Acta Physica Polonica B **53**, 6-A3 (2022)

RE-EVALUATION OF THE ISOSCALAR MIXING ANGLE WITHIN SELECTED MESONIC NONETS*

Xue-Chao Feng†

College of Physics and Electronic Engineering
Zhengzhou University of Light Industry, 450002 Zhengzhou, China

Ke-Wei Wei

School of Science, Henan University of Engineering
451191 Zhengzhou, China

Received 18 November 2021, accepted 10 May 2022, published online 14 June 2022

Based on the relations from the meson–meson mass mixing matrix, the mixing angles of the isoscalar state have been re-evaluated via mass relations and the latest experimental results. The results in the present work are compared with the values from different theoretical models and the quarkonia content of the isoscalar state is presented. In order to check the validity of the analysis, some predictions on the decays of the isoscalar state are presented. These predictions may be useful for the phenomenological analysis for meson nonet in future experiments.

DOI:10.5506/APhysPolB.53.6-A3

1. Introduction

Quantum Chromodynamics (QCD) was proposed in the 1970s as the basic theory to describe the hadrons and their strong interactions. However, understanding of the strong interactions is far from complete. One of the open problems is the difficulty in interpreting the nature of the experimental data from the first principles. Building models, which capture the most important features of strong QCD, is one way to resolve this problem. Therefore, the spectroscopy and the phenomenological description of conventional mesons become important and a series of theoretical models are built to investigate the meson properties in the hadronic physics [1–5].

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† Corresponding author: `fxchao@zzuli.edu.cn`

In the quark model, conventional mesons are bound states of quarks q and antiquarks $\bar{q}'$ (the flavors of q and q' may be different and the spin is 1/2). The quark and antiquark spins can couple to give a total spin 0 and 1. The total spin couples the orbital angular momentum resulting in the total angular momentum J. Therefore, meson parity and charge conjugation are determined by $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, respectively. The mesons are classified in J^{PC} multiplets, namely, $\rho(770)$, $K^*(892)$, $\omega(782)$, and $\phi(1020)$ as multiplets of $J^{PC} = 1^{--}$, $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, and $f_2'(1525)$ as multiplets of $J^{PC} = 2^{++}$, $\rho_3(1690)$, $K_3^*(1780)$, $\omega_3(1670)$, and $\phi_3(1850)$ as multiplets of $J^{PC} = 3^{--}$, *etc.* According to the «Review of Particle Physics» by the Particle Data Group (PDG) in 2020, the nine $q\bar{q}'$ combinations containing the light quark up, down, and strange quarks are grouped into an octet and a singlet of light quark mesons [6]

$$3 \otimes \bar{3} = 8 \oplus 1\,.$$

Due to the SU(3)-symmetry breaking, the isoscalar physical states appear as mixtures of the singlet and octet members. The singlet–octet mixing is also called SU(3) mixing. Considering of the mixing can provide clues for testing QCD and the constituent quark model and the study of mixing has became the focus of many studies in the literature in the last two decades [7, 8]. In the present work, we re-evaluate the mixing angles and the decays of meson nonet via the latest experimental results. The phenomenological description is meaningful for understanding the nature of new resonances.

2. State mixing and determination of the mixing angles

As discussed in the above section, mesons are $q\bar{q}'$ bound states of quark q and antiquark $\bar{q}'$ in the quark model. In general, the two bare isoscalar states can mix, which results in two physical isoscalar states. In the non-strange $F_\mathrm{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and strange $F_\mathrm{s} \equiv s\bar{s}$ basis, the mass-squared matrix describing the mixing of the two physical isoscalar states can be written as [7, 8]

$$M^2 = \begin{pmatrix} M_{F_\mathrm{n}}^2 + 2A & \sqrt{2}AX \\ \sqrt{2}AX & M_{F_\mathrm{s}}^2 + AX^2 \end{pmatrix}, \tag{1}$$

where M_{F_n} and M_{F_s} are the masses of bare states F_n and F_s, which is a widely adopted assumption [9–11]. A is the mixing parameter which describes the $q\bar{q} \leftrightarrow q'\bar{q}'$ transition amplitudes [12, 13]. X is a phenomenological parameter which describes the SU(3) broken ratio of the non-strange and strange quark propagators via the constituent quark-mass ratio. In the present work, the constituent quark masses in different phenomenological models are shown in Table 1. In Table 1, $m_{\rm n}$ and $m_{\rm s}$ denote the constituent quarks masses (where and below n denotes up and down quark).

Table 1. Constituent quark masses (in MeV) in different phenomenological models.

Mass	$m_{\rm n}(n=u,d)$	$m_{\rm s}$	$X=\frac{m_{\rm n}}{m_{\rm s}}$
Ref. [4]	220	419	0.53
Ref. [14]	229	460	0.63
Ref. [15]	360	540	0.67
Ref. [16]	337.5	486	0.70
Ref. [17]	311	487	0.64
Ref. [18]	310	483	0.64
Average values	321	491.2	0.66

In a meson nonet, the isoscalar physical states φ and φ' are the eigenstates of the mass-squared matrix, and the masses square of M_φ^2 and $M_{\varphi'}^2$ are the eigenvalues, respectively. In the present work, φ is mainly a non-strange component and φ' is mainly a strange component. The physical states φ and φ' can be related to the $F_{\rm s}$ and $F_{\rm n}$ by

$$\begin{pmatrix} |\varphi\rangle \\ |\varphi'\rangle \end{pmatrix} = U \begin{pmatrix} F_{\rm n} \\ F_{\rm s} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} F_{\rm n} \\ F_{\rm s} \end{pmatrix}, \tag{2}$$

where β is the mixing angle in the basis of $F_{\rm s}$ and $F_{\rm n}$. The unitary matrix U can be described as

$$M^2 = U^\dagger \begin{pmatrix} M_\varphi^2 & 0 \\ 0 & M_{\varphi'}^2 \end{pmatrix} U\,. \tag{3}$$

In addition to relation (2), the mix of the physical isoscalar states can be also described as

$$\begin{pmatrix} |\varphi\rangle \\ |\varphi'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_8 \end{pmatrix} \tag{4}$$

with

$$\phi_1 = \frac{\left(u\bar{u}+d\bar{d}+s\bar{s}\right)}{\sqrt{3}}\,, \qquad \phi_8 = \frac{\left(u\bar{u}+d\bar{d}-2s\bar{s}\right)}{\sqrt{6}}\,,$$

where θ is the SU(3) singlet–octet mixing angle.

With the help of

$$\begin{pmatrix} \phi_1 \\ \phi_8 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} F_{\rm n} \\ F_{\rm s} \end{pmatrix}, \tag{5}$$

the following relation is obtained:

$$\begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix}. \tag{6}$$

From Eqs. (1), (2), and (6), the following relations are obtained:

$$\begin{aligned} M_{F_{\rm n}}^2 + 2A &= \left(\sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta\right)^2 M_{\varphi'}^2 \\ &\quad + \left(\sqrt{\frac{2}{3}}\cos\theta + \sqrt{\frac{1}{3}}\sin\theta\right)^2 M_{\varphi}^2, \end{aligned} \tag{7}$$

$$\begin{aligned} M_{F_{\rm s}}^2 + AX^2 &= \left(\sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta\right)^2 M_{\varphi}^2 \\ &\quad + \left(\sqrt{\frac{2}{3}}\cos\theta + \sqrt{\frac{1}{3}}\sin\theta\right)^2 M_{\varphi'}^2, \end{aligned} \tag{8}$$

$$\begin{aligned} \sqrt{2}AX &= \left(\sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta\right)\left(\sqrt{\frac{2}{3}}\cos\theta + \sqrt{\frac{1}{3}}\sin\theta\right) \\ &\quad \times \left(M_{\varphi}^2 - M_{\varphi'}^2\right). \end{aligned} \tag{9}$$

One can see from the above relation that the mixing angle and corresponding physical states are correlated. Inserting the masses of corresponding physical states and the constituent quark-mass ratio into relations (7), (8), and (9), we can obtain the SU(3) singlet–octet mixing angle and the results are shown in Table 2. In the present work, considering the fact that $F_{\rm n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ is the orthogonal partner of the isovector state of a meson nonet, one can expect that $F_{\rm n}$ degenerates with the isovector state $M_{F_{\rm n}} = M_{I=1}$ [9–11, 19]. Here and below, all the masses used as an input for our calculation are taken from the PDG (Table 3).

From Table 2, we find that the SU(3) singlet–octet mixing angles of 1^3S_1, 1^3P_2, and 1^3D_3 are consistent with existing experimental results and other theoretical predictions [6, 20–22]. Moreover, we also present the mixing angles η and η' in the 1^1S_0 meson nonet. The nature of η and η' meson is a

Table 2. The SU(3) singlet–octet mixing angles for 1^3S_1, 1^3P_2, 1^3D_3, 1^1S_0, and 2^1S_0 meson nonets.

J^{PC}, $N^{2S+1}L_J$	1^{--}, 1^3S_1	2^{++}, 1^3P_2	3^{--}, 1^3D_3	0^{-+}, 1^1S_0	0^{-+}, 2^1S_0
$\theta(^\circ)$	35.9 ± 0.1	31.1 ± 0.2	32.4 ± 0.7	-4.2 ± 0.3	34.4 ± 13.8

Table 3. The parameters in relations (7), (8), and (9).

J^{PC}, $N^{2S+1}L_J$	M_{F_n} [MeV]	M_{F_s} [MeV]	A [MeV^2]
1^{--}, 1^3S_1	782.66	1018.09	0.005799
2^{++}, 1^3P_2	1275.5	1523.84	0.053521
3^{--}, 1^3D_3	1667	1857.74	0.035731
0^{-+}, 1^1S_0	978.78	639.02	0.324512
0^{-+}, 2^1S_0	1294	1477.11	0.601166

longstanding subject in hadron physics, which can provide important information of the low energy dynamics of QCD. Nonetheless, there is apparent disagreement for the 1^1S_0 meson nonet. The reason for that may be that we have not taken into account the mixing with glueball in our calculation. In Refs. [23, 24], authors indicate that the η' may have a large glueball content.

Based on relations (2) and (4), we obtain the quarkonia contents for the 1^3S_1, 1^3P_2, 1^3D_3, 1^1S_0, and 2^1S_0 meson nonet in the non-strange $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and strange $s\bar{s}$ basis. The results are shown in Table 4.

Table 4. The quarkonia content of the isoscalar state for the 1^3S_1, 1^3P_2, 1^3D_3, 1^1S_0, and 2^1S_0 meson nonet.

J^{PC}, $N^{2S+1}L_J$	φ	φ'	$\beta(^\circ)$	$\cos\beta$	$\sin\beta$
1^{--}, 1^3S_1	$\omega(782)$	$\phi(1020)$	-0.6	0.9999	-0.0105
2^{++}, 1^3P_2	$f_2(1270)$	$f_2'(1525)$	4.2	0.9971	0.0733
3^{--}, 1^3D_3	$\omega_3(1670)$	$\phi_3(1850)$	2.9	0.9987	0.0506
0^{-+}, 1^1S_0	$\eta'(958)$	η	-39.5	0.7716	-0.6361
0^{-+}, 2^1S_0	$\eta(1295)$	$\eta(1475)$	-0.9	0.9998	-0.0157

3. Decays and mixing

In order to check the consistency of our results in Table 4, we compared the decays of isoscalar of meson nonet with the experimental data. According to Refs. [22, 25], we have the following relations:

For the 1^3S_1 meson state

$$\frac{\Gamma(\phi \to \pi\gamma)}{\Gamma(\omega \to \pi\gamma)} = \frac{\sin^2 \beta_{(1^3S_1)}}{\cos^2 \beta_{(1^3S_1)}} \left(\frac{(M_\phi^2 - M_\pi^2)M_\omega}{(M_\omega^2 - M_\pi^2)M_\phi} \right)^3 , \tag{10}$$

$$\frac{\Gamma(\phi \to \pi^+\pi^-)}{\Gamma(\omega \to \pi^+\pi^-)} = \frac{\sin^2 \beta_{(1^3S_1)}}{\cos^2 \beta_{(1^3S_1)}} \left(\frac{\sqrt{M_\phi^2 - 4M_{\pi^\pm}^2}}{\sqrt{M_\omega^2 - 4M_{\pi^\pm}^2}} \right)^3 . \tag{11}$$

For the 1^3P_2 meson state

$$\frac{\Gamma(f_2(1270) \to \gamma\gamma)}{\Gamma(a_2(1320) \to \gamma\gamma)} = \frac{1}{9} \left(5\cos^2 \beta_{(1^3P_2)} + \sqrt{2}\sin^2 \beta_{(1^3P_2)}\right)^2 \times \left(\frac{M_{f_2(1270)}}{M_{a_2(1320)}} \right)^3 , \tag{12}$$

$$\frac{\Gamma(f_2'(1525) \to \gamma\gamma)}{\Gamma(a_2(1320) \to \gamma\gamma)} = \frac{1}{9} \left(5\sin^2 \beta_{(1^3P_2)} - \sqrt{2}\cos^2 \beta_{(1^3P_2)}\right)^2 \times \left(\frac{M_{f_2'(1525)}}{M_{a_2(1320)}} \right)^3 . \tag{13}$$

For the 2^1S_0 meson state

$$\frac{\Gamma(\eta_{(1295)} \to \gamma\gamma)}{\Gamma(\pi_{(1300)} \to \gamma\gamma)} = \frac{1}{9} \left(5\cos^2 \beta_{(2^1S_0)} + \sqrt{2}\sin^2 \beta_{(2^1S_0)}\right)^2 \left(\frac{M_{\eta 1295}}{M_{\pi 1300}} \right)^3 , \tag{14}$$

$$\frac{\Gamma(\eta_{(1475)} \to \gamma\gamma)}{\Gamma(\pi_{(1300)} \to \gamma\gamma)} = \frac{1}{9} \left(5\sin^2 \beta_{(2^1S_0)} - \sqrt{2}\cos^2 \beta_{(2^1S_0)}\right)^2 \left(\frac{M_{\eta 1475}}{M_{\pi 1300}} \right)^3 , \tag{15}$$

$$\frac{\Gamma(\eta(1295) \to \omega\gamma)}{\Gamma(\eta(1295) \to \rho\gamma)} = \frac{1}{9} \left(\frac{M_{\eta(1295)}^2 - M_\omega^2}{M_{\eta(1295)}^2 - M_\rho^2} \right)^3 \times \left(\frac{\cos \beta_{(1^3S_1)} \cos \beta_{(2^1S_0)} - 2\sin \beta_{(1^3S_1)} \sin \beta_{(2^1S_0)}}{\cos \beta_{(2^1S_0)}} \right)^2 , \tag{16}$$

$$\frac{\Gamma(\eta(1475)\to\omega\gamma)}{\Gamma(\eta(1475)\to\rho\gamma)} = \frac{1}{9}\left(\frac{M^2_{\eta(1475)}-M^2_\omega}{M^2_{\eta(1475)}-M^2_\rho}\right)^3 \times\left(\frac{\cos\beta_{(1^3S_1)}\sin\beta_{(2^1S_0)}+2\sin\beta_{(1^3S_1)}\cos\beta_{(2^1S_0)}}{\sin\beta_{(2^1S_0)}}\right)^2, \tag{17}$$

$$\frac{\Gamma(\eta(1295)\to\phi\gamma)}{\Gamma(\eta(1295)\to\rho\gamma)} = \frac{1}{9}\left(\frac{M^2_{\eta(1295)}-M^2_\phi}{M^2_{\eta(1295)}-M^2_\rho}\right)^3 \times\left(\frac{\sin\beta_{(1^3S_1)}\cos\beta_{(2^1S_0)}+2\cos\beta_{(1^3S_1)}\sin\beta_{(2^1S_0)}}{\cos\beta_{(2^1S_0)}}\right)^2, \tag{18}$$

$$\frac{\Gamma(\eta(1475)\to\phi\gamma)}{\Gamma(\eta(1475)\to\rho\gamma)} = \frac{1}{9}\left(\frac{M^2_{\eta(1475)}-M^2_\phi}{M^2_{\eta(1475)}-M^2_\rho}\right)^3 \times\left(\frac{\sin\beta_{(1^3S_1)}\sin\beta_{(2^1S_0)}-2\cos\beta_{(1^3S_1)}\cos\beta_{(2^1S_0)}}{\sin\beta_{(2^1S_0)}}\right)^2. \tag{19}$$

The predicted results of (10)–(19) are determined as shown in Table 5.

Table 5. The predicted results in our framework (10)–(19).

Relation	This work	Expt. [6]	Relation	This work	Expt. [6]
(10)	4.23×10^{-4}		(11)	3.48×10^{-4}	0.0024 ± 0.0006
(12)	2.60	3.03 ± 0.40	(13)	0.19	0.0081 ± 0.0011
(14)	2.76		(15)	0.29	
(16)	0.11		(17)	0.59	
(18)	1.88×10^{-4}		(19)	1.76×10^{3}	

4. Conclusions

In summary, we have studied the mixing angles of the isoscalar state based on the relations from the meson–meson mass mixing matrix. In order to check the consistency of our results, we compared the values given in different references. On the one hand, from Table 2, we can see that the mixing angles of 1^3S_1, 1^3P_2, and 1^3D_1 meson nonet in this work are in agreement with Refs. [6, 20]. Moreover, the decays of the isoscalar state of meson nonet are presented in Table 5. On the other hand, we also find that the calculated results for the pseudoscalar meson are inconsistent with values from other theoretical models. The reason for this may be that we have not considered the mixing between η and η' with the pseudoscalar glueball. In the past forty years, the quarkonia–glueball structure of η and η' has been discussed many times [9, 26, 27]. Therefore, we speculate that the η' may have a large glueball content.

In our work, we did not discuss the mixing angles between $f_1(1285)$ and $f_1(1420)$, and between $h_1(1170)$ and $h_1(1415)$. These two mixing angles are related to the mixing between K_{1A} and K_{1B}. The K_{1A} and K_{1B} mixing is investigated in Refs. [28–31]. Although we have not calculated these parameters now, with the further enrichment of experimental data, we can still do a lot of analysis by using the relationship in the future. In addition, if the mixing angle can be determined, the relations we get can also be used to analyze the mass of some physical states.

This project was supported by the Zhengzhou University of Light Industry Foundation of China (grants No. 2009XJJ011 and 2012XJJ008) and the Key Project of Scientific and Technological Research of the Education Department of Henan Province (grant No. 13B140332).

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