# ASYMPTOTIC NORMALIZATION COEFFICIENTS FOR THE $^{17}\mathrm{F} \to {}^{16}\mathrm{O} + p$ CONFIGURATION FROM THE $^{16}\mathrm{O}(^{10}\mathrm{B}, {}^{9}\mathrm{Be})^{17}\mathrm{F}$ REACTION AND ESTIMATION OF THE $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ ASTROPHYSICAL $S$-FACTOR

F.Kh. Ergashev, S.V. Artemov, O.R. Tojiboev
A.A. Karakhodzhaev

Institute of Nuclear Physics, 100214 Tashkent, Uzbekistan

K. Rusek, A. Trzcińska, M. Wolińska-Cichocka, E. Piasecki

Heavy Ion Laboratory University of Warsaw, 02-093 Warsaw, Poland

N. Burtebayev, S.B. Sakuta, Maulen Nassurlla
Marzhan Nassurlla

Institute of Nuclear Physics, 050032 Almaty, Kazakhstan

R.F. Rumi

Institute of Microelectronics and Semiconductors
National University of Uzbekistan, Tashkent

*Received 20 May 2022, accepted 7 September 2022, published online 7 October 2022*

Suitability of the $^{16}\mathrm{O}(^{10}\mathrm{B}, {}^{9}\mathrm{Be})^{17}\mathrm{F}$ reaction near the Coulomb barrier energy for extraction of the ANC values and indirect determination of the astrophysical $S$-factor for proton radiative capture by $^{16}\mathrm{O}$ nucleus is studied. New experimental data on this reaction to the ground ($5/2^+$) and excited ($1/2^+$, $E^* = 0.495$ MeV) states of $^{17}\mathrm{F}$ at the $^{10}\mathrm{B}$ beam energy of 41.3 MeV from the U-200P cyclotron (HIL, University of Warsaw) are presented. The ANC squared values $1.03 \pm 0.13$ fm$^{-1}$ and $5430 \pm 950$ fm$^{-1}$ for the ground and first excited states of the $^{17}\mathrm{F}$ nucleus, respectively, were obtained using the modified DWBA analysis. The value $S(0) = 9.0 \pm 1.5$ keVb was found for the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ reaction using the ANC values.

DOI:10.5506/APhysPolB.53.9-A5

## 1. Introduction

The reactions involving the nucleons transfer in the interactions of light ions near the Coulomb barrier are very informative on research of the structure of nuclear single-particle states [1]. As a rule, these are peripheral processes that make it possible to obtain reliable values of asymptotic normalization coefficients (ANCs) of the corresponding configurations used in calculations of nuclear astrophysical processes [2–4].

In particular, the radiative proton capture $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ plays an important role in the CNO cycle, compensating for the loss of the CN catalyst in the main CN cycle due to the $^{15}\mathrm{N}(p,\gamma)^{16}\mathrm{O}$ reaction by a sequence of reactions: $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}(\beta^{+}\nu)^{17}\mathrm{O}(p,\alpha)^{14}\mathrm{N}$ [5, 6].

In the range of low relative energies ($E_p < 2.5$ MeV), the $S$-factor of the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ reaction is completely determined by the direct mechanism of proton capture and can be calculated using the ANC values for the proton bound states in $^{17}\mathrm{F}$ nucleus.

To date, there are several works in which the ANC values for bound states $^{17}\mathrm{F}$ (g.s., $5/2^{+}$ and $E^{*} = 0.495$ MeV, $1/2^{+}$ state) were found from the analysis of the $^{16}\mathrm{O}(^{3}\mathrm{He},d)^{17}\mathrm{F}$ reaction [7, 8] and $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ radiative capture [9]. In Ref. [10] (see also [11]), a bibliography of experimental data on the cross sections for the radiative capture of a proton by the $^{16}\mathrm{O}$ nucleus is given, and a detailed analysis of these data is carried out. Also, in [12] (and the references therein), a review of theoretical works on the calculations of these ANCs is given and the theoretical estimates were made within the method of analytic continuation of the scattering data to the negative-energy region.

In general, the spread of these values is rather large amounting to about 20% for the $s_{1/2}$ state ($E^{*} = 0.495$ MeV) which is stable with respect to the proton emission. This uncertainty does not satisfy the requirements for testing various astrophysical scenarios.

The purpose of this work is to measure the differential cross sections (DCSs) of the proton transfer $^{16}\mathrm{O}(^{10}\mathrm{B},^{9}\mathrm{Be})^{17}\mathrm{F}$ reaction at the energy near the Coulomb barrier to check their suitability for obtaining the ANCs $C^2_{^{17}\mathrm{F}\to^{16}\mathrm{O}+p} = C^2_{16\,1}$ and $C^2_{^{17}\mathrm{F}^{*}\to^{16}\mathrm{O}+p} = C^2_{16\,1^{*}}$ values, and to extract them by means of the widely used now modified distorted waves Born approximation (MDWBA) — see, for example, [3, 13] and references therein.

## 2. Experiment

Measurements of the DCS of the $^{16}\mathrm{O}(^{10}\mathrm{B},^{9}\mathrm{Be})^{17}\mathrm{F}$ reaction were carried out with the $^{10}\mathrm{B}$ ion beam of the U-200P cyclotron of the Heavy Ion Laboratory (University of Warsaw) at the energy $E_{^{10}\mathrm{B}} = 41.3$ MeV.

The reaction ($^{10}$B, $^{9}$Be) is rather convenient for the indicated above purposes. First, the ANC value for the $^{10}\mathrm{B} \to {}^{9}\mathrm{Be} + p$ configuration is well known [1]. Secondly, the projectile ($^{10}$B) is lighter than the target ($^{16}$O), and there is no kinematic compression of the angular distributions in the laboratory system, which provides a fairly good angular resolution. Finally, the $^{9}$Be nucleus does not have nucleon-stable excited states. Therefore, there is no interference between the $^{17}$F and $^{9}$Be levels in the energy spectrum of $^{9}$Be. In addition, the selected beam energy ($E_{^{10}\mathrm{B}} = 41.3$ MeV) is close to the Coulomb barrier, which allows us to expect the peripherality dominance in the proton transfer process.

The experimental technique was described in [4], so here it is presented only briefly. The targets were self-supporting $Al_2O_3$ films $\sim 0.15$ mg/cm$^2$ thick. The target thicknesses were measured by the energy loss of $\alpha$-particles from the Ra-226 $\alpha$-source with an error of $\sim$ 6–7%. The charged reaction products were recorded and identified by $\Delta E$–$E$ telescopes installed in the experimental chamber of the ICARE multi-detector facility, which includes systems of remotely controlled platforms with telescopes and a target device [14, 15]. Four $\Delta E$–$E$ telescopes were used, consisting of silicon detectors ($E$) about 500 $\mu$m thick and ionization chambers ($\Delta E$) filled with isobutane at the pressure of $\sim$ 55 Torr. Standard CAMAC electronics and the MIDAS and SMAN data acquisition systems [16] were used.

The total energy resolution was about 500 keV and was mainly determined by the energy spread of the primary beam and by the target thickness.

A typical two-dimensional ($\Delta E$–$E$)-spectrum obtained at the interaction of $^{10}$B ions with an $Al_2O_3$ target is shown in Fig. 1. It can be seen that the *loci* corresponding to the registration of the $^{9}$Be and $^{10}$B particles are well separated from neighboring ones.

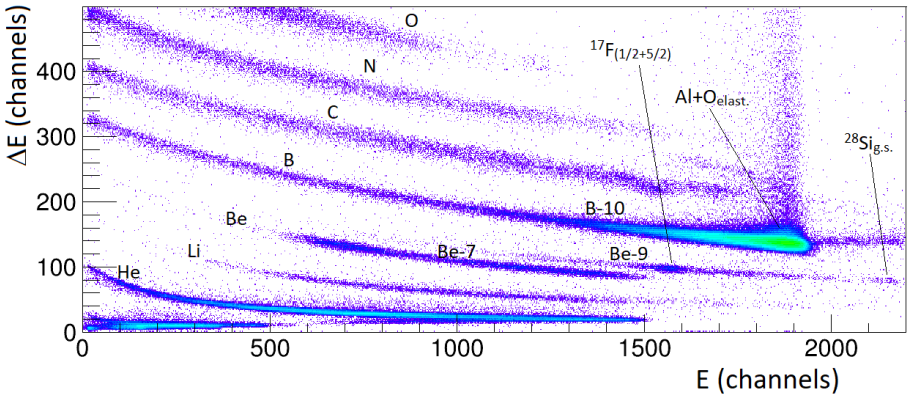


Fig. 1. Two-dimensional ($\Delta E$–$E$)-spectrum measured at the angle of 7° for the $Al_2O_3$ target irradiated with $^{10}$B ions.

The two-dimensional spectra were processed using the ROOT software [17] to obtain the angular distributions of the experimental differential cross sections for the angles of the forward hemisphere.

The energy spectrum of scattered $^{10}$B is shown in Fig. 2. The $^{10}$B ions scattered on the $^{16}$O and $^{27}$Al nuclei are kinematically separated only at angles $\theta_{\text{lab}} > 22°$. In Fig. 3, the energy spectrum of $^{9}$Be from the ($^{10}$B, $^{9}$Be) reaction is presented. As can be seen, the peaks corresponding to the formation of the $^{17}$F nucleus in the ground ($5/2^+$) and first excited ($1/2^+$) states are not resolved in our measurements. The processing of the energy spectra is given below.

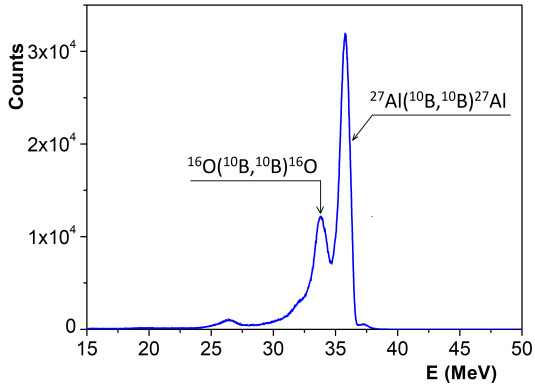


Fig. 2. Fragment of the energy spectrum of $^{10}$B particles scattered by the $Al_2O_3$ target at an angle of 22°.

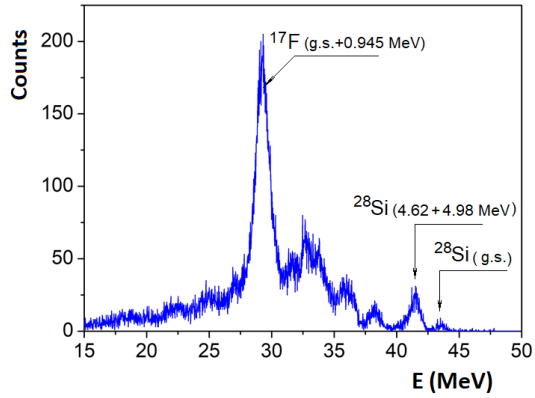


Fig. 3. Energy spectrum of the $^{9}$Be particles from the ($^{10}$B, $^{9}$Be) reaction measured at an angle $\theta_{\text{lab}} = 22°$ with the $Al_2O_3$ target.

### 3. Elastic $^{16}$O + $^{10}$B scattering and optical potentials

In the region of angles ($\theta > 22^\circ$), where the peaks of elastic scattering by $^{16}$O and $^{27}$Al are resolved, the elastic $^{16}$O + $^{10}$B scattering cross sections were extracted directly from the spectra. For smaller angles, these peaks were separated in proportion to the cross sections calculated with global OPs for $^{27}$Al + $^{10}$B and $^{16}$O + $^{10}$B scattering [18] taking into account the precisely known ratio of $^{27}$Al and $^{16}$O nuclei in $Al_2O_3$ target. It was assumed that the uncertainties of the parameters of the nuclear part of the OP have an insignificant effect on the DCS values determined in this way, due to the Rutherford scattering dominances in the region of small angles. The obtained cross sections are in good agreement with the directly measured cross sections in the angular region, where the peaks are kinematically separated (Fig. 4). The angular distribution of the DCSs of elastic $^{16}$O+$^{10}$B scattering is compared with the DCSs calculated by the optical model using the OP parameters obtained in [19] at a close to our beam energy $E_{^{10}\mathrm{B}} = 41.6$ MeV (see also [20]). The angular distribution is well described by these OP parameters, and our DCS values within the error coincide with the experimental ones obtained in [19]. A good agreement with the data from [19] confirms the correctness of the absolute normalization of the $^{16}$O($^{10}$B, $^{9}$Be)$^{17}$F reaction experimental data obtained in our measurements, the error of which, according to our estimates, does not exceed 9%. Other sets of OPs found in the literature for the input channel were rejected by the quality of the fit to the obtained data on $^{16}$O + $^{10}$B elastic scattering.

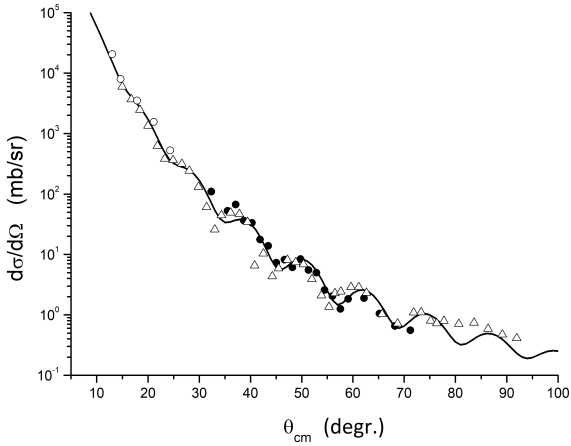


Fig. 4. Angular distribution of the $^{16}$O + $^{10}$B elastic scattering. The open and black circles are our experimental DCSs obtained in the regions of unresolved and kinematically separated peaks of scattering by $^{27}$Al and $^{16}$O, respectively. The open triangles and the solid line are the experimental DCSs and the optical model calculation using the OP from [19].

The OPs for the $^{17}F + {}^{9}Be$ output channel were deduced from the data on the scattering of nuclei neighboring in mass and charge. The selected sets of the OP parameters are presented in Table 1.

Table 1. OP parameters used in the DCS calculations of the $^{16}O(^{10}B, {}^{9}Be)^{17}F$ reaction. Letters A and B denote the OP sets for the entrance and exit channels, respectively. The subscripts $D$ and $V$ denote the surface and volume form of the imaginary OP. The index C refers to the Coulomb potential.

| Set | $V$ [MeV] | $r_V$ [fm] | $a_V$ [fm] | $W_V(W_D)$ [MeV] | $r_W(r_D)$ [fm] | $a_W(a_D)$ [fm] | $r_C$ [fm] | Ref. |
|---|---|---|---|---|---|---|---|---|
| A | 49.03 | 2.276 | 0.523 | 44.37 | 2.421 | 0.302 | 2.276 | [19] |
| B1 | 70.90 | 1.259 | 1.123 | 147.9 | 1.728 | 0.673 | 1.188 | [21] |
| B2 | 134.0 | 1.744 | 0.694 | 11.50 | 2.353 | 0.694 | 2.261 | [22] |
| B3 | 262.8 | 1.270 | 0.726 | 11.26 | 1.640 | 0.600 | 1.556 | [23] |
| | | | | (48.375) | (1.200) | (0.843) | | |

## 4. Processing of the experimental reaction spectra

As can be seen from the energy spectrum of $^{9}Be$ (see Fig. 3), the peaks of ground $(5/2^+)$ and first excited $(1/2^+)$ states of the $^{17}F$ nucleus are not resolved. In addition, in this region of the spectrum, there is some background due to the $^{27}Al(^{10}B, {}^{9}Be)^{28}Si$ reaction with the population of a large number of highly excited states of the $^{28}Si$ nucleus [24]. However, it is small compared to the peak corresponding to the $^{17}F$ nucleus, and (as it will be shown below — see Fig. 5) does not exceed 20–50% relative to the number of events related to the $1/2^+$ channel of the $^{16}O(^{10}B, {}^{9}Be)^{17}F$ reaction (in which the DCS is minimal) — see also Fig. 5. The background substrate was taken into account in the form of a trapezoid with the bases to the right and left of the oxygen peak corresponding to the experimentally observed background levels. The correctness of this estimate was verified using the DWBA calculations of the background intensity with the "experimental" spectroscopic factors from [25] (and references therein) for strongly populating states of $^{28}Si$. It turned out that the experimentally observed background level (immediately to the left and to the right of the oxygen peak) agrees well with these estimates, and the estimated background substrate located directly below the oxygen peak rather well corresponds to the approximation by trapezoid. Thus, the total $^{17}F$ peak after background subtraction contains fairly reliable information about the summed $(5/2^+)$ and $(1/2^+)$ cross sections, and it is necessary to separate their contributions.

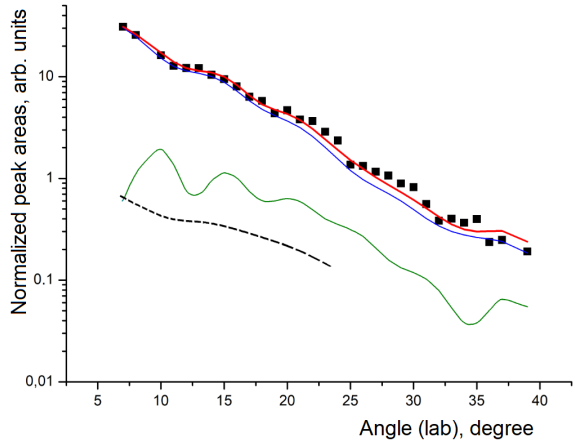


Fig. 5. Results of separation of the peaks corresponding to the overlapped $5/2^+$ and $1/2^+$ channels of the $^{16}$O($^{10}$B, $^{9}$Be)$^{17}$F reaction.

For this reason, an artificial separation of the total number $S_i(\theta_i)$ of events in each unresolved peak (normalized to the integral of the $^{10}$B ion beam at each angle of measurement $\theta_i$) can be carried out in proportion to the calculated (within the DWBA approach) single-particle DCS values $\sigma^{\rm DW}_{5/2}(\theta_i)$ and $\sigma^{\rm DW}_{1/2}(\theta_i)$ of proton transfer to the considered states of $^{17}$F. The rationale for the procedure is as follows. The shape of the angular distribution of each of the reaction channels to the $5/2^+$ and $1/2^+$ states should correspond to the behavior of single-particle DCSs for simple proton stripping calculated within the DWBA, provided that this process strongly dominates in both cases. In such a case, only the ambiguity of the OP parameters used can lead to some distortions.

Obviously, the $S_i$ values, being normalized also to the surface density of $^{16}$O nuclei in the target and the solid angle of the detector, have the meaning of the summed experimental DCSs, $\sigma_i(\theta_i)$ in the laboratory system. Therefore, the procedure for separating the peak areas is reduced to determining the contribution of the DCS of each reaction channel to the summed cross section.

Thus, assuming that the averaged over all combinations of the selected OPs (see Table 1) (and converted to the laboratory system) DCSs $\bar{\sigma}^{\rm DW}_{5/2}(\theta_i)$ and $\bar{\sigma}^{\rm DW}_{1/2}(\theta_i)$ adequately determine the experimental angular distributions, the weight factors $a_{5/2}$ and $a_{1/2}$ were found by the $\chi^2$ method for the functional $\sum_i \{\sigma_i(\theta_i) - a_{5/2} \times \bar{\sigma}^{\rm DW}_{5/2}(\theta_i) - a_{1/2} \times \bar{\sigma}^{\rm DW}_{1/2}(\theta_i)\}^2$. Here, the summation is carried out over all measurement angles $\theta_i$. Calculations of the single particle DCSs are made using the DWUCK5 code [26] under the assumption of proton transfer with the orbital moments $\ell_p = 2$ to the state $1d_{5/2}$ and with

$\ell_p = 0$ to the state $2s_{1/2}$. For the potentials of the proton bound states in the Woods–Saxon form, the "standard" values of the geometric parameters $r_0 = 1.25$ fm and $a = 0.65$ fm were used. Obviously, the factors $a_{5/2}$ and $a_{1/2}$ also determine the ratio of normalized peak areas $S_{5/2}(\theta_i)$ and $S_{1/2}(\theta_i)$ for each channel in the summed peak. The results of such a procedure are demonstrated in Fig. 5. The experimental points (squares) are the normalized (see above) summed events in the unresolved oxygen peaks, which are actually the sums of experimental DCS for $5/2^+$ and $1/2^+$ reaction channels in the laboratory coordinate system. Blue and green curves are the separated $5/2^+$ and $1/2^+$ DCSs defined by $\chi^2$ fitting to the experimental points *i.e.* $a_{5/2} \times \bar{\sigma}^{\mathrm{DW}}_{5/2}(\theta_i)$ and $a_{1/2} \times \bar{\sigma}^{\mathrm{DW}}_{1/2}(\theta_i)$. The red curve is the resulting summed DCS. It should be noted that this separation procedure is successful due to the significantly different shapes of the angular dependence of the DCS for the processes being separated. The dashed curve is the DWBA estimate of the summed DCS of the reactions on the $^{27}$Al nuclei (background level).

The obtained in such a way "simulated" experimental values (converted to the c.m. system), $\sigma^{\mathrm{exp}}_{5/2}(\theta_i)$ and $\sigma^{\mathrm{exp}}_{1/2}(\theta_i)$, are shown in Figs. 6 and 8 together with the calculated angular distributions (see below).

The errors of the DCS values are determined by the statistical error of the $S_i(\theta_i)$ values, the ambiguities of the OP, and the error of the $\chi^2$ fitting. The main contribution to the errors comes from the peak's separation procedure. The total errors of the obtained cross sections at small angles (in the region of normalization of the theoretical cross sections) for the transitions to the $1d_{5/2}$ and $2s_{1/2}$ states of the $^{17}$F nucleus are $\sim$ 8–15% and $\sim$ 13–20%, respectively.

The procedure proposed in [27] was used to evaluate the validity of this separation method. In this procedure, the peaks were assumed to be Gaussian, and the positions of the peaks corresponding to the ground $5/2^+$ and excited $1/2^+$ states of the final $^{17}$F nucleus were fixed in accordance with the energy calibration of the spectrum scale. At each angle, the half-widths of the peak were fixed equal to the half-width of the elastic scattering peak from the same measurement. The background substrate was taken in the form of a Gaussian segment with a large half-width and with a shifted center of Gaussian, so that it visually passed through the readings to the right and left of the peak (see Fig. 5). The heights of the Gaussians corresponding to the $5/2^+$ and $1/2^+$ states were fitted by the $\chi^2$ method to the shape of the experimental peak after the background subtraction. This procedure makes it possible to obtain separation with acceptable errors only in the range of angles 20°–25° lab. ($\sim$ 30°–40° c.m.), where the ratio DCS$(1/2^+)$/DCS$(5/2^+)$ turned out to be maximum ($\sim$ 0.15–0.20). The area ratios of the separated peaks found by this "direct" method coincide within an error of 20–30% with the results of the first method.

First of all, after obtaining cross sections for two separated reaction channels in this way, it is necessary to evaluate the contribution of other reaction mechanisms. The Fresco program [28] was used to estimate the effects of channel reaction coupling (CRC) in the transition to the ground $(5/2^+)$ and excited $(1/2^+)$ states of $^{17}$F. In addition to transitions to the $^{17}$F states in the proton transfer reaction, the coupling scheme also included elastic and inelastic scattering in the $^{16}\mathrm{O} + {}^{10}\mathrm{B}$ system with the excitation of the $E^* = 0.718$ MeV $(1^+)$ and $E^* = 6.13$ MeV $(3^-)$ states in the $^{10}$B and $^{16}$O nuclei, respectively. All combinations of the OPs for entrance and exit reaction channels from Table 1 were probed.

The results of the calculation with the A+B1 OP combination are presented in Fig. 6 together with the obtained experimental DCSs. For the ground state, calculations show that the CRC effects are negligible at angles less than $\sim 25°$, and the one-step peripheral proton stripping dominates. For the first excited state, the estimation of the CRC contribution is 3–7% at the second maximum of the angular distribution for different sets of OP. This result, as well as the agreement with the results of direct separation of the peaks areas, confirm the validity of the method used for obtaining the experimental DCSs.

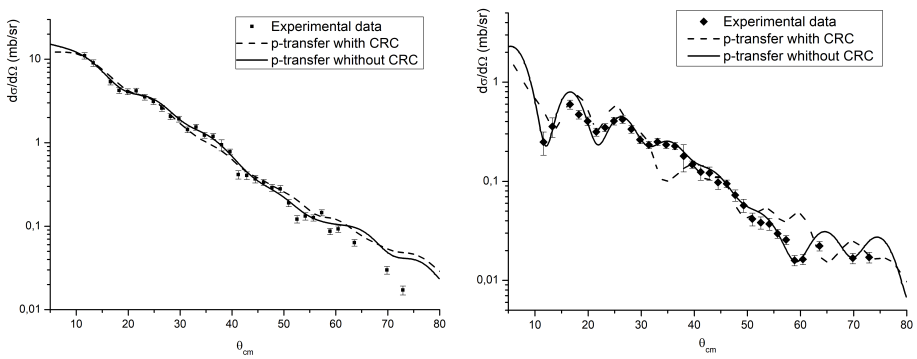


Fig. 6. A comparison with the experiment of the cross sections of the proton transfer to the ground (left) and first excited (right) states calculated by the CRC method with (dashed lines) and without (solid lines) couplings.

## 5. Extraction of the ANC values

One can see that the found weight coefficients $a_{5/2}$ and $a_{1/2}$ have the meaning of the product of spectroscopic factors (SF) $Z_{^{10}\mathrm{B}\to{}^{9}\mathrm{Be}+p} \times Z_{^{17}\mathrm{F}_{5/2}\to{}^{9}\mathrm{Be}+p}$ and $Z_{^{10}\mathrm{B}\to{}^{9}\mathrm{Be}+p} \times Z_{^{17}\mathrm{F}_{1/2}\to{}^{9}\mathrm{Be}+p}$, but in the laboratory coordinate system. Besides, the coefficients are determined by comparing the cal-

culated cross sections of both reaction channels at the same angles, whereas in the center-of-mass system, these angles are different. Additionally, as the results were obtained averaged over all measurement angles and over the used parameters of the OP, this makes it difficult to evaluate the errors in the obtained SF.

On the other hand, the SF values strongly depend on the geometric parameters of the (Woods–Saxon) bound-state potential, especially in the case of a peripheral reaction. Therefore, if the $^{16}\mathrm{O}(^{10}\mathrm{B},^{9}\mathrm{Be})^{17}\mathrm{F}$ reaction turns out to be peripheral, it is expedient to extract the ANC values within the framework of MDWBA in a similar way as it was done in [4]. In this case, using the normalization of the calculated DCSs to the experimental ones only for several small angles, where the contribution of the CRC effects is minimal, it is possible to reduce the corresponding errors.

The DCS $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{\mathrm{MDW}}$ in MDWBA for the peripheral reaction $\mathrm{A}(x,y)\mathrm{B}$ of proton transfer to core A with the formation of the nucleus B, has the form of

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{\mathrm{MDW}} = \left\{\frac{C^2_{x\to y+p}}{b^2_{x\to y+p}}\right\}\left\{\frac{C^2_{\mathrm{B}\to\mathrm{A}+p}}{b^2_{\mathrm{B}\to\mathrm{A}+p}}\right\}\sigma^{\mathrm{DW}}\left(E,\theta;b_{x\to y+p},b_{\mathrm{B}\to\mathrm{A}+p}\right). \quad (1)$$

Here, $\sigma^{\mathrm{DW}}\left(E,\theta;b_{x\to y+p},b_{\mathrm{B}\to\mathrm{A}+p}\right)$ is the single-particle DCS, calculated within the framework of the usual DWBA. For the reaction considered here, $x = {}^{10}\mathrm{B}$, $y = {}^{9}\mathrm{Be}$; A and B are the nuclei $^{16}\mathrm{O}$ and $^{17}\mathrm{F}$ in ground or $E^* = 0.495$ MeV state. Accordingly, the values of $C^2_{^{10}\mathrm{B}\to{}^{9}\mathrm{Be}+p}\left(=C^2_{9\,1}\right)$, $C^2_{^{17}\mathrm{F}\to{}^{16}\mathrm{O}+p}\left(=C^2_{16\,1}\right)$, and $C^2_{^{17}\mathrm{F}^*\to{}^{16}\mathrm{O}+p}\left(=C^2_{16\,1^*}\right)$ are the squared ANCs for overlap functions $^{10}\mathrm{B}=\{^{9}\mathrm{Be}+p\}$, $^{17}\mathrm{F}=\{^{16}\mathrm{O}+p\}$, and $^{17}\mathrm{F}^*=\{^{16}\mathrm{O}+p\}$; $b^2_{^{10}\mathrm{B}\to{}^{9}\mathrm{Be}+p}\left(=b^2_{9\,1}\right)$, $b^2_{^{17}\mathrm{F}\to{}^{16}\mathrm{O}+p}\left(=b^2_{16\,1}\right)$, and $b^2_{^{17}\mathrm{F}^*\to{}^{16}\mathrm{O}+p}\left(=b^2_{16\,1^*}\right)$ are the corresponding single-particle squared ANCs of the model wave functions of the proton binding in the $^{10}\mathrm{B}$, $^{17}\mathrm{F}$, and $^{17}\mathrm{F}^*$ nuclei.

Thus, to find the ANCs $C^2_{16\,1}$ and $C^2_{16\,1^*}$, one should be convinced of the peripherality of proton transfer in these reactions, which in the MDWBA is determined by the behavior of the function $R\left(E,\theta;b_{x\to y+p},b_{\mathrm{B}\to\mathrm{A}+p}\right) = R\left(b_{16\,1}\right)$

$$R\left(b_{16\,1}\right) = \left\{\frac{C^2_{9\,1}}{b^2_{9\,1}}\right\}\frac{\sigma^{\mathrm{DW}}\left(E,\theta;b_{9\,1},b_{16\,1}\right)}{b^2_{16\,1}}. \quad (2)$$

For a peripheral transfer process, the value $R\left(b_{16\,1}\right)$ at fixed values $E$, $\theta$, and $b_{9\,1}$ (which is assumed to be known) should remain constant if the values $b_{16\,1}$ are changed by varying the shape of the corresponding proton binding potential in the nucleus $^{17}\mathrm{F}$. To do this, one should vary the geometric parameters $r_0$ and $a$ of the Woods–Saxon potentials within physically reasonable limits (while adjusting the potential depths each time to the experimental proton binding energies in $^{17}\mathrm{F}$).

Figure 7 shows the behavior of the test functions $R\,(b_{16\,1})$ and $R\,(b_{16\,1^*})$ at the scattering angle $\theta_{\rm cm} = 10^\circ$ when the geometric parameters $r_0$ and $a$ of the bound-state potential vary within 1.1÷1.4 fm and 0.5÷0.8 fm, respectively.

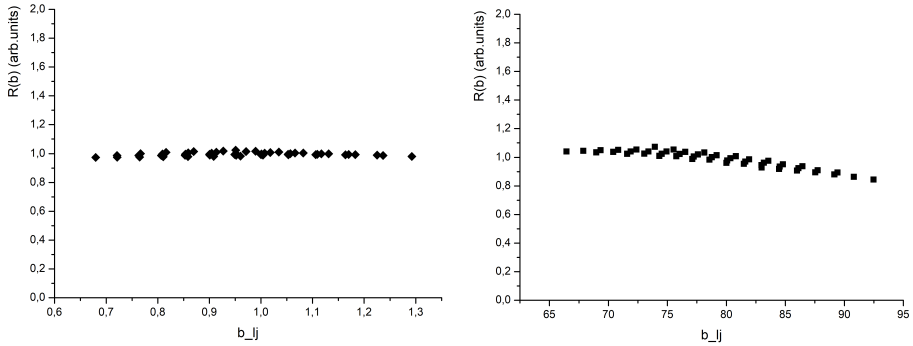


Fig. 7. The behavior of the test functions $R\,(b_{16\,1})$ and $R\,(b_{16\,1^*})$ for the $^{16}$O($^{10}$B, $^{9}$Be$_{\rm gs}$)$^{17}$F$_{\rm gs}$ (left) and $^{16}$O($^{10}$B, $^{9}$Be$_{\rm gs}$)$^{17}$F$^*$ reactions.

Moreover, the values $R(b)$ have some distribution relative to the $R(b)$ value at the standard values of the geometric parameters, which can be approximated by a Gaussian. The half-widths of these distributions are $\sim 2.5\%$ and $\sim 5\%$ for the ground and excited states, respectively. Thus, the proton transfer process for both channels of the $^{16}$O($^{10}$B, $^{9}$Be)$^{17}$F reaction can be considered peripheral, and in accordance with the MDWBA, the normalization of the calculated DCS in the region of small angles should be determined by the product of the squares of the corresponding ANCs.

The squared ANCs $C^2_{16\,1}$ and $C^2_{16\,1^*}$ have been extracted from normalization of the calculated in Section 4 single-particles DCSs to the experimental ones (see Fig. 8) using formula (1) for different sets of OPs from Table 1 at the values $C^2_{9\,1} = 4.35 \pm 0.39$ fm$^{-1}$ from Ref. [1] and calculated value $b^2_{9\,1} = 10.92$ fm$^{-1}$.

The normalization was made for the first 6 experimental $\theta_i$ points for the $5/2^+$ state and the second-fifth points for the $1/2^+$ state of the angular distributions, and the weighted means as well as their uncertainties for the $C^2_{16\,1}$ and $C^2_{16\,1^*}$ values are presented in Table 2. For the $1/2^+$ state, the ANC values are given taking into account the contribution of the CRC effects as it was done in [4]. The values in brackets are the uncertainty connected with experimental error and derived from all experimental points (the first one) as well as the ambiguities associated with the uncertainty in the $R$-function for each experimental point and the error of the $C^2_{9\,1}$ value (the second one).

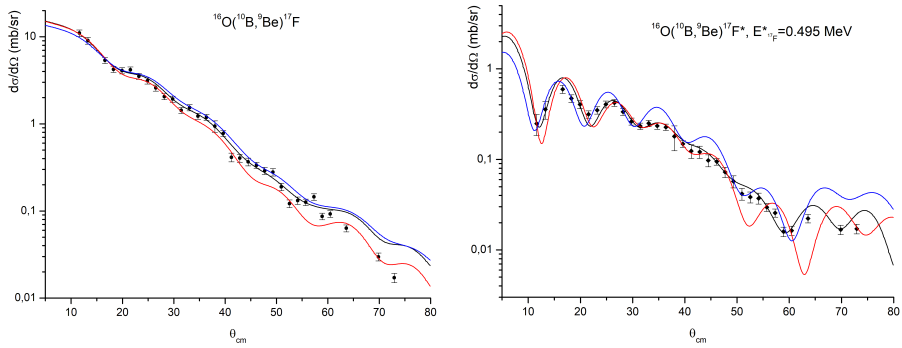


Fig. 8. The experimental (points) and calculated angular distributions (solid lines) of $^{9}$Be from the $^{16}$O($^{10}$B, $^{9}$Be)$^{17}$F reaction with the proton transfer to the $1d_{5/2}$ (left) and $2s_{1/2}$ states of $^{17}$F nuclei. Red, blue, and black curves are the MDWBA calculations with OP sets A+B1, A+B2 and A+B3, respectively (see Table 1).

Table 2. The weighted mean values of the squared ANCs for the ground and first excited states of the $^{17}$F nucleus.

| Set | $C^2_{^{17}\mathrm{F}\to{}^{16}\mathrm{O}+p}$ | $C^2_{^{17}\mathrm{F}^*\to{}^{16}\mathrm{O}+p}$ |
|---|---|---|
| A+B1 | $1.012 \pm 0.132$ (0.091; 0.0956) | $5121 \pm 853$ (598; 609) |
| A+B2 | $1.041 \pm 0.131$ (0.090; 0.0958) | $4566 \pm 735$ (550; 488) |
| A+B3 | $1.032 \pm 0.131$ (0.089; 0.0957) | $6604 \pm 1264$ (962; 821) |
| Averaged mean | $1.028 \pm 0.131$ (0.090; 0.0957) | $5430 \pm 950$ (703; 639) |

Thus, the values of the squared ANCs obtained from the normalization of the calculated curves to the experimental DCSs in the area of the forward angles (with reasonable rounding of the obtained values) are as follows: $C^2_{16\,1} = 1.03 \pm 0.13$ fm$^{-1}$ and $C^2_{16\,1^*} = 5430 \pm 950$ fm$^{-1}$ for the ground and first excited states of the $^{17}$F nucleus, respectively.

## 6. Discussion of the results and estimation of the astrophysical *S*-factor

As mentioned above, the ANC values of the proton bound states in $^{17}$F nucleus was previously obtained from the analysis of the $^{16}$O($^{3}$He, $d$)$^{17}$F reaction at energies of $\sim$ 20–40 MeV by two groups of researchers [7] and [30], as well as from the analysis of experimental astrophysical $S$-factors at energies above 0.2 MeV [9].

For the $5/2^+$ ground state, the values of the "experimentally determined" squared ANC $C^2_{16\,1}$ obtained from the proton transfer reactions have a relatively small spread, and overlap within their errors. Thus, the value of the ANC square, recalculated through the value of the corresponding nuclear vertex constant [7], turned out to be 1.02 fm$^{-1}$; in [30], this value was found equal to $1.08 \pm 0.1$ fm$^{-1}$. The value 1.04 fm$^{-1}$ is proposed in [3] from a comparison of neutron and proton ANCs for mirror states, and in [31] also using the relationship between the ANC of mirror nuclei $^{17}$O and $^{17}$F, the value $C^2_{16\,1} = 0.98 \pm 0.14$ is obtained from the analysis of the $^{16}\mathrm{O}(d,p)^{17}\mathrm{O}$ reaction. It should be noted that the value $C^2_{16\,1} = 1.03$ fm$^{-1}$ obtained in our work is in good agreement with the above values and is close to the average value for all the given data.

In [9] and [10], an analysis of experimental data directly from the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ reaction at relative energies below the first resonance in the $^{16}\mathrm{O}+p$ system ($E^r_{\mathrm{lab}} = 2660$ keV) was done. The ANC values were obtained as fitting parameters at model description of the data. The modified two-body approach [32] was used in [9], and the value $C^2_{16\,1} = 1.09 \pm 0.11$ fm$^{-1}$ was obtained from the analysis of Morlock data [33]. In [10], the available experimental data on the direct radiative capture of $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ were scrupulously reviewed, some systematic corrections were made to the experimental data from [33] and [34], and the uncertainties were carefully evaluated. From the analysis of these data within the framework of the R-matrix approach, the values $C_{16\,1}$ were found to be $1.13 \pm 0.01$ fm$^{-1/2}$ and $1.19 \pm 0.02$ fm$^{-1/2}$, respectively. This gives an average value of $C^2_{16\,1} \approx 1.35$ fm$^{-1}$, which is somewhat higher than the values from [9] and from the proton transfer reactions.

The theoretically calculated values are $\approx 0.95$ and 1.20 fm$^{-1}$ [35], which were obtained using a microscopic model (with the V2 and Minnesota forms of the nucleon–nucleon potential, respectively) are significantly different from the totality of "experimentally determined" values presented above, whereas the theoretical and "experimentally determined" averaged values are close. The spread of $C^2_{16\,1}$ values in [12], where the methods of analytic continuation of the scattering data to the negative-energy region were used, is fairly large. Only one of the obtained $C^2_{16\,1}$ values (equal to 0.7744 fm$^{-1}$) is relatively close to the "experimentally determined" ones.

For the excited state $1/2^+$, the value of the "experimentally determined" squared ANC, recalculated through the corresponding value of the nuclear vertex constant from [7], $C^2_{16\,1^*}$ is equal to 5294 fm$^{-1}$, and these values from [30] and [9] are $6490 \pm 680$ fm$^{-1}$ and $5700 \pm 225$ fm$^{-1}$, respectively. The values $C^2_{16\,1^*} = 6720 \pm 990$ fm$^{-1}$ obtained in [31] and $C^2_{16\,1^*} \approx 6667$ fm$^{-1}$ which corresponds to the results of [10] are maximal among the

"experimentally determined" squared ANCs. The equally weighted evaluation of the above "indirect determined" values of ANC $C^2_{16\,1^*}$ shows that their spread lies within ±13%.

Despite rather large experimental errors in our measurement, the value of 5430 fm$^{-1}$ found by us agrees reasonably with the available values.

The theoretically calculated in [35] values of $C^2_{16\,1^*}$ are 8306 and 7468 fm$^{-1}$, and in [12], they are 7944 fm$^{-1}$ and 10377 fm$^{-1}$. In contrast to the ANC for the ground state of the $^{17}$F nucleus, all of them are overestimated relative to the "indirectly determined" values by approximately 40% and more. In [36], the source-term approach to calculate one-nucleon overlap functions was used. In this work, the values $C^2_{16\,1}$ and $C^2_{16\,1^*}$ are significantly underestimated relative to the experimental ones, amounting to 0.58 fm$^{-1}$ and 2101 fm$^{-1}$, respectively.

For the precise determination of the total astrophysical factor in the low-energy region, it is the $C^2_{16\,1^*}$ ANC that is decisive. To clarify this value, it is expedient to measure the total cross section of the $^{16}$O$(p,\gamma)^{17}$F reaction at $E_p$ less than 300 keV with smaller errors than in the earliest work [37], for example, by the activation method and subsequent analysis within a two-body potential model, which includes the smallest number of model parameters.

The SF values $Z_{16\,1} = 1.14 \pm 0.14$ and $Z_{16\,1^*} = 0.87 \pm 0.15$ were evaluated via the obtained here ANC values using the relationship between ANC, SF, and single-particle ANC for an arbitrary two-particle nuclear configuration $\alpha \to \beta + \gamma$ [29]

$$Z_{\alpha\to\beta+\gamma} = \frac{C^2_{\alpha\to\beta+\gamma}}{b^2_{\alpha\to\beta+\gamma}}\,.$$

The single particle ANCs, $b_{16\,1} = 0.95$ fm$^{-1/2}$ and $b_{16\,1^*} = 78.8$ fm$^{-1/2}$ were calculated at the standard values of the geometric parameters ($r_0 = 1.25$ fm and $a = 0.65$ fm) of the proton bound state potential. One can see that the obtained values correlate rather well with the values that can be obtained through the "experimental" values of the ANC given in the discussed above publications. Slightly smaller values $Z_{16\,1} = 1.0$ and $Z_{16\,1} = 0.95$ were found in [38] and [6], respectively, from the analysis of the $^{16}$O$(^3$He$, d)^{17}$F reaction.

In general, the consistency between our and the literature values of the SF additionally indicates the correctness of the method for extracting the experimental DCSs given in Section 4.

The ANC squared values obtained by us were used to estimate the astrophysical $S$-factor of the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ proton radiative capture process. When using the approximating expression: $S_{16\,1}(E=0) = (0.37 \times C^2_{16\,1} + 1.58 \times 10^{-3} \times C^2_{16\,1^*})$ keVb, proposed in [35], we got the value $S(0) = 9.0 \pm 1.5$ keVb. This value is slightly lower than the value $S(0) = 10.6 \pm 0.8$ keVb, accepted in the review [39], but does not contradict it within the limits of errors.

## 7. Summary and conclusions

In this work, new experimental data on the elastic $^{16}\mathrm{O} + {}^{10}\mathrm{B}$ scattering and the $^{16}\mathrm{O}(^{10}\mathrm{B}, {}^{9}\mathrm{Be})^{17}\mathrm{F}$ reaction with $^{10}$B beam at the energy of 41.3 MeV accelerated by the U-200P cyclotron of the Heavy Ion Laboratory (University of Warsaw) are presented. The differential cross sections were measured at the angles of the forward hemisphere.

The angular distributions of the $^{16}\mathrm{O}(^{10}\mathrm{B}, {}^{9}\mathrm{Be})^{17}\mathrm{F}$ reaction with transitions to the ground ($5/2^+$) and excited ($1/2^+$, $E^* = 0.495$ MeV) states of $^{17}$F were analyzed within the Born approximation of the modified distorted wave method.

Using the Fresco program, the effects of coupling between the channels of elastic and inelastic scattering and the proton transfer to the states of the $^{17}$F nucleus on the reaction cross sections were estimated. This made it possible to extract the squared ANC values from the analysis of these data.

The ANC squared values obtained by us were used to estimate the astrophysical $S$-factor of the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ proton radiative capture process. We got the value $S(0) = 9.0 \pm 1.5$ keVb which is slightly lower than the previously accepted value $S(0) = 10.6 \pm 0.8$ keVb, but within the limits of errors does not contradict it. Thus, the $^{16}\mathrm{O}(^{10}\mathrm{B}, {}^{9}\mathrm{Be})^{17}\mathrm{F}$ reaction is a convenient tool for the correct extraction of the ANC values and indirectly determination of the $S$-factor for the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ proton radiative capture process.

This research is funded by the Ministry of Education and Science of the Republic of Kazakhstan (grant #BR10965191, program "Complex research in nuclear and radiation physics, high energy physics and cosmology for development of the competitive technologies") and the European Union's Horizon 2020 Research and Innovation Programme under grant agreement #654002.

## REFERENCES


[1] R. Yarmukhamedov, K.I. Tursunmakhatov, N. Burtebayev, «Asymptotic theory of charged particle transfer reactions at low energies and nuclear astrophysics», *Int. J. Mod. Phys.: Conf. Ser.* **49**, 1960016 (2019).

[2] R.E. Tribble *et al.*, «Indirect techniques in nuclear astrophysics: a review», *Rep. Prog. Phys.* **77**, 106901 (2014).

[3] A.M. Mukhamedzhanov, L.D. Blokhintsev, «Asymptotic normalization coefficients in nuclear reactions and nuclear astrophysics», *Eur. Phys. J. A* **58**, 29 (2022).

[4] S.V. Artemov *et al.*, «Asymptotic normalization coefficient for $^{12}\mathrm{C} + p \to {}^{13}\mathrm{N}$ from the $^{12}\mathrm{C}(^{10}\mathrm{B}, {}^{9}\mathrm{Be})^{13}\mathrm{N}$ reaction and the $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$ astrophysical $S$-factor», *Eur. Phys. J. A* **58**, 24 (2022).

[5] C. Barbieria, B.K. Jennings, «Study of the $^{16}\mathrm{O}(p,\gamma)$ Reaction at Astrophysical Energies», *Nucl. Phys. A* **758**, 395 (2005).

[6] C. Rolfs, «Spectroscopic factors from radiative capture reactions», *Nucl. Phys. A* **217**, 29 (1973).

[7] S.V. Artemov *et al.*, «Analysis of the reactions $(^3\mathrm{He}, d)$ on $1p$-shell nuclei by a method combining DWBA and dispersion theory», *Phys. Atom. Nuclei* **59**, 428 (1996).

[8] C.A. Gagliardi *et al.*, «Tests of transfer reaction determinations of astrophysical $S$ factors», *Phys. Rev. C* **59**, 1149 (1999).

[9] S.V. Artemov, S.B. Igamov, K.I. Tursunmakhatov, R. Yarmukhamedov, «Determination of Nuclear Vertex Constants (Asymptotic Normalization Coefficients) for the Virtual Decays $^3\mathrm{He} \to d + p$ and $^{17}\mathrm{F} \to {}^{16}\mathrm{O} + p$ and Their Use for Extrapolating Astrophysical $S$-Factors of the Radiative Proton Capture by the Deuteron and the $^{16}\mathrm{O}$ Nucleus at Very Low Energies», *Bull. Russ. Acad. Sci.: Physics* **73**, 165 (2009).

[10] C. Iliadis *et al.*, «New reaction rate for $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ and its influence on the oxygen isotopic ratios in massive AGB stars», *Phys. Rev. C* **77**, 045802 (2008).

[11] P. Mohr, C. Iliadis, «Recommended cross-section of the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ reaction below 2.5 MeV: A potential tool for quantitative analysis and depth profiling of oxygen», *Nucl. Instrum. Methods Phys. Res. A* **688**, 62 (2012).

[12] L.D. Blokhintsev, A.S. Kadyrov, A.M. Mukhamedzhanov, D.A. Savin, «Extrapolation of scattering data to the negative-energy region. III. Application to the $p$–$^{16}\mathrm{O}$ system», *Phys. Rev. C* **98**, 064610 (2018).

[13] O.R. Tojiboev, R. Yarmukhamedov, S.V. Artemov, S.B. Sakuta, «Asymptotic normalization coefficients for $^7\mathrm{Be} + p \to {}^8\mathrm{B}$ from the peripheral $^7\mathrm{Be}(d,n)^8\mathrm{B}$ reaction and their astrophysical application», *Phys. Rev. C* **94**, 054616 (2016).

[14] M. Rousseau *et al.*, «Highly deformed $^{40}\mathrm{Ca}$ cofigurations in $^{28}\mathrm{Si} + {}^{12}\mathrm{C}$», *Phys. Rev. C* **66**, 034612 (2002).

[15] C. Bhattacharya *et al.*, «Deformation effects in $^{56}$Ni nuclei produced in $^{28}$Si + $^{28}$Si at 112 MeV», *Phys. Rev. C* **65**, 014611 (2001).

[16] M. Kowalczyk, «SMAN: A Code for Nuclear Experiments», Warsaw University report, 1998.

[17] ROOT, A Data Analysis Framework, http://root.cern.ch/drupal/

[18] R.O. Akyuz, A. Winther, «Parameterization of the Nucleus–Nucleus Potential», in: R.A. Broglia, C.H. Dasso, R. Ricci (Eds.) «Nuclear Structure and Heavy-ion Reactions. Proceedings of the International School of Physics "E. Fermi", 1979» Course LXXVII, *North-Holland*, Amsterdam 1981, p. 491.

[19] L.A. Parks, D.P. Stanley, L.H. Courtney, K.W. Kemper, «Quadrupole contributions to $^{10,11}$B + $^{16}$O elastic scattering», *Phys. Rev. C* **21**, 217 (1980).

[20] N. Burtebayev *et al.*, «Effect of the transfer reactions for $^{16}$O + $^{10}$B elastic scattering», *Acta Phys. Pol. B* **50**, 1423 (2019).

[21] J.S. Winfield *et al.*, «Spectroscopic factor discrepancies in ($^{9}$Be, $^{10}$B) for different ejectile excitations», *Nucl. Phys. A* **437**, 65 (1985).

[22] A.T. Rudchik *et al.*, «The $^{7}$Li($^{18}$O, $^{16}$N)$^{9}$Be reaction and optical potential of $^{16}$N + $^{9}$Be *versus* $^{16}$O + $^{9}$Be», *Nucl. Phys. A* **860**, 8 (2011).

[23] Yongli Xu *et al.*, «Global optical model potential for the weakly bound projectile $^{9}$Be», *Phys. Rev. C* **99**, 034618 (2019).

[24] R.B. Firestone, C.M. Baglin, «Table of Isotopes: 1998 Update. 8$^{\text{th}}$ Edition», *Wiley*, 1998.

[25] H. Nann, «Level structure of $^{28}$Si by the $^{27}$Al($^{3}$He, $d$)$^{28}$Si reaction», *Nucl. Phys. A* **376**, 61 (1982).

[26] P.D. Kunz, «Computer code DWUCK5», http://spot.colorado.edu/~kunz/DWBA.html

[27] M. Wojdyr, «Fityk: a general-purpose peak fitting program», *J. Appl. Cryst.* **43**, 1126 (2010), reprint.

[28] I.J. Thompson, «Fresco», Department of Physics, University of Surrey, July 2006, Guild ford GU27XH, England, version Fresco 2.0, http://www.fresco.org.uk/

[29] L.D. Blokhintsev, I. Borbely, E.I. Dolinskii, «Yadernye Vershinnye Konstanty», *Fiz. Elem. Chastits At. Yadra* **8**, 1189 (1977).

[30] V. Kroha *et al.*, «Asymptotic normalization constants in nuclear astrophysics», *Czechoslov. J. Phys.* **51**, 471 (2001).

[31] Guo Bing *et al.*, «Test of Determination of $(p, \gamma)$ Astrophysical $S$-Factors Using the Asymptotic Normalization Coefficients from Neutron Transfer Reactions», *Chinese Phys. Lett.* **24**, 2544 (2007).

[32] S.B. Igamov, R. Yarmukhamedov, «Modfied two-body potential approach to the peripheral direct capture astrophysical $a + A \to B + \gamma$ reaction and asymptotic normalization coefficients», *Nucl. Phys. A* **781**, 247 (2007).

[33] R. Morlock *et al.*, «Halo Properties of the First 1/2$^{+}$ State in $^{17}$F from the $^{16}$O$(p, \gamma)^{17}$F Reaction», *Phys. Rev. Lett.* **79**, 3837 (1997).

[34] H.C. Chow, G.M. Griffiths, T.H. Hall, «The $^{16}O(p,\gamma)^{17}F$ Direct Capture Cross Section with an Extrapolation to Astrophysical Energies», *Can. J. Phys.* **53**, 1672 (1975).

[35] D. Baye, P. Descouvemont, M. Hesse, «Microscopic analysis of extranuclear capture on the $^{16}O(p,\gamma)^{17}F$ reaction», *Phys. Rev. C* **58**, 545 (1998).

[36] N.K. Timofeyuk, «Properties of one-nucleon overlap functions for $A \geq 16$ double-closed-shell nuclei in the source-term approach», *Phys. Rev. C* **84**, 054313 (2011).

[37] R.E. Hester, R.E. Pixley, W.A.S. Lamb, «Radiative Capture of Protons in Oxygen at 140 to 170 kev», *Phys. Rev.* **111**, 1604 (1958).

[38] J. Vernotte *et al.*, «Spectroscopic factors from one-proton stripping reactions on *sd*-shell nuclei: experimental measurements and shell-model calculations», *Nucl. Phys. A* **571**, 1 (1994).

[39] E.G. Adelberger *et al.*, «Solar fusion cross sections. II. The *pp* chain and CNO cycles», *Rev. Mod. Phys.* **83**, 195 (2011).