TOWARDS THE ASSIGNMENTS FOR 1^1D_2 AND 1^3D_2 MESON NONETS*

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In this work, we investigate the mass spectrum of 1^1D_2 and 1^3D_2 meson nonets in the framework of the meson mass matrix and Regge phenomenology. The results are compared with the values from different phenomenological models and may be useful for the assignment of the 1^1D_2 and 1^3D_2 meson nonets in the future.

1. Introduction

Quantum Chromodynamics (QCD) was established half a century ago as a theory of strong interactions. In the past few years, QCD has achieved great success and continued to provide satisfactory explanations for many puzzling experimental results. However, we also need to face the fact objectively that there is still a long way to obtain a comprehensive understanding of strong interactions. One of the problems to be solved is that perturbative QCD is not applicable at low energy, and it is difficult to produce accurate results from non-perturbative calculations [1]. In this case, it is necessary to build models that can reveal the most important features of QCD, *e.g.*, quenched lattice QCD [2, 3], the Dyson–Schwinger formalism [4], constituent quark models [5], and light cone QCD [6, 7]. In addition, since mesons are an ideal laboratory for studying strong interactions in the strongly coupled non-perturbative regime, the study of the meson spectrum is of great significance for better understanding the dynamics of strong interactions [8–11].

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In this work, we will discuss the assignment of the 1^1D_2 and 1^3D_2 meson nonets. The masses and widths of the 1^1D_2 and 1^3D_2 meson nonets are listed in Table 1. Here, the values are taken from PDG [12].

Table 1. The masses and widths of 1^3D_2 and 1^1D_2 meson nonets (in units of MeV). $\rho_2(???)$, $\omega_2(???)$, and $\phi_2(???)$ remain to be found in experiments.

State (1^1D_2)	Mass	Width	State (1^3D_2)	Mass	Width
$\pi_2(1670)$	$1670.6^{+2.9}_{-1.2}$	258^{+8}_{-9}	$\rho_2(???)$		
$\eta_2(1645)$	1617 ± 5	181 ± 11	$\omega_2(???)$		
$\eta_2(1870)$	1842 ± 8	225 ± 14	$\phi_2(???)$		
$K_2(1770)^{\dagger}$	1773 ± 8	186 ± 14	$K_2(1820)^{\dagger}$	1819 ± 12	264 ± 34

For the 1^1D_2 meson nonet, the states $\pi_2(1670)$, $\eta_2(1645)$, and $\eta_2(1870)$ are well established. The state $\pi_2(1670)$ was observed in the $\pi^+p \to p\pi^+\pi^+\pi^-$ reaction half a century ago [13]. In 2002, the E852 Collaboration performed a partial-wave analysis of the $\pi^-p \to \pi^+\pi^-\pi^-p$ reaction and the $\pi_2(1670)$ was confirmed [14]. In 2005, three isovector 2^{-+} states $\pi_2(1670)$, $\pi_2(1880)$, and $\pi_2(1970)$ were seen in the $\omega\rho^-$ decay channel [15]. In Ref. [16], the $\pi_2(1670)$ state was measured with mass 1642^{+12}_{-1} MeV and width 311^{+12}_{-23} MeV. For the isoscalar, the Crystal Barrel Collaboration observed $\eta_2(1645)$ and $\eta_2(1870)$ in the $p\bar{p} \to \eta\pi^0\pi^0\pi^0$ reaction [17]. Subsequently, the $\eta_2(1645)$ and $\eta_2(1870)$ were studied by the WA102 Collaboration in the $a_2(1320)\pi$, $f_2(1270)\pi$, $a_0(1320)\pi$ channels [18, 19]. For the kaon sector, the assignment of $K_2(1770)$ remains interesting. The $K_2(1770)$ was first reported in the $K^-p \to pK^-\pi^+\pi^-$ and $K^-p \to p\bar{K}^0\pi^-\pi^0$ [20] reactions. Moreover, the evidence for two $J^P = 2^-$ strange mesons was presented in the $K^-p \to K^-\pi^+\pi^-\pi^0p$ reaction [21]. Unlike the $\pi_2(1670)$, $\eta_2(1645)$, and $\eta_2(1870)$, the kaon has non-diagonal C-parity, the spin-singlet (1^1D_2) and spin-triplet (1^3D_2) can mix to produce the physical states $K_2(1770)$ and $K_2(1820)$ [22]. The cognition of the 1^3D_2 meson nonet is even lower than that of the 1^1D_2 meson nonet. Since 1^3D_2 meson has the same spin angular momentum and orbital momentum as 1^1D_2 and 1^3D_1 meson, the mass of the corresponding states may have a large overlap. Moreover, the large decay widths also make it very difficult to find these states. Till now, only the kaon $K_2(1820)$ is listed in Table 1, the other members are still not established in the experiments. Therefore, both the theoretical and experimental analyses of the assignment for 1^1D_2 and 1^3D_2 meson nonets are required.

In this work, the mass spectrum of 1^1D_2 and 1^3D_2 meson nonets is investigated in the framework of the meson mass mixing matrix and Regge phenomenology. This work is organized as follows. The mass mixing matrix is described in Section 2. The mass relations are derived from Regge phenomenology in Section 3. The summary is given in Section 4.

2. Mass mixing matrix of isoscalar states

In the quark model, mesons exist as $q\bar{q}'$ bound states of quark q and antiquark $\bar{q}'$ (the flavors of q and $\bar{q}'$ may be different). The $q\bar{q}'$ bound states can be classified based on the representations of SU(3) flavor group. In general, states with the same J^P and additive quantum numbers can mix, so the bare isoscalar states can mix and form two physical states [23–25].

In the $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ basis, the mass matrix describing the mix of the physical isoscalar states can be expressed as

$$M^2 = \begin{pmatrix} M_{n\bar{n}}^2 + 2A_{nn} & \sqrt{2}A_{ns} \\ \sqrt{2}A_{ns} & M_{s\bar{s}}^2 + A_{ss} \end{pmatrix} \tag{1}$$

with

$$A_{nn} = \frac{\zeta}{m_n m_n}, \qquad A_{ns} = \frac{\zeta}{m_n m_s}, \qquad A_{ss} = \frac{\zeta}{m_s m_s}, \tag{2}$$

where $M_{n\bar{n}}$ and $M_{s\bar{s}}$ are the masses of states $n\bar{n}$ and $s\bar{s}$, respectively; A_{nn}, A_{ns} and A_{ss} are the mixing parameters, which describe the $q\bar{q} \leftrightarrow q'\bar{q}'$ transition amplitudes [26, 27]; ζ is an SU(3)-invariant phenomenological parameter, m_n, m_s are the masses constituent quarks (n denotes the up or down quark).

For a given meson nonet, the physical isoscalar states Φ and Φ' are the eigenstates of the mass matrix M^2

$$UM^2U^\dagger = \begin{pmatrix} M_\Phi^2 & 0 \\ 0 & M_{\Phi'}^2 \end{pmatrix}, \tag{3}$$

where U is the unitary matrix, M_Φ^2 and $M_{\Phi'}^2$ are masses of states Φ and Φ', respectively.

From relations (1), (2), and (3), one has

$$M_{n\bar{n}}^2 + \frac{2\zeta}{m_n^2} + M_{s\bar{s}}^2 + \frac{\zeta}{m_s^2} = M_\Phi^2 + M_{\Phi'}^2, \tag{4}$$

$$\left(M_{n\bar{n}}^2 + \frac{2\zeta}{m_n^2}\right)\left(M_{s\bar{s}}^2 + \frac{\xi}{m_s^2}\right) - \frac{2\xi}{m_n^2 m_s^2} = M_\Phi^2 M_{\Phi'}^2. \tag{5}$$

Relations (4) and (5) are the trace and determinant of relation (3), respectively.

Applying relations (4) and (5) for the 1^1D_2 and 1^1P_1 meson nonets, we obtain the masses $M_{s\bar{s}(1^1D_2)} = 1849.0 \pm 8.1$ MeV or 1609.1 ± 5.7 MeV and $M_{s\bar{s}(1^1P_1)} = 1423.9 \pm 8.7$ MeV or 1156.4 ± 8.1 MeV. The masses $M_{s\bar{s}(1^1D_2)} = 1609.1 \pm 5.7$ MeV, and $M_{s\bar{s}(1^1P_1)} = 1156.4 \pm 8.1$ MeV should be discarded since they are smaller than the masses of the isovector states $\pi_2(1670)$ and $b_1(1235)$. In this work, we use the quark masses $m_n = 0.29$ GeV, $m_s = 0.46$ GeV as extracted from meson spectroscopy [28, 29].

Apart from the mixing of isoscalar states, for the kaons of 1^1D_2 and 1^3D_2, the spin-singlet and spin-triplet can mix to form physical states $K_2(1770)$ and $K_2(1820)$. The mixing can be expressed as

$$K_2(1773) = K_2\left(1^1D_2\right)\cos\theta_K + K_2\left(1^3D_2\right)\sin\theta_K, \tag{6}$$

$$K_2(1820) = -K_2\left(1^1D_2\right)\sin\theta_K + K_2\left(1^3D_2\right)\cos\theta_K, \tag{7}$$

where θ_K is the (1^1D_2)-(1^3D_2) mixing angle. The same analysis process is used for axial-vector mesons [30]. The masses of $K_2(1^1D_2)$ and $K_2(1^3D_2)$ can be related to $M_{K_2(1773)}$ and $M_{K_2(1820)}$ by the following relation:

$$\begin{pmatrix} \cos\theta_K & \sin\theta_K \\ -\sin\theta_K & \cos\theta_K \end{pmatrix} \begin{pmatrix} M^2_{K_2(1^1D_2)} & \xi \\ \xi & M^2_{K_2(1^1D_2)} \end{pmatrix} \begin{pmatrix} \cos\theta_K & -\sin\theta_K \\ \sin\theta_K & \cos\theta_K \end{pmatrix}$$

$$= \begin{pmatrix} M^2_{K_2(1773)} & 0 \\ 0 & M^2_{K_2(1820)} \end{pmatrix}, \tag{8}$$

where ξ represents a parameter describing the $K_2(1^1D_2)$ and $K_2(1^3D_2)$ mixing. From relations (6), (7), and (8), the mixing angle θ_K and the mass relations between $M_{K_2(1^1D_2)}$ and $M_{K_2(1^3D_2)}$ are obtained, i.e.,

$$M^2_{K_2(1^1D_2)} = M^2_{K_2(1773)}\cos^2\theta_K + M^2_{K_2(1820)}\sin^2\theta_K, \tag{9}$$

$$M^2_{K_2(1^3D_2)} = M^2_{K_2(1773)}\sin^2\theta_K + M^2_{K_2(1820)}\cos^2\theta_K, \tag{10}$$

$$\cos\left(2\theta_K\right) = \frac{M^2_{K_2(1^1D_2)} - M^2_{K_2(1^3D_2)}}{M^2_{K_2(1773)} - M^2_{K_2(1820)}}, \tag{11}$$

$$M^2_{K_2(1^1D_2)} + M^2_{K_2(1^3D_2)} = M^2_{K_2(1773)} + M^2_{K_2(1820)}. \tag{12}$$

As described in the previous section, based on the mass mixing matrix of isoscalar states and the mixing of the spin-singlet and spin-triplet, we build a bridge that connects the masses of meson nonet members. In the subsequent analysis, we will determine the masses of the unknown meson states by applying relations (4), (5), and (11) to the known meson masses.

3. Regge phenomenology and mass relations

In this section, we use the Regge theory for the assignment of meson multiplets. The Regge theory, which connects the high-energy behavior of the scattering amplitude with singularities in the complex angular momentum plane of the partial wave amplitudes, was developed in the 1960s [31]. Recently, the Regge theory has attracted renewed attention in that it can be used to predict the meson masses or to determine the quantum numbers of newly observed states in experiments [9, 28, 32–35]. Regge theory indicates that mesons are associated with Regge poles, which move in the complex angular momentum plane as a function of energy. The plots of Regge trajectories of hadrons in the (J, M^2) plane are usually called the Chew–Frautschi plots, where J and M are the total spins and the masses of the hadrons, respectively. According to the Chew–Frautschi conjecture, the poles fall onto linear trajectories in the (J, M^2) plane

$$J \;=\; \alpha_{n\bar{n}N}(0) + \alpha'_{n\bar{n}N} M^2_{n\bar{n}N}\,, \tag{13}$$

$$J \;=\; \alpha_{n\bar{s}N}(0) + \alpha'_{n\bar{s}N} M^2_{n\bar{s}N}\,, \tag{14}$$

$$J \;=\; \alpha_{s\bar{s}N}(0) + \alpha'_{s\bar{s}N} M^2_{s\bar{s}N}\,, \tag{15}$$

where N is the radial quantum number. The α' and α are the slope and intercept of the Regge trajectory, respectively. In this work, the intercept and slope can be expressed as

$$\alpha_{n\bar{n}N}(0) \;+\; \alpha_{s\bar{s}N}(0) = 2\alpha_{n\bar{s}N}(0)\,, \tag{16}$$

$$\frac{1}{\alpha'_{n\bar{n}N}} \;+\; \frac{1}{\alpha'_{s\bar{s}N}} = \frac{2}{\alpha'_{n\bar{s}N}}\,. \tag{17}$$

The intercept relation was derived from the dual-resonance model [36], and is satisfied in two-dimensional QCD [37], the dual-analytic model [38], the quark bremsstrahlung model [39]. The slope relation (17) was obtained in the framework of topological expansion and the $q\bar{q}$-string picture of hadrons [40].

From relations (13)–(17), one has

$$M^2_{n\bar{n}N}\alpha'_{n\bar{n}N} + M^2_{s\bar{s}N}\alpha'_{s\bar{s}N} = M^2_{n\bar{s}N}\alpha'_{n\bar{s}N}\,. \tag{18}$$

Based on the assumption that the slopes of parity partner trajectories coincide and are independent of charge conjugation [9, 41], that is to say,

$$\left\{ \begin{array}{l} \alpha'_{n\bar{n}}(1^1P_1) = \alpha'_{n\bar{n}}(1^1D_2) = \alpha'_{n\bar{n}}(1^3D_2) \\[2mm] \alpha'_{n\bar{s}}(1^1P_1) = \alpha'_{n\bar{s}}(1^1D_2) = \alpha'_{n\bar{s}}(1^3D_2) \\[2mm] \alpha'_{s\bar{s}}(1^1P_1) = \alpha'_{s\bar{s}}(1^1D_2) = \alpha'_{s\bar{s}}(1^3D_2) \end{array} \right. , \tag{19}$$

we have the following relations, which are related to meson multiplets with the same Regge slopes by eliminating the slopes:

$$\frac{4M^2_{n\bar{s}(1^1D_2)}M^2_{n\bar{n}(1^1P_1)} - 4M^2_{n\bar{n}(1^1D_2)}M^2_{n\bar{s}(1^1P_1)}}{M^2_{n\bar{n}(1^1D_2)}M^2_{s\bar{s}(1^1P_1)} - M^2_{s\bar{s}(1^1D_2)}M^2_{n\bar{n}(1^1P_1)}}$$
$$= \frac{M^2_{n\bar{s}(1^1D_2)}\left(M^2_{n\bar{n}(1^1P_1)} - M^2_{s\bar{s}(1^1P_1)}\right) - M^2_{n\bar{s}(1^1P_1)}\left(M^2_{n\bar{n}(1^1D_2)} - M^2_{s\bar{s}(1^1D_2)}\right)}{M^2_{n\bar{s}(1^1D_2)}M^2_{s\bar{s}(1^1P_1)} - M^2_{s\bar{s}(1^1D_2)}M^2_{n\bar{s}(1^1P_1)}},$$
$$(20)$$

$$\frac{4M^2_{n\bar{s}(1^3D_2)}M^2_{n\bar{n}(1^1P_1)} - 4M^2_{n\bar{n}(1^3D_2)}M^2_{n\bar{s}(1^1P_1)}}{M^2_{n\bar{n}(1^3D_2)}M^2_{s\bar{s}(1^1P_1)} - M^2_{s\bar{s}(1^3D_2)}M^2_{n\bar{n}(1^1P_1)}}$$
$$= \frac{M^2_{n\bar{s}(1^3D_2)}\left(M^2_{n\bar{n}(1^1P_1)} - M^2_{s\bar{s}(1^1P_1)}\right) - M^2_{n\bar{s}(1^1P_1)}\left(M^2_{n\bar{n}(1^3D_2)} - M^2_{s\bar{s}(1^3D_2)}\right)}{M^2_{n\bar{s}(1^3D_2)}M^2_{s\bar{s}(1^1P_1)} - M^2_{s\bar{s}(1^3D_2)}M^2_{n\bar{s}(1^1P_1)}}.$$
$$(21)$$

Inserting the masses of states into relations (11), (20), (21), with the aid of $M^2_{s\bar{s}(1^3D_2)} = 2M^2_{n\bar{s}(1^3D_2)} - M^2_{n\bar{n}(1^3D_2)}$ [24, 42], we obtain the values of $M_{n\bar{s}(1^1D_2)}$, $M_{n\bar{s}(1^3D_2)}$, $M_{n\bar{n}(1^3D_2)}$, $M_{s\bar{s}(1^3D_2)}$ and list the results in Tables 3 and 4. In this work, we take

$$M^2_{n\bar{s}(1^1P_1)} = \frac{1}{2}M^2_{K_1(1270)} + \frac{1}{2}M^2_{K_1(1400)}, \qquad (22)$$

the K_{1A} and K_{1B} are nearly equal 45° mixtures of the $K_1(1270)$ and K_1 (1400) [43].

Apart from the ground meson, the radial excitations can be estimated in the framework of Regge phenomenology. Based on the assumption that the ground and the radial excitation have the same slopes [34], we obtain the following from relations (13), (14), and (15):

$$M^2_{n\bar{n}N}\alpha'_{n\bar{n}1} - M^2_{n\bar{n}1}\alpha'_{n\bar{n}1} = \alpha_{n\bar{n}1}(0) - \alpha_{n\bar{n}N}(0), \qquad (23)$$
$$M^2_{s\bar{s}N}\alpha'_{s\bar{s}1} - M^2_{s\bar{s}1}\alpha'_{s\bar{s}1} = \alpha_{s\bar{s}1}(0) - \alpha_{s\bar{s}N}(0), \qquad (24)$$
$$M^2_{n\bar{s}N}\alpha'_{n\bar{s}1} - M^2_{n\bar{s}1}\alpha'_{n\bar{s}1} = \alpha_{n\bar{s}1}(0) - \alpha_{n\bar{s}N}(0). \qquad (25)$$

In Refs. [44, 45], Filipponi *et al.* indicate that the values of $\alpha_{n\bar{n}1}(0) - \alpha_{n\bar{n}N}(0)$, $\alpha_{n\bar{s}1}(0) - \alpha_{n\bar{s}N}(0)$, and $\alpha_{s\bar{s}1}(0) - \alpha_{s\bar{s}N}(0)$ depend on the constituent quark masses through the combination $m_i + m_j$ (m_i and m_j are the constituent masses of quark and antiquark). In this case, a factor $f_{i\bar{j}}(m_i + m_j)$ is introduced into relations (23)–(25) [32, 46]. Then, these relations are

expressed as

$$M^2_{n\bar{n}N} = M^2_{n\bar{n}1} + \frac{(N-1)}{\alpha'_{n\bar{n}}}(1 + f_{n\bar{n}}(m_n + m_n)), \qquad (26)$$

$$M^2_{n\bar{s}N} = M^2_{n\bar{s}1} + \frac{(N-1)}{\alpha'_{n\bar{s}}}(1 + f_{n\bar{s}}(m_n + m_s)), \qquad (27)$$

$$M^2_{s\bar{s}N} = M^2_{s\bar{s}1} + \frac{(N-1)}{\alpha'_{s\bar{s}}}(1 + f_{s\bar{s}}(m_s + m_s)). \qquad (28)$$

In this work, the parameters used as input are taken from our previous work (Table 2) [46].

Table 2. The slopes [GeV^{-2}] and the parameters f_{nn}, f_{ns} and f_{ss} [GeV^{-1}] of relations (19), (20), and (21).

Parameters	$\alpha'_{n\bar{n}}$	$\alpha'_{n\bar{s}}$	$\alpha'_{s\bar{s}}$	f_{nn}	f_{ns}	f_{ss}
Value	0.7218	0.6613	0.76902	0.3556	0.1376	0.2219

The results are shown in Tables 3, 4 and Figs. 1 and 2.

Table 3. The radial excitation masses of the 1^1D_2 multiplet in this work are compared with the other predictions (in units of MeV). This work (1) and (2) are the solutions of relation (20). The mass used as input for our calculation is shown in boldface.

1^1D_2	N	This work (1)	This work (2)	[47]	[48]	[49]	[50]	[51]
	1	**$1670.6^{+2.9}_{-1.2}$**				1643		1680
$M_{n\bar{n}}$	2	2111.9±2.3				1960		2130
	3	2476.1±2.0				2216		
	1	1761.7 ± 5.6	1767.4 ± 5.9			1709	1778	1780
$M_{n\bar{s}}$	2	2188.7 ± 4.6	2193.3 ± 4.8			2066	2121	2230
	3	2545.0 ± 3.8	2549.0 ± 4.1				2380	
	1	1849.0 ± 8.1		1825	1893	1909		1890
$M_{s\bar{s}}$	2	2262.4 ± 6.7		2282	2336	2321		
	3	2611.1 ± 5.8		2685	2723	2662		

Table 4. The radial excitation masses of the 1^3D_2 multiplet in this work are compared with the other predictions (in units of MeV). This work (1) and (2) are the solutions of relation (21).

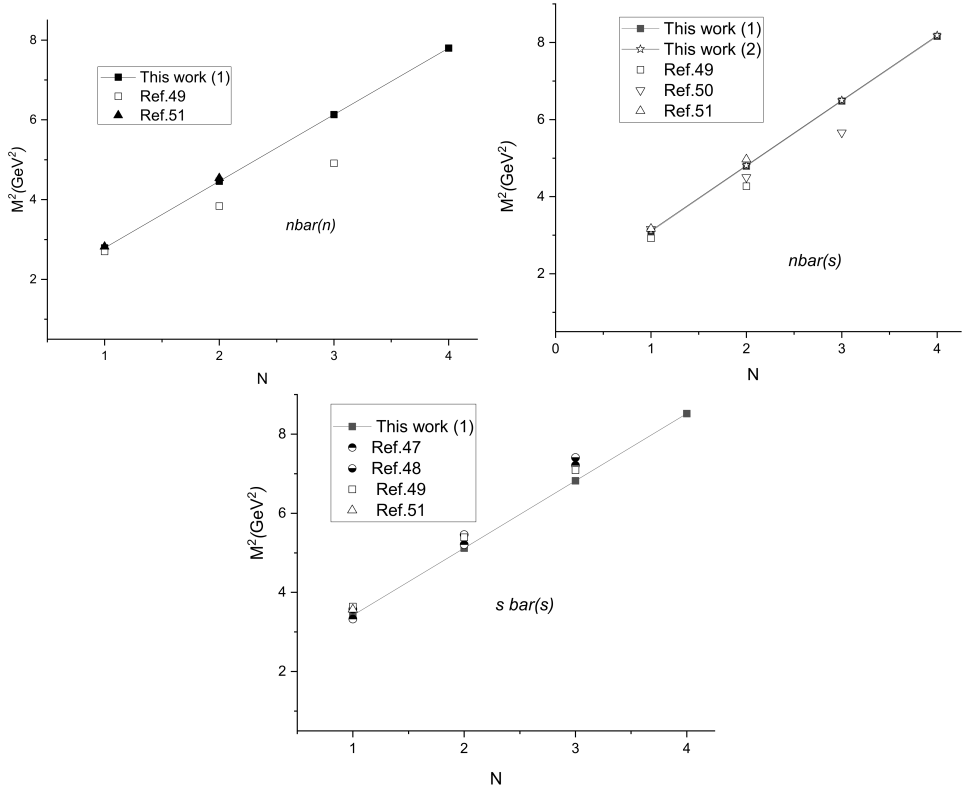| 1^3D_2	N	This work (1)	This work (2)	[47]	[48]	[49]	[50]	[51]
	1	1687.7 ± 15.5	1692.3 ± 15.6			1661		1700
$M_{n\bar{n}}$	2	2125.9 ± 12.3	2129.6 ± 12.4			1983		2150
	3	2488.1 ± 10.6	2491.2 ± 10.6			2241		
	1	1824.8 ± 9.6	1829.7 ± 9.8			1824	1789	1810
$M_{n\bar{s}}$	2	2236.8 ± 7.9	2243.8 ± 8.0			2163	2131	2260
	3	2589.1 ± 6.8	2592.6 ± 7.0				2388	
	1	1952.4 ± 13.5	1957.5 ± 13.6	1840	1904			1910
$M_{s\bar{s}}$	2	2347.7 ± 11.3	2351.9 ± 11.4	2297	2348			
	3	2685.3 ± 9.9	2689.0 ± 10.1	2701	2734			

Fig. 1. The radial excitation of N^1D_2 meson nonet.
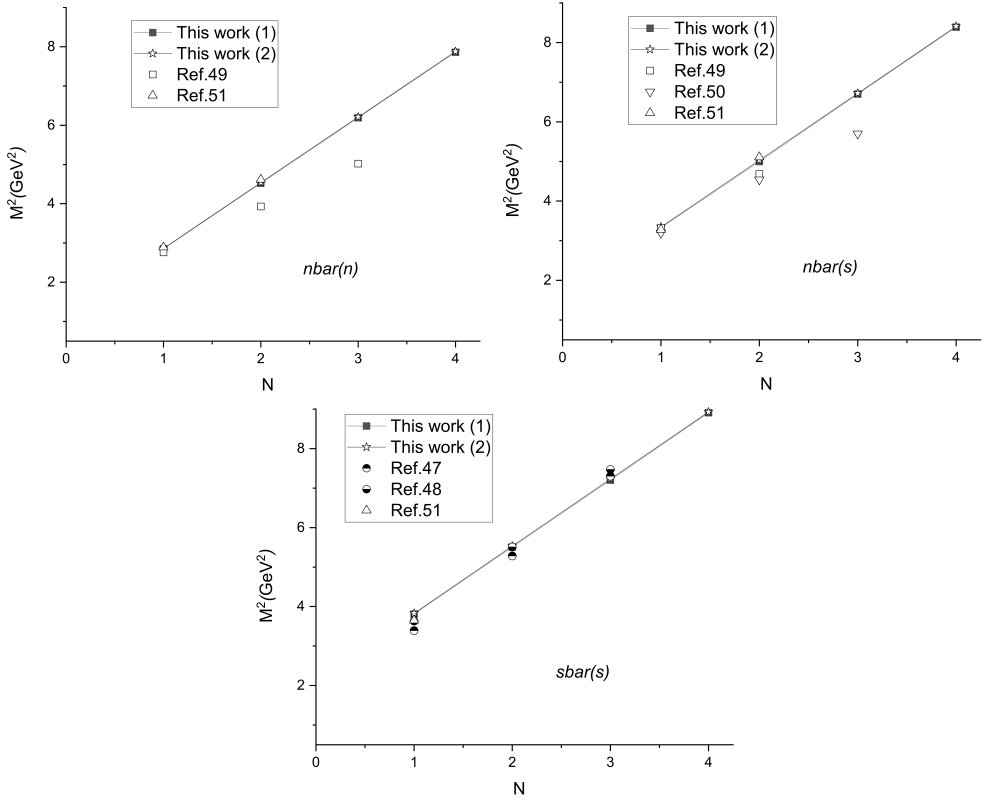
Fig. 2. The radial excitation of N^1D_2 meson nonet.

4. Conclusion

In this work, we investigate the mass spectrum of 1^1D_2 and 1^3D_2 meson nonets in the framework of the meson mass matrix and Regge phenomenology. The mass of $1^1D_2(s\bar{s})$ is determined to be 1849.0 MeV, which agrees with the state $\eta_2(1870)$ with a mass of 1842 ± 8 MeV in PDG. The consistency of masses also indicates that $\eta_2(1870)$ is the candidate for 1^1D_2 meson nonet with a small mixing with $n\bar{n}$ component, the corresponding results need to be further confirmed in the experiment. Apart from the ground meson, the masses of radial excitations of 1^1D_2 are obtained. There are five isovector states with $J^{PC} = 2^{-+}$ ($\pi_2(1670)$, $\pi_2(1880)$, $\pi_2(2050)$, $\pi_2(2100)$, $\pi_2(2285)$) listed in PDG. The $\pi_2(2100)$ is omitted from the summary table, the states $\pi_2(2050)$ and $\pi_2(2285)$ are classified as "further states". In this work, the isovector of 2^1D_2 mass is determined to be 2111.9 MeV, which is consistent with the mass of $\pi_2(2100)$ (2090 ± 29 MeV). We suggest that $\pi_2(2100)$ would be the plausible candidate for 2^1D_2 meson state. For the 1^3D_2 me-

son nonet, only the kaon $K_2(1820)$ is found, and it will mix with $K_2(1770)$ state. In PDG, the four states $\rho_2(1940)$, $\omega_2(1975)$, $\omega_2(2195)$, $\rho_2(2225)$ with $J^{PC} = 2^{--}$ are interpreted as "further states". In this work, we find that there is still a large deviation between our results and the masses of these states. Till now, since the experimental information of 1^3D_2 meson nonet is not sufficient, it is necessary to conduct a systematic and complete study for these states. The results may be useful for the discovery of unobserved states in the experiments (BESIII, CMD-3, and COMPASS).

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