COMPARISON OF VALENCE-CORRELATION SCHEMES IN NUCLEI HAVING VALENCE NUCLEONS IN THE SAME SHELL

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The variation of excitation energies of yrast 2^+ states and ratios E_{4^+}/E_{2^+} with N_pN_n in the mass range of $A = 6$–136 having valence nucleons in the same shell is studied. The mass range $A = 6$–136 was considered due to the availability of data when valence protons and neutrons fill in the same major shell. It has been noticed that the depression in E_{2^+} and ascension in R appear in **isobars** that differ from the predictions of the N_pN_n scheme in which **isotones** are expected. A new correlation scheme, the product of valence nucleon N_{T}, and holes $\bar{N}_{\mathrm{T}}$, *i.e.*, $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ is proposed to explain the experimental observations.

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1. Introduction

The nucleons within an atomic nucleus are arranged in a shell-like structure. This was initially inferred by observing an abrupt change in the variation of several nuclear observables at magic numbers when the proton and neutron numbers take on specified values: 2, 8, 20, 28, 50, 82, and 126 [1]. One such observable is the increased excitation energy of yrast 2^+ state, E_{2^+} in even–even nuclei at magic numbers. Employing valence correlation schemes (VCSs) [2–11], nuclear observables such as E_{2^+}, $R = E_{4^+}/E_{2^+}$, and $B(\mathrm{E2}, 0^+ \to 2^+)$ are correlated with the number of valence nucleons.

In the N_pN_n scheme [2–6], spectroscopic observables of nuclei depend on the number of valence protons N_p and neutrons N_n. The N_p and N_n are counted to the nearest closed shells, irrespective of these valence nucleons being particles or holes. As an example, the variation in E_{2^+} with neutron number N and corresponding valence neutron N_n is presented in Fig. 1. With the filling of valence protons and neutrons in different shells, $Z = 50$–82 and $N = 82$–126, the minima in E_{2^+} appear for $N = 104$, which corresponds

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to the maximum valence neutrons, $N_{n\text{-max}} = 22$ based on the N_pN_n scheme (see Fig. 1 (a)). However, when valence protons and neutrons fill the same shell, Z, $N = 50$–82 (see Fig. 1 (b)), the minima in E_{2^+} do not appear at $N_{n\text{-max}} = 16$ or $N = 66$.

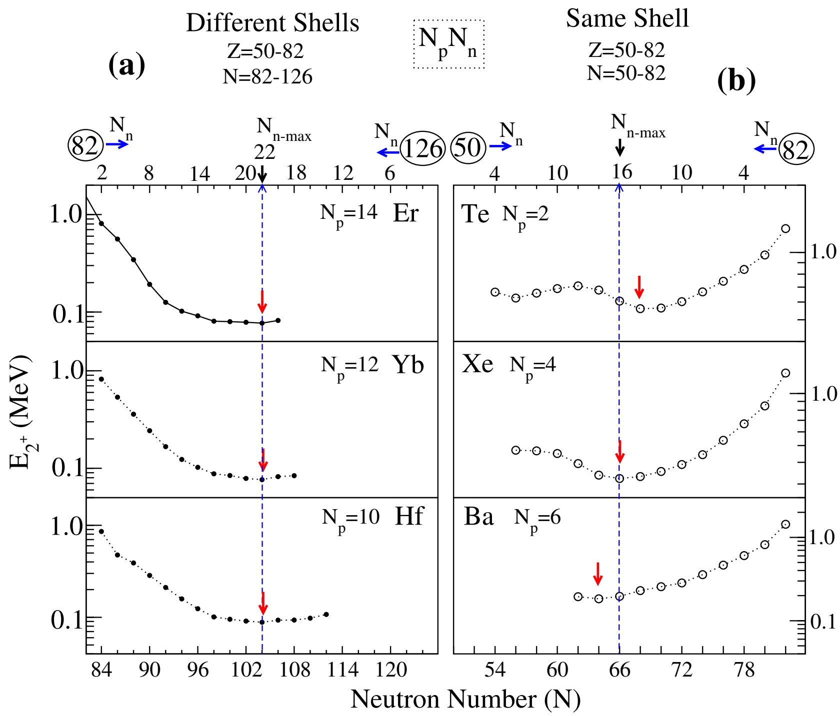

Fig. 1. The excitation energies of yrast 2^+ states shown with neutron number N (bottom axis) and corresponding valence neutron N_n (upper axis). The dashed lines indicate the $N_{n\text{-max}}$ and arrows indicate the minima in E_{2^+} for an isotopic chain. (a) When valence protons and neutrons fill in different major shells, the minima in E_{2^+} appear at $N_{n\text{-max}}$ that means systematics followed N_pN_n scheme. (b) When valence protons and neutrons fill the same major shell, the minima in E_{2^+} appear at different N_n values. Data were taken from Ref. [12]. The dotted lines are drawn for visual aid.

Figure 2 presents the variation in E_{2^+} and R with N_pN_n in the mass range of $A = 6$–136, having valence protons and neutrons in the same shell. From the close analysis it turns out that the minima in E_{2^+} and maxima in R do not appear at $(N_pN_n)_{\max}$ in most of the isotopic chains such as Ne, Mg, Zn, Ge, Se, and Te. Therefore, the deviation from N_pN_n scheme invigorates an alternative valence-correlation scheme, which is the main objective of the present paper.

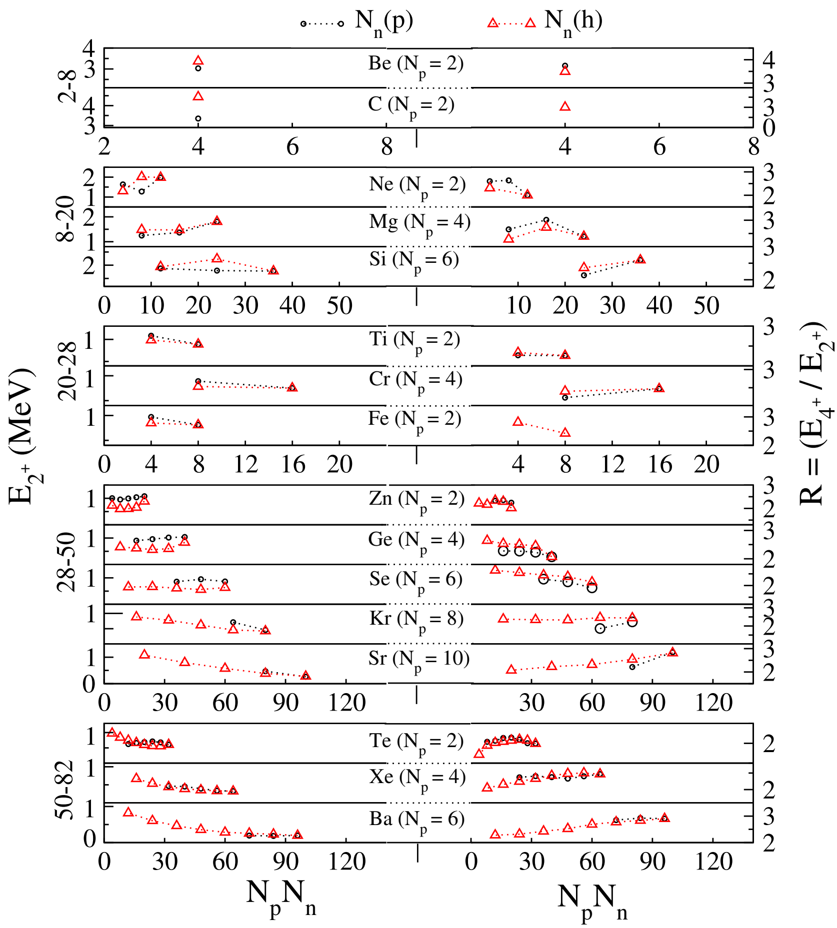

Fig. 2. The variation in E_{2^+} (left panels) and R (right panels) with N_pN_n, when protons and neutrons fill in the same shell. Data were taken from Ref. [12]. Two branches are shown when neutrons are treated as a particle (with circle) and hole (with triangle). The dotted lines are drawn for visual aid.

To know the exact value of N_n for an isotope which has minimum E_{2^+} or maximum R value in its isotopic chain, the E_{2^+} and R data were plotted with respect to N_n, N_ν, and corresponding neutron number N in Fig. 3. The N_ν is the number of valence neutrons counted as $N_\nu = N - N_{\text{core}}$ (defined in Section 2 and listed in Table 1, as an example of $^{26}_{10}\text{Ne}_{16}$; $N = 16$, $N_n = 4$, and $N_\nu = 8$). On a close analysis of Fig. 3, it seems that local depression in E_{2^+} and ascension in R appear in nuclei with different N_n (instead of $N_{n\text{-max}}$) for different isotopic chains in a considered shell. These different N_n values for a given shell correspond to the **isobars** with atomic mass $A = 28$, 76, and

120 (see Fig. 3) for shell $Z, N = 8$–20, 28–50, and 50–82, respectively. Apart from the above systematics, the sum of $N_\pi + N_\nu$ is also fixed for a given shell, where N_π is the number of valence protons counted as $N_\pi = Z - N_{\rm core}$ (defined in Section 2, and $N_\pi + N_\nu = N_{\rm T}$). However, according to the N_pN_n scheme, minima in E_{2^+} and maxima in R are expected for **isotones** with $N = 14$, 38/40, and 66 or $N_{n\text{-max}} = 6$, 10, and 16.

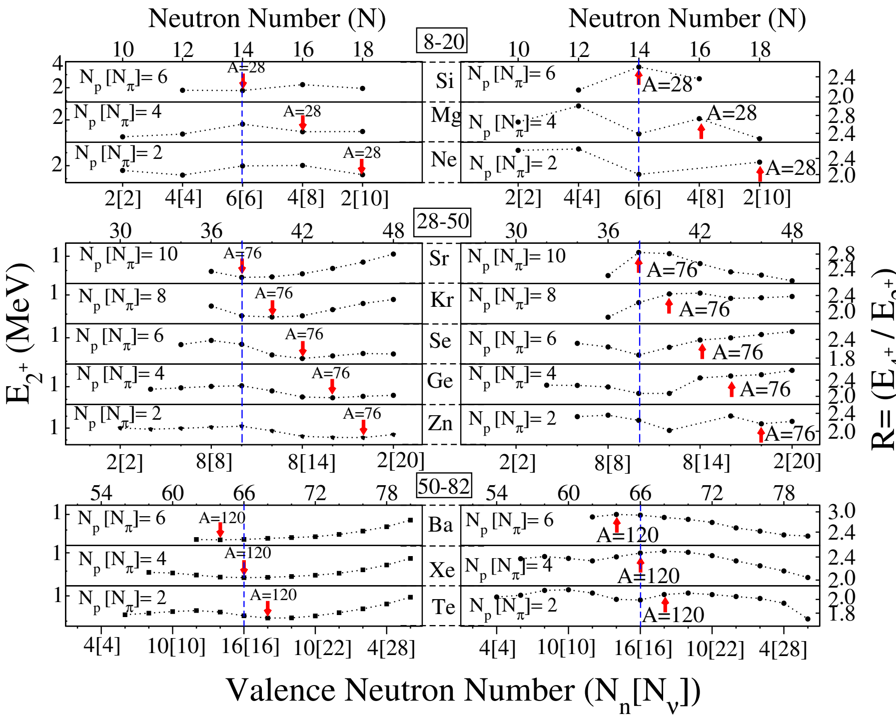

Fig. 3. Variation in E_{2^+} (left panels) and R (right panels) shown with the valence neutron N_n or N_ν (bottom axis of panels) and the corresponding neutron number N (upper axis of panels), for shells $Z, N = 8$–20, 28–50, and 50–82. The dashed lines indicate the $N_{n\text{-max}}$ and arrows indicate the depression in E_{2^+} and ascension in R. The dotted lines are drawn for visual aid.

2. Proposed valence-correlation scheme $N_{\rm T}\bar{N}_{\rm T}$

The N_pN_n scheme [2–6] is the product of the number of valence protons N_p and the number of valence neutrons N_n, relative to the nearest closed shell. Note that earlier, the midshell nucleons were counted as particles and past midhell was counted as holes. Above the midshell, the N_p or N_n are counted relative to the next higher magic number.

For the sake of clarity, here we first define the notation and the proposed correlation scheme $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ employed in the present work. Alternatively, the valence proton N_π and valence neutron N_ν can also be calculated by subtraction of the atomic number Z and neutron number N from the down magic numbers $Z_{\text{core-d}}$, $N_{\text{core-d}}$, and the upper magic numbers $Z_{\text{core-up}}$ and $N_{\text{core-up}}$ as given below. The down magic number $Z_{\text{core-d}}$, $N_{\text{core-d}}$ and the upper magic number $Z_{\text{core-up}}$ and $N_{\text{core-up}}$ for the shell Z, $N = 50$–82 are for example: down magic number; $Z_{\text{core-d}} = N_{\text{core-d}} = 50$, and upper magic number; $Z_{\text{core-up}} = N_{\text{core-up}} = 82$. The valence nucleon particles can be calculated as

$$N_\pi = Z - Z_{\text{core-d}}\,, \tag{1}$$

$$N_\nu = N - N_{\text{core-d}}\,. \tag{2}$$

Similarly, the valence hole can be calculated as

$$\bar{N}_\pi = Z_{\text{core-up}} - Z\,, \tag{3}$$

$$\bar{N}_\nu = N_{\text{core-up}} - N\,. \tag{4}$$

Further, N_π and N_ν are the number of valence protons and valence neutrons of the considered nuclei, respectively. The $\bar{N}_\pi$ and $\bar{N}_\nu$ are the number of holes in that nuclei. The total valence nucleons N_{T} and total hole positions $\bar{N}_{\mathrm{T}}$ are given by $N_{\mathrm{T}} = N_\pi + N_\nu$ and $\bar{N}_{\mathrm{T}} = \bar{N}_\pi + \bar{N}_\nu$.

Note that the midshell concept is not applied for counting the particles or holes in the $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ scheme. The valence particles or holes are correlated in both schemes. The valence nucleons can be considered as $N_p = N_\pi$, $N_n = N_\nu$ up to mid-shell and $N_p = N_{\pi\text{-max}} - N_\pi$, $N_n = N_{\nu\text{-max}} - N_\nu$ beyond that. The $N_{\pi\text{-max}}$ ($N_{\nu\text{-max}}$) is the total number of states in the shell for protons (neutrons).

As an analogy to the N_pN_n scheme, we propose the $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$, product of valence nucleon and the hole positions. A difference can be seen in Table 1 as an example of neon isotopes, when protons and neutrons fill the same shell Z, $N = 8$–20. One can notice that the $(N_pN_n)_{\text{max}}$ and $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\text{max}}$ correspond to different isotopes ${}^{24}_{10}\mathrm{Ne}_{14}$ and ${}^{28}_{10}\mathrm{Ne}_{18}$, respectively. We observe very interesting features for the neon isotopic chain as the depression in E_{2^+} and ascension in R appear with the $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\text{max}} = 144$ instead of $(N_pN_n)_{\text{max}} = 12$.

The E_{2^+} and R ratios systematics with N_pN_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ are displayed in Figs. 4–7 for a number of isotopes in the mass range of $A = 4$–136, which are discussed in detail as follows.

Table 1. The $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ and N_pN_n values for neon isotopes when protons and neutrons fill the same major shell Z, $N = 8$–20. The maximum values of $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ and N_pN_n correspond to different isotopes $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max} \to {}^{28}_{10}\mathrm{Ne}_{18}$, $(N_pN_n)_{\max} \to {}^{24}_{10}\mathrm{Ne}_{14}$. The N_π/N_ν and $\bar{N}_\pi/\bar{N}_\nu$ are occupied and unoccupied spaces, respectively.

Neon isotopes shell (8–20)								
Nuclei	N_p	N_n	N_pN_n	N_π	N_ν	N_{T}	$\bar{N}_{\mathrm{T}}$	$N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$
${}^{20}_{10}\mathrm{Ne}_{10}$	2	2	4	2	2	4	20	80
${}^{22}_{10}\mathrm{Ne}_{12}$	2	4	8	2	4	6	18	108
${}^{24}_{10}\mathrm{Ne}_{14}$	2	6	**12**	2	6	8	16	128
${}^{26}_{10}\mathrm{Ne}_{16}$	2	4	8	2	8	10	14	140
${}^{28}_{10}\mathrm{Ne}_{18}$	2	2	4	2	10	12	12	**144**

3. Comparison of N_pN_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ schemes with E_{2^+} and R ratios systematics

In this section, we present our results and discussion based on the proposed $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ scheme and compared with the N_pN_n scheme when protons and neutrons fill the same shell. The mass region is divided into $Z, N = 8$–20, 28–50; 20–28 and 50–82 shells. To show the depression/ascension in a line, the data were plotted with N_n instead of the N_pN_n because the maximum value of N_pN_n is different for different isotopic chains for a given shell. However, the $N_{n\text{-max}}$ is the same for all isotopic chains for a given shell. The number of particles (p) and number of holes (h) is equal at $N_{n\text{-max}}$ *i.e.*, $N_n(\mathrm{p}) = N_n(\mathrm{h})$. On the other hand, the maximum value of $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ is the same for all isotopic chains in a given shell *i.e.*, $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max}$ and $N_{\mathrm{T}} = \bar{N}_{\mathrm{T}}$.

3.1. Shells $Z, N = 8$–20 and 28–50

The depression in E_{2^+} and ascension in R are not consistently appearing at $N_{n\text{-max}}$ (see Fig. 4 and 5). However, the local depression in E_{2^+} and ascension in R appear at $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max}$. The $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ scheme is showing consistent results. Cakirli and Casten [7] divided valence space into three regions based on particle (p) and hole (h) concept in N_pN_n scheme: particle–particle (p–p), particle–hole (p–h), and hole–hole (h–h) regions. The R values are larger for the p–p and h–h regions as compared to the p–h region. Similarly, two cases are expected in the $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ scheme: (1) $N_{\mathrm{T}} > \bar{N}_{\mathrm{T}}$; (2) $N_{\mathrm{T}} < \bar{N}_{\mathrm{T}}$. The R values appear larger for the $N_{\mathrm{T}} > \bar{N}_{\mathrm{T}}$ case.

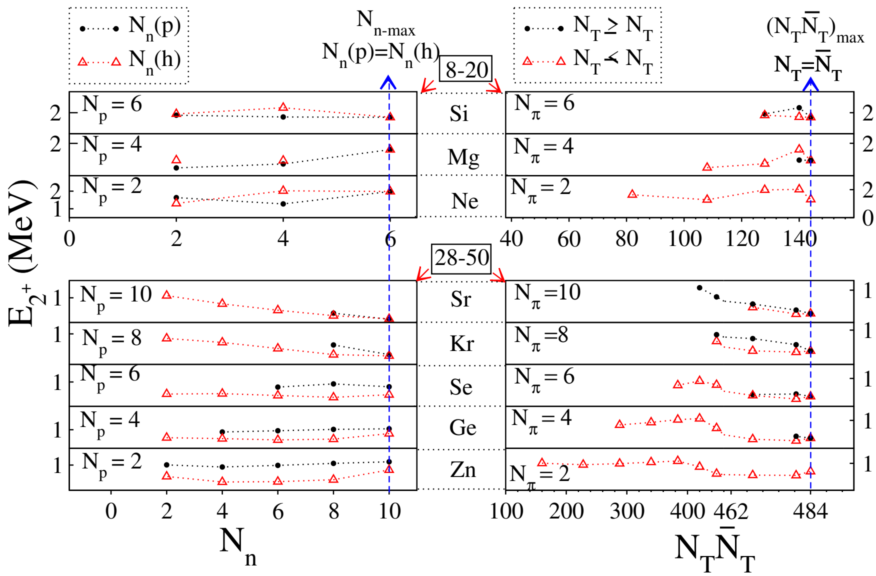

Fig. 4. A comparison of the N_pN_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ schemes of $Z/N = 8$–20 and $Z/N = 28$–50 shells shown for the E_{2^+} with respect to N_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$. The minima in 2^+ energy do not appear at $N_{n\text{-max}}$ for many isotopic chains. However, depression appears at the $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max}$. Dashed lines indicate $N_{n\text{-max}}$ and $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max}$.

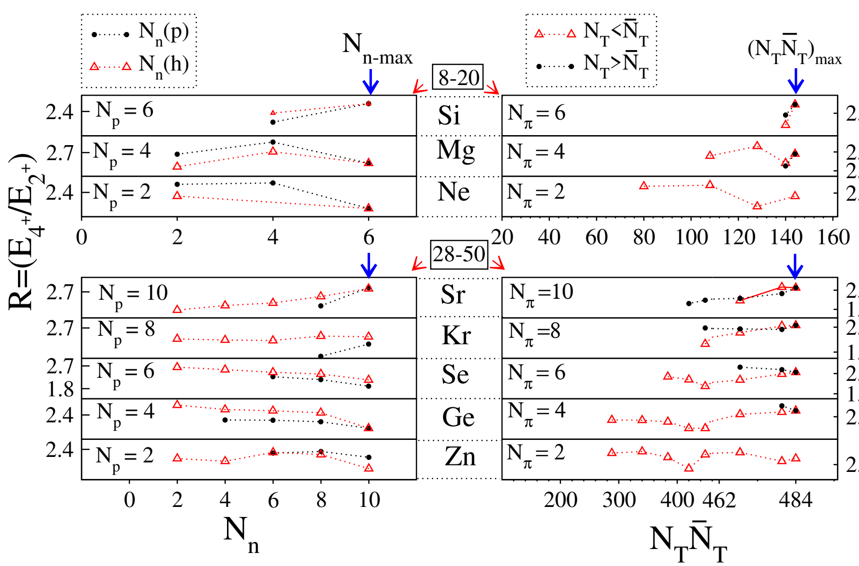

Fig. 5. A comparison of N_pN_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ schemes for Z, $N = 8$–20 and Z, $N = 28$–50 shells shown for the ratio R with respect to N_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$. The maxima in ratio R does not appear at $\mathrm{N}_{n\text{-max}}$ for many isotopic chains. However, ascension in R appears at $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max}$. Arrows indicate $N_{n\text{-max}}$ and $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max}$.

3.2. Shell $Z, N = 20$–28

Figure 6 shows the E_{2^+} systematics with respect to N_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$. The minima in E_{2^+} are consistently appearing at $N_{n\text{-max}}$, not for $(N_{\mathrm{T}}\bar{N}_{\mathrm{T}})_{\max}$. The E_{2^+} and R systematics of $1f_{7/2}$ shell is in accordance with the N_pN_n scheme.

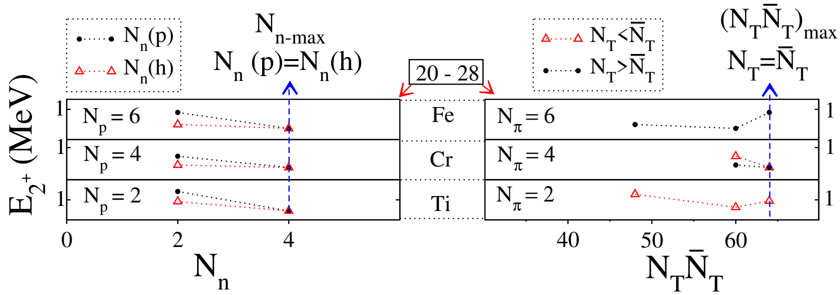

Fig. 6. A comparison of the N_pN_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ schemes for Z, $N = 20$–28 shell shown for the E_{2^+} with respect to N_n and $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$. The minima in 2^+ energy appear at $N_{n\text{-max}}$.

3.3. Shell $Z, N = 50$–82

The results for this shell seem to be not consistent with either N_pN_n or $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ schemes (see the upper panels in Fig. 7). However, if we assume Z, $N = 50$–70 (effective valence region) then the results are consistent and in accordance with the $N_{\mathrm{T}}\bar{N}_{\mathrm{T}}$ scheme (see the bottom panels in Fig. 7). A theoretical calculation is required for an effective valence region. Although some weak indication of effective valence region (subshell closure) $Z = 50$–70 [13], $N = 50$–70 [14] was discussed in Refs. [13, 14]. The occupation probabilities of various neutron and proton subshells for the ground states calculated from a Hartree–Fock–Bogoliubov wave function generated for $^{114\text{-}130}$Xe isotopes reveal that the four protons for the entire set of xenon isotopes are spread over nearly four states namely $Z = 70$, $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, and $1g_{7/2}$. The quenching of the spin-orbit splitting in neutron-rich nuclei has been obtained in many mean-field calculations and also experimentally observed in nuclei, for example, ^{98}Ni, $N = 70$ [14]. There is no sharp experimental evidence for Z, $N = 50$–70 subshell closure. The present observation of the depression in E_{2^+} and the ascension in R within the valence region Z, $N = 50$–70 could be a weak sign of subshell closure.

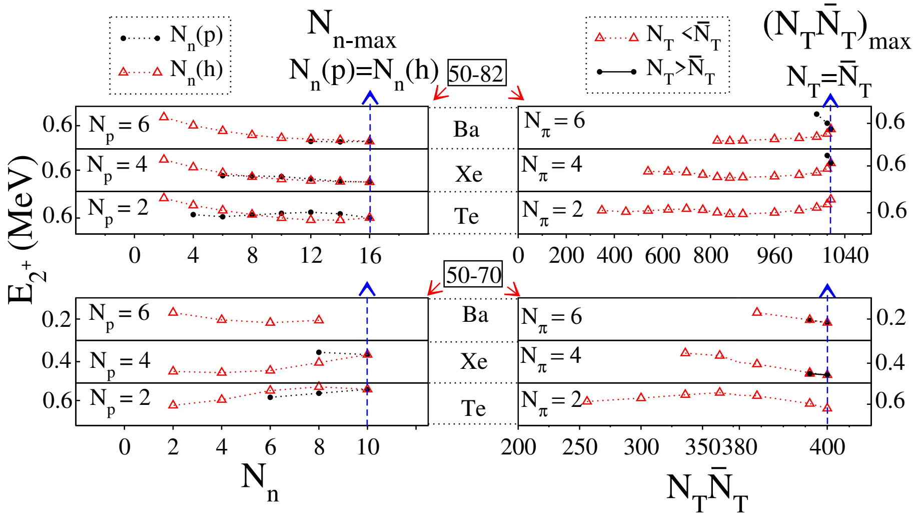

Fig. 7. A comparison of the N_pN_n and $N_\mathrm{T}\bar{N}_\mathrm{T}$ schemes for Z, $N = 50$–82 shell and assumed subshell Z, $N = 50$–70 shown for E_{2^+} with respect to N_n and $N_\mathrm{T}\bar{N}_\mathrm{T}$. The minima in 2^+ energy do not appear at $N_{n\text{-max}}$ for many isotopic chains when we consider shell Z, $N = 50$–82. However, depression appears at the $(N_\mathrm{T}\bar{N}_\mathrm{T})_\mathrm{max}$ when assuming the subshell as Z, $N = 50$–70.

4. *R* values compared when valence nucleons filling in the same shell with normalize N_pN_n and $N_\mathrm{T}\bar{N}_\mathrm{T}$

In order to compare the R values for different isotopic chains when valence nucleons fill the same shell, the R values are shown in Fig. 8 with $\frac{N_pN_n}{(N_{p\text{-max}}+N_{n\text{-max}})^2}$ and $\frac{N_\mathrm{T}\bar{N}_\mathrm{T}}{(N_{\pi\text{-max}}+N_{\nu\text{-max}})^2}$. A clear difference appears in both panels. According to the N_pN_n scheme, the R values are proportional to N_pN_n which means that the higher values of R (bump) must appear at the higher value of $\frac{N_pN_n}{(N_{p\text{-max}}+N_{n\text{-max}})^2}$. However, a pattern opposite to expectation has appeared. Similarly, in the case of $N_\mathrm{T}\bar{N}_\mathrm{T}$ phenomenology, the R values are proportional to $N_\mathrm{T}\bar{N}_\mathrm{T}$ which means that the higher values of R (bump) must appear at the higher value of $\frac{N_\mathrm{T}\bar{N}_\mathrm{T}}{(N_{\pi\text{-max}}+N_{\nu\text{-max}})^2}$. The pattern appeared as expected.

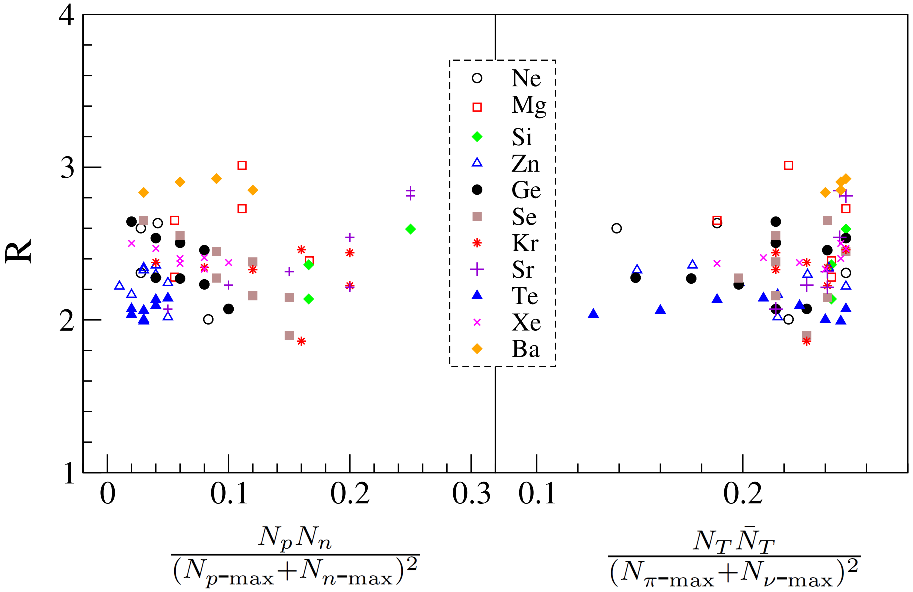

Fig. 8. R values are shown with $\frac{N_{\rm T}\bar{N}_{\rm T}}{(N_{\pi\text{-total}}+N_{\nu\text{-total}})^2}$ and $\frac{N_p N_n}{(N_{p\text{-max}}+N_{n\text{-max}})^2}$ for different isotopic chains when valence nucleons fill the same valence regions, $Z, N = 8$–20, 28–50, and 50–70.

5. Conclusions

The minima in excitation energies E_{2^+} and the maxima in $R = E_{4^+}/E_{2^+}$ appear in nuclei with the maximum of N_pN_n when valence protons and neutrons fill in **different shells**. In other words, the minima in E_{2^+} systematics appear for **isotones** in the N_pN_n scheme. However, when valence protons and neutrons fill in the **same shell**, the depression in E_{2^+} and ascension in R systematics appear in most of the cases for **isobars**, so this is not in accordance with the N_pN_n scheme. In analogy to the N_pN_n scheme, a new valence correlation scheme $N_{\rm T}\bar{N}_{\rm T}$ being the product of valence nucleon $N_{\rm T}$ and the holes $\bar{N}_{\rm T}$ has been introduced. The number of valence protons N_π and neutrons N_ν can be accommodated in the considered orbit or shell *i.e.*, $N_\pi + N_\nu = N_{\rm T}$. Presently, the variation of excitation energies of yrast 2^+ states and ratios R has been studied in a number of isotopic chains upon the availability of experimental data in the mass range of $A = 6$–136. It has been observed that the depression in E_{2^+} and ascension in R can be correlated with $(N_pN_n)_{\rm max}$ and $(N_{\rm T}\bar{N}_{\rm T})_{\rm max}$ depending on the shell considered. A detailed theoretical calculation in favour of the above-mentioned observed features in E_{2^+} and R may further enrich our understanding of the subject.

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