Vol. 52 (2021) *Acta Physica Polonica B* No 8

CONFORMAL TRANSFORMATIONS OF CONSERVATION EQUATIONS IN SPIN HYDRODYNAMICS*

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(Received March 31, 2021; accepted April 27, 2021)

We study and analyse the conformal transformations of different conservation laws in the spin hydrodynamics framework.

DOI:10.5506/APhysPolB.52.1081

1. Introduction

Relativistic fluid dynamics has been proved to be a successful theory for ultra-relativistic heavy-ion collisions [1–4]. Spin polarization measurements of Λ hyperons done recently indicated that we should include space-time evolution of spin in the framework of relativistic fluid dynamics [5–10]. Formulation of such a spin fluid dynamics formalism was studied first in Ref. [11], which gave rise to many studies [12–22]. Contrary to theoretical studies done on spin, considered spin-vorticity coupling to be the reason for the spin polarization at freeze-out [23–67], the spin hydrodynamic formalism has been developed on the basis on conservation laws and local thermodynamic equilibrium, introducing new dynamic quantity namely, spin polarization tensor. In this work, we use the Gubser symmetry [68–70] arguments to find the conformal transformations and criteria for conformal invariance of the conservation laws which are used in the spin hydrodynamics framework [19, 21, 71]. Throughout the article, we use natural units, *i.e.*, $c = \hbar = k_{\rm B} = 1$.

* Presented at XXVII Cracow Epiphany Conference on *Future of Particle Physics*, Cracow, Poland, January 7–10, 2021.

2. Conservation laws

Baryon number conservation law is given by $d_\alpha N^\alpha(x) = 0$, where for perfect-fluid case, net baryon current N^α is defined as

$$N^\alpha = \mathcal{N} U^\alpha = 4 \sinh\left(\frac{\mu}{T}\right) \, \mathcal{N}_{(0)} \, U^\alpha \,, \tag{1}$$

with $\mathcal{N}$ and U^α being the net baryon density and fluid flow vector, respectively. μ, T, and $\mathcal{N}_{(0)}$ are baryon chemical potential, temperature, and number density for ideal relativistic gas of classical massive particles [1], respectively.

For perfect-fluid dynamics case, energy and linear momentum conservation law is expressed as

$$d_\alpha T^{\alpha\beta}(x) \equiv d_\alpha \left[(\mathcal{E} + \mathcal{P}) \, U^\alpha U^\beta + \mathcal{P} g^{\alpha\beta} \right] = 0 \,, \tag{2}$$

with the energy density $\mathcal{E} = 4\cosh(\frac{\mu}{T})\mathcal{E}_{(0)}$ and pressure $\mathcal{P} = 4\cosh(\frac{\mu}{T})\mathcal{P}_{(0)}$, with $\mathcal{E}_{(0)}$ and $\mathcal{P}_{(0)}$ being the energy density and pressure for ideal relativistic gas of classical massive particles [1].

Total angular momentum consists of orbital angular momentum $L^{\alpha,\beta\gamma}$ and spin angular momentum $S^{\alpha,\beta\gamma}$ as

$$J^{\alpha,\beta\gamma} \;=\; L^{\alpha,\beta\gamma} + S^{\alpha,\beta\gamma} = x^\beta T^{\alpha\gamma} - x^\gamma T^{\alpha\beta} + S^{\alpha,\beta\gamma} \,, \tag{3}$$

where the total angular momentum conservation law is $d_\alpha J^{\alpha,\beta\gamma} = d_\alpha S^{\alpha,\beta\gamma} + 2T^{[\beta\gamma]} = 0$. Symmetric $T^{\mu\nu}$ implies the conservation of spin separately, where the spin tensor is given as [19, 72]

$$S^{\alpha,\beta\gamma}_{\rm GLW} = \mathcal{C} U^\alpha \omega^{\beta\gamma} + \mathcal{A} U^\alpha U^{[\beta} k^{\gamma]} + \mathcal{B} \left[U^{[\beta} \Delta^{\alpha\delta} \omega^{\gamma]}{}_{\delta} + U^\alpha \Delta^{\delta[\beta} \omega^{\gamma]}{}_{\delta} + \Delta^{\alpha[\beta} k^{\gamma]} \right] , \tag{4}$$

where $\omega^{\alpha\beta}(x)$ is spin polarization tensor and $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are defined as $\mathcal{A} = 2\mathcal{C} - 3\mathcal{B}$, $\mathcal{B} = -\frac{\mathcal{E}+\mathcal{P}}{2Tz^2}$, and $\mathcal{C} = \frac{\mathcal{P}}{4T}$.

3. Gubser symmetry and conformal weights

Gubser symmetry [68, 69] consists of two special conformal transformations, *i.e.*, $(\mathrm{SO}(3)_q)$, boosts in the direction of η, *i.e.*, $(\mathrm{SO}(1,1))$ and reflection in the r–ϕ plane, *i.e.*, (Z_2). For a system to conserve conformal symmetry, it needs to be invariant under Weyl rescaling [68, 69, 73–75] which implies that tensors of the type of (m,n) transform homogeneously as

$$\mathcal{X}^{\mu_1\ldots\mu_m}_{\nu_1\ldots\nu_n}(x) \;\to\; \Omega^{\Delta_\mathcal{X}} \mathcal{X}^{\mu_1\ldots\mu_m}_{\nu_1\ldots\nu_n}(x) \to \mathrm{e}^{-\varphi(x)\Delta_\mathcal{X}} \mathcal{X}^{\mu_1\ldots\mu_m}_{\nu_1\ldots\nu_n}(x) \,, \tag{5}$$

where $\Delta_{\mathcal{X}} = [\mathcal{X}] + m - n$ is the conformal weight of the quantity $\mathcal{X}$ with $[\mathcal{X}]$ being its mass dimension, and m and n represent the contravariant and covariant indices, respectively. For example, the rank 2 dimensionless metric tensor $g_{\mu\nu}$ transforms under Weyl rescaling as [69, 73]

$$g_{\mu\nu} \to \Omega^{-2}\, g_{\mu\nu}\,. \tag{6}$$

Using Eq. (6) and the normalization of flow vector, one can find $\Delta_{U^\mu} = 1$. Energy density and pressure have mass dimension $[\mathcal{E}] \equiv [\mathcal{P}] = 4$, hence one can obtain their conformal weight to be $\Delta_{\mathcal{E}} = \Delta_{\mathcal{P}} = 4$. Similarly, net baryon density has conformal weight $\Delta_{\mathcal{N}} = 3$, and temperature and baryon chemical potential have the same conformal weight $\Delta_T = \Delta_\mu = 1$, as both have the same mass dimension, which is 1. Conformal weight of energy-momentum tensor can be known using the conformal weight of $\mathcal{E}$, $\mathcal{P}$, and U^μ, which results in $\Delta_{T^{\alpha\beta}} = 6$, since $\Delta_{x^\beta} = 0$. One can see from Eq. (3) that spin tensor should have the same conformal weight as $T^{\mu\nu}$, so canonical spin tensor [76], GLW spin tensor [72] in Eq. (4), and HW spin tensor [54, 58], expressed respectively below, should have the same conformal weights

$$S_{\rm C}^{\alpha,\beta\gamma} = \frac{i}{8}\bar{\psi}\left\{\gamma^\alpha, \left[\gamma^\beta, \gamma^\gamma\right]\right\}\psi\,, \tag{7}$$

$$S_{\rm GLW}^{\alpha,\beta\gamma} = \frac{i}{4m}\left(\bar{\psi}\sigma^{\beta\gamma}\partial^\alpha\psi - \partial_\rho\epsilon^{\beta\gamma\alpha\rho}\bar{\psi}\gamma^5\psi\right), \tag{8}$$

$$S_{\rm HW}^{\alpha,\beta\gamma} = S_{\rm C}^{\alpha,\beta\gamma} - \frac{1}{4m}\left(\bar{\psi}\sigma^{\beta\gamma}\sigma^{\alpha\rho}\partial_\rho\psi + \partial_\rho\bar{\psi}\sigma^{\alpha\rho}\sigma^{\beta\gamma}\psi\right), \tag{9}$$

with $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$, where, spinor ψ and dual spinor $\bar{\psi} \equiv \psi^\dagger\gamma_0$ have conformal weight $\Delta_\psi = \Delta_{\bar{\psi}} = \frac{3}{2}$ and gamma matrix have conformal weight $\Delta_{\gamma^\mu} = 1$ [77, 78], hence $\Delta_{S_{\rm GLW}^{\alpha\beta\gamma}} = \Delta_{S_{\rm C}^{\alpha\beta\gamma}} = \Delta_{S_{\rm HW}^{\alpha\beta\gamma}} = 6$. In a similar way, one can obtain conformal weight of net baryon number current, $\Delta_{N^\alpha} = 4$, using conformal weight of $\mathcal{N}$ and Eq. (1). We summarize below the transformation rules under Weyl rescaling (for 4D spacetime) using Eq. (5)

$$N^\alpha \to \Omega^4\, N^\alpha\,, \qquad T^{\alpha\beta} \to \Omega^6\, T^{\alpha\beta}\,, \qquad S^{\alpha\beta\gamma} \to \Omega^6\, S^{\alpha\beta\gamma}\,. \tag{10}$$

4. Conformal invariance of laws of conservation

For 4D conformal fluid dynamics, we intend to find the conformal transformations of the laws of conservation for net baryon number, energy and linear momentum, and spin, which are given below as

$$d_\alpha N^\alpha(x) = \partial_\alpha N^\alpha + \Gamma^\alpha_{\alpha\beta}N^\beta = 0\,, \tag{11}$$

$$d_\alpha T^{\alpha\beta}(x) = \partial_\alpha T^{\alpha\beta} + \Gamma^\alpha_{\alpha\lambda}T^{\lambda\beta} + \Gamma^\beta_{\alpha\lambda}T^{\alpha\lambda} = 0\,, \tag{12}$$

$$d_\alpha S^{\alpha\beta\gamma}(x) = \partial_\alpha S^{\alpha\beta\gamma} + \Gamma^\alpha_{\alpha\lambda}S^{\lambda\beta\gamma} + \Gamma^\beta_{\alpha\lambda}S^{\alpha\lambda\gamma} + \Gamma^\gamma_{\alpha\lambda}S^{\alpha\beta\lambda} = 0\,, \tag{13}$$

respectively, where $\Gamma^{\beta}_{\alpha\lambda}$ are Christoffel symbols [71], and the conformal transformation of the Christoffel symbols is [71, 75, 79]

$$\Gamma^{\beta}_{\lambda\alpha} = \hat{\Gamma}^{\beta}_{\lambda\alpha} + \delta^{\beta}_{\lambda}\partial_{\alpha}\varphi + \delta^{\beta}_{\alpha}\partial_{\lambda}\varphi - \hat{g}_{\lambda\alpha}\hat{g}^{\beta\sigma}\partial_{\sigma}\varphi\,, \tag{14}$$

with δ^{β}_{λ} denoting the Kronecker delta function and φ being the function of spacetime coordinates. For 4D spacetime, net baryon number (11) conservation equation is conformal-frame independent [68, 69, 73–75], and since net baryon number has conformal weight $\Delta_{N^{\alpha}} = 4$, one obtains

$$d_{\alpha}N^{\alpha} = \Omega^{4}\hat{d}_{\alpha}\hat{N}^{\alpha}\,. \tag{15}$$

Now putting Eq. (10) and Eq. (14) into Eq. (12), we obtain the transformation of conservation of energy and linear momentum [74, 75]

$$d_{\alpha}T^{\alpha\beta} = \Omega^{6}\left[\hat{d}_{\alpha}\hat{T}^{\alpha\beta} - \hat{T}^{\lambda}{}_{\lambda}\hat{g}^{\beta\delta}\partial_{\delta}\varphi\right]. \tag{16}$$

One observes that $\hat{T}^{\alpha\beta}$ should have trace 0 to be conserved in de Sitter spacetime [80–82], and using Eq. (10) and Eq. (14) in Eq. (13), spin conservation law transforms as

$$d_{\alpha}S^{\alpha\beta\gamma} = \Omega^{6}\left[\hat{d}_{\alpha}\hat{S}^{\alpha\beta\gamma}\left(\hat{S}_{\lambda}{}^{\lambda\gamma}\hat{g}^{\beta\sigma} + \hat{S}^{\alpha\beta}{}_{\alpha}\hat{g}^{\sigma\gamma}\right)\partial_{\sigma}\varphi\right]. \tag{17}$$

We find from the above equation that the spin tensor must satisfy the condition $\hat{S}_{\alpha}{}^{\alpha\beta} = 0$ in order to have conformal invariance of spin conservation law. It is easy to see that this condition is not satisfied by GLW (8) and HW (9) definitions, and hence, conformal invariance of Eq. (17) breaks.

5. Summary

We have analysed the properties of the laws of conservation for net baryon number, energy-linear momentum, and spin with respect to the conformal transformations. We found the condition required for the conformal invariance of the spin conservation law, which the GLW and HW spin tensors explicitly break.

I am grateful to Radosław Ryblewski and Gabriel Sophys for their fruitful collaboration and thank W. Florkowski, D. Sénéchal and M. Shokri for inspiring discussions. Supported in part by the National Science Center, Poland (NCN) grants No. 2016/23/B/ST2/00717 and No. 2018/30/E/ST2/00432.

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