$j-1$ ANOMALOUS STATES AND ELECTROMAGNETIC TRANSITION RATES IN THE NEUTRON MID-SHELL Ag NUCLEI

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The neutron mid-shell silver nuclei exhibit anomalous ordering of the $(7/2^+, 9/2^+)$ states. The two states are part of the $\pi g_{9/2}^{-3}$ multiplet split under an unusually strong QQ interaction, leading to a rearrangement of the multiplet structure typical of the semi-magic Ag nuclei. To study the anomaly evolution with the neutron number, a systematic study of the electromagnetic transition rates was performed, showing that the M1 component of the $9/2^+_1 \to 7/2^+_1$ transition is typically hindered by at least two orders of magnitude with respect to the single-particle estimates, while the B(E2) component is enhanced. The experimental electromagnetic transition rates are systematically consistent with the three-hole valence-shell cluster and quadrupole vibrator coupling formalism developed in the 1970s.

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1. Introduction

The silver nuclei are placed three-proton holes away from the $Z = 50$ shell closure suggesting that the $\pi g_{9/2}^{-3}$ configuration plays an important role in the formation of their low-lying positive-parity states. Indeed, this is the case of ${}^{97}_{47}\mathrm{Ag}_{50}$ [1], but in the heavier silver nuclei, the structure rapidly changes, leading to reordering of the $\pi g_{9/2}^{-3}$ multiplet levels [2], giving rise to one of the most prominent effects in silver structure, known as the $j-1$ anomaly [3].

In the Nuclear Shell Model [4], the j^{-3} split seniority scheme arises from the residual interaction between the valence particles, where the energy splitting can be calculated as $\langle j^3\alpha; JM|H|j^3\alpha; JM\rangle = 3\sum_{J'}[j^2(J')jJ|\}j^3J]^2 A_{J'}$. Here, $A_{J'}$ are the two-body matrix elements which can be deduced from the neighbouring 2-hole even–even nuclei following the Talmi procedure, for example. In the case of $\pi g_{9/2}^{-3}$ configuration, the multiplet comprises states with angular momenta from $J^\pi = 3/2^+$ to $21/2^+$, except for the $19/2^+$ state. In this approach, the sole seniority $v = 1$ state $9/2^+$ is the lowest energy, while the $7/2^+$ state appears several hundreds of keV higher. Indeed, such an ordering is observed in ^{97}Ag [1, 5, 6]. Hence, the j^{-3} coupling scheme arising from three holes on the same-j single-particle orbit describes well the ^{97}Ag spectrum [2]. However, in the heavier than ^{97}Ag silver isotopes, the level ordering changes, and in the middle of the isotopic chain, the $7/2^+$ level appears below $9/2^+$. In the 1950s, Flowers [7] argued that such an ordering cannot be reproduced without breaking the jj-coupling approximation and introducing $g_{7/2}^n$ admixtures to the wave functions. Further, a three-hole cluster plus vibrator model was developed in the 1970s [8] to describe the anomaly in the neutron mid-shell Ag nuclei. In this model, the magnitude of the $(7/2^+, 9/2^+)$ splitting is controlled by the cluster–core interaction. In Ref. [9], it was stressed that the anomalous ordering is due to unusually strong QQ residual interaction. The anomalous features are not unique for the Ag nuclei. Similar effects are observed in other isotopic and isotonic chains [2, 9, 10], but are most prominent in the medium-mass and heavy Ag nuclei [2]. Both, the single-j shell model and the three-hole cluster plus vibrator model approaches give a clear indication of M1 transition strength. In the shell-model approach, where all three particles are on the same-j orbit, the M1 transitions are strictly forbidden between states with different seniorities [4], while in the three-hole cluster plus vibrator model, the $B(\mathrm{M1}; 9/2^+ \to 7/2^+)$ for the stable Ag is hindered by two orders of magnitude with respect to the single-particle estimate. The three-hole cluster plus vibrator model suggests that $B(\mathrm{E2}; 9/2^+ \to 7/2^+)$ is enhanced. Clearly, the degree to which the M1 component is hindered can be used as a measure for the purity of the wave functions of the states involved and it is indicative of the evolution of the anomaly. This work presents a systematical study of the $9/2^+ \to 7/2^+$ electromagnetic transition strengths in the neutron mid-shell silver nuclei.

2. Nuclear data

Throughout the silver isotopic chain, a number of $\Delta J = 1$ transitions connect the supposed members of the $\pi g_{9/2}^{-3}$ multiplet. Given that the M1 transitions are forbidden in the single-j shell model approach, any deviation from this rule would hint at more complex wave functions [8]. In order to obtain the electromagnetic transition strengths experimentally, nuclear data on level half-lives ($T_{1/2}$), transition energies, intensities, mixing ratios (δ), and conversion electron coefficients (α) are needed. However, not until recently, the level energy data on Ag nuclei was incomplete. The closest to the stable silver isotopes ^{111,113}Ag had unknown medium- and high-spin yrast sequences. The positive-parity states in the neutron-rich and neutron-deficient Ag spectra were also unknown. With the advent of the radioactive beams, the yrast states were established for all Ag nuclei, from the semi-magic $^{97}_{47}\mathrm{Ag}_{50}$ to the nearly semi-magic $^{127}_{47}\mathrm{Ag}_{80}$, allowing for a more detailed systematical study of the Ag properties (Ref. [2] and references therein). The new and exhaustive experimental data set shows a strong correlation between the $j-1$ anomaly and the energy of the first phonon in the Cd core nuclei [2], supporting the core-coupled interpretation of these states. Such an interpretation is also in line with the Alaga model [8], given that the core's states are built on $\pi g_{9/2}^{-2}$ excitations. In order to get a deeper insight into the structure of the positive-parity states in Ag, nuclear matrix elements have to be analysed. However, these calculations depend also on nuclear lifetimes and γ-ray mixing ratios which are generally missing in the nuclear databases. For example, the lifetime of the $9/2^+$ state, which in the neutron mid-shell Ag nuclei is placed above the $7/2^+$ state, is known only for a few nuclei placed close to the line of β-stability. Most of the $9/2^+ \to 7/2^+$ transitions in Ag are low-energy which makes δ measurements challenging. Furthermore, silver nuclei are, in general, difficult to produce. Regarding the higher-spin states, the lifetimes of $11/2^+$ and $13/2^+$ members of the $\pi g_{9/2}^{-3}$ multiplet have been measured only in ^{101}Ag.

A way around the missing data problem is, instead of analysing the individual $B(\mathrm{M1})$ and $B(\mathrm{E2})$ values, to analyse the trend of the ratio

$$R = \frac{B(\mathrm{M1})}{B(\mathrm{E2})} = 2.3 \times 10^{-12} \times \frac{E_\gamma^2 A^{4/3}}{\delta^2}\,, \tag{1}$$

where $B(\mathrm{M1})$ and $B(\mathrm{E2})$ are in the Weisskopf units, A is the atomic mass number, and E_γ is the transition energy in keV. The advantage of this approach is that R can be deduced even if $T_{1/2}$ is unknown, allowing to extend the range of the systematics. Thus, in the case of hindered M1 and enhanced E2 components, the R-value would be much smaller than unity.

In the present work, we shall consider two cases, ^{111}Ag and ^{115}Ag, for which no lifetime data is presently available. However, δ can be deduced from intensity balances to the parent state, allowing the calculation of R

from Eq. (1). The mixing ratio can be calculated from

$$\delta^2_{\mathrm{E2/M1}} = \frac{\alpha_{\mathrm{exp}} - \alpha_{\mathrm{M1}}}{\alpha_{\mathrm{E2}} - \alpha_{\mathrm{exp}}} \tag{2}$$

by using the conversion coefficients for pure M1 and E2, α_{M1}, and α_{E2} calculated with BrIcc [11], and α_{exp} deduced from the experimental data.

^{111}Ag is a neutron-rich nucleus having two neutrons more than the last stable isotope ^{109}Ag. Due to of its position on the Segré chart, it cannot be produced through the conventional fusion–evaporation reactions nor in spontaneous fission. However, ^{111}Ag can be obtained as a daughter from ^{111}Pd β-decay. Historically [12, 13], the source was produced via thermal neutron capture on the stable ^{110}Pd isotope. ^{111}Ag was also populated in various transfer reactions: $^{109}\mathrm{Ag}(t,p)$ [14], $^{110}\mathrm{Pd}(^{3}\mathrm{He},pn\gamma)$ [15], $^{110}\mathrm{Pd}(^{3}\mathrm{He},d)$ [16], and $^{112}\mathrm{Cd}(d,^{3}\mathrm{He})$ [17], giving access to a number of low-spin states. More recently, its higher spin states were excited via $^{168}\mathrm{Er}(^{30}\mathrm{Si},\mathrm{F}\gamma)$ induced fission reaction [18]. The partial decay scheme of the ^{111}Pd $5/2^+$ state is presented in Fig. 1 according to the data in Ref. [19]. The mixing ratio of

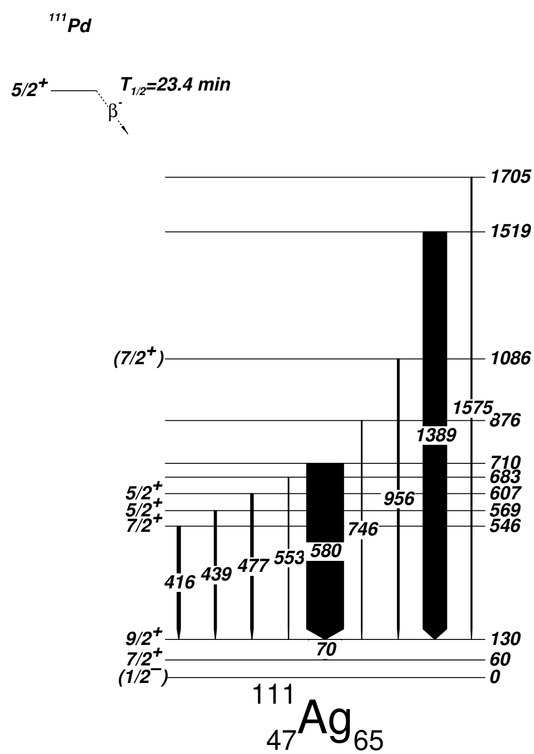

Fig. 1. ^{111}Ag partial level scheme from ^{111}Pd 23.4 min β-decay. γ-ray energies and intensities are rounded of from the values given in the latest nuclear data evaluation [19]. The arrow thicknesses are proportional to the γ-ray intensities.

the 70 keV $9/2^+ \to 7/2^+$ transition was not measured directly, but it can be deduced from the data in Ref. [19] under the assumption of no direct feeding to the $9/2^+$ daughter state, based on the large parent–daughter states spin difference. Thus, the net side feeding to this particular level is zero, leading to $\alpha_{\rm exp} = I_{70\gamma}/TI_{\rm in} - 1$, where $I_{70\gamma} = 90$ (3) is the γ-ray intensity of the $9/2^+ \to 7/2^+$ transition and $TI_{\rm in} = 197$ (6) is the total intensity of all γ-ray transitions feeding the $9/2^+$ state, according to the data in Ref. [19]. Thus, the experimental electron conversion coefficient for the 70 keV transition is $\alpha_{\rm exp} = 1.19$ (10) and $\delta^2 = 0.0135$ (27), calculated from Eq. (2). Then, the R-value 0.0005 (9), obtained in the present study, suggests that the M1 component of the $9/2^+ \to 7/2^+$ transition is hindered by at least two orders of magnitude and the E2 component is enhanced by the same degree. While this evaluation gives only a tentative estimation of R, it shows that dedicated experiment is necessary in order to obtain a more precise value. Nevertheless, the new value is consistent with the structure of the light Ag nuclei, as shown in Table 1 and Fig. 3.

Table 1. Level energies $E_{\rm i,f}$ and spin and parities $J^\pi_{\rm i,f}$, γ-ray energies E_γ, and mixing ratios δ are from Refs. [19–23], unless otherwise specified. The ^{111,115}Ag mixing ratios are deduced in the present work. The uncertainties on the last significant digits are denoted in italics.

	^{105}Ag$_{58}$	^{107}Ag$_{60}$	^{109}Ag$_{62}$	^{111}Ag$_{64}$	^{115}Ag$_{68}$
Ref.	[20]	[21]	[22]	[19]	[23]
$E_{\rm i}$	53	125	133	130	167
$J^\pi_{\rm i}$	$9/2^+$	$9/2^+$	$9/2^+$	$9/2^+$	$9/2^+$
$E_{\rm f}$	25	93	88	60	41
$J^\pi_{\rm f}$	$7/2^+$	$7/2^+$	$7/2^+$	$7/2^+$	$7/2^+$
E_γ	27.67 *1*	32.46 *2*	44.77 *13*	70.44 *5*	125.52 *10*
δ	0.044 *8*	0.074 *14*	0.14 *6*	0.15 *11*	0.6 *4*
$R \times 10^4$	4.5 *17*	2.2 *8*	1.3 *12*	5 *9*	0.6 *8*

Let us now consider the 126 keV $9/2^+ \to 7/2^+$ transition in ^{115}Ag. The nucleus was produced as a fragment from ^{252}Cf spontaneous fission experiment [24] performed at the Argonne National Laboratory, where the γ-rays were detected by the Gammasphere spectrometer comprising 51 HPGe detectors with anti-Compton shields. ^{115}Ag partial decay scheme as observed in this work is shown in Fig. 2. The electron conversion coefficient of the 126.1 keV transition $\alpha_{\rm exp} = 0.318(65)$ was deduced from the intensity balance to the 167 keV level, performed on energy spectra gated on the 859 keV

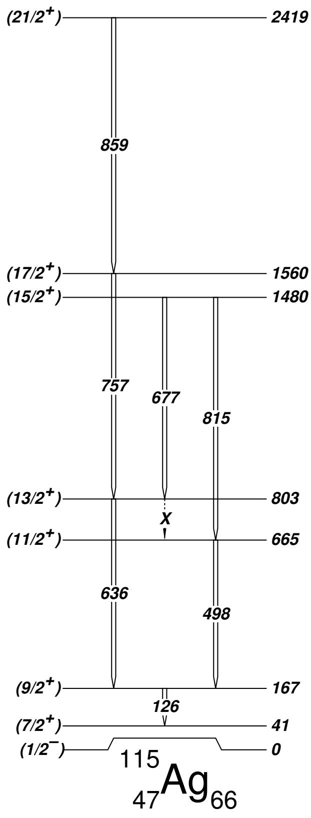

Fig. 2. ^{115}Ag partial level scheme as observed in the present study.

and 757 keV γ-rays. Then $\delta^2_{\mathrm{E2/M1}} = 0.29$ (26) was deduced from Eq. (2) and $\alpha_{\mathrm{M1}} = 0.224$ (4) and $\alpha_{\mathrm{E2}} = 0.643$ (10) are calculated with BrIcc [11]. The R-value of 7×10^{-5} (6), obtained from Eq. (1), suggests that the E2 component is enhanced by two or three orders of magnitude, while the M1 is probably hindered by two orders of magnitude with respect to the Weisskopf estimates, similarly to the ^{111}Ag case. This is only a tentative interpretation and to get more detailed information on the electromagnetic transition strengths lifetime and dedicated conversion electron experiments are needed. However, the two new R-values in ^{111,115}Ag allow for a wider systematical study and interpretation of the low-lying ^{111,115}Ag states in light of the data in the neighbouring nuclei. The systematic studies presented in Fig. 3 shows a gradual decrease of R from 0.00046 (17) in ^{105}Ag to 0.00006 (8) in ^{115}Ag. This result resembles the trend of the $\Delta E = E_{9/2^+} - E_{7/2^+}$, discussed in Ref. [2], and further supports the role of the core at low excitation energies in the silver nuclei.

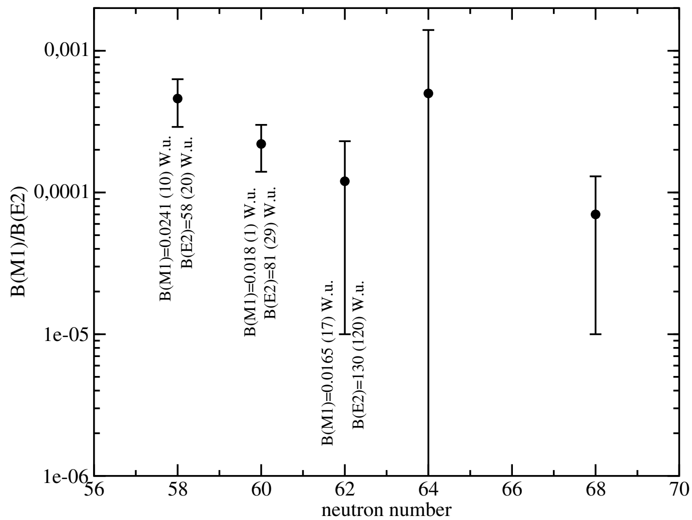

Fig. 3. Evolution of the $R = B(\mathrm{M1})/B(\mathrm{E2})$ ratio with the neutron number for the $9/2^+ \to 7/2^+$ transitions in the neutron mid-shell Ag nuclei. For ^{105,107,109}Ag, the ratio $R = B(\mathrm{M1})/B(\mathrm{E2})$ is calculated with data from Refs. [20–22]. For ^{111,115}Ag, R is calculated with γ-ray energy data from Refs. [19, 23], while δ is deduced in the present work.

3. Conclusion

The anomalous behaviour of the $(j, j-1)$ states, observed in the odd-mass Ag nuclei is related to the re-ordering of the $(9/2^+, 7/2^+)$ doublet from the $\pi g_{9/2}^{-3}$ multiplet of states with spins from $3/2^+$ to $21/2^+$, where M1 transitions are forbidden. To study the evolution of the multiplet with the neutron number, it is instructive to study the evolution of the reduced electromagnetic transition probabilities. However, lifetime and mixing ratio data are only available for less exotic Ag nuclei which makes it difficult to extend systematic studies over the whole silver isotopic chain. Nevertheless, a more complete analysis can be done by using the $R = B(\mathrm{M1})/B(\mathrm{E2})$ ratio instead of the individual $B(\mathrm{M1})$ values since the ratio does not depend on the initial level half-life. When $B(\mathrm{M1})$ and $B(\mathrm{E2})$ are in the Weisskopf units, R can hint at deviations from the single-particle approach but also can shed light on the M1 hindrance. Indeed, the present analysis, performed for the neutron mid-shell Ag nuclei, shows that the M1 component of the $9/2^+ \to 7/2^+$ transition is hindered by two orders of magnitude, while the $B(\mathrm{E2})$ component is enhanced by the same order.

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