits SSB so that the phase transition occurs earlier, when the quanta of the symmetric phase have a tiny but still positive mass squared. One then discovers that, in the two easily available alternative schemes (the simple 1-loop and/or Gaussian approximation), $V_{\text{eff}}(\varphi)$ has *two* distinct mass scales [8, 9]: *(i)* a mass m_h^2, defined by its quadratic shape at the minimum, and *(ii)* a mass M_h^2 entering the zero-point energy which determines its depth. Always considered as being the same mass, in these approximations, one finds instead $M_h^2 \sim Lm_h^2 \gg m_h^2$, where $L = \ln(\Lambda_{\text{s}}/M_h)$ and Λ_{s} is the ultraviolet cutoff of the scalar sector. Since vacuum stability depends on the much larger M_h, and not on m_h, SSB could originate within the pure scalar sector regardless of the other parameters of the theory (*e.g.* the vector boson and top-quark mass).

It is obvious that the quadratic shape and depth of the potential are different quantities. For a more formal derivation, one should recall that the derivatives of the effective potential produce (minus) the n-point functions at zero external momentum. Therefore m_h^2, which is $V_{\text{eff}}''(\varphi)$ at the minimum, is directly the 2-point, self-energy function $|\Pi(p = 0)|$. On the other hand, the zero-point energy is (one-half of) the trace of the logarithm of

the inverse propagator $G^{-1}(p) = (p^2 - \Pi(p))$. Therefore, after subtracting constant terms and quadratic divergences, matching the 1-loop zero-point energy ("zpe") gives the relation

$$\text{zpe} \sim -\frac{1}{4} \int\limits_{p_{\min}}^{p_{\max}} \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\Pi^2(p)}{p^4} \sim -\frac{\langle \Pi^2(p) \rangle}{64\pi^2} \ln \frac{p_{\max}^2}{p_{\min}^2} \sim -\frac{M_h^4}{64\pi^2} \ln \frac{\Lambda_s^2}{M_h^2} . \quad (1)$$

This shows that M_h^2 effectively includes the contribution of all momenta and reflects a typical average value $|\langle \Pi(p) \rangle|$ at larger p^2. In this sense, the m_h–M_h two-mass structure resembles the two branches (phonons and rotons) in the energy spectrum of superfluid He-4 which is usually considered the non-relativistic analog of the broken phase.

Note that the non-uniform scaling of the two masses with Λ_s is crucial not to run in contradiction with the "triviality" of Φ^4. This implies a continuum limit with a Gaussian set of Green's functions and, thus, with just a massive free-field propagator. With this constraint, a cutoff theory can either predict $m_h \to M_h$ or a non-uniform scaling when $\Lambda_s \to \infty$. The single-mass limit will then depend on the unit scale, m_h or M_h, chosen for measuring momenta. Namely: (a) m_h is the unit scale so that M_h and the higher branch simply decouple, (b) M_h is the unit scale so that, when $\Lambda_s \to \infty$, the phase space of the lower branch becomes smaller and smaller until ideally shrinking to the zero-measure set $p_\mu = 0$ which is transformed into itself under the Lorentz Group. This means that the lower branch merges into the vacuum state and the only remaining excitation is the higher branch with mass M_h.

The existence of this two-mass structure in the cutoff theory was checked with lattice simulations of the Euclidean propagator [8] in the Ising limit which is a convenient laboratory to exploit the non-perturbative aspects of the theory. It corresponds to a Φ^4 with an infinite bare coupling $\lambda_0 = +\infty$, as if one were sitting precisely at the Landau pole. For a given non-zero, low-energy coupling $\lambda \sim 1/L$, this represents the best possible definition of the local limit with a cutoff. Then, once m_h^2 is directly computed from the $p^2 \to 0$ limit of $G(p)$ and M_h^2 is extracted from its behaviour at higher p^2, the lattice data are consistent with a transition between two different regimes and with the expected increasing logarithmic trend $M_h^2 \sim L m_h^2$.

If, for finite Λ_s, the scalar propagator really interpolates between two vastly different scales, by increasing the energy there should be a transition from a relatively low value, *e.g.* $m_h = 125$ GeV, to a much larger M_h. At the same time, differently from m_h, the larger mass M_h would remain finite in units of the weak scale $\langle \Phi \rangle \sim (G_{\text{Fermi}}\sqrt{2})^{-1/2} \sim 246.2$ GeV in the continuum limit. Therefore, one could write a proportionality relation, say $M_h = K\langle \Phi \rangle$, and extract the constant K from a fit to the same lattice data. By expressing

the proportionality relation in terms of some constant c_2, say

$$M_h^2 = m_h^2 L \, (c_2)^{-1} \tag{2}$$

and replacing the leading-order estimate $\lambda \sim 16\pi^2/(3L)$ in the relation $\lambda = 3m_h^2/\langle\Phi\rangle^2$, one obtains $K \sim (4\pi/3)\,(c_2)^{-1/2}$. Since from our lattice propagator [8] we found $(c_2)^{-1/2} = 0.67 \pm 0.01$ (stat.) ± 0.02 (sys.) this gives the leading-order estimate $M_h = 690 \pm 10$ (stat.) ± 20 (sys.) GeV. Instead, with the next-to-leading m_h–$\langle\Phi\rangle$ relation and the same c_2, we obtained $M_h = 750 \pm 10$ (stat.) ± 20 (sys.) GeV [8]. The two values could then be summarized into a final estimate $M_h \sim 720 \pm 30$ GeV which accounts for this theoretical uncertainty and updates the previous work of Refs. [13, 14].

I emphasize that, by accepting the "triviality" of the theory in 4D, the cutoff-independent combination $3M_h^2/\langle\Phi\rangle^2 = 3K^2$ cannot represent a measure of observable interactions. This $3K^2 = O(10)$ is clearly quite distinct from the other coupling $\lambda = 3m_h^2/\langle\Phi\rangle^2 \sim 1/L$ but should not be viewed as a coupling constant which produces *observable* interactions in the broken-symmetry phase. Since M_h reflects the magnitude of the vacuum energy density, it would be natural to consider $3K^2 \sim \lambda L$ as a *collective* self-interaction of the vacuum condensate which persists in the $\Lambda_s \to \infty$ limit. This original view [15, 16] can intuitively be formulated in terms of a scalar condensate whose increasing density $\sim L$ [17] compensates for the decreasing strength $\lambda \sim 1/L$ of the two-body coupling. On the other hand, $\lambda \sim 1/L$ remains as the appropriate coupling to describe the *individual* interactions of the elementary excitations of the vacuum, *i.e.* the Higgs field and the Goldstone bosons. In this way, consistently with the "triviality" of Φ^4 theory, their interactions will become weaker and weaker for $\Lambda_s \to \infty$. On this basis, the heavier state would couple to longitudinal vector bosons with the same typical strength $\lambda = 3m_h^2/\langle\Phi\rangle^2 \sim 1/L$ of the low-mass state and would thus represent a relatively narrow resonance.

In the following, I will first re-discuss in Sections 2, 3 and 4 the main theoretical and numerical arguments in favour of the m_h–M_h picture. Later on, in Sections 5 and 6, I will concentrate on more phenomenological aspects and argue that the hypothetical, heavier M_h would naturally fit with some excess of 4-lepton events observed by ATLAS around 680 GeV. Finally, in the conclusive Section 7, after a short summary, I will mention a work of van der Bij [18] where a Higgs field propagator with more than one peak was also considered. This brings in contact with the possible effects of a two-mass structure on radiative corrections.

2. SSB and the effective potential

To start with, let us recall the description of SSB as a second-order transition and follow the Particle Data Group (PDG) [19] where the scalar potential is expressed as

$$V_{\mathrm{PDG}}(\varphi) = -\tfrac{1}{2}m_{\mathrm{PDG}}^2\varphi^2 + \tfrac{1}{4}\lambda_{\mathrm{PDG}}\varphi^4\,. \tag{3}$$

By fixing $m_{\mathrm{PDG}} \sim 88.8$ GeV and $\lambda_{\mathrm{PDG}} \sim 0.13$, this has a minimum at $|\varphi| = \langle\Phi\rangle \sim 246$ GeV and quadratic shape $V_{\mathrm{PDG}}''(\langle\Phi\rangle) = (125\text{ GeV})^2$. As a built-in relation, the second derivative of the potential also determines its depth, *i.e.* the vacuum energy $\mathcal{E}_{\mathrm{PDG}}$

$$\mathcal{E}_{\mathrm{PDG}} = -\tfrac{1}{2}m_{\mathrm{PDG}}^2\langle\Phi\rangle^2 + \tfrac{1}{4}\lambda_{\mathrm{PDG}}\langle\Phi\rangle^4 = -\tfrac{1}{8}(125\,\mathrm{GeV}\langle\Phi\rangle)^2 \sim -1.2\times 10^8\,\mathrm{GeV}^4\,. \tag{4}$$

However, as anticipated in Introduction, recent lattice simulations of Φ^4 in 4D [10–12] indicate instead a (weakly) first-order transition. SSB would then emerge as a true instability of the symmetric vacuum at $\varphi = 0$. Its quanta have a tiny and still positive mass squared $V_{\mathrm{eff}}''(\varphi = 0) = m_\Phi^2 > 0$ but, nevertheless, below a critical value of $0 \leq m_\Phi^2 < m_{\mathrm{c}}^2$, their attractive, long-range interaction [17] can destabilize this symmetric vacuum. The lowest energy state of the massless theory at $m_\Phi^2 = 0$ would then correspond to the broken-symmetry phase, as suggested by Coleman and Weinberg [7] in their original 1-loop calculation.

In this interpretation of SSB, the dynamics of the symmetric phase represents the primary Φ^4 sector and its degree of locality Λ_{s} is the ultimate cutoff scale of the theory (as with the hard-sphere core of He-4 atoms in superfluid helium). We are thus lead to identify Λ_{s} as the Landau pole where the bare coupling $\lambda_0 \to +\infty$. This corresponds precisely to the Ising limit and provides the best possible definition of a local Φ^4 theory for any non-zero low-energy coupling $\lambda \sim 1/\ln\Lambda_{\mathrm{s}} \ll 1$ [1]. This latter coupling is instead appropriate for low-energy physics, as in the original Coleman–Weinberg calculation of the effective potential for field values $|\varphi| \ll \Lambda_{\mathrm{s}}$ [2]. The Coleman–Weinberg calculation represents the simplest scheme which is consistent with a weak

---

[1] This is true at 1-loop. Beyond 1-loop, the standard perturbation theory gives contradictory indications (Landau pole in odd orders *versus* spurious ultraviolet fixed points in even orders). Borel re-summation procedures [20–22], yielding a positive, monotonically increasing β-function, support again the idea of the Landau pole.

[2] By assuming a Landau pole, the λ in the effective potential is naturally interpreted as the small coupling at a scale $\mu \sim |\varphi| \ll \Lambda_{\mathrm{s}}$. However, rejecting the Landau pole, from the resulting trend $\lambda \sim 1/\ln\Lambda_{\mathrm{s}}$, this λ could also be interpreted as an infinitesimal "asymptotically free" bare coupling. In the more general context of the ϵ-expansion, the two points of view might reflect the existence of two separate Φ^4 theories living in $D = 4 \pm \epsilon$ space-time dimensions [23, 24].

first-order picture. We will first reproduce below this well-known computation and discuss afterward its general validity. After subtraction of constant terms and quadratic divergences, the effective potential is

$$V_{\text{eff}}(\varphi) = \frac{\lambda}{4!}\varphi^4 + \frac{\lambda^2}{256\pi^2}\varphi^4 \left[\ln\left(\frac{1}{2}\lambda\varphi^2 \Big/ \Lambda_s^2\right) - \frac{1}{2}\right] \tag{5}$$

and its first few derivatives are

$$V'_{\text{eff}}(\varphi) = \frac{\lambda}{6}\varphi^3 + \frac{\lambda^2}{64\pi^2}\varphi^3 \ln\left(\frac{1}{2}\lambda\varphi^2 \Big/ \Lambda_s^2\right) \tag{6}$$

and

$$V''_{\text{eff}}(\varphi) = \frac{\lambda}{2}\varphi^2 + \frac{3\lambda^2}{64\pi^2}\varphi^2 \ln\left(\frac{1}{2}\lambda\varphi^2 \Big/ \Lambda_s^2\right) + \frac{\lambda^2\varphi^2}{32\pi^2}. \tag{7}$$

By introducing the mass squared parameter $M^2(\varphi) \equiv \frac{1}{2}\lambda\varphi^2$, the same potential can be expressed as a classical background + zero-point energy of a particle with mass $M(\varphi)$, i.e.

$$V_{\text{eff}}(\varphi) = \frac{\lambda\varphi^4}{4!} - \frac{M^4(\varphi)}{64\pi^2} \ln\frac{\Lambda_s^2\sqrt{e}}{M^2(\varphi)}. \tag{8}$$

Thus, non-trivial minima of $V_{\text{eff}}(\varphi)$ occur at those points $\varphi = \pm v$, where

$$M_h^2 = \frac{\lambda v^2}{2} = \Lambda_s^2 \exp\left(-\frac{32\pi^2}{3\lambda}\right) \tag{9}$$

with a quadratic shape

$$m_h^2 \equiv V''_{\text{eff}}(\pm v) = \frac{\lambda^2 v^2}{32\pi^2} = \frac{\lambda}{16\pi^2}M_h^2 \sim \frac{M_h^2}{L} \ll M_h^2, \tag{10}$$

where $L \equiv \ln\frac{\Lambda_s}{M_h}$. Notice that the energy density depends on M_h and *not* on m_h, because

$$\mathcal{E} = V_{\text{eff}}(\pm v) = -\frac{M_h^4}{128\pi^2}, \tag{11}$$

therefore, the critical temperature at which symmetry is restored, $k_B T_c \sim M_h$, and the stability of the broken phase depends on the larger M_h and not on the smaller m_h.

Now, one may object to the above straightforward minimization procedure that the 1-loop calculation is just the first term of an infinite series and should be further "improved". As it is well known, in this conventional view, the 1-loop minimum disappears and one would again predict a second-order

transition, just the result that we know to be in contrast with most recent lattice simulations. Therefore, one should look at the calculation in the different perspective of Eq. (8). This has the qualitatively different meaning of a classical background + zero-point energy with a φ-dependent mass and, as such, is consistent by itself without any need of being further improved.

To confirm the validity of this interpretation, one can compare with other approximation schemes, for instance the Gaussian approximation [25, 26] which has a variational nature and explores the Hamiltonian in the class of the Gaussian functional states. It also represents a very natural alternative because, at least in the continuum limit, a Gaussian structure of Green's functions fits with the generally accepted "triviality" of the theory in 4D. This other calculation produces a result in agreement with the one-loop potential [15, 16]. This is not because there are no non-vanishing corrections beyond 1-loop; there is actually an infinite resummation of terms. The point, however, is that those additional terms do not alter the functional form of the result which is the same as in Eq. (8)

$$V_{\text{eff}}^{\text{G}}(\varphi) = \frac{\hat{\lambda}\varphi^4}{4!} - \frac{\Omega^4(\varphi)}{64\pi^2} \ln \frac{\Lambda_{\text{s}}^2 \sqrt{e}}{\Omega^2(\varphi)} \tag{12}$$

with

$$\hat{\lambda} = \frac{\lambda}{1 + \frac{\lambda}{16\pi^2} \ln \frac{\Lambda_{\text{s}}}{\Omega(\varphi)}} \qquad \text{and} \qquad \Omega^2(\varphi) = \frac{\hat{\lambda}\varphi^2}{2} \, . \tag{13}$$

This explains why the one-loop potential can also admit a non-perturbative interpretation. It is the prototype of the Gaussian and of the *infinite number* of "post-Gaussian" calculations [27, 28] where higher-order contributions are effectively reabsorbed into the same basic structure: classical background + zero-point energy with a φ-dependent mass.

3. Eliminating Λ_{s} in the M_h–$\langle \Phi \rangle$ relation

The effective potential of Section 2 provides a different path to renormalization. Since, for any non-zero λ, there is a finite Landau pole, one could, in fact, consider the whole set of pairs $(\Lambda_{\text{s}},\lambda),(\Lambda_{\text{s}}',\lambda'), (\Lambda_{\text{s}}'',\lambda'')\ldots$ with different Landau poles and corresponding low-energy couplings. By considering this whole set of parameters, and imposing some symmetry principle, one can minimize the influence of the cutoff on observable quantities and even consider the $\Lambda_{\text{s}} \to \infty$ limit.

The basic constraint on the equivalent (Λ_s, λ) pairs, consists in requiring the same vacuum energy Eq. (11) or, equivalently, the same mass scale Eq. (9), namely

$$\left(\Lambda_s \frac{\partial}{\partial \Lambda_s} + \Lambda_s \frac{\partial \lambda}{\partial \Lambda_s} \frac{\partial}{\partial \lambda} \right) \mathcal{E}(\lambda, \Lambda_s) = 0 \,. \tag{14}$$

With the definition

$$\Lambda_s \frac{\partial \lambda}{\partial \Lambda_s} \equiv -\beta(\lambda) = -\frac{3\lambda^2}{16\pi^2} + O\left(\lambda^3\right) \,, \tag{15}$$

this gives $|\mathcal{E}| \sim \mathcal{I}_1^4$, where $\mathcal{I}_1$ is the first RG-invariant[3]

$$\mathcal{I}_1 = M_h = \Lambda_s \exp\left(\int^\lambda \frac{\mathrm{d}x}{\beta(x)} \right) \sim \Lambda_s \exp\left(-\frac{16\pi^2}{3\lambda} \right) \,. \tag{16}$$

The above relations derive from the more general invariance of the effective potential in the three-dimensional space $(\varphi, \lambda, \Lambda_s)$

$$\left(\Lambda_s \frac{\partial}{\partial \Lambda_s} + \Lambda_s \frac{\partial \lambda}{\partial \Lambda_s} \frac{\partial}{\partial \lambda} + \Lambda_s \frac{\partial \varphi}{\partial \Lambda_s} \frac{\partial}{\partial \varphi} \right) V_{\text{eff}}(\varphi, \lambda, \Lambda_s) = 0 \,. \tag{17}$$

In fact, at the minima $\varphi = \pm v$, where $(\partial V_{\text{eff}}/\partial \varphi) = 0$, Eq. (14) is a direct consequence of Eq. (17). Another consequence of this analysis is that, besides a first invariant mass scale $\mathcal{I}_1 = M_h$, by introducing an anomalous dimension for the vacuum field

$$\Lambda_s \frac{\partial \varphi}{\partial \Lambda_s} \equiv \gamma(\lambda)\varphi \,, \tag{18}$$

there is a second invariant, namely

$$\mathcal{I}_2(\varphi) = \varphi \exp\left(\int^\lambda \mathrm{d}x \frac{\gamma(x)}{\beta(x)} \right) \tag{19}$$

which introduces a particular normalization of φ. This had to be expected because from Eq. (9) the cutoff-independent combination is $\lambda v^2 \sim M_h^2 = \mathcal{I}_1^2$ and not v^2 itself implying $\gamma = \beta/(2\lambda)$. This particular definition of the

---

[3] Note the minus sign in the definition of the β-function. This is because, in our coupling constant $\lambda \equiv \lambda(\mu, \Lambda_s)$, at $\mu \sim \varphi$, we are differentiating with respect to the cutoff and not with respect to μ. Thus, at fixed μ, λ has to decrease by increasing Λ_s.

average vacuum field[4] is then the natural candidate to represent the weak scale [8]

$$\mathcal{I}_2(v) = \langle \Phi \rangle \sim 246.2 \text{ GeV} \tag{20}$$

so that the minimization of the effective potential can be expressed as a proportionality relation of the two invariants $\mathcal{I}_1$ and $\mathcal{I}_2$ through some constant K, *i.e.*

$$M_h = K \langle \Phi \rangle . \tag{21}$$

On the other hand, the second derivative at the minima, $m_h^2 \equiv (\partial^2 V_{\text{eff}}/\partial \varphi^2)$ at $\varphi = \pm v$, remains as a cutoff-dependent quantity.

With such a guiding principle from the effective potential, one deduces that M_h and $\langle \Phi \rangle$ scale uniformly with Λ_s. The constant K could then be extracted from a lattice simulation of the propagator, by combining the measured M_h/m_h ratio with a theoretical m_h–$\langle \Phi \rangle$ relation. The main ingredients of this analysis will be reported in Section 4.

4. Lattice simulation of the propagator

To show that the existence of two mass scales in the broken phase is not just speculation, let us now compare with lattice simulations of the propagator. These were performed [8] in the 4D Ising limit of the theory which has always been considered a convenient laboratory to exploit the non-perturbative aspects of the theory. As anticipated, it corresponds to a Φ^4 with an infinite bare coupling $\lambda_0 = +\infty$, as if one were sitting precisely at the Landau pole. In this sense, for any finite cutoff Λ_s, it provides the best definition of the local limit for a given non-zero, low-energy coupling $\lambda \sim 1/L$ (where $L = \ln(\Lambda_s/M_h)$).

Let us start from the traditional Euclidean lattice action of Φ^4 theory

$$S = \sum_x \left[\frac{1}{2} \sum_{\mu=1}^{4} (\partial_\mu \Phi_0)^2 + \frac{1}{2} m_0^2 \Phi_0^2(x) + \frac{\lambda_0}{4!} \Phi_0^4(x) \right] , \tag{22}$$

where $\partial_\mu \Phi_0 = \Phi_0(x + \hat{\mu}) - \Phi_0(x)$ and the lattice spacing is taken $a = 1$. Analogously, all masses are given in units of $1/a$ and the cutoff is $\Lambda_s = \pi/a$. In this action, let us perform the following changes of variables [29] $\Phi_0 = \sqrt{2\kappa} \Phi$, $m_0^2 = (1 - 2g)/\kappa - 8$ and $\lambda_0 = 6g/\kappa^2$ so that we obtain

$$S = \sum_x \left[-2\kappa \sum_{\mu=1}^{4} \Phi(x + \hat{\mu})\Phi(x) + \Phi^2(x) + g \left(\Phi^2(x) - 1 \right)^2 \right] . \tag{23}$$

---

[4] This somewhat resembles the definition of the physical gluon condensate in QCD which is $\langle g^2 F_{\mu\nu}^a F^{a\mu\nu} \rangle$ and not just $\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle$.

Then, for $\lambda_0 \to +\infty$, the lattice field can only take the values $\Phi(x) = \pm 1$ and one gets the Ising limit

$$S_{\text{Ising}} = -2\kappa \sum_x \sum_{\mu=1}^{4} \Phi(x + \hat{\mu}) \Phi(x) \tag{24}$$

the broken-symmetry phase corresponding to $\kappa > \kappa_c$, with $\kappa_c = 0.0748474(3)$ [10, 11]. With this lattice action, we computed the lattice field vacuum expectation value

$$v = \langle |\Phi| \rangle, \qquad \Phi \equiv \frac{1}{V_4} \sum_x \Phi(x) \tag{25}$$

and the connected propagator

$$G(x) = \langle \Phi(x)\Phi(0) \rangle - v^2, \tag{26}$$

where $\langle \ldots \rangle$ denotes averaging over the lattice configurations.

By computing the Fourier transformed connected propagator as a function of the lattice momentum $\hat{p}_\mu = 2 \sin p_\mu / 2$, the extraction of m_h^2 is straightforward because its inverse is just the zero-momentum propagator or susceptibility χ

$$2\kappa\chi \equiv 2\kappa G(p=0) = \frac{1}{m_h^2}. \tag{27}$$

Instead, to extract M_h, the propagator data were first fitted to the 2-parameter form

$$G_{\text{fit}}(p) = \frac{Z_{\text{prop}}}{\hat{p}^2 + m_{\text{latt}}^2} \tag{28}$$

in terms of the squared lattice momentum $\hat{p}^2$. The data were then re-scaled by $(\hat{p}^2 + m_{\text{latt}}^2)$ so that deviations from a flat plateau are immediately visible. This defines the standard single-particle mass and the difference from unity of the height of the plateau, which is Z_{prop}, measures the residual coupling to multi-particle states.

The results in Fig. 1 for $\kappa = 0.074$, *i.e.* close to the critical point but still in the symmetric phase, show that, there, a single lattice mass works remarkably well in the whole range of momentum down to $p = 0$. Thus, single-particle states and multi-particle states are very weakly coupled in the whole momentum range. At least with a lattice mass of about 0.2, one is then close enough to the "trivial" continuum limit of Φ^4 in 4D.

According to the standard point of view, in the broken phase, there should be no particular difference. However, it was already observed in Refs. [30] and [29] that, in this case, the mass from the higher-momentum fit cannot describe the data in the $p \to 0$ limit where the deviations from
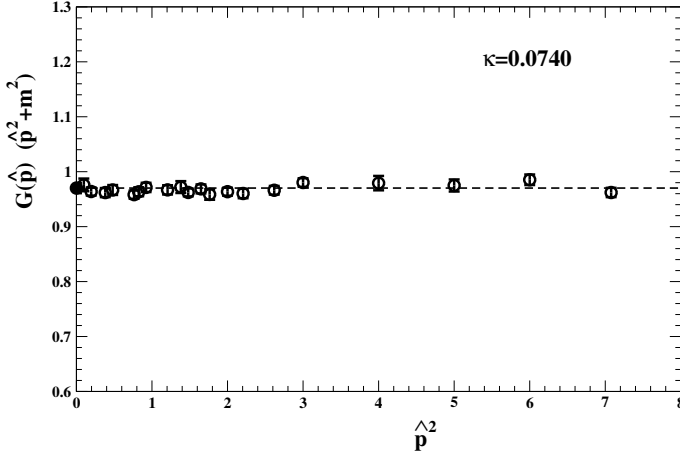
Fig. 1. The lattice data of Ref. [8] for the re-scaled propagator in the symmetric phase at $\kappa = 0.074$ as a function of the square lattice momentum $\hat{p}^2$. The fitted mass from high $\hat{p}^2$, $m_{\text{latt}} = 0.2141(28)$, describes well the data down to $\hat{p} = 0$. The dashed line indicates the value of $Z_{\text{prop}} = 0.9682(23)$ and the $\hat{p} = 0$ point is $2\kappa\chi m_{\text{latt}}^2 = 0.9702(91)$.

constancy become highly significant. As a further check, one more simulation was performed in Ref. [8] on a large 76^4 lattice for $\kappa = 0.0749$, which is even closer to the critical point. As one can see from Fig. 2, the mass parameter $m_{\text{latt}} \equiv M_h$, obtained from the fit to the propagator for $\hat{p}^2 > 0.1$, cannot reproduce the data for $\hat{p}^2 < 0.1$. In this low-momentum range, in fact, the data select a smaller mass, which is very close to the inverse susceptibility $m_h = (2\kappa\chi)^{-1/2} = 0.0769(8)$, see Fig. 3.

The difference between m_h and M_h determines zero-momentum peaks, see Fig. 2, which increase for $\kappa \to \kappa_c$. The observed values, $M_h^2/m_h^2 = 1.24(5)$, $1.31(5)$, $1.47(9)$, respectively for $\kappa = 0.0751$, 0.07504, 0.0749 [8], are consistent with the expected logarithmic trend $M_h^2 = Lm_h^2(c_2)^{-1}$ in Eq. (2), so that, as anticipated in Introduction, one can fit these data and obtain $(c_2)^{-1/2} = 0.67 \pm 0.01$ (stat.) ± 0.02 (sys.). With this value, and the leading-order estimate $\lambda = 16\pi^2/(3L)$ in the relation $\lambda = 3m_h^2/\langle\Phi\rangle^2$, one finds $M_h = 690 \pm 10$ (stat.) ± 20 (sys.) GeV. Instead, with the next-to-leading m_h–$\langle\Phi\rangle$ relation and the same c_2, we obtained $M_h = 750 \pm 10$ (stat.) ± 20 (sys.) GeV [8]. The two values were then summarized into a final estimate $M_h \sim 720 \pm 30$ GeV which accounts for the theoretical uncertainty and updates the previous work of Refs. [13, 14].
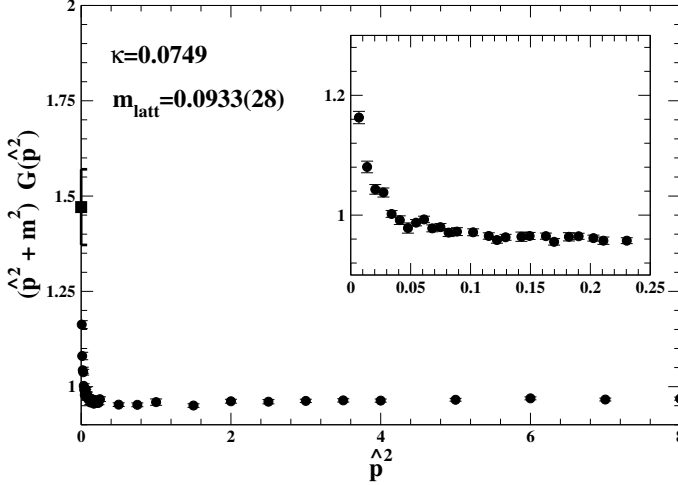
Fig. 2. The propagator data of Ref. [8], for $\kappa = 0.0749$, rescaled with the lattice mass $M_h \equiv m_{\text{latt}} = 0.0933(28)$ obtained from the fit to all data with $\hat{p}^2 > 0.1$. The peak at $p = 0$ is $M_h^2/m_h^2 = 1.47(9)$ as computed from the fitted M_h and $m_h = (2\kappa\chi)^{-1/2} = 0.0769(8)$.
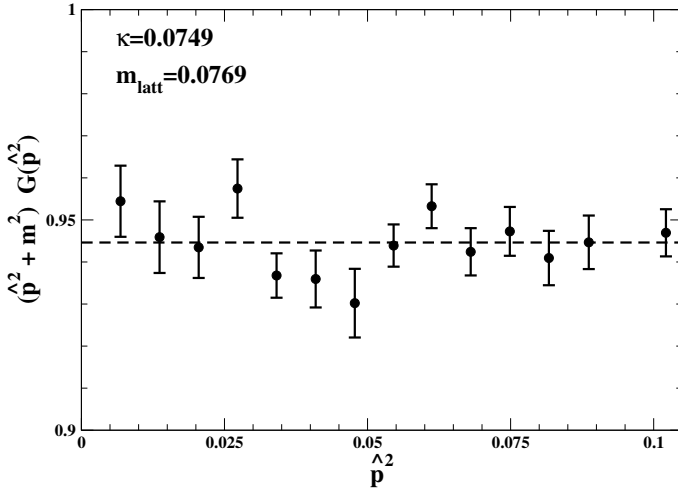

Fig. 3. The propagator data of Ref. [8] at $\kappa = 0.0749$ for $\hat{p}^2 < 0.1$. The lattice mass used here for the rescaling was fixed at the value $m_h = (2\kappa\chi)^{-1/2} = 0.0769(8)$.

5. The role of m_h and M_h in the observable interactions

The lattice simulations in the previous section, supporting the idea of a scalar propagator which interpolates between a mass scale m_h and a larger M_h with $M_h^2 \sim L m_h^2$, are consistent with the picture deduced from

the effective potential. To understand the interplay of the two masses and their role in the observable processes, it is convenient to first follow Ref. [31] where the phenomenology of a heavy but weakly interacting Higgs resonance was first considered.

Differently from here, where m_h and M_h are assumed to coexist, in Ref. [31], there was adopted the ideal single-mass limit (b) of Introduction which (with the exception of the Lorentz-invariant, zero-measure set $p_\mu = 0$) effectively reproduces a standard propagator with mass M_h. However, the problem was the same as considered here: a Λ_s-independent scaling law $M_h = K\langle\Phi\rangle$. This opens a corner of parameter space, namely large K but $M_h \ll \Lambda_s$, that does not exist in the conventional view. However, for this reason, the constant $3K^2$ is now basically different from the coupling λ defined through the β-function

$$\ln\frac{\mu}{\Lambda_s} = \int\limits_{\lambda_0}^{\lambda} \frac{dx}{\beta(x)}\,.\tag{29}$$

For $\beta(x) = 3x^2/(16\pi^2) + O(x^3)$, whatever the contact coupling λ_0 at the asymptotically large Λ_s, at finite scales $\mu \sim M_h$ this gives $\lambda \sim 16\pi^2/(3L)$ with $L = \ln(\Lambda_s/M_h)$. As anticipated, this λ (and not the Λ_s-independent $3K^2$) is the appropriate coupling to describe the infinitesimal interactions of the fluctuations of the broken phase.

Defining $M_W = g_{\text{gauge}}\langle\Phi\rangle/2$ and with the notations of [32], a convenient parametrization [31] of these residual interactions in the scalar potential is ($r = M_h^2/4M_W^2 = K^2/g_{\text{gauge}}^2$)

$$U_{\text{scalar}} = \frac{1}{2}M_h^2 h^2 + \epsilon_1 r g_{\text{gauge}} M_W h \left(\chi^a\chi^a + h^2\right) + \frac{1}{8}\epsilon_2 r g_{\text{gauge}}^2 \left(\chi^a\chi^a + h^2\right)^2\,.\tag{30}$$

The two parameters ϵ_1 and ϵ_2, which are usually set to unity, account for $\lambda \neq 3K^2$, *i.e.*

$$\epsilon_1^2 = \epsilon_2 = \frac{\lambda}{3K^2} \sim 1/L\,.\tag{31}$$

But what about the full gauge theory? At first sight, the original calculation [33] in the unitary gauge could give the impression that $W_L W_L$ scattering is indeed governed by the large coupling $3K^2 = 3M_h^2/\langle\Phi\rangle^2$. To find the answer, let us recall the basics of the calculation. One starts from a tree-level amplitude A_0 which is formally $O(g_{\text{gauge}}^2)$ but ends up with

$$A_0(W_L W_L \to W_L W_L) \sim \frac{3M_h^2 g_{\text{gauge}}^2}{4M_W^2} = 3K^2\,.\tag{32}$$

Here, the factor g^2_{gauge} comes from the vertices. The $1/M^2_W$ derives from the external longitudinal polarizations $\epsilon^{(L)}_\mu \sim (k_\mu/M_W)$ and M^2_h emerges after expanding the propagator

$$\frac{1}{s-M^2_h} \to \frac{1}{s}\left(1 + \frac{M^2_h}{s} + \dots\right). \tag{33}$$

While the leading $1/s$ contribution cancels against a similar term from the other diagrams (which otherwise would give an amplitude growing with s), the M^2_h from the 2nd-order term is effectively "promoted" to coupling constant reproducing the same result of a pure $\lambda\Phi^4$ with contact coupling $\lambda_0 = 3K^2$ at some large scale Λ_s.

However, in Φ^4, this is just the tree approximation with the same coupling at all momentum scale. To find the $\chi\chi \to \chi\chi$ amplitude at some scale μ, we should instead use Eq. (29) and let the coupling evolve, from λ_0 to λ, as previously done in Eq. (30), *i.e.*

$$A(\chi\chi \to \chi\chi)\Big|_{g_{\text{gauge}}=0} \sim \lambda \sim \frac{1}{\ln(\Lambda_s/\mu)}. \tag{34}$$

Thus, recalling that the Equivalence Theorem is valid to all orders in the scalar self-interactions but to the lowest order in g_{gauge} [34–36], we obtain the result anticipated in Introduction

$$A(W_\text{L}W_\text{L} \to W_\text{L}W_\text{L}) = \left[1 + O\left(g^2_{\text{gauge}}\right)\right] A(\chi\chi \to \chi\chi)\Big|_{g_{\text{gauge}}=0} = O(\lambda). \tag{35}$$

This analysis, from our present perspective where m_h and M_h coexist and could be experimentally determined, shows that at $\mu \sim M_h$, the supposed strong interactions proportional to $\lambda_0 = 3K^2$ are actually controlled by the much smaller coupling

$$\lambda = \frac{3m^2_h}{\langle\Phi\rangle^2} = 3K^2 \frac{m^2_h}{M^2_h} \sim 1/L. \tag{36}$$

Analogously, the conventional very large width into longitudinal vector bosons computed with $\lambda_0 = 3K^2$, say $\Gamma^{\text{conv}}(M_h \to W_\text{L}W_\text{L}) \sim M^3_h/\langle\Phi\rangle^2$, should instead be rescaled by $\epsilon^2_1 = (\lambda/3K^2) = m^2_h/M^2_h$. This gives

$$\Gamma(M_h \to W_\text{L}W_\text{L}) \sim \frac{m^2_h}{M^2_h} \Gamma^{\text{conv}}(M_h \to W_\text{L}W_\text{L}) \sim M_h \frac{m^2_h}{\langle\Phi\rangle^2}, \tag{37}$$

where M_h indicates the available phase space in the decay and $m^2_h/\langle\Phi\rangle^2$ the interaction strength. Through the decays of the heavier state, the coupling $\lambda = 3m^2_h/\langle\Phi\rangle^2$ could thus become visible confirming that m_h and M_h represent excitations of the same field.

6. Comparison with 4-lepton data

Suppose to take seriously the idea of a second heavier excitation of the Higgs field with mass $M_h \sim 700$ GeV. Are there experimental indications for such resonance? Furthermore, what kind of phenomenology should we expect? Finally, what about the identification $m_h \sim 125$ GeV, implicitly assumed for our lower mass scale? In the following, I will summarize the results of Ref. [37], where these questions were addressed in connection with a certain excess of 4-lepton events observed by ATLAS [38, 39] for invariant mass $\mu_{4l} \sim 700$ GeV $(l = e, \mu)$.

Of course, the 4-lepton channel is just one possible decay channel and, for a comprehensive analysis, one should also look at the other final states. For instance, at the 2-photon channel that, in the past, has been showing some intriguing signal for the close energy of 750 GeV. However, the 4-lepton channel is experimentally clean and, for this reason, is considered the "golden" channel for a heavy Higgs resonance. Moreover, the bulk of the effect can be analysed at an elementary level. Thus, it makes sense to start from here.

The main new aspect is the strong reduction of the conventional width in Eq. (37). For $M_h = 700$ GeV, where $\Gamma^{\text{conv}}(M_h \to ZZ) \sim 56.7$ GeV [40, 41], fixing $m_h = 125$ GeV gives

$$\Gamma(M_h \to ZZ) \sim \frac{m_h^2}{(700\ \text{GeV})^2}\ 56.7\ \text{GeV} \sim 1.8\ \text{GeV}. \tag{38}$$

Afterward, by maintaining the other contributions for $M_h = 700$ GeV [40, 41]

$$\Gamma(M_h \to \text{fermions} + \text{gluons} + \text{photons} \ldots) \sim 28\ \text{GeV} \tag{39}$$

and with the same ratio $\Gamma(M_h \to WW)/\Gamma(M_h \to ZZ) \sim 2.03$, we find a total width

$$\Gamma(M_h \to \text{all}) \sim 28\ \text{GeV} + 3.03\ \Gamma(M_h \to ZZ) \sim 33.5\ \text{GeV} \tag{40}$$

and a fraction $B(M_h \to ZZ) \sim (1.8\ /\ 33.5) \sim 0.054$.

Now, the production cross section $\sigma(pp \to M_h)$. Here, the main contributions are the basic Gluon–Gluon Fusion (GGF) and Vector–Boson Fusion (VBF) processes, where two gluons or two vector bosons VV (WW or ZZ) fuse to produce the heavy state M_h, *i.e.*

$$\sigma(pp \to M_h) \sim \sigma(pp \to M_h)_{\text{GGF}} + \sigma(pp \to M_h)_{\text{VBF}}. \tag{41}$$

For the GGF term, I will consider two estimates: $\sigma(pp \to M_h)_{\text{GGF}} =$ 800(80) fb from Ref. [40] and $\sigma(pp \to M_h)_{\text{GGF}} = 1078(150)$ fb from Ref. [41]. These refer to $\sqrt{s} = 14$ TeV and should be rescaled by about -12% for $\sqrt{s} = 13$ TeV.

About VBF, I observe that the $VV \to M_h$ process is the inverse of the $M_h \to VV$ decay so that $\sigma(pp \to M_h)_{\text{VBF}}$ can be expressed [42] as a convolution with the parton densities of the same Higgs resonance decay width. Thus, with a large $3K^2$ coupling to longitudinal W's and Z's and conventional width $\Gamma^{\text{conv}}(M_h \to WW + ZZ) \sim 172$ GeV, the VBF mechanism would become sizeable. However, this coupling is not present in our picture, where instead $\Gamma(M_h \to WW + ZZ) \sim 5.5$ GeV. Therefore, the VBF will correspondingly be reduced from its conventional estimate $\sigma^{\text{conv}}(pp \to M_h)_{\text{VBF}} = 250 \div 300$ fb by the small ratio $(5.5/172) \sim 0.032$. This gives $\sigma(pp \to M_h)_{\text{VBF}} \lesssim 10$ fb and can be neglected.

In the end, for a relatively narrow resonance, the effects of its virtuality should be small. Thus, one can approximate the resonance cross section by on-shell branching ratios as

$$\sigma_{\text{R}}(pp \to 4l) \sim \sigma(pp \to M_h) \, B(M_h \to ZZ) \, 4B^2 \left(Z \to l^+ l^-\right) . \qquad (42)$$

Altogether, for $B(M_h \to ZZ) \sim 0.054$ and $4B^2(Z \to l^+ l^-) \sim 0.0045$, the expected peak cross section and numbers of events (for efficiency ~ 0.98) are reported in Table I.

TABLE I

For $M_h = 700$ GeV and $m_h = 125$ GeV, we report our predictions for the peak cross section $\sigma_{\text{R}}(pp \to 4l)$ and the number of events at two values of the luminosity. The two total cross sections are our extrapolation to $\sqrt{s} = 13$ TeV of the values in [40] and [41] for $\sqrt{s} = 14$ TeV. As explained in the text, only the GGF mechanism is relevant in our model.

$\sigma(pp \to M_h)$	$\sigma_{\text{R}}(pp \to 4l)$	n_{4l} (36.1 fb^{-1})	n_{4l} (139 fb^{-1})
700(70) fb	0.17(2) fb	6.0 ± 0.6	23.2 ± 2.3
950(150) fb	0.23(4) fb	8.1 ± 1.3	31.5 ± 5.0

For the smaller statistics of 36.1 fb^{-1}, see Fig. 4a of [38], these predictions can be directly compared, and are well consistent with the measured value $n_{4l} \sim (6 \pm 3)$ for $\mu_{4l} = 700$ GeV. Instead, to compare with the larger ATLAS sample [39] of 139 fb^{-1}, a different treatment is needed. In fact, in their Fig. 2d there are now *three* points in the relevant energy region: at $\mu_{4l} \sim 635$ GeV, where $n_{4l} \sim 7.0 \pm 3.0$, at $\mu_{4l} \sim 665$ GeV, where $n_{4l} \sim 16.5 \pm 4.0$, and at $\mu_{4l} \sim 695$ GeV, where $n_{4l} \sim 9.0^{+2.2}_{-3.0}$. Therefore, by defining $\mu_{4l} = E$ and $s = E^2$, we have assumed that these 4-lepton events derive from the

interference of a resonating amplitude $A^{\mathrm{R}}(s) \sim 1/(s - M_R^2)$ and a slowly varying background $A^{\mathrm{B}}(s)$. For positive interference below the peak, setting $M_R^2 = M_h^2 - iM_h\Gamma_h$, this gives a total cross section

$$\sigma_{\mathrm{T}} = \sigma_{\mathrm{B}} - \frac{2\left(s - M_h^2\right)\,\Gamma_h M_h}{\left(s - M_h^2\right)^2 + \left(\Gamma_h M_h\right)^2}\,\sqrt{\sigma_{\mathrm{B}}\sigma_{\mathrm{R}}} + \frac{\left(\Gamma_h M_h\right)^2}{\left(s - M_h^2\right)^2 + \left(\Gamma_h M_h\right)^2}\sigma_{\mathrm{R}}\,,$$

(43)

where, in principle, both the average background σ_{B} and the resonating σ_{R} could be treated as free parameters. By converting event numbers into cross sections, the best fit is at $M_h = 682$ GeV, see Fig. 4. Since in our model, for small changes of the mass, Γ_h varies linearly with M_h, the width was constrained by the relation $(\Gamma_h/M_h) = (33.5/700)$. ATLAS estimate of the background is also shown as a dashed line.
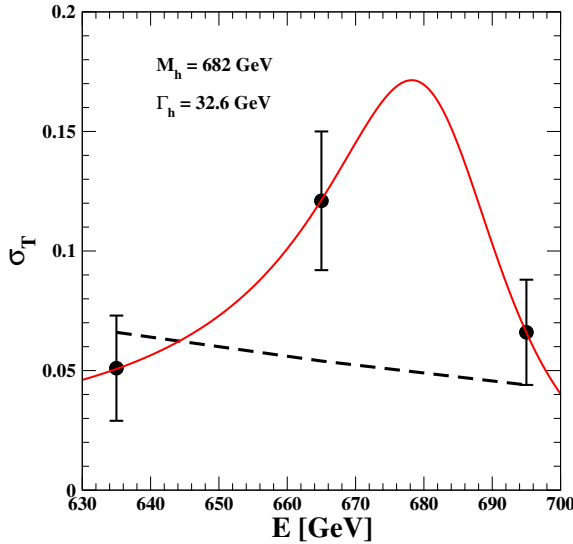

Fig. 4. For $M_h = 682$ GeV and $\Gamma_h = 32.6$ GeV, a fit (in solid red) with Eq. (43) to the ATLAS data converted into cross sections in fb (the black dots). The fitted parameters are $\sigma_{\mathrm{R}} = 0.154 \pm 0.054$ fb and $\sigma_{\mathrm{B}} = 0.009^{+0.017}_{-0.008}$ fb, or total fitted background $N_{\mathrm{B}}^{\mathrm{fit}} = 4^{+7}_{-4}$. Errors from fit give $\chi^2 = 2.41$ with $\chi^2_{\min} = 0$. The dashed line is ATLAS background estimate.

With this theoretical input the fit yields a small average background $\sigma_{\mathrm{B}} = 0.009^{+0.017}_{-0.008}$ fb, equivalent to total background events $N_{\mathrm{B}}^{\mathrm{fit}} = 4^{+7}_{-4}$, and a σ_{R} consistent with Table I. Therefore, if the observed number of events $N_{\mathrm{obs}} \sim 32 \pm 7$ is expressed as $N_{\mathrm{obs}} = N_{\mathrm{R}} + N_{\mathrm{B}}$, the fit would give a minimum number of extra non-background events $N_{\mathrm{R}}^{\mathrm{fit}} \sim 21 \pm 7$, *i.e.* a genuine non-zero signal at the 3-sigma level. To better understand the relation between

the strength of the signal and resonance width, I have performed other fits by first decreasing the width. This can give a background closer to ATLAS estimate $N_{\rm B}({\rm ATLAS}) \sim 22$. In fact, an acceptable fit is still obtained with $\Gamma_h = 14$ GeV, $M_h = 689$ GeV and a larger fitted background $N_{\rm B}^{\rm fit} = 16^{+4}_{-12}$, see Fig. 5. At the opposite side, a good fit can also be obtained for $\Gamma_h = 50$ GeV and $M_h = 677$ GeV, see Fig. 6, but, in this case, the fitted number of background events $N_{\rm B}^{\rm fit} = 1^{+5}_{-1}$ is much, much lower than the ATLAS estimate.
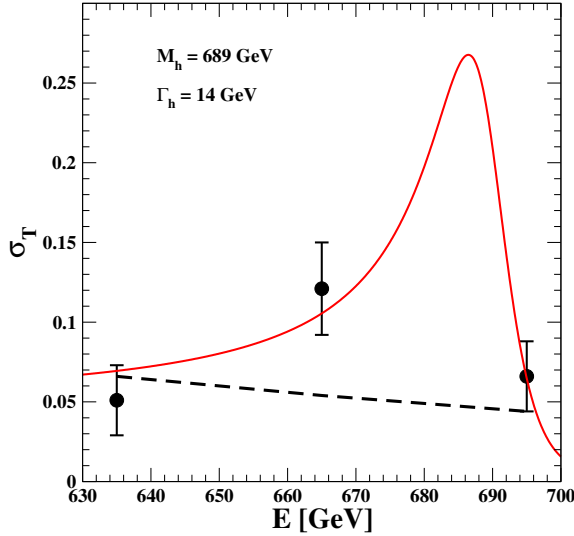

Fig. 5. As in Fig. 4 for $M_h = 689$ GeV and $\Gamma_h = 14$ GeV. Fitted parameters are $\sigma_{\rm R} = 0.19^{+0.16}_{-0.13}$ fb and $\sigma_{\rm B} = 0.04^{+0.01}_{-0.03}$ fb, or total fitted background $N_{\rm B}^{\rm fit} = 16^{+4}_{-12}$. Errors give $\chi^2 = 2.41$ with $\chi^2_{\rm min} = 1$. The dashed line is ATLAS background estimate.

Conclusion: with Eq. (43) and the average background as a free parameter, we find an excellent fit of the ATLAS data, see Fig. 4, with our reference Γ_h/M_h ratio, a $\sigma_{\rm R}$ as in Table I and a small background. Acceptable fits can however be obtained with smaller widths and a background closer to the ATLAS value, see Fig. 5. Altogether, the observed range of the various parameters can be approximated as: $M_h \sim (680 \pm 10)$ GeV, $\Gamma_h \sim (32 \pm 18)$ GeV, $\sigma_{\rm R} \sim (0.15 \pm 0.05)$ fb, $\sigma_{\rm B} \sim 0.01^{+0.03}_{-0.01}$ fb. In particular, M_h stays in the theoretical band $M_h = 690 \pm 10$ (stat.) ± 20 (sys.) GeV obtained when our lattice data [8] are combined with the leading-order m_h–$\langle \Phi \rangle$ relation.

Therefore, for the special role of the 4-lepton channel, further checks of the background and further statistical tests are needed. For instance, with the 36.1 fb^{-1} luminosity, the deep diving of the local p_0 at the 3.6 sigma level

[43] was already suggesting a new narrow resonance at 700 GeV. It remains to be seen if an unambiguous answer could still be obtained with the present LHC configuration[5] or must be postponed to the high-luminosity phase.
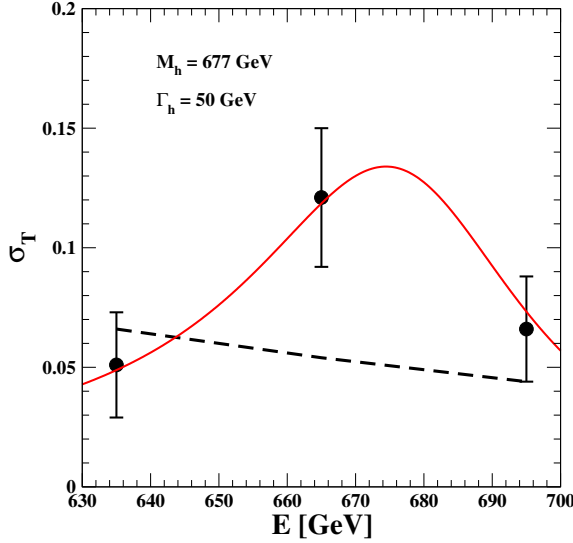

Fig. 6. As in Fig. 4 for $M_h = 677$ GeV and $\Gamma_h = 50$ GeV. Fitted parameters are $\sigma_R = 0.131 \pm 0.036$ fb and $\sigma_B = 0.002^{+0.012}_{-0.002}$ fb, or total fitted background $N_B^{\text{fit}} = 1^{+5}_{-1}$. Errors give $\chi^2 = 2.41$ with $\chi^2_{\min} = 0.1$. The dashed line is ATLAS background estimate.

7. Summary and outlook

After the observation of the narrow scalar resonance at 125 GeV, Veltman's original idea of a naturally large Higgs particle mass seems to be ruled out. Yet, perhaps, the last word has not been said. If SSB in Φ^4 theories is really a weak first-order phase transition, as indicated by most recent lattice simulations, one should consider approximations to the effective potential which are consistent with this scenario. Then, by combining analytic cal-

---

[5] To this end, 4-lepton data from CMS would be crucial. At present, there are no data with the full statistics of 139 fb^{-1} but partial results are given in previous reports. For instance, for 12.9 fb^{-1}, in Fig. 3 (right) of CMS PAS HIG-16-033 of 2016/08/04, at $\mu_{4l} = 660$ GeV, the event number $n_{4l} = 4^{+3}_{-2}$ is much larger than the estimated background of $0.4 \div 0.5$ events. This would give a rough estimate $\sigma_R(pp \to 4l) \sim (0.3 \pm 0.2)$ fb, consistently with our Table I. More extensive data were also reported in CMS PAS HIG-18-001 of 2018/06/03. However, the very compressed scale, see Figs. 2 (left) and 9, prevents to extract the numerical values in the relevant region around 680 GeV. Finally, in Fig. 6 of CMS PAS HIG-19-001 of 2019/03/22, the data stop at $\mu_{4l} = 500$ GeV.

culations and lattice simulations, it becomes conceivable that, besides the known resonance with mass $m_h \sim 125$ GeV, a new excitation with mass $M_h \sim 700$ GeV might show up. The peculiarity, though, is that the heavier state should couple to longitudinal vector bosons with the same typical strength of the low-mass state and would thus represent a relatively narrow resonance. In this case, such hypothetical new resonance would naturally fit with some excess of 4-lepton events observed by ATLAS around 680 GeV. Analogous data from CMS are needed to definitely confirm or disprove this interpretation.

However, before concluding, I will discuss the possible implications that a two-mass structure of the Higgs field may have for radiative corrections. Our lattice simulations in Section 4 indicate two regimes of the inverse propagator $G^{-1}(p)$. This behaves as $(p^2 + m_h^2)$ in the low-p^2 limit, see Fig. 3 for $p^2 < 0.1$, and as $(p^2 + M_h^2)$ at larger p^2, see Fig. 2 for $p^2 > 0.1$. Extrapolating the observed scaling $M_h^2 \sim m_h^2 \ln(\Lambda_s/M_h)$ to very large values of Λ_s gives the idea, in Minkowski space, of two vastly different mass-shell regions, as when the spectral density has not the standard single-peak structure.

By expressing $G^{-1}(p) = p^2 - \Pi(p)$, the observed $m_h - M_h$ difference confirms the point of view of Introduction that, in the broken-symmetry phase, the self-energy function $\Pi(p)$ exhibits a non-trivial momentum dependence. This can be interpreted as the coexistence, in the cutoff theory, of two kinds of quasi-particles associated respectively with the quadratic shape and depth of the effective potential. The analogy with superfluid He-4, where the observed energy spectrum arises by combining the two quasi-particle spectra of phonons and rotons, would then suggest a model propagator

$$G(p) = \frac{1 - I(p)}{2} \frac{Z_1}{p^2 + m_h^2} + \frac{1 + I(p)}{2} \frac{Z_2}{p^2 + M_h^2}, \tag{44}$$

where the interpolating function $I(p)$ depends on an intermediate momentum scale p_0 and tends to $+1$ for large $p^2 \gg p_0^2$ and to -1 when $p^2 \to 0$. For instance, by fixing sharply the central values of Section 4, $Z_1 = 0.945$, $m_h = 0.0769$, $Z_2 = 0.966$, $M_h = 0.0933$, the form $I(p) = \tanh(p^2 - p_0^2)/r$ gives a good fit to the lattice data, $\chi^2 = 29/(40-6)$, for $p_0^2 = 0.087$ and $r = 0.0043$. Moreover, small changes of the Z's and of the masses can induce sizeable changes of p_0^2 and r indicating that the crossover region may be wider than with a simple step function. Moreover, any trial form for $I(p)$ in Eq. (44) introduces a model dependence that could obscure the significance of the results. Thus, while the lattice data indicate that, in Minkowski space, the spectral density has not the standard single-peak structure, in practice, performing the analytic continuation in a non-perturbative regime, and in a numerical simulation where only a discrete set of data is available, is a difficult task [44].

Therefore, to get an intuitive insight on the possible effects on radiative corrections, I will refer to a work of van der Bij [18], where a propagator which resembles Eq. (44) was also considered. To this end, he starts from two observations. First, renormalizability, by itself, does not mean a single-particle peak but only a spectral density which falls off sufficiently fast at infinity. Second, the Higgs field fixes the vacuum state of the theory which determines the masses of all other particles. Therefore, while Goldstone bosons are well understood, the Higgs field itself remains different and it is not unreasonable to expect a spectral density which is not a single δ-function. Here, he does not mention the two-branch spectrum of superfluid He-4 but the idea of the SSB vacuum as some kind of medium seems implicit in this remark. With a definite example [45], he then considers the possibility that the physical Higgs boson is actually a mixture of two states with a spectral density approximated by two δ-function peaks. The resulting propagator structure can be written as $(-1 \leq \eta \leq 1)$

$$G(p) \sim \frac{1 - \eta}{2} \frac{1}{p^2 + m_h^2} + \frac{1 + \eta}{2} \frac{1}{p^2 + M_h^2} \tag{45}$$

and could be used in the analysis of the ρ-parameter [46, 47]. Since the two-loop correction [48] is completely negligible for masses below 1 TeV, one can restrict to the one-loop level, where the two branches Eq. (45) do not mix, as when replacing in the main logarithmic term an effective mass $m_{\text{eff}} \sim \sqrt{m_h M_h} \, (M_h/m_h)^{\eta/2}$. In our case, this would be between $m_h = 125$ GeV and $M_h \sim 700$ GeV so that it becomes important to understand how well the mass parameter obtained indirectly from radiative corrections agrees with the $m_h = 125$ GeV, measured directly at the LHC.

Here, after the (partial) reassessment of the NuTeV anomaly [49, 50], only two measurements give sharp indications. Namely, $A_{\text{FB}}(b)$ which favours a large effective mass and A_{LR} from SLD which goes in the opposite direction. This is well illustrated in the PDG review [19]. In fact, from the experimental set $(A_{\text{LR}}, M_Z, \Gamma_Z, m_t)$, one would predict the pair $[m_{\text{eff}} = 38^{+30}_{-21}$ GeV, $\alpha_{\text{s}}(M_Z) = 0.1182(47)]$. While, from the set $(A_{\text{FB}}(b, c), M_Z, \Gamma_Z, m_t)$, the other pair $[m_{\text{eff}} = 348^{+187}_{-124}$ GeV, $\alpha_{\text{s}}(M_Z) = 0.1278(50)]$.

These two extreme cases show that, at this level of precision, we should try to evaluate the uncertainty induced by strong interactions. This enters indirectly, for instance in the M_W–M_Z interdependence, through the contribution of the hadronic vacuum polarization to $\Delta\alpha(M_Z)$, but also directly through the value of $\alpha_{\text{s}}(M_Z)$. More precisely, in the two examples considered above, this uncertainty enters through $r(M_Z)$, the strong-interaction

correction to the quark–parton model in $\sigma(e+e^- \to \text{hadrons})$ at the center-of-mass energy $Q = M_Z$. Through the total W and Z widths, and the LEP1 peak cross sections, this affects all quantities, even the pure leptonic widths and asymmetries.

Since a small coupling does not guarantee, by itself, a good convergence of the perturbative expansion, one should seriously consider that, even at large center-of-mass energies, the experimental quantity $r^{\text{EXP}}(Q)$ obtained from the data can sizeably differ from its theoretical prediction $r^{\text{TH}}(Q) = 1 + \alpha_s(Q)/\pi + \ldots$ computed from the first few terms. In this case, for a real precision test, instead of treating $\alpha_s(M_Z)$ as a free parameter, one could extrapolate $r^{\text{EXP}}(Q)$ toward $Q = M_Z$ and use this value to extract the EW corrections from experiments.

As pointed out in Ref. [51], in fact, there is some excess in the data so that, to extrapolate correctly from PETRA, PEP and TRISTAN toward the Z peak, one should replace in r^{TH} (34 GeV) a considerably larger $\alpha_s(34 \text{ GeV}) \sim 0.17$ instead of the canonical 0.14 predicted from the deep inelastic scattering. This $\Delta\alpha_s = +0.03$ is a small $+1\%$ effect in the QCD correction but is visible in the slope of the γ–Z interference. On the Z peak, the effect is smaller because we are now speaking of a shift from $\alpha_s(M_Z) = 0.118$ to $\alpha_s(M_Z) = 0.128$ which is just a $+0.3\%$ effect in $r^{\text{TH}}(M_Z)$. Nevertheless, the Higgs mass parameter extracted from the LEP1 data would be considerably increased [52, 53].

Later on, some excess in the total hadronic cross section had also been observed at LEP2 [54–56] so that the whole issue of $\sigma(e+e^- \to \text{hadrons})$ was reconsidered by Schmitt [57] in a thorough analysis of all data in the range of $20 \text{ GeV} \leq Q \leq 209 \text{ GeV}$. His conclusion was that, individually, none of the measurements shows a significant discrepancy. However, when taken together, there is an overall excess at the 4-sigma level. If translated into the QCD correction, this corresponds to replacing the higher range of values $\alpha_s(M_Z) \gtrsim 0.128$ in $r^{\text{TH}}(M_Z)$ and, if used to evaluate the EW corrections, would increase the value of m_{eff} obtained from many experimental quantities. For instance, from the set $(A_{\text{LR}}, A_{\text{FB}}(b,c), M_Z, \Gamma_Z, m_t)$.

In this sense, the present view that the Higgs mass parameter extracted indirectly from radiative corrections agrees perfectly with the $m_h = 125$ GeV measured directly at the LHC, is not free of ambiguities and one could in the end discover other motivations for a new resonance, quite independently of the effective potential and/or of lattice simulations of the propagator. This emphasizes once more the importance of new, combined LHC measurements, starting from the "golden" 4-lepton channel around 700 GeV.

Dedication

This paper is dedicated to the memory of Professor M.J.G. Veltman. Perhaps now, that he is no longer with us, we can better realize how much his moral legacy has transcended the purely scientific.

REFERENCES

[1] M.J.G. Veltman, «Second Threshold in Weak Interactions», *Acta Phys. Pol. B* **8**, 475 (1977).

[2] F. Englert, R. Brout, «Broken Symmetry and the Mass of Gauge Vector Mesons», *Phys. Rev. Lett.* **13**, 321 (1964).

[3] P.W. Higgs, «Broken symmetries, massless particles and gauge fields», *Phys. Lett.* **12**, 132 (1964).

[4] S.L. Glashow, «Partial-symmetries of weak interactions», *Nucl. Phys.* **22**, 579 (1961).

[5] ATLAS Collaboration (G. Aad *et al.*), «Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC», *Phys. Lett. B* **716**, 1 (2012).

[6] CMS Collaboration (S. Chatrchyan *et al.*), «Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC», *Phys. Lett. B* **716**, 30 (2012).

[7] S.R. Coleman, E.J. Weinberg, «Radiative Corrections as the Origin of Spontaneous Symmetry Breaking», *Phys. Rev. D* **7**, 1888 (1973).

[8] M. Consoli, L. Cosmai, «The mass scales of the Higgs field», *Int. J. Mod. Phys. A* **35**, 2050103 (2020).

[9] M. Consoli, L. Cosmai, «Spontaneous Symmetry Breaking and Its Pattern of Scales», *Symmetry* **12**, 2037 (2020).

[10] P.H. Lundow, K. Markström, «Critical behavior of the Ising model on the four-dimensional cubic lattice», *Phys. Rev. E* **80**, 031104 (2009).

[11] P.H. Lundow, K. Markström, «Non-vanishing boundary effects and quasi-first-order phase transitions in high dimensional Ising models», *Nucl. Phys. B* **845**, 120 (2011).

[12] S. Akiyama, Y. Kuramashi, T. Yamashita, Y. Yoshimura, «Phase transition of four-dimensional Ising model with higher-order tensor renormalization group», *Phys. Rev. D* **100**, 054510 (2019).

[13] P. Cea, M. Consoli, L. Cosmai, «Indications on the Higgs boson mass from lattice simulations», *Nucl. Phys. B Proc. Suppl.* **129–130**, 780 (2004).

[14] P. Cea, L. Cosmai, «The Higgs Boson: From the Lattice to LHC», *ISRN High Energy Phys.* **2012**, 1 (2012).

[15] M. Consoli, P.M. Stevenson, «The non-trivial effective potential of the 'trivial' $\lambda\Phi^4$ theory: A lattice test», *Z. Phys. C* **63**, 427 (1994).

[16] M. Consoli, P.M. Stevenson, «Mode-dependent field renormalization and triviality in $\lambda\varphi^4$», *Phys. Lett. B* **391**, 144 (1997).

[17] M. Consoli, P.M. Stevenson, «Physical Mechanisms Generating Spontaneous Symmetry Breaking and a Hierarchy of Scales», *Int. J. Mod. Phys. A* **15**, 133 (2000).

[18] J.J. van der Bij, «Physics After the Discovery of the Higgs Boson», *Acta Phys. Pol. B Proc. Suppl.* **11**, 397 (2018).

[19] Particle Data Group (M. Tanabashi *et al.*), «Review of Particle Physics», *Phys. Rev. D* **98**, 030001 (2018).

[20] D. Shirkov, unpublished.

[21] S. Gorishnii, S. Larin, F. Tkachov, K. Chetyrkin, «Five-loop renormalization group calculations in the $g\phi^4$ theory», *Phys. Lett. B* **132**, 351 (1983).

[22] D. Kazakov, «The method of uniqueness, a new powerful technique for multiloop calculations», *Phys. Lett. B* **133**, 406 (1983).

[23] P.M. Stevenson, «Dimensional continuation and the $(\lambda\varphi^4)_4$ theories», *Z. Phys. C* **35**, 467 (1987).

[24] M. Consoli, P.M. Stevenson, «Landau Poles, Ultraviolet Fixed Points, and "Triviality" in the Perturbative Expansion of $(\lambda\Phi^4)_4$», *Mod. Phys. Lett. A* **11**, 2511 (1996).

[25] T. Barnes, G.I. Ghandour, «Renormalization of trial wave functionals using the effective potential», *Phys. Rev. D* **22**, 924 (1980).

[26] P.M. Stevenson, «Gaussian effective potential. II. $\lambda\varphi^4$ field theory», *Phys. Rev. D* **32**, 1389 (1985).

[27] I. Stancu, P.M. Stevenson, «Second-order corrections to the Gaussian effective potential of $\lambda\varphi^4$ theory», *Phys. Rev. D* **42**, 2710 (1990).

[28] P. Cea, L. Tedesco, «Perturbation theory with a variational basis: The generalized Gaussian effective potential», *Phys. Rev. D* **55**, 4967 (1997).

[29] P.M. Stevenson, «Comparison of perturbative RG theory with lattice data for the 4d Ising model», *Nucl. Phys. B* **729**, 542 (2005).

[30] P. Cea, M. Consoli, L. Cosmai, P.M. Stevenson, «Further Lattice Evidence for a Large Rescaling of the Higgs Condensate», *Mod. Phys. Lett. A* **14**, 1673 (1999).

[31] P. Castorina, M. Consoli, D. Zappalà, «An alternative heavy Higgs mass limit», *J. Phys. G: Nucl. Part. Phys.* **35**, 075010 (2008).

[32] M.J.G. Veltman, F.J. Yndurain, «Radiative corrections to WW scattering», *Nucl. Phys. B* **325**, 1 (1989).

[33] B.W. Lee, C. Quigg, H.B. Thacker, «Weak interactions at very high energies: The role of the Higgs-boson mass», *Phys. Rev. D* **16**, 1519 (1977).

[34] J.M. Cornwall, D.N. Levin, G. Tiktopoulos, «Derivation of gauge invariance from high-energy unitarity bounds on the matrix», *Phys. Rev. D* **10**, 1145 (1974); *Erratum ibid.* **11**, 972 (1975).

[35] M.S. Chanowitz, M.K. Gaillard, «The TeV physics of strongly interacting W's and Z's», *Nucl. Phys. B* **261**, 379 (1985).

[36] J. Bagger, C. Schmidt, «Equivalence theorem redux», *Phys. Rev. D* **41**, 264 (1990).

[37] M. Consoli, L. Cosmai, `arXiv:2007.10837 [hep-ph]`.

[38] ATLAS Collaboration (M. Aaboud *et al.*), «Search for heavy ZZ resonances in the $\ell^+\ell^-\ell^+\ell^-$ and $\ell^+\ell^-\nu\bar{\nu}$ final states using proton–proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector», *Eur. Phys. J. C* **78**, 293 (2018).

[39] ATLAS Collaboration (G. Aad *et al.*), «Search for heavy resonances decaying into a pair of Z bosons in the $\ell^+\ell^-\ell'^+\ell'^-$ and $\ell^+\ell^-\nu\bar{\nu}$ final states using 139 fb^{-1} of proton–proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector», *Eur. Phys. J. C* **81**, 332 (2021), `arXiv:2009.14791 [hep-ex]`.

[40] A. Djouadi, «The anatomy of electroweak symmetry breaking: Tome I: The Higgs boson in the Standard Model», *Phys. Rep.* **457**, 1 (2008).

[41] LHC Higgs Cross Section Working Group (S. Dittmaier *et al.*), «Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables», `arXiv:1101.0593 [hep-ph]`.

[42] G.L. Kane, W. Repko, W. Rolnick, «The effective $W^{\pm}$, Z^0 approximation for high energy collisions», *Phys. Lett. B* **148**, 367 (1984).

[43] D. Denysiuk, Ph.D. thesis, IRFU, Saclay, 2017.

[44] D. Dudal, O. Oliveira, M. Roelfs, P. Silva, «Spectral representation of lattice gluon and ghost propagators at zero temperature», *Nucl. Phys. B* **952**, 114912 (2020).

[45] A. Hill, J.J. van der Bij, «Strongly interacting singlet-doublet Higgs model», *Phys. Rev. D* **36**, 3463 (1987).

[46] D.A. Ross, M.J.G. Veltman, «Neutral currents and the Higgs mechanism», *Nucl. Phys. B* **95**, 135 (1975).

[47] M.J.G. Veltman, «Limit on mass differences in the Weinberg model», *Nucl. Phys. B* **123**, 89 (1977).

[48] J. van der Bij, M.J.G. Veltman, «Two-loop large Higgs mass correction to the ρ-parameter», *Nucl. Phys. B* **231**, 205 (1984).

[49] W. Bentz, I.C. Cloet, J.T. Londergan, A.W. Thomas, «Reassessment of the NuTeV determination of the weak mixing angle», *Phys. Lett. B* **693**, 462 (2010).

[50] P. Coloma *et al.*, «Curtailing the dark side in non-standard neutrino interactions», *J. High Energy Phys.* **1704**, 116 (2017).

[51] V. Branchina, M. Consoli, R. Fiore, D. Zappalà, «Combined fit to $R(e^+e^- \to$ hadrons) and data from the CERN e^+e^- collider LEP», *Phys. Rev. D* **46**, 75 (1992).

[52] M. Consoli, Z. Hioki, «Remarks on the Value of the Higgs Mass from the Present LEP Data», *Mod. Phys. Lett. A* **10**, 845 (1995).

[53] M. Consoli, Z. Hioki, «Indications on the Higgs-boson Mass from the LEP Data», *Mod. Phys. Lett. A* **10**, 2245 (1995).

[54] J. Erler, «Constraining Electroweak Physics», in: S.A. Abel, A.E. Faraggi, J. Santiago-Perez, V. Sanz (Eds.) «String Phenomenology 2003: Proceedings of the 2nd International Conference Durham, UK, 4 July–4 August, 2003», *World Scientific*, Singapore 2003.

[55] G. Wilkinson, «LEP2 $e^+e^- \to f\bar{f}$, $\gamma\gamma$ (γ): results and interpretations», arXiv:hep-ex/0205103.

[56] S. Wynhoff, «Standard Model physics results from LEP-2», in: 5th International Symposium on Radiative Corrections: Applications of Quantum Field Theory to Phenomenology, RADCOR 2000, SLAC-R-579, Stanford, 2001, arXiv:hep-ex/0101016.

[57] M. Schmitt, «Apparent Excess in $e^+e^- \to$ hadrons», arXiv:hep-ex/0401034.