# MODIFICATIONS ON THE PROPERTIES OF $D^*_{s0}(2317)$ AS FOUR-QUARK STATE IN THERMAL MEDIUM*

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The low mass of $D^*_{s0}(2317)$ presents problems for the conventional quark model, leading to consideration of other options regarding a multi-quark system. Here, we investigate the scalar open-charm state $D^*_{s0}(2317)$ and its bottom partner by the Thermal QCD Sum Rules (TQCDSR) method using the two-point correlation function with contributions of the nonperturbative condensates up to dimension six. Our calculations indicate that the variations in mass and decay constant values are stable through temperatures up to $T \cong 100$ MeV and fall after this point. At the critical temperature, the values of mass and decay constant change up to $94\%, 71\%$ of their values in vacuum in the molecular scenario, and $94\%, 75\%$ in the diquark–antidiquark scenario. Besides, we find more dramatic changes in mass and decay constant above $T_{\rm c}$, and hence we interpret it as the hadrons starting to disappear. Also, the detailed search for hot medium effects on the hadronic parameters of open-charm meson $D^*_{s0}(2317)$ and the bottom partner may have implications for the QCD phase diagram derived from heavy-ion collision experiments. Finally, these results may help distinguish conventional quark model mesons from exotic states.

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## 1. Introduction

As first seen in $D^*_{s0}(2317) \to D_s\pi^0$ by BaBar (2003) [1], $D_{s1}(2460) \to D^*_s\pi^0$ by CLEO (2003) [2], and confirmed by Belle (2004)[3], a clear experimental proof for the inner structure of $D^*_{s0}(2317)$ is still unavailable [4].

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Experiments have observed a narrow mass below the $DK$ threshold for the $D^*_{s0}(2317)$ state shown in Fig. 1, while a number of models such as the Quark Model [5] and Lattice Theory find it above, close to the $DK$ threshold [6, 7].

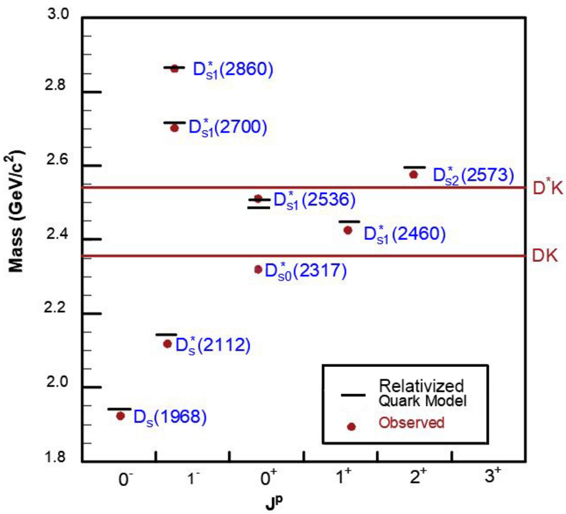


Fig. 1. Open-charm meson at the $DK$ threshold [5, 8].

It also has a very small width and only the upper limit has been measured with the following experimental mass value:

$$M_{D^*_{s0}(2317)} = (2317.7 \pm 0.6)\ \text{MeV}\,, \qquad \Gamma_{D^*_{s0}(2317)} < 3.8\ \text{MeV}\,.$$

However, this width value is in disagreement with the Heavy Quark Symmetry (HQS) estimation expecting to create a broad 1/2 doublet with $J^P = 0^+, 1^+$ [9].

The decay modes of $D^*_{s0}(2317)$ were reported, respectively, by the CLEO and BaBar collaborations [2, 10] at the 90% confidence level

$$\frac{\Gamma(D^*_{s0}(2317) \to D^*_s(2112)\gamma)}{\Gamma(D^*_{s0}(2317) \to D_s\pi^0)} \begin{cases} < 0.052 & \text{CLEO [2]}\,, \\ < 0.14 & \text{BaBar [10]}\,, \end{cases} \tag{1}$$

and the branching ratio as follows:

$$\frac{\mathcal{B}(D^*_{s0}(2317) \to D^*_s(2112)\gamma)}{\mathcal{B}(D^*_{s0}(2317) \to D_s\pi^0)} \begin{cases} < 0.059 & \text{CLEO [2]}\,, \\ < 0.18 & \text{Belle [11]}\,, \\ < 0.16 & \text{BaBar [12]}\,. \end{cases} \tag{2}$$

On the other hand, for the first time in 2018, the absolute branching fraction $\mathcal{B}(D^*_{s0}(2317) \to D^\pm_s\pi^0)$ was measured as $1.00^{+0.00}_{-0.14}(\text{stat.})^{+0.00}_{-0.14}(\text{syst.})$ with a statistical significance of $5.8\sigma$ in the BES III detector at a center-of-mass energy of $\sqrt{s} = 4.6$ GeV [13]. It predicts that $D^*_{s0}(2317)$ should have a branching fraction of $\gamma D_s^{*-}$ at around 15% or even larger, but agrees well with the calculation in the molecular picture [14] which shows that the branching fraction of $\pi^0 D_s^-$ is between (93–100)%.

Much work has also investigated heavy-light, narrow open-charm systems both in conventional and nonconventional frameworks by using the Chiral Unitarity Model (CUM)[15, 16], Light-cone QCD Sum Rules (LCQCDSR) [17], Effective Lagrangian Approach (ELA)[14, 18], QCD Sum Rules (QCDSR) [19–23], Lattice QCD (LQCD)[24, 25], and Nonrelativistic Constituent Quark Model (NRCQM)[26]. Nevertheless, many of the mesons in the open-charm sector are not well described by the Quark Model. This situation has opened a discussion of their inner structures. The uncertainty with conventional $c\bar{s}$ interpretation motivates many authors to hypothesize that the $D^*_{s0}(2317)$ state might be a molecule [15, 27] or a diquark–antidiquark state [28, 29]. Further, the existence of a $DK$ pole at this energy has been recently confirmed with Lattice calculations of scattering amplitudes. This state may be responsible for the bump near the $DK$ threshold around 2.4 GeV [30].

As the mass of $D^*_{s0}(2317)$ is about 40 MeV below the threshold of $DK$, it is most likely thought to be a $DK$ hadronic molecule [27], and such a selection naturally manifests the anomalous mass of $D^*_{s0}(2317)$. In Ref. [31], a model is proposed to calculate the two-body nonleptonic decays $B \to D^{(*)} D_s(2317)(D_s(2460))$, presuming that $D_s(2317)$ and $D_s(2460)$ are $DK$ and $D^*K$ molecules. The coupled channel prediction showed that the mass of $D^*_{s0}(2317)$ may result from the strong coupling of the P-wave charmed-strange mesons to the $DK$ [32]. The decay behaviors of the $D^*_{s0}(2317)$ was explored in the $DK$ hadronic molecular picture [14]. In Ref. [33], the $D^*_{s0}(2317)$ was considered as a kaonic molecule. Studies using the Bethe–Salpeter approach [34] and the potential model pointed out that the $D^*_{s0}(2317)$ could be a $DK$ hadronic molecule [35].

Another possible explanation for the open-charm system $D^*_{s0}(2317)$ is that it may be a diquark–antidiquark state. The QCDSR calculations also supported the idea that the $D^*_{s0}(2317)$ could be a diquark–antidiquark state [19, 24]. In Ref. [19], using four different possible currents and including condensates up to dimension twelve, it is found that the $D^*_{s0}(2317)$ state might be a $0^+$ diquark–antidiquark state defined by scalar–scalar or axial–axial currents in the QCDSR approach. Also, Bracco *et al.* [36] obtained an extremely narrow width ($\Gamma(D_0^{(1s)} \to D_s\pi) = 8$ KeV) for $D_s^+(2317)$ in full QCD.

To clarify this contradictory situation, we investigate the $D^*_{s0}(2317)$ state in the hot medium. Variations in the mass and decay constants of any hadron with temperature indicate that the QCD vacuum changes drastically. The general opinion in the literature is that at the extreme conditions, hadrons cannot survive as bound states [37], instead they dissociate to a new Quark–Gluon Plasma (QGP) state [38], and in this case, chiral symmetry is partially restored. The features of the deconfined state of matter, as well as the phase boundaries from hadronic-to-quark–gluon degrees of freedom, are still discussed near the so-called critical temperature $T_{\rm c} \cong 155$ MeV [39–41]. Lattice QCD calculations indicate that the chiral symmetry restoration occurs at about the same critical temperature and energy density as the deconfinement phase transition at the vanishing baryon chemical potential [40–43].

However, there is no well-defined separation of phases through the crossover nature of transition, thermal properties of hadrons change expeditiously in the vicinity of $T_{\rm c}$. As the temperature rises, the quark condensate values are predicted to lessen from a nonvanishing value in vacuum to $\langle q\bar{q}\rangle \approx 0$ which coincides with the chiral symmetry restoration. Therefore, it is vital to find experimental observables sensitive to the quark condensates since $\langle q\bar{q}\rangle$ is not a measurable quantity. In this context, to get clear signals from QGP, many heavy-ion experiments are continued at the Nuclotron-based Ion Collider facility (NICA), Future Facility for Antiproton and Ion Research (FAIR) as well as RHIC at Large Hadron Collider (LHC).

Briefly, the heavy-ion collision experiments at extreme conditions help understand the characteristics of hot hadronic matter, properties of QCD vacuum, chiral phase transitions, and deconfinement. Though the hadrons produced in these experiments have a very short lifetime, they interact with other particles in the fireball-medium where the baryon density is expected to reach a very high value and temperature comes close to the critical one. To explain the data precisely in these experiments, the medium modifications of hadron parameters must be known. It is the main motivation for studying the thermal behaviors of hadrons.

In this paper, by adopting the QCDSR approach to finite temperature we evaluate the mass and decay constant of $D_{s0}^*(2317)$ treating it as a four-quark content taking into account quark, gluon, and quark–gluon mixed condensates up to dimension six, and assuming that the quark–hadron duality is valid as well. Note that the vacuum condensate expressions are displaced with their thermal versions. This analysis can give us some hints about the nature of the $D_{s0}^*(2317)$ and may provide knowledge about the systematics of strong interactions in the hot medium.

This article is arranged as follows. In Section 2, we obtain the TQCDSR to evaluate the mass and decay constant of $D_{s0}^*(2317)$. Section 3 is devoted to a numerical analysis where we first present input parameters used in the computation and give numerical results for the mass and decay constant of the considered state. These results are confronted with the data obtained from other theoretical models and available experimental data in the vacuum limit. Finally, the appendix consists of the explicit expressions of the obtained thermal spectral densities $\rho^{\rm QCD}(s,T)$ in the TQCDSR theory.

## 2. Finite temperature sum rules for the $D^*_{s0}(2317)$ state

The QCDSR technique can be used to obtain the mass and decay constant of the $D^*_{s0}(2317)$ state. In QCDSR, the correlation function's dependence on momentum allows us to extract the physical properties of any hadron by calculating the correlator in two different momentum regions. The first is called the physical side and is described by hadronic observables such as the mass, residue, and coupling constant of relevant hadrons. The second is the QCD side calculated in a deep space-like region ($q^2 \ll \Lambda^2_{\rm QCD}$) in terms of QCD parameters such as the mass of quarks, and quark–gluon condensates. After getting correlation functions by converging these two regions onto each other, we equalize the two expressions under the assumption of quark–hadron duality which effectively sweeps the contributions of higher states under the carpet.

To explore deviations of mass and decay constants of $D^*_{s0}(2317)$ dependent on the increase of temperature, we extend the QCDSR technique to TQCDSR. The calculation is initiated by writing down the correlation function as in the QCDSR model [44–47]

$$\Pi(q,T) = i\int {\rm d}^4x\ {\rm e}^{iq\cdot x}\langle\omega|\mathcal{T}\left\{J(x)J^\dagger(0)\right\}|\omega\rangle\,, \qquad (3)$$

where $\mathcal{T}$ represents the time ordering operator, $\omega$ denotes the hot medium, $T$ is the temperature, $J(x)$ is the interpolating current accompanying the $D^*_{s0}(2317)$ resonance, and $q$ is the transferred momentum. In the thermal equilibrium, the thermal average of any operator $\mathcal{A}$ can be written as follows:

$$\langle\mathcal{A}\rangle = {\rm Tr}\,\left({\rm e}^{-\beta\mathcal{H}}\mathcal{A}\right)\Big/{\rm Tr}\,\left({\rm e}^{-\beta\mathcal{H}}\right)\,, \qquad (4)$$

where $\mathcal{H}$ is the QCD Hamiltonian as it is well known from statistical mechanics and the thermodynamic $\beta$ is a numerical quantity related to the thermodynamic temperature of a system: $\beta = 1/T$.

### *2.1. Physical side*

At low momentum ($q^2 \leq 0$ for $q^2$ space-like) or large distance, in Eq. (3), the interpolating current $J$ and its conjugate $J^\dagger$ are interpreted as the annihilation and creation operators of the hadron. The correlation function is saturated with a complete set of hadrons having the same quark content and quantum numbers. This interpretation of the correlator is called the *physical side*.

To extract the TQCDSR expressions, we initially compute the correlation function in connection with the physical degrees of freedom. By inserting a complete set of hadronic states between interpolating currents which have the same quantum numbers with the related particle and integrating over-x the following equality is obtained as (for brevity, we will use $\mathcal{D}$ to represent the $D^*_{s0}(2317)$ state in formulas):

$$\Pi^{\mathrm{Phys}}(q,T) = \frac{\langle\omega|J|\mathcal{D}(q)\rangle\langle\mathcal{D}(q)|J^\dagger|\omega\rangle}{m^2_{\mathcal{D}}(T)-q^2} + \dots\,, \tag{5}$$

where $m_{\mathcal{D}}(T)$ is the temperature-dependent ground-state mass of the scalar $D^*_{s0}(2317)$ particle and dots symbolize contributions of the higher states and continuum which are parametrized through the continuum threshold parameter $s_0$. The definition of the temperature-dependent decay constant $f_{\mathcal{D}}(T)$ with the matrix element is written as

$$\langle\omega|J|\mathcal{D}(q)\rangle = \sqrt{2} f_{\mathcal{D}}(T) m^4_{\mathcal{D}}(T)\,. \tag{6}$$

Thus, the correlation function for the physical side can be written for the ground-state mass and decay constant in the form of

$$\Pi^{\mathrm{Phys}}\left(q^2,T\right) = \frac{2m^8_{\mathcal{D}}(T) f^2_{\mathcal{D}}(T)}{m^2_{\mathcal{D}}(T)-q^2}\,. \tag{7}$$

After isolating the ground-state contributions from the pole terms by taking the derivative, *i.e.* applying the Borel transformation, the physical side is found as

$$\widehat{\mathcal{B}}\left(q^2\right)\Pi^{\mathrm{Phys}}\left(q^2,T\right) = 2m^8_{\mathcal{D}}(T) f^2_{\mathcal{D}}(T)\ \mathrm{e}^{-m^2_{\mathcal{D}}(T)/M^2}\,, \tag{8}$$

where $M$ is a sum rule parameter known as Borel mass providing elimination of the contributions emerging from excited resonances and continuum states.

### *2.2. QCD side*

For the high momentum ($q^2 \geq 0$) or short distance, the correlation function is evaluated with the help of Wilson's Operator Product Expansion (OPE) due to the complex structure of the QCD vacuum. Employing OPE, the contributions of quark, gluon, and mixed condensates can be included in the calculation of the correlator. This second way of getting the correlation function is called *QCD side*.

In this section, our purpose is to define the QCD side in which the correlation function is determined in terms of quark and gluon degrees of freedom. In the first step of the calculation, by selecting the appropriate currents, heavy- and light-quark fields are contracted and after some lengthy calculations, the correlation function is obtained.

The scalar molecular current defining the $D^*_{s0}(2317)$ state can be addressed in the following form [48]:

$$J(x)=\left(i\bar{d}^a\gamma_5 c^a\right)\left(i\bar{s}^b\gamma_5 d^b\right), \tag{9}$$

where $a$ and $b$ are color indices.

Also, the $D^*_{s0}(2317)$ state can be regarded as diquark–antidiquark and the current $J(x)$ can be defined by the following expression [29]:

$$J(x)=\frac{\varepsilon^{ijk}\varepsilon^{mnk}}{\sqrt{2}}\left[\left(u_i^T C\gamma_5 c_j\right)\left(\bar{u}_m\gamma_5 C\bar{s}_n^T\right)+(u\to d)\right]. \tag{10}$$

Here, $i,j,k,m,n$ are color indexes, $\varepsilon$ is the antisymmetric Levi-Civita tensor, and $C$ is the charge conjugation matrix.

We can formulate the correlation function $\Pi^{\mathrm{QCD}}(q^2,T)$ as dispersion integral

$$\Pi^{\mathrm{QCD}}\left(q^2,T\right)=\int\limits_{\mathcal{M}^2}^{\infty}\frac{\rho^{\mathrm{QCD}}(s,T)}{s-q^2}\mathrm{d}s+\widetilde{\Gamma}\left(q^2,T\right), \tag{11}$$

where $\mathcal{M}^2=(m_c+m_s+2m_q)^2$ with $q=u$ or $d$ quark, $\rho^{\mathrm{QCD}}(s,T)$ is the spectral density, and $\widetilde{\Gamma}(q^2,T)$ symbolize the contributions coming directly from the calculation of the correlation function. The spectral densities are calculated as an imaginary part of the correlation function with the relation $\rho^{\mathrm{QCD}}(s,T)=\frac{1}{\pi}\mathrm{Im}[\Pi^{\mathrm{QCD}}]$.

Inserting the interpolating currents Eq. (9) and Eq. (10) for the molecular and diquark–antidiquark states into the correlation function (Eq. (3)) and using the Wick theorem, we obtain the QCD side of the correlation function in terms of the heavy- and light-quark propagators in the molecular and diquark–antidiquark scenarios, respectively,

$$\begin{aligned}\Pi^{\mathrm{QCD}}\left(q^2,T\right) &= i\int \mathrm{d}^4x\ \mathrm{e}^{iq\cdot x}\Big[\mathrm{Tr}\left(\gamma_5 S_d^{a'a}(-x)\gamma_5 S_c^{aa'}(x)\right)\\ &\quad\times\mathrm{Tr}\left(\gamma_5 S_s^{b'b}(-x)\gamma_5 S_d^{bb'}(x)\right)\Big], \end{aligned}\tag{12}$$

$$\begin{aligned}\Pi^{\mathrm{QCD}}\left(q^2,T\right) &= \frac{i}{2}\varepsilon^{ijk}\varepsilon^{imn}\varepsilon^{i'j'k'}\varepsilon^{i'm'n'}\int \mathrm{d}^4x\ \mathrm{e}^{iq\cdot x}\Big[\mathrm{Tr}\left(\gamma_5\widetilde{S}_u^{jj'}(x)\gamma_5 S_c^{kk'}(x)\right)\\ &\quad\times\mathrm{Tr}\left(\gamma_5\widetilde{S}_s^{nn'}(-x)\gamma_5 S_u^{m'm}(-x)\right)+(u\to d)\Big], \end{aligned}\tag{13}$$

and we employ the short-hand notation $\widetilde{S}^{jj'}(x)=CS^{jj'T}(x)C$ in Eq. (13). Meanwhile, at finite temperature, due to a failure of the Lorentz invariance with the preferred reference frame and unveiling of the residual $\mathcal{O}(3)$ symmetry, the additional operators emerge in the short-distance expansion of the product of two quark bilinear operators and, consequently, the thermal heavy- and light-quark propagators contain new terms compared with the vacuum quark propagators [49]. Also, we replaced the vacuum condensates with their thermal averages. The general definition of the thermal heavy quark propagator for the charm quark $S_c^{ij}(x)$ can be written as [45]

$$S_c^{ij}(x) = i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-ik\cdot x}\left[\frac{\delta_{ij}\left(\not{k}+m_c\right)}{k^2-m_c^2} - \frac{gG_{ij}^{\alpha\beta}}{4}\right.$$
$$\left.\times\frac{\sigma_{\alpha\beta}\left(\not{k}+m_c\right)+\left(\not{k}+m_c\right)\sigma_{\alpha\beta}}{(k^2-m_c^2)^2} + \frac{g^2}{12}G^A_{\alpha\beta}G^{\alpha\beta}_A\delta_{ij}m_c\frac{k^2+m_c\not{k}}{(k^2-m_c^2)^4}+\ldots\right]. \quad (14)$$

Here, $G_{ij}^{\alpha\beta}$ is the short-hand notation of the external gluon field which is defined as

$$G_{ij}^{\alpha\beta} \equiv G_A^{\alpha\beta}\lambda_{ij}^A/2\,,$$

where $\lambda_A^{ij}$ are the standard Gell-Mann matrices with the number of gluon flavours $A = 1, 2\ldots 8$ being $i$ and $j$ color indices. In Eq. (14), the first term gives the perturbative contribution to the considered parameter, while the others, *i.e.* nonperturbative terms contain gluonic additives. In the nonperturbative terms, the gluon field strength tensor $G^A_{\alpha\beta}G^{\alpha\beta}_A(0)$ is fixed at $x=0$ and the thermal light-quark propagator $S_q^{ij}(x)$ is expressed as [50]

$$\begin{aligned} S_q^{ij}(x) \;=\; & i\frac{\not{x}}{2\pi^2x^4}\delta_{ij} - \frac{m_q}{4\pi^2x^2}\delta_{ij} - \frac{\langle\bar{q}q\rangle_T}{12}\delta_{ij} - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle_T\left[1-i\frac{m_q}{6}\not{x}\right]\delta_{ij} \\ & +\frac{i}{3}\left[\not{x}\left(\frac{m_q}{16}\langle\bar{q}q\rangle_T - \frac{1}{12}\left\langle u^\mu\Theta^f_{\mu\nu}u^\nu\right\rangle\right)+\frac{1}{3}(u\cdot x)\not{u}\left\langle u^\mu\Theta^f_{\mu\nu}u^\nu\right\rangle\right]\delta_{ij} \\ & -\frac{ig_sG_{ij}^{\mu\nu}}{32\pi^2x^2}\left(\not{x}\sigma_{\mu\nu}+\sigma_{\mu\nu}\not{x}\right)\,, \end{aligned} \quad (15)$$

where $m_q$ indicates the light-quark mass, $\langle\bar{q}q\rangle_T$ is the light-quark condensate as a function of temperature, $m_0^2 \equiv \langle 0|\bar{q}g\sigma Gq|0\rangle/\langle 0|\bar{q}q|0\rangle$ is extracted from sum rules of the nucleon channel [51] as the ratio of dimension-5 mixed quark–gluon condensate to dimension-3 chiral condensate, $u_\mu$ is the four-velocity of matter in the hot medium which is $u_\mu = (1,0,0,0)$ in the matter rest frame, and $\Theta^f_{\mu\nu}$ is the fermionic part of the energy momentum tensor. Additionally, the following gluon condensate expression depending on the gluonic part of the energy-momentum tensor $\Theta^g_{\lambda\sigma}$ is used [49]:

$$\langle \mathrm{Tr}^c G_{\alpha\beta} G_{\mu\nu} \rangle = \frac{1}{24} \left( g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu} \right) \left\langle G^a_{\lambda\sigma} G^{a\lambda\sigma} \right\rangle + \frac{1}{6} \left[ g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu} \right. \\ \left. -2 (u_\alpha u_\mu g_{\beta\nu} - u_\alpha u_\nu g_{\beta\mu} - u_\beta u_\mu g_{\alpha\nu} + u_\beta u_\nu g_{\alpha\mu}) \right] \left\langle u^\lambda \Theta^g_{\lambda\sigma} u^\sigma \right\rangle . \qquad (16)$$

After inserting the quark propagators into Eq. (12) and Eq. (13) and performing the trace procedure, the correlation function in the coordinate space is extracted. Then we encounter terms in the form of $\frac{1}{(x^2)^m}$ in the correlator. In order to transfer the calculations to momentum space, the following formula is used (note that $x$ comes to the exponential term):

$$\frac{1}{(x^2)^m} = \int \frac{\mathrm{d}^D p}{(2\pi)^D} \mathrm{e}^{-ip\cdot x} i (-1)^{m+1} 2^{D-2m} \pi^{D/2} \\ \times \frac{\Gamma[D/2-m]}{\Gamma[m]} \left( -\frac{1}{p^2} \right)^{D/2-m} \qquad (17)$$

so that the resultant expressions have three 4-integrals: first one over 4-momentum $p$ coming from the above formula, the second one initially existing in the correlation function in Eq. (3) (integral over 4-position $x$), and another over 4-momentum $k$ emerging from the heavy-quark propagators in Eq. (14). We start with taking the 4-integral over $x$, which leads us to a Dirac delta function. We use this function to perform an integral over $k$. The remaining integral over 4-momentum $p$ is made by applying the Feynman parametrization, and to solve the resultant 4-integrals, we employ

$$\int \mathrm{d}^4 \ell \frac{(\ell^2)^m}{(\ell^2 + \Delta)^n} = \frac{i \pi^2 (-1)^{m-n} \Gamma[m+2] \Gamma[n-m-2]}{\Gamma[2] \Gamma[n] (-\Delta)^{n-m-2}} . \qquad (18)$$

With the help of below relation, the imaginary part corresponding to different structures is derived

$$\Gamma \left[ \frac{D}{2} - n \right] \left( -\frac{1}{\Delta} \right)^{D/2-n} = \frac{(-1)^{n-1}}{(n-2)!} (-\Delta)^{n-2} \ln[-\Delta] , \qquad n \geq 2 . \qquad (19)$$

The next step to extract the thermal mass and decay constant sum rules of the $D^*_{s0}(2317)$ state is to impose the Borel transformation (*i.e.* omitting the continuum contribution taking derivative) to the invariant amplitude $\Pi^{\mathrm{QCD}}(q^2, T)$, and selecting the same structures in both physical and QCD sides, then equalizing the obtained expressions with the related part of $\widehat{\mathcal{B}}(q^2)\ \Pi^{\mathrm{Phys}}(q, T)$, lastly thermal decay constant sum rule is found as

$$f_{\mathcal{D}}(T) = \sqrt{ \frac{ \int_{\mathcal{M}^2}^{s_0(T)} \mathrm{d}s \rho^{\mathrm{QCD}}(s,T) \mathrm{e}^{-s/M^2} + \widehat{\mathcal{B}} \widetilde{\Gamma}\,(q^2, T) }{ 2 m^8_{\mathcal{D}}(T) \mathrm{e}^{-m^2_{\mathcal{D}}(T)/M^2} } } . \qquad (20)$$

To determine the thermal mass sum rule of the $D_{s0}^*(2317)$ state, one must take the derivative of Eq. (20) according to $(-1/M^2)$ and thus we get the following analytic expression for the mass sum rule depending on temperature:

$$m_{\mathcal{D}}(T) = \sqrt{\frac{\frac{\mathrm{d}}{\mathrm{d}(-1/M^2)}\left(\int_{\mathcal{M}^2}^{s_0(T)} \mathrm{d}s\ \rho^{\mathrm{QCD}}(s,T)\ \mathrm{e}^{-s/M^2} + \widehat{\mathcal{B}}\widetilde{\Gamma}\left(q^2,T\right)\right)}{\int_{\mathcal{M}^2}^{s_0(T)} \mathrm{d}s\ \rho^{\mathrm{QCD}}(s,T)\ \mathrm{e}^{-s/M^2} + \widehat{\mathcal{B}}\widetilde{\Gamma}\left(q^2,T\right)}}\,. \tag{21}$$

$s_0(T)$ in Eqs. (20) and (21) represent the thermal continuum threshold parameter which is related to $s_0(0)$ as introduced in Eq. (28) whose task is to separate the contributions coming from the ground state and higher states. The two auxiliary parameters, $s_0$ and the Borel mass, of the sum rule method will be explained in detail in Section 3.

Now, we will perform numerical analysis of the obtained sum rules for the $D_{s0}^*(2317)$ state taking into account both molecular and diquark–antidiquark scenarios.

## 3. Numerical calculation and analysis

The TQCDSR calculations for the mass and decay constant of the open-charm system $D_{s0}^*(2317)$ involve some parameters, for example, quark, gluon, and mixed vacuum condensates, and quark masses as well. The values of these input parameters are compiled in Table I.

TABLE I

Input parameters.

| Parameters | Values |
|---|---|
| $m_u$ | $2.16^{+0.49}_{-0.26}$MeV [8] |
| $m_d$ | $4.67^{+0.48}_{-0.17}$MeV [8] |
| $m_s$ | $93^{+11}_{-5}$MeV [8] |
| $m_c$ | $1.27 \pm 0.02$ GeV [8] |
| $\langle 0\|\bar{q}q\|0\rangle$ | $(-275(5))^3$ MeV$^3$ [52] |
| $\langle 0\|\bar{s}s\|0\rangle$ | $(-296(11))^3$ MeV$^3$ [52] |
| $\left\langle \frac{\alpha_s G^2}{\pi}\right\rangle$ | $0.028(3)$ GeV$^4$ [53] |
| $m_0^2$ | $(0.8 \pm 0.1)$ GeV$^2$ [44, 45] |

In addition to these parameters, the temperature-dependent quark, gluon condensates, and also energy density must be determined. For the quark condensate, the fit function provided from Ref. [52], which intersects with the Lattice QCD data, is utilized representing the $u$ and $d$ quarks with $q$

$$\frac{\langle\bar{q}q\rangle_T}{\langle 0|\bar{q}q|0\rangle} = -6.534\times 10^{-4}\mathrm{e}^{0.040T} + 1.015\,, \tag{22}$$

and for the $s$ quark

$$\frac{\langle\bar{s}s\rangle_T}{\langle 0|\bar{s}s|0\rangle} = -2.169\times 10^{-5}\mathrm{e}^{0.516T} + 1.002\,. \tag{23}$$

Meanwhile the fit function is valid up to a temperature of $T = 180$ MeV [54] and $\langle 0|\bar{q}q|0\rangle$ represents the condensate of light quarks at vacuum.

The gluonic and fermionic pieces of the energy density can be parametrized as in Ref. [54] using the Lattice QCD data presented in Ref. [55]

$$\begin{aligned} \left\langle u^\mu \theta^f_{\mu\nu} u^\nu \right\rangle_T &= \left(0.009\,\mathrm{e}^{24.876T} + 0.024\right)\, T^4\,, \\ \left\langle u^\mu \theta^g_{\mu\nu} u^\nu \right\rangle_T &= \left(0.091\,\mathrm{e}^{21.277T} - 0.731\right)\, T^4\,. \end{aligned} \tag{24}$$

The temperature-dependent gluon condensate $\langle G^2\rangle_T$ is defined as in Ref. [52]

$$\delta\left\langle \frac{\alpha_{\mathrm{s}} G^2}{\pi}\right\rangle_T = -\frac{8}{9}\left[\delta T^\mu_\mu(T) - m_u\delta\langle\bar{u}u\rangle_T - m_d\delta\left\langle\bar{d}d\right\rangle_T - m_s\delta\langle\bar{s}s\rangle_T\right]\,, \tag{25}$$

where the vacuum subtracted values of the considered quantities are used as $\delta f(T) \equiv f(T) - f(0)$ and $\delta T^\mu_\mu(T) = \varepsilon(T) - 3p(T)$: $\varepsilon(T)$ is the energy density and $p(T)$ is the pressure. Taking into account the recent Lattice calculations [55, 56], we get the fit function of $\delta T^\mu_\mu(T)$ as [54]

$$\frac{\delta T^\mu_\mu(T)}{T^4} = \left(0.020\,\mathrm{e}^{29.412T} + 0.115\right)\,. \tag{26}$$

Moreover, we use the following expression for the temperature-dependent strong coupling [57, 58]:

$$g_s^{-2}(T) = \frac{11}{8\pi^2}\ln\left(\frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}}\right) + \frac{51}{88\pi^2}\ln\left[2\ln\left(\frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}}\right)\right]\,, \tag{27}$$

where $\Lambda_{\overline{\mathrm{MS}}} \simeq T_{\mathrm{c}}/1.14$.

The continuum threshold as a function of temperature belonging to the $D^*_{s0}(2317)$ state is another auxiliary parameter that needs to be determined. The expression of the continuum threshold in terms of temperature is used as follows (for details, see [59–61]):

$$\frac{s_0(T)}{s_0(0)} = \left[\frac{\langle\bar{q}q\rangle_T}{\langle 0|\bar{q}q|0\rangle}\right]^{2/3}\,, \tag{28}$$

where $s_0(T)$ is defined with $s_0(0)$ in the vacuum threshold. It is not random and $s_0(0)$ depends on the mass of the first excited state of the $D^*_{s0}(2317)$. For this reason, the chosen interval for the $s_0$ is relatively weakly dependent on the physical quantities of the state under study. According to the QCDSR technique, the physical quantities should not rely on the auxiliary parameters $M^2$ and $s_0$. However in reality, these quantities are sensitive to the choice both of $M^2$ and $s_0$. Therefore, the parameters $M^2$ and $s_0$ should be settled to minimize the dependence of $m_\mathcal{D}$ and $f_\mathcal{D}$ on them. The convergence of the OPE plus suppression of the contributions coming from the higher states and continuum are a must for the sake of fixing the working region of the Borel parameter $M^2$. As shown in Fig. 2, $M^2_{\rm max}$ should be 1.6 GeV$^2$. To determine the working region of the continuum threshold, we use $\sqrt{s_0} = m_\mathcal{D} + (0.3; 0.5)$ in the classical conjecture.

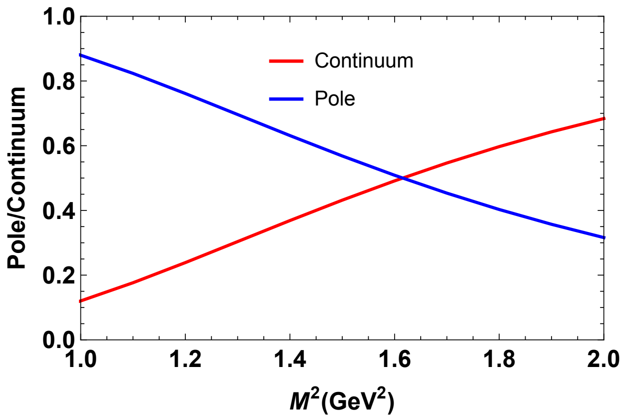


Fig. 2. The relative pole and continuum contributions of the $J^P = 0^+$ open-charm state with $\sqrt{s} = 2.81$ GeV at the diquark–antidiquark assumption as an example.

The analysis carried out takes into account all of the aforementioned constraints, allowing us to fix the continuum threshold and the Borel parameter as 1.2 GeV$^2 \leq M^2 \leq 1.6$ GeV$^2$ and 6.8 GeV$^2 \leq s_0 \leq 7.9$ GeV$^2$. Note that the dependence of the mass and decay constant on $M^2$ is stable in this interval, ensuring the obtained sum rules will give accurate results.

To show the independence of physical quantities from $M^2$ and $s_0$, we present the 3D plot of the mass *versus* continuum threshold and the Borel parameter $M^2$ in the diquark–antidiquark picture in Fig. 3 and we see the stability of mass sum rule according to these parameters, which is the main criterion of QCDSR.

At zero temperature, the open-charm system $D^*_{s0}(2317)$ has been investigated with many theoretical models both in the molecular and diquark–antidiquark pictures [27–36]. Our results in $T = 0$ limit are presented below

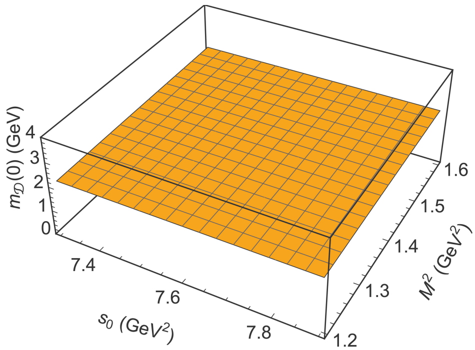


Fig. 3. $3D$ graph of the vacuum mass of the $D^*_{s0}(2317)$ state *versus* continuum threshold $s_0$ and Borel mass $M^2$ in the diquark–antidiquark picture.

both in molecular and diquark–antidiquark scenarios

$$
\begin{aligned}
m^{\rm mol}_{\mathcal{D}} &= 2316^{+33}_{-34}\ \text{MeV}\,, \qquad f^{\rm mol}_{\mathcal{D}} = 82.1^{+2.1}_{-2.1}\ \text{keV}\,,\\
m^{\rm di}_{\mathcal{D}} &= 2317^{+33}_{-34}\ \text{MeV}\,, \qquad f^{\rm di}_{\mathcal{D}} = 93.1^{+2.3}_{-2.4}\ \text{keV}\,.
\end{aligned}
$$

The last step is to look for variations of the mass and decay constant of the $D^*_{s0}(2317)$ state in terms of the temperature. In this context, the changes in the mass and decay constants are drawn as a function of the temperature for molecular and diquark–antidiquark assumptions in Figs. 4 and 5, respectively.

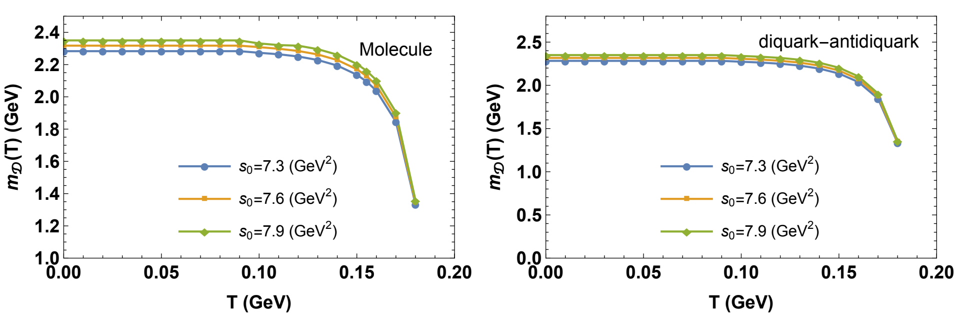


Fig. 4. The temperature-dependent mass $m_{\mathcal{D}}(T)$ of the $D^*_{s0}(2317)$ state in the molecular and diquark–antidiquark pictures for different fixed values of $s_0(0)$.

As a by-product, we replace the $c$ quark in the interpolating current with the $b$ quark and deduce the bottom partner of $D^*_{s0}(2317)$, which we will call this resonance as $B_{sJ}$ from now on. This state may have the mass and decay constant values in the following range in both molecular and

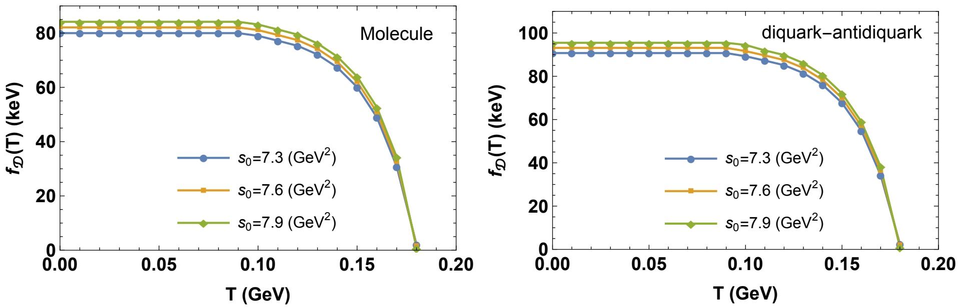


Fig. 5. The temperature-dependent decay constant $f_{\mathcal{D}}(T)$ of the $D^*_{s0}(2317)$ state in the molecular and diquark–antidiquark pictures for different fixed values of $s_0(0)$.

diquark—antidiquark pictures:

$$
\begin{aligned}
m^{\rm mol}_{B_{sJ}} &= (5140\text{–}5458)\ \text{MeV}\,, \qquad f^{\rm mol}_{B_{sJ}} = (33.2\text{–}53.3)\ \text{keV}\,,\\
m^{\rm di}_{B_{sJ}} &= (5142\text{–}5460)\ \text{MeV}\,, \qquad f^{\rm di}_{B_{sJ}} = (38.3\text{–}61.5)\ \text{keV}\,,
\end{aligned}
$$

which are consistent with the estimations in Ref. [48] within the limits of uncertainties.

## 4. Conclusion

Theoretical and experimental disagreement regarding the mass and decay width of open-charm meson $D^*_{s0}(2317)$ have stimulated exotic interpretations of this state in the literature. Its measured mass and width do not match the predictions from the potential-based quark models. Therefore, it is important to make further studies and future experiments of this bound state to understand these uncertainties.

In this sense, we examined the $D^*_{s0}(2317)$ state in the molecular and diquark–antidiquark pictures by calculating its spectroscopic parameters using the TQCDSR method. We can summarize our results as follows:

— Our numerical calculations show that mass and decay constant parameters are practically independent of temperature at least up to the temperature of 100 MeV, but after that point, they begin to drop as the temperature increases.

— Near the critical temperature, the decay constant reaches 71% and 74% of its vacuum value in molecular and diquark–antidiquark pictures, respectively, while the masses decrease by 94% in both pictures. As for the mass, our result is in good agreement with Ref. [8]. Besides, we find more dramatic changes in the mass and decay constants above the critical temperature, and hence we interpret it as the hadrons starting to disappear.

— Our results do not give any definite information as to whether the $D^*_{s0}(2317)$ resonance is molecular or diquark–antidiquark structure since they are within the uncertainties of the TQCDSR theory.

— According to a recent study [62], the compact tetraquark and loosely bound molecule are produced at different medium temperatures: a tetraquark is formed in QGP above the critical temperature, while a molecule is formed only in the hadronic medium after the kinetic freeze-out. Moreover, in Ref. [37], Bicudo *et al.* conclude that an effective gluon mass remains finite at $T = T_{\rm c}$.

— When we compare the molecular and tetraquark representations by looking at the graphs, it is seen that the mass decreases at lower temperatures and in a larger amount in the molecular representation. This is consistent with the expectation of a loosely bound molecule structure that is formed later in the fireball evolution than the tetraquark. That is the production of molecule freezes out later in the hadronic evolution.

— Although no scalar particle has yet been detected in the bottom-strange meson list of the PDG, we predict that it could be found in experiments soon.

As a result, the noticeable decline in the values of mass and decay constant can be conceived as a manifestation of the QGP phase transition. The QGP provides a very good ground to understand the nonperturbative aspects of strong interactions, early universe, and astrophysical processes such as neutron stars. In short, the investigation of QGP helps to reveal the nature of both micro- and macro-scale events in the universe. For this reason, searches related to thermal properties of hadrons and also exotic candidates can provide valuable hints and information for future experiments such as CMS, LHCb, and PANDA.

## Appendix A

*The thermal spectral densities*

Here, we present the results of our evaluations for the spectral density

$$\rho^{\rm QCD}(s,T) = \rho^{\rm pert}(s) + \sum_{k=3}^{6} \rho^k(s,T)\,. \tag{A.1}$$

This is essential for our calculations of the mass and decay constant as a function of the temperature belonging to the $D^*_{s0}(2317)$ resonance via the TQCDSR. Here, we only present the diquark–antidiquark picture results since formulas are too long to present them fully. $\rho^k(s,T)$ denote the non-perturbative contributions to $\rho^{\rm QCD}(s,T)$ for different operator dimensions. The explicit form for $\rho^{\rm pert}(s)$ and $\rho^k(s,T)$ expressed with the integrals over the Feynman parameter $z$ as follows:

$$\begin{aligned}\rho^{\rm pert}(s) \;=\; & \int_0^1 {\rm d}z \frac{-z^2\left(m_c^2+s\beta\right)^2}{3\times 2^{11}\pi^6\beta^3}\left(m_c^4 z^2 - 4m_c^3 z\left(m_d+m_u\right)+4m_c^2\beta z\right. \\ & \times\left[2m_s(m_d+m_u)+sz\right]-4m_c\beta\left[9m_s\left(m_d^2+m_u^2\right)+sz\right. \\ & \left.\left.\times(m_d+m_u)\right]+s\beta^2 z\left[20m_s(m_d+m_u)+13sz\right]\right)\Theta[L(s,z)]\,,\end{aligned} \quad \text{(A.2)}$$

$$\begin{aligned}\rho^3(s,T) \;=\; & \int_0^1 {\rm d}z \frac{-z}{2^7\pi^4\beta^2}\Big[\langle\bar{d}d\rangle\Big(m_c^5 z+2m_c^4\beta z(m_d-m_s)+2m_c^3\beta\big(-m_d^2 \\ & +4m_d m_s+sz\big)+2m_c^2\beta^2\left[2m_d^2 m_s+3sz(m_d-m_s)\right]+m_c s\beta^2 \\ & \times\left(-2m_d^2+8m_d m_s+sz\right)+2s\beta^3\left[3m_d^2 m_s+2sz(m_d-m_s)\right]\Big) \\ & +m_c^5\langle\bar{u}u\rangle z-2m_c^4\beta z\Big(\langle\bar{s}s\rangle(m_d-m_s+m_u)+\langle\bar{u}u\rangle(m_s-m_u)\Big) \\ & +2m_c^3\beta\Big(2m_d^2\langle\bar{s}s\rangle-m_d m_s\langle\bar{s}s\rangle+m_u\big[-m_s\langle\bar{s}s\rangle+4m_s\langle\bar{u}u\rangle \\ & +2m_u\langle\bar{s}s\rangle-m_u\langle\bar{u}u\rangle\big]+s\langle\bar{u}u\rangle z\Big)+2m_c^2\beta^2\Big(2m_s m_u^2\langle\bar{u}u\rangle \\ & -3sz\big[\langle\bar{s}s\rangle(m_d-m_s+m_u)+\langle\bar{u}u\rangle(m_s-m_u)\big]\Big)+m_c s\beta^2 \\ & \times\Big(4m_d^2\langle\bar{s}s\rangle-2m_d m_s\langle\bar{s}s\rangle-2m_u\big[m_s\big(\langle\bar{s}s\rangle-4\langle\bar{u}u\rangle\big)+m_u\big(\langle\bar{u}u\rangle \\ & -2\langle\bar{s}s\rangle\big)\big]+s\langle\bar{u}u\rangle z\Big)-2s\beta^3\Big(2sz\big[\langle\bar{s}s\rangle\big(m_d-m_s+m_u\big)+\langle\bar{u}u\rangle \\ & \times\big(m_s-m_u\big)\big]-3m_s m_u^2\langle\bar{u}u\rangle\Big)\Big]\,\Theta[L(s,z)]\,,\end{aligned} \quad \text{(A.3)}$$

$$\begin{aligned}
\rho^4(s,T) \;=\; & \int_0^1 \mathrm{d}z\Big[\frac{1}{3^2\times 2^{12}\beta^3\pi^6}z\Big(-16\beta^2\Big[6szm_c^2\Big(8\beta\pi^2\left\langle u^\mu\theta^f_{\mu\nu}u^\nu\right\rangle \\
& \times(-3+16z)+g_s^2\langle u^\mu\theta^g_{\mu\nu}u^\nu\rangle[3+4z(-3+2z)]\Big)6\beta zm_c^4 \\
& \times\Big(16\pi^2\left\langle u^\mu\theta^f_{\nu}u^\nu\right\rangle+g_s^2\langle u^\mu\theta^g_{\mu\nu}u^\nu\rangle\Big)-6\beta s(-1+3z) \\
& \times\Big[16\pi^2\left\langle u^\mu\theta^f_{\mu\nu}u^\nu\right\rangle+\langle u^\mu\theta^g_{\mu\nu}u^\nu\rangle g_s^2\Big]-6\beta m_c^3(m_d+m_u) \\
& \times\Big(16\pi^2\left\langle u^\mu\theta^f_{\mu\nu}u^\nu\right\rangle+\langle u^\mu\theta^g_{\mu\nu}u^\nu\rangle g_s^2\Big)+\beta s\Big(2sz\big[8\beta\pi^2\left\langle u^\mu\theta^f_{\mu\nu}u^\nu\right\rangle \\
& \times(-3+50z)+g_s^2\langle u^\mu\theta^g_{\mu\nu}u^\nu\rangle[6+z(-34+25z)]\big]-9m_dm_szg_s^2 \\
& \times\big[-16\beta\pi^2\langle u^\mu\theta^f_{\mu\nu}u^\nu\rangle+\langle u^\mu\theta^g_{\mu\nu}u^\nu\rangle\big]-9m_sm_u\Big[-16\beta\pi^2 \\
& \times\left\langle u^\mu\theta^f_{\mu\nu}u^\nu\right\rangle+zg_s^2\langle u^\mu\theta^g_{\mu\nu}u^\nu\rangle\big]\Big)\Big]+G^2g_s^2\Big[-2(6-5z)^2zm_c^4 \\
& +2\beta^2sm_c(m_d+m_u)\big[-36+z(36+z)\big]+72\beta^2m_c^3(m_d+m_u) \\
& +12\beta^3sz\Big(4s(3-2z)+9m_s(m_d+m_u)\Big)+\beta zm_c^2\Big[-147sz^2 \\
& -72\Big(3s+m_s(m_d+m_u)\Big)+4z\Big(90s+17m_s(m_d+m_u)\Big)\Big]\Big]\Big) \\
& \times\Theta[L(s,z)]\,,
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\rho^5(s,T) \;=\; & \int_0^1 \mathrm{d}z\Big[\frac{m_0^2}{3\times 2^7\beta\pi^4}\Big[3\langle\bar{u}u\rangle zm_c^3+\langle\bar{d}d\rangle\Big(3zm_c^3+2\beta zm_c^2(2m_d-3m_s) \\
& +\beta m_c(3sz-m_d^2+6m_dm_s)+\beta^2(6szm_d-9szm_s+2m_d^2m_s)\Big) \\
& -2\beta zm_c^2\Big(3\langle\bar{s}s\rangle m_d-2\langle\bar{s}s\rangle m_s+3\langle\bar{u}u\rangle m_s+3\langle\bar{s}s\rangle m_u-2\langle\bar{u}u\rangle m_u\Big) \\
& +\beta^2\Big(2\langle\bar{u}u\rangle m_sm_u^2-3sz\big[3\langle\bar{s}s\rangle m_d-2\langle\bar{s}s\rangle m_s+3\langle\bar{u}u\rangle m_s+3\langle\bar{s}s\rangle m_u \\
& -2\langle\bar{u}u\rangle m_u\big]\Big)+\beta m_c\Big(3s\langle\bar{u}u\rangle z+3\langle\bar{s}s\rangle m_d^2-\langle\bar{s}s\rangle m_dm_s+m_u \\
& \times\big[-\langle\bar{s}s\rangle m_s+6\langle\bar{u}u\rangle m_s+3\langle\bar{s}s\rangle m_u-\langle\bar{u}u\rangle m_u\big]\Big)\Big]\Theta[L(s,z)]\,,
\end{aligned} \tag{A.5}$$

$$\rho^6(s,T) = \int_0^1 dz\Big[\frac{1}{9^2\times 2^5\pi^4}\Big[-54\langle\bar{d}d\rangle\pi^2\langle\bar{s}s\rangle\Big(4zm_c^2+m_c(-4m_d+m_s) +2\beta\big[2sz+m_d(-m_d+m_s)\big]\Big)+\langle\bar{d}d\rangle^2\Big(-54\pi^2(m_c+\beta m_d) \times(m_d-2m_s)+g_s^2\Big[-4zm_c^2+m_cm_d-2\beta(2sz+m_dm_s)\Big]\Big) +g_s^2\Big(-4\left(\langle\bar{s}s\rangle^2+\langle\bar{u}u\rangle^2\right)zm_c^2+\langle\bar{u}u\rangle^2m_cm_u+\langle\bar{s}s\rangle^2m_c(m_d+m_u) -2\beta\Big[2s\left(\langle\bar{s}s\rangle^2+\langle\bar{u}u\rangle^2\right)z+\langle\bar{u}u\rangle^2m_sm_u\Big]\Big)-54\pi^2\langle\bar{u}u\rangle \times\Big(4\langle\bar{s}s\rangle zm_c^2+m_c\Big[m_s\big(\langle\bar{s}s\rangle-2\langle\bar{u}u\rangle\big)+m_u(-4\langle\bar{s}s\rangle+\langle\bar{u}u\rangle)\Big] +\beta\Big[4s\langle\bar{s}s\rangle z+m_u[2(\langle\bar{s}s\rangle-\langle\bar{u}u\rangle)m_s+(-2\langle\bar{s}s\rangle+\langle\bar{u}u\rangle)m_u]\Big]\Big)\Big] \times\Theta[L(s,z)]\Big], \tag{A.6}$$

$$\widehat{\mathcal{B}}\widetilde{\Gamma}\left(q^2,T\right) = \left[\frac{1}{3\times 2^5\pi^2}m_cm_s\langle\bar{s}s\rangle\Big(m_d^2\langle\bar{d}d\rangle+m_u^2\langle\bar{u}u\rangle\Big)\,e^{-m_c^2/M^2}\right] -\int_0^1 dz\left[\frac{z^2}{3\times 2^{11}\beta^2\pi^6}G^2g_s^2m_c^3m_s\left(m_d^2+m_u^2\right)\,e^{-m_c^2/M^2\beta}\right], \tag{A.7}$$

where $\Theta$ denotes the unit step function, $L(s,z)=sz(1-z)-zm_c^2$, and $\beta=z-1$.

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