THE RIGOROUS TEST OF THE GENERALIZED BRINK–AXEL HYPOTHESIS IN THE $A = 138$ NUCLEAR MASS REGION

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The generalized Brink–Axel (gBA) hypothesis suggests that the γ-ray strength function (γSF) of a nucleus only depends on the γ-ray energy, and not on the properties of the initial and final excitation energy levels between which the nucleus decays. This hypothesis has been tested in various studies and it is still controversial. In this study, the gBA hypothesis was tested in the $A = 138$ nuclear mass region by rigorously investigating the dependence of the γSF of ^{138}La on both initial and final excitation energies. The results showed that the shape and absolute value of the γSF are independent of the initial and final excitation energy. Therefore, the results of this work are in support of the generalized Brink–Axel hypothesis.

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1. Introduction

A γ-ray strength function, $f(E_\gamma)$, is an average electromagnetic quantity of nuclei. In particular, it quantifies the probability of a nucleus to emit or absorb a γ ray of a given energy, E_γ, when it is excited to the quasi-continuum, where the density of quantum states is very high such that their wave functions overlap. The *downward* strength, $f(E_\gamma)\downarrow$ is related to the nuclear level density and average radiative width through $f(E_\gamma)\downarrow = f_{J^\pi}(E_\gamma) = \bar{\Gamma}(E_i, E_\gamma)\rho_{J^\pi}(E_i)/E_\gamma^{2\lambda+1}$, where $\bar{\Gamma}(E_i, E_\gamma)$, $\rho_{J^\pi}(E_i)$, and λ are the average radiative width of primary γ rays emitted from the initial energy $E_{\rm i}$, the nuclear level density of the initial states with the spin and parity of J^π and multipolarity of transitions, respectively. On the other hand, the *upward* strength, $f(E_\gamma)\uparrow$ is related to the average photoneutron cross section, $\langle\sigma(E_\gamma)\rangle$, by $\langle\sigma(E_\gamma)\rangle = 3\pi^2\hbar^2c^2 f(E_\gamma)E_\gamma$, where c and $\hbar$ are speed of light and Planck's constant, respectively. According to the detailed-balance principle, the *downward* strength and *upward* strength are equivalent provided the same states are populated [1].

According to the Brink hypothesis, the photo-absorption cross section of the giant electric dipole resonance (GDR) only depends on the photon energy and not on the properties of initial and final states [2]. This hypothesis has now been generalized to include both absorption and emission of γ rays between resonant states [3, 4]. This version of the hypothesis is referred to as the generalized Brink–Axel (gBA) hypothesis.

According to the literature [5], the generalized Brink–Axel hypothesis has not been experimentally tested, thoroughly, across the nuclear chart, and has been controversial for many years. Even studies that have tested it, theoretically and experimentally, are in disagreement. For instance, the experimental studies of Refs. [5–8] yielded results that support the gBA. On the other hand, it is clearly violated in the experimental studies of Refs. [9–12].

Thus, it is clear that there is still a need to further experimentally test the gBA across the nuclear chart. This has not been done in the $A = 138$ mass region, which is the focus area of this work. Although the work of Ref. [13] attempted to test it by investigating the dependence of the γ-ray strength function of ^{138}La on the initial energy. Their work was not very sensitive to the properties of the initial energy since it used the standard Oslo Method [14], which requires very wide excitation energy bins and hence, allowed them to test only two excitation energy bins of 1 MeV and 1.6 MeV. It is also not able to test the dependence of the γSF on the final excitation energy. Thus, in this work, we tested the generalized Brink–Axel hypothesis by investigating the dependence of the γ-ray strength function of ^{138}La on the initial excitation energy and final excitation energy, using the more effective recently developed approach [5], which allows the extraction of γSFs at different initial and final excitation energy bins which are 105 keV wide.

2. Data analysis methods

The experimental data used in this study were taken from Refs. [13, 15], who measured it at the cyclotron laboratory of the University of Oslo. In particular, ^{4}He–γ coincidence events were produced using ^{139}La(^{3}He, ^{4}He)^{138}La reaction, with the beam energy of 38 MeV and target thickness of 2.5 mg/cm^2. The γ rays of ^{138}La nucleus were measured in the CACTUS array (26 $5'' \times 5''$ NaI(Tl) detectors with the total efficiency of 14.1% at 1.3 MeV) [16], while charged ^{4}He particles were measured in the SiRi array (64 ΔE–E silicon detector telescopes with the efficiency of 6%) [17], which was positioned at forward angles to cover the angular range of 40° to 54° with respect to the beam axis. These coincidence events were analyzed, using the offline time window of 50 ns, for a different research purpose and produced various 2D histograms such as the primary γ matrix of ^{138}La nucleus. This matrix has been used in this work as input data and is shown in figure 1. A primary γ matrix is a 2D histogram that contains particle–γ coincidence events in the form of nuclear excitation energy *vs.* primary γ energy. Basically, each primary γ matrix contains primary γ spectra, $g(E_{\rm i}, E_\gamma)$, at different excitation energies $E_{\rm i}$. The $g(E_{\rm i}, E_\gamma)$ are normalized such that $P(E_\gamma, E_{\rm i}) = g(E_{\rm i}, E_\gamma)/\sum_{E_\gamma} g(E_{\rm i}, E_\gamma)$, which is the probability of a nucleus to emit a γ ray of energy E_γ at initial excitation energy $E_{\rm i}$.

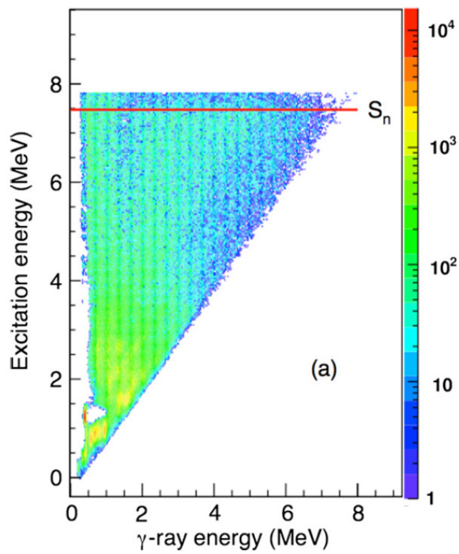

Fig. 1. Primary γ matrix of ^{138}La.

The dependence of the γ-ray strength function, of ^{138}La, on the initial and final excitation energy was studied using the new method of Ref. [5]. In particular, when the nuclear level density, ρ, is known from the standard Oslo Method, the γ transmission coefficient, $\mathcal{T}$, as a function of initial excitation energy is given by [5]

$$\mathcal{T}(E_{\rm i}, E_\gamma) = \frac{N(E_{\rm i}) P(E_\gamma, E_{\rm i})}{\rho(E_{\rm i} - E_\gamma)}\,, \tag{1}$$

where $E_{\rm f} = E_{\rm i} - E_\gamma$ is the final excitation energy level and $N(E_{\rm i})$ is the normalization factor given by

$$N(E_{\rm i}) = \frac{\int_0^{E_{\rm i}} \mathcal{T}(E_\gamma)\rho(E_{\rm i} - E_\gamma)\,{\rm d}E_\gamma}{\int_0^{E_{\rm i}} P(E_\gamma, E_{\rm i})\,{\rm d}E_\gamma}\,. \tag{2}$$

By substituting $E_{\rm f} = E_{\rm i} - E_\gamma$ in Eq. (1), we obtain the γ transmission coefficient as a function of final excitation energy as [5]

$$\mathcal{T}(E_{\rm f}, E_\gamma) = \frac{N(E_{\rm f}+E_\gamma)P(E_\gamma, E_{\rm f}+E_\gamma)}{\rho(E_{\rm f})}\,. \tag{3}$$

The details of the γ-ray strength function, $f(E_\gamma)$, at different initial and final excitation energies are obtained by transforming $\mathcal{T}(E_{\rm i}, E_\gamma)$ and $\mathcal{T}(E_f, E_\gamma)$ from Eqs. (1) and (3) into $f(E_\gamma)$ using the expression

$$f(E_\gamma) = \frac{\mathcal{T}(E_\gamma)}{2\pi E_\gamma^3}\,. \tag{4}$$

The exponent of 3 in the denominator results from the assumption that primary γ transitions are dominated by dipole transitions. This assumption was also proven true in other studies [18].

3. Results and discussion

In this section, we discuss our results on the dependence of the γ-ray strength function (γSF) of ^{138}La on the initial excitation energy and final excitation energy. Although in this paper the results are only shown for six $E_{\rm i}$ and six $E_{\rm f}$, similar results were also obtained at other $E_{\rm i}$ and $E_{\rm f}$. Figure 2 shows the γSF which was obtained from the experimental primary γ matrix, in figure 1, through Eqs. (1), (2), and (4) at different initial excitation energy bins. Similarly, figure 3 shows the γ-ray strength function of ^{138}La at different final excitation energy bins.

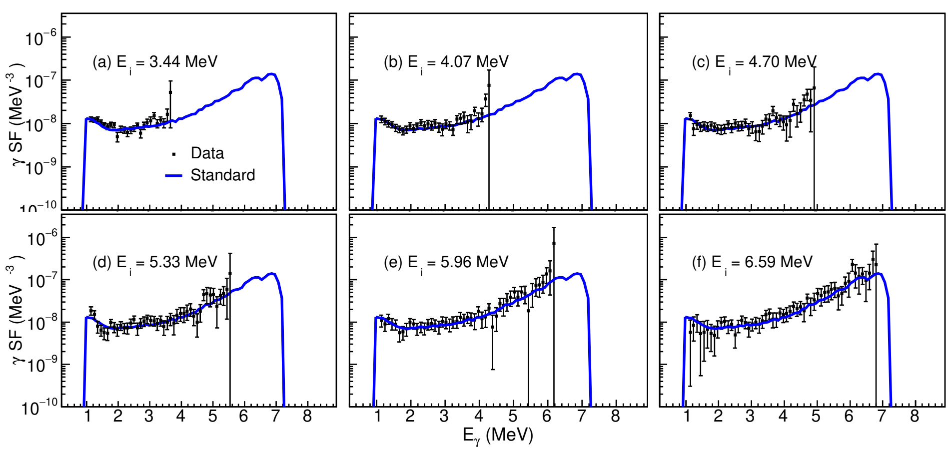

Fig. 2. The γ-ray strength function of ^{138}La at different initial excitation energies. Each excitation energy bin is 105 keV wide.

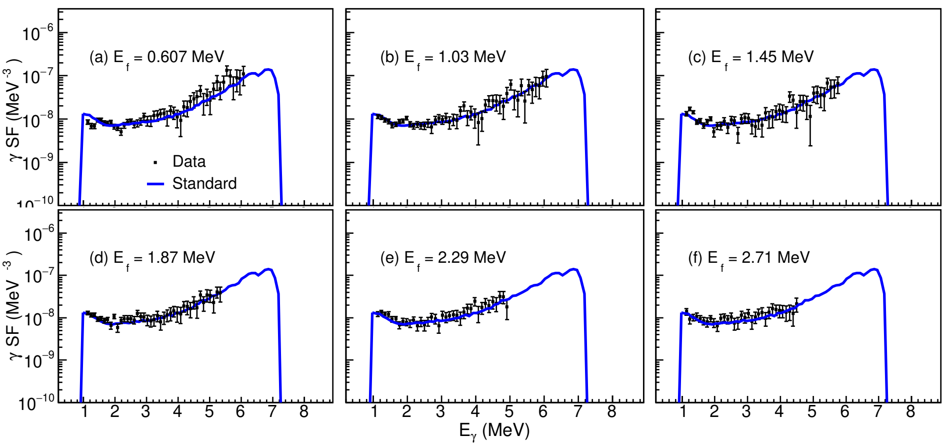

Fig. 3. The γ-ray strength function of ^{138}La at different final excitation energies. The width of each excitation energy bin is 105 keV.

In particular, figure 2 (a), (b), (c), (d), (e), and (f) shows the γ-ray strength function at $E_{\rm i} = 3.44$, 4.07, 4.70, 5.33, 5.96, and 6.59 MeV, respectively. The blue curve is the γSF of ^{138}La obtained with the standard Oslo Method in the 3.5 to 7.1 MeV excitation energy region of the primary γ-ray matrix that is depicted in figure 1 [13, 15]. It drops sharply to zero at $E_\gamma \approx 1$ MeV and $E_\gamma \approx 7.1$ MeV because these are minimum and maximum γ ray energies between which the standard Oslo Method was applied. It is also clear that the γSFs obtained at different $E_{\rm i}$ are very similar and agree with the results of the standard Oslo Method. This observation is consistent with the work of Ref. [19]. Furthermore, in figure 3, the γSFs at $E_{\rm f} = 0.607$, 1.03, 1.45, 1.87, 2.29, and 2.71 MeV are also compared to the γSF results of the standard Oslo Method shown as the blue curve. This comparison also shows that the γSF at all $E_{\rm f}$ are in excellent agreement with the blue curve. These results are similar to the findings of Ref. [5]. Thus, it is clear that the results of this work show that the generalized Brink–Axel hypothesis does hold, within the estimated experimental error bars, in the ^{138}La case.

4. Summary and conclusions

The dependence of the γ-ray strength function of ^{138}La on the initial and final excitation energy levels was investigated using the experimental primary γ-ray spectra and the recently developed data analysis method, which is the modification of the well-known Oslo Method. The γSF obtained at various initial excitation energies and final excitation energies are in agreement, within the experimental error bars, with the γ-ray strength function that was obtained using the standard Oslo Method in the 3.5 MeV to 7.1 MeV excitation energy region of the ^{138}La nucleus. This means that the shape and absolute value of the γSF of ^{138}La do not depend on the initial or final excitation energy. It is, therefore, concluded that the generalized Brink–Axel hypothesis is not violated in the $A = 138$ nuclear mass region.

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