SCHWARZSCHILD-LIKE FIVE-DIMENSIONAL TOPOLOGICALLY CHARGED WORMHOLE WITHOUT EXOTIC MATTER

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In this article, we study the characteristics of a five-dimensional topologically charged Schwarzschild-like wormhole — a non-vacuum solution derived from Einstein's field equations while adhering to the weak energy conditions. Notably, this five-dimensional wormhole extends the framework of the Klinkhamer four-dimensional vacuum-defect by introducing a topological charge. Our investigation focuses on elucidating the distinct features and implications of this five-dimensional wormhole with topological charge. As a specific case, we examine a particular instance of this topologically charged defect wormhole and provide a detailed analysis of the obtained results.

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1. Introduction

With the intricate topology of space-time, the Einstein field equations give rise to a fascinating set of solutions known as wormholes [1]. A wormhole possesses a tunnel-like structure capable of linking two distinct universes or connecting widely separated regions within the same universe. The first traversable wormhole solution, characterized by spherical symmetry and incorporating a massless scalar field with negative energy density within the framework of the Einstein field equations, was reported in [2]. A similar solution was also independently discovered in a different work [3]. However, the attraction of wormhole studies gained significant interest after the seminal work presented by Morris *et al.* [4, 5]. They considered static and spherically symmetric wormholes, conducted a meticulous study, and obtained the presence of exotic matter that violates the energy condition, namely, the null energy condition (NEC) [1] at the wormhole throat, a condition that states that any null observer should measure a non-negative average energy density in the space-time. This new class of wormhole solutions [4, 5] describes the potential travel of human beings through the tunnel of the wormholes, provided there is no horizon throughout the tunnel, and near the throat of the wormhole, the material must exhibit radial tension exceeding the mass–energy density. The inclusion of exotic matter raises concerns about the stability of such wormhole models. Studies have shown that the Morris–Thorne wormhole [4], as well as other wormhole models [2, 3], are unstable against Gaussian pulses in either exotic or normal massless Klein–Gordon fields, as reported in [6]. Additionally, research presented in Ref. [7] indicates that more general static, spherically symmetric wormhole solutions to the Einstein field equations coupled to a massless ghost scalar field are unstable with respect to linear fluctuations.

Numerous authors have attempted to minimize the violation of energy conditions, as discussed in Refs. [8–14]. Moreover, attempts have also been made by several researchers to construct traversable wormholes both in general relativity and modified theories of gravity without the requirement for negative energy density (see, for example, Refs. [15–19]). However, to comprehend the extensive delineation of the physical aspects of wormholes, we suggest readers studying different wormhole configurations, such as rotating wormholes [20–26], dynamical wormholes [27, 28], and wormholes with a cosmological constant Λ [29, 30].

Topological defects are theoretical entities believed to have formed during the early stages of the universe's evolution, as extensively discussed in various references [31–36]. These defects could have originated during early phase transitions in particle physics models, such as those associated with the breaking of grand unification symmetry or spontaneously broken gauge theories, as proposed in numerous field theory models [37–39]. Cosmic strings and global monopoles, among various types of topological defects, are particularly intriguing in gravitation and cosmology. Global monopoles are spherically symmetric objects that arise from the self-coupling triplet of scalar fields ϕ^a. These scalar fields undergo a spontaneous breaking of the global O(3) gauge symmetry, resulting in structures akin to cosmic strings with U(1) symmetry, exhibiting distinct properties. Notably, studies of global monopoles have revealed that they possess a negative gravitational potential [40]. The concept of global monopole charge has also found relevance in cosmological contexts, as evidenced by research described in [41–43]. Moreover, recent investigations into wormholes within the Milky Way galaxy, taking into account the presence of global monopole charge, have been reported in [44].

In the paper by Klinkhamer [45], a degenerate, traversable wormhole-defect space-time was initially introduced, notable for its adherence to the weak energy condition. Expanding on this groundwork, the same author, in a subsequent work [46], proposed the concept of vacuum-defect wormholes, and further extended the approach to encompass wormholes characterized by multiple vacuum-defect regions. Remarkably, these models inherently satisfy the energy conditions without necessitating any additional constraints. In a parallel exploration of satisfying energy conditions, a distinct avenue was pursued by introducing topologically charged wormhole space-times in works such as [24, 47, 48]. Notably, these studies ensured the satisfaction of the weak energy condition within the confines of four-dimensional relativity theory. In a separate contribution by Wang [49], a four-dimensional traversable wormhole, referred to as a Schwarzschild-type defect wormhole, was constructed. The line-element describing this Schwarzschild-type defect wormhole in the coordinate system (t, ξ, θ, ϕ) is given by $(\hbar = c = 1 = G)$

$$\begin{aligned} \mathrm{d}s^2 \;=\; & -\left(1-\frac{2\,M}{\sqrt{\xi^2+b^2}}\right)\mathrm{d}t^2+\left[\left(1-\frac{2M}{\sqrt{\xi^2+b^2}}\right)\left(1+\frac{b^2}{\xi^2}\right)\right]^{-1} \\ & \times \mathrm{d}\xi^2+\left(\xi^2+a^2\right)\,\mathrm{d}\Omega^2\,, \end{aligned} \tag{1}$$

where $\mathrm{d}\Omega^2 = (\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$, $b > 2\,M > 0$, and $a > 0$. The physical and geometrical properties have been also discussed in detail in [49].

Extending the concept of vacuum-defect wormhole into higher dimensions, an important contribution was presented in Ref. [50]. This work introduced a higher-dimensional version of the vacuum-defect wormhole, introducing a mass parameter M, and χ as the fifth spatial coordinate into the framework. The simplest example of a higher-dimensional wormhole is the five-dimensional wormhole model, and the line-element describing this defect wormhole is given by

$$\begin{aligned} \mathrm{d}s^2 \;=\; & -\mathrm{d}t^2+\left[\mathrm{d}\chi+\sqrt{\frac{2M}{\chi\sqrt{1+\xi^2/b^2}}}\mathrm{d}t\right]^2 \\ & +\left[\left(1-\frac{2M}{\sqrt{\xi^2+b^2}}\right)\left(1+\frac{b^2}{\xi^2}\right)\right]^{-1}\,\mathrm{d}\xi^2 \\ & +\left[b^2+\frac{M}{M+\delta M}\left(\chi^2-b^2\right)+\xi^2\right]\,\mathrm{d}\Omega^2\,. \end{aligned} \tag{2}$$

Recently, another five-dimensional vacuum defect traversable wormhole without the mass term M has been reported in [51]. This five-dimensional vacuum defect is a direct extension of the Klinkhamer vacuum-defect wormhole [46]. The metric describing this intriguing five-dimensional vacuum-defect wormhole is given by

$$ds^2 = -dt^2 + \left(1 + \frac{b^2}{\xi^2}\right)^{-1} d\xi^2 + \left(\xi^2 + b^2\right)\left(d\Omega^2 + \cos^2\theta\, d\chi^2\right) . \quad (3)$$

In this study, our objective is to conduct a comprehensive analysis of a five-dimensional traversable wormhole featuring topological charge. This particular configuration is attained through the inclusion of a parameter denoted as M, which signifies the mass associated with the system. The topologically charged wormhole under consideration represents a non-vacuum solution derived from the Einstein field equations, accounting for the presence of matter–energy components. Remarkably, our investigation reveals that the matter–energy distribution adheres to the energy conditions. Consequently, we present an example of a non-exotic matter traversable wormhole within the five-dimensional framework. This exploration contributes to the understanding of complex gravitational structures, shedding light on viable solutions that satisfy fundamental physical principles.

2. Analysis of a Schwarzschild-like five-dimensional topologically charged wormhole

In this section, we embark on an exploration of a five-dimensional traversable wormhole that introduces a mass term M and incorporates topological charge into the existing wormhole metric (3). To achieve this, we adopt the following ansatz, describing a five-dimensional topologically charged Schwarzschild-like defect wormhole in the coordinate chart $(t, \xi, \theta, \phi, \chi)$, given by

$$\begin{aligned} ds^2 = & -\left(1 - \frac{2\,M}{\sqrt{\xi^2 + b^2}}\right) dt^2 + \left[\left(1 - \frac{2M}{\sqrt{\xi^2 + b^2}}\right)\left(1 + \frac{b^2}{\xi^2}\right)\right]^{-1} \frac{d\xi^2}{\alpha^2} \\ & + \left(\xi^2 + b^2\right)\left(d\Omega^2 + \cos^2\theta\, d\chi^2\right) . \end{aligned} \quad (4)$$

The coordinates are in the ranges of

$$t \in (-\infty, \infty)\,, \quad \xi \in (-\infty, \infty)\,, \quad \theta \in [0, \pi)\,, \quad \phi \in (-\pi, \pi), \quad \chi \in [0, \infty)\,. \quad (5)$$

In the limit of $M \to 0$ and $\alpha \to 1$, this metric reduces to a five-dimensional extension of the vacuum-defect wormhole [51]. Moreover, in the particular case, $\chi = 0$, one will recover the topologically charged Schwarzschild-like wormhole (see [47] for details discussion).

The non-zero components of the Einstein tensor G^{A}_{B}, where $A = 0, 1, 2, 3, 4$ for this space-time (4) are given by

$$G^{t}_{t} = \frac{3\left(1-\frac{2M}{\sqrt{\xi^2+b^2}}\right)^{-1}}{\xi^2+b^2}\left[-1+\alpha^2+\frac{2M^2\alpha^2}{\xi^2+b^2}+\frac{M}{\sqrt{\xi^2+b^2}}\left(2-3\alpha^2\right)\right],$$
$$G^{\xi}_{\xi} = -3\alpha^2\left[1-\alpha^2+\frac{M\alpha^2}{\sqrt{\xi^2+b^2}}\right],$$
$$G^{\theta}_{\theta} = \frac{\left(-1+\alpha^2\right)}{\xi^2+b^2} = G^{\phi}_{\phi} = G^{\chi}_{\chi}. \tag{6}$$

The Kretschmann scalar curvature, $\mathcal{K} = R^{ABCD}\,R_{ABCD}$, the Ricci scalar, $R = g_{AB}\,R^{AB}$, and the Ricci invariant $\mathcal{R} = R^{AB}\,R_{AB}$ for this metric (4) are given by

$$\mathcal{K} = \frac{4\left[3\left(\xi^2+b^2\right)\left(-1+\alpha^2\right)^2+2M\alpha^2\left(11M\alpha^2-6\sqrt{b^2+\xi^2}\left(-1+\alpha^2\right)\right)\right]}{\left(\xi^2+b^2\right)^3},$$
$$R = \frac{4M\alpha^2}{\left(\xi^2+b^2\right)^{3/2}}+\frac{6\left(1-\alpha^2\right)}{\xi^2+b^2},$$
$$\mathcal{R} = \frac{2\left[6\left(\xi^2+b^2\right)\left(-1+\alpha^2\right)^2+M\alpha^2\left(7M\alpha^2-12\sqrt{b^2+\xi^2}\left(-1+\alpha^2\right)\right)\right]}{\left(\xi^2+b^2\right)^3}. \tag{7}$$

Given the five-dimensional wormhole space-time described by the line-element (4) and characterized by a non-zero Einstein tensor $G^{A}_{\;,,B}$ as presented in Eq. (5), we opt for an anisotropic fluid as the matter content. The energy-momentum tensor for this anisotropic fluid takes the following form:

$$T^{A}_{B} = \mathrm{diag}(-\rho, p_\xi, p_t, p_t, p_\chi), \tag{8}$$

where ρ represents the energy density and p_ξ, p_t, p_χ represent the pressure components along ξ-direction, tangential directions, and χ-direction, respectively.

Solving the field equations $G^{A}_{B} = T^{A}_{B}$ and using Eqs. (6) and (8), we obtain the energy-density expression given by

$$\rho = \frac{3\left(1-\frac{2M}{\sqrt{\xi^2+b^2}}\right)^{-1}}{\xi^2+b^2}\left[\left(1-\frac{2M}{\sqrt{\xi^2+b^2}}\right)-\alpha^2\left\{1+\frac{2M^2}{\xi^2+b^2}-\frac{3M}{\sqrt{\xi^2+b^2}}\right\}\right] \tag{9}$$

and the pressure components are given by

$$p_\xi = -3\,\alpha^2 \left[1-\alpha^2+\frac{M\,\alpha^2}{\sqrt{\xi^2+b^2}}\right],$$
$$p_t = \frac{\left(-1+\alpha^2\right)}{\xi^2+b^2} = p_\chi\,. \tag{10}$$

At the wormhole throat $\xi = 0$, these physical quantities are finite given by

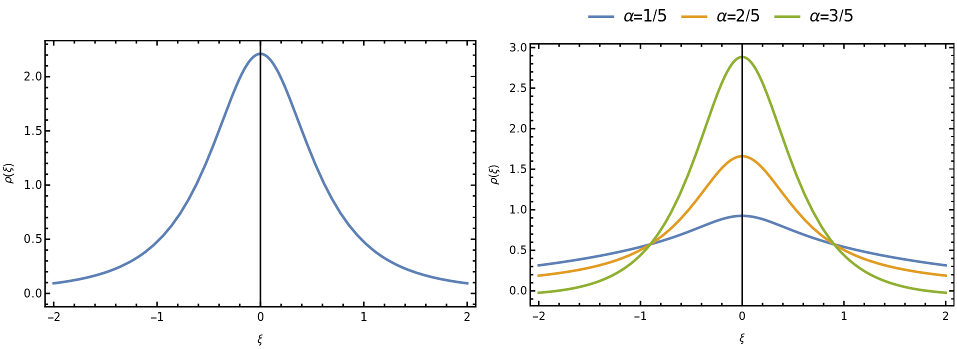
$$\rho|_{\xi=0} = \frac{3}{b^3\,(b-2\,M)}\left[b\,(b-2\,M)-\alpha^2\,\left(b^2+2\,M^2-3\,M\,b\right)\right],$$
$$p_\xi|_{\xi=0} = -\frac{3\,\alpha^2}{b}\left(b-b\,\alpha^2+M\,\alpha^2\right),$$
$$p_t|_{\xi=0} = \frac{\left(-1+\alpha^2\right)}{b^2}\,. \tag{11}$$

Also. the scalar quantities associated with the space-time curvature from equation (6) in that case become

$$\mathcal{K}|_{\xi=0} = \frac{4\left[3\,b^2\left(-1+\alpha^2\right)^2+2\,M\,\alpha^2\left(11\,M\,\alpha^2+6\,b-6\,b\,\alpha^2\right)\right]}{b^6},$$
$$R|_{\xi=0} = \frac{4\,M\,\alpha^2}{b^3}+\frac{6\left(1-\alpha^2\right)}{b^2},$$
$$\mathcal{R}|_{\xi=0} = \frac{2\left[6\,b^2\left(-1+\alpha^2\right)^2+M\,\alpha^2\left(7\,M\,\alpha^2+12\,b-12\,b\,\alpha^2\right)\right]}{b^6}\,. \tag{12}$$

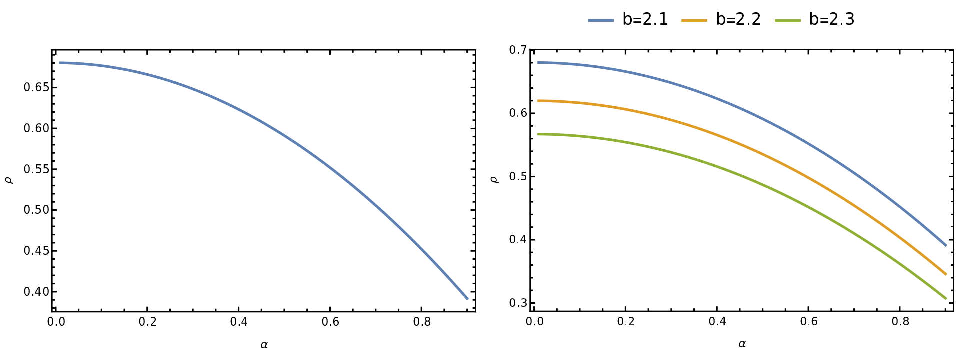
From the preceding analysis, it is evident that various scalar quantities linked to the curvature of space-time, as well as the energy density and pressure components of the anisotropic fluid representing the matter content, remain finite at the wormhole throat $\xi = 0$ and for ξ tending towards $\pm, \infty$. Below, we discuss a special case that corresponds to $\alpha \to 1$, that is, without any topological charge.

We have illustrated the energy density of the matter content through Figs. 1 and 2 depicting its variation concerning two parameters: the distance parameter ξ (Fig. 1) and the topological parameter α (Fig. 2). In the left panels, we have fixed $b = 2.1\,M > 2\,M$, while in the right panels, we have varied both the topological parameter α and the parameter $b > 2\,M$. Notably, we observe that in Fig. 1 (b), the energy density $\rho(\xi)$ gradually increases as the topological parameter α rises. Furthermore, at the wormhole throat $\xi = 0$, we note that the energy density $\rho|_{\xi=0}$ gradually decreases with increasing values of $b > 2\,M$.



(a) $\alpha = 1/2$ and $b/M = 2.1$ (b) $b/M = 2.1$

Fig. 1. The energy density $\rho(\xi)$ with ξ.



(a) $b/M = 2.1$ (b) $M = 1$

Fig. 2. The energy density $\rho|_{\xi=0}$ with α.

Special case that corresponds to $\alpha \to 1$

In this particular scenario, we examine the special case where $\alpha \to 1$, implying the absence of any topological charge. For this specific condition, the five-dimensional line-element ansatz describing a traversable defect wormhole, derived from Eq. (4), can be expressed as

$$\begin{aligned} \mathrm{d}s^2 \;=\; & -\left(1 - \frac{2\,M}{\sqrt{\xi^2 + b^2}}\right)\mathrm{d}t^2 + \left(1 - \frac{2\,M}{\sqrt{\xi^2 + b^2}}\right)^{-1}\left(1 + \frac{b^2}{\xi^2}\right)^{-1}\mathrm{d}\xi^2 \\ & + \left(\xi^2 + b^2\right)\left(\mathrm{d}\Omega^2 + \cos^2\theta\,\mathrm{d}\chi^2\right) . \end{aligned} \tag{13}$$

For the above five-dimensional traversable defect wormhole metric (13), the nonzero components of the Einstein tensor G^A_B are given by

$$G^t_t = -\frac{3\,M}{(\xi^2+b^2)^{3/2}} = G^\xi_\xi\,. \tag{14}$$

The various scalar quantities, such as the Ricci scalar, the Kretschmann scalar, and the Ricci invariant are given by

$$\begin{aligned} R &= \frac{4\,M}{(\xi^2+b^2)^{3/2}}\,,\\ \mathcal{K} &= \frac{11\,R^2}{2}\,,\\ \mathcal{R} &= \frac{7\,R^2}{8}\,. \end{aligned} \tag{15}$$

One can see that these scalar quantities associated with the space-time curvature are finite at the wormhole throat $\xi = 0$ and vanish for $\xi \to \pm\infty$.

Since this wormhole space-time (13) is a non-vacuum solution of the field equations with the non-zero components given in Eq. (14), one can consider the same matter–energy content having energy-momentum tensor given in Eq. (8). Therefore, comparing Eqs. (14) with the energy-momentum tensor (8), we obtain different physical parameters associated with the fluid as follows:

$$\begin{aligned} \rho &= \frac{3\,M}{(\xi^2+b^2)^{3/2}} > 0\,,\\ p_\xi &= -\frac{3\,M}{(\xi^2+b^2)^{3/2}}\,,\\ p_t &= 0 = p_\chi\,. \end{aligned} \tag{16}$$

From the preceding discussion, it is evident that the energy density is positive due to the condition $M > 0$, thereby satisfying both the weak and null energy conditions since $\rho + p_\xi = 0$ [52]. A noteworthy observation is that when the mass parameter M tends to zero, the space-time described by Eq. (13) transforms into a five-dimensional vacuum-defect wormhole [51].

3. Conclusions

Numerous traversable wormhole models have been developed within the framework of general relativity in both four and higher dimensions. Researchers have also explored such models in various modified gravity theories, including $f(R)$ gravity, $f(T)$ gravity, $f(R,T)$ gravity, and $f(Q)$, analyzing outcomes to minimize the violation of energy conditions and eliminate the need for non-exotic matter (for a comprehensive review, see Ref. [53]).

In our study, we conducted a thorough investigation into a distinct type of space-time: a Schwarzschild-like five-dimensional charged wormhole. Our analysis yielded significant insights, confirming that scalar quantities associated with space-time curvature remain finite at the throat of the wormhole. We focused on the matter–energy distribution inherent in this non-vacuum solution, as derived from the Einstein field equations. Our findings demonstrated that the energy density of the anisotropic fluid representing matter content is non-negative everywhere, including the wormhole throat, thereby satisfying the weak energy condition. The physical quantities associated with the energy-momentum tensor remain finite everywhere including the wormhole throat. Finally, we conducted an in-depth analysis of a special scenario where $\alpha \to 1$, corresponding to a five-dimensional Schwarzschild-like traversable defect wormhole without charge. Through this analysis, we illustrated that the matter–energy distribution adheres to the energy condition, providing a comprehensive understanding of the physical characteristics of this particular traversable wormhole space-time.

It is worthwhile mentioning that the author in Ref. [49] introduced a Schwarzschild-type traversable defect wormhole in four dimensions. Building upon this foundation, our present paper extends the concept of the four-dimensional defect wormhole to a higher dimension, specifically exploring a five-dimensional scenario with the inclusion of a topological charge. We firmly believ that the findings presented in these works are not only interesting but also hold significant relevance in the field.

REFERENCES

[1] M. Visser, «Lorentzian Wormholes. From Einstein to Hawking», *American Institute of Physics Melville*, NY, USA 1995, https://link.springer.com/book/9781563966538.

[2] H.G. Ellis, «Ether flow through a drainhole: A particle model in general relativity», *J. Math. Phys.* **14**, 104 (1973).

[3] K. Bronnikov, «Scalar-Tensor Theory and Scalar Charge», *Acta Phys. Pol. B* **4**, 251 (1973).

[4] M.S. Morris, K.S. Thorne, «Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity», *Am. J. Phys.* **56**, 395 (1988).

[5] M.S. Morris, K.S. Thorne, U. Yurtsever, «Wormholes, Time Machines, and the Weak Energy Condition», *Phys. Rev. Lett.* **61**, 1446 (1988).

[6] H.-A. Shinkai, S.A. Hayward, «Fate of the first traversible wormhole: Black-hole collapse or inflationary expansion», *Phys. Rev. D* **66**, 044005 (2002).

[7] J.A. Gonzalez, F.S. Guzman, O. Sarbach, «Instability of wormholes supported by a ghost scalar field: I. Linear stability analysis», *Class. Quantum Grav.* **26**, 015010 (2009).

[8] S. Kar, «Evolving wormholes and the weak energy condition», *Phys. Rev. D* **49**, 862 (1994).

[9] S. Kar, D. Sahdev, «Evolving Lorentzian wormholes», *Phys. Rev. D* **53**, 722 (1996).

[10] A. DeBenedictis, A. Das, «On a general class of wormhole geometries», *Class. Quantum Grav.* **18**, 1187 (2001).

[11] F.S.N. Lobo, «Phantom energy traversable wormholes», *Phys. Rev. D* **71**, 084011 (2005).

[12] S. Sushkov, «Wormholes supported by a phantom energy», *Phys. Rev. D* **71**, 043520 (2005).

[13] F. Rahaman, M. Kalam, M. Sarker, K. Gayen, «A theoretical construction of wormhole supported by phantom energy», *Phys. Lett. B* **633**, 161 (2006).

[14] S.D. Forghani, H. Mazharimousavi, M. Halilsoy, «Fate of a thin-shell wormhole powered by Morris–Thorne wormhole», *Eur. Phys. J. Plus* **133**, 497 (2018).

[15] M.S. Churilova, R.A. Konoplya, Z. Stuchlík, A. Zhidenko, «Wormholes without exotic matter: quasinormal modes, echoes and shadows», *J. Cosmol. Astropart. Phys.* **2021**, 010 (2021).

[16] N. Godani, G.C. Samanta, «Traversable wormholes supported by non-exotic matter in general relativity», *New Astronomy* **84**, 101534 (2021).

[17] K.A. Bronnikov, A.M. Galiakhmetov, «Wormholes without exotic matter in Einstein–Cartan theory», *Gravit. Cosmol.* **21**, 283 (2015).

[18] M. Zubair, R. Saleem, Y. Ahmad, G. Abbas, «Exact wormholes solutions without exotic matter in $f(R,T)$ gravity», *Int. J. Geom. Methods Mod. Phys.* **16**, 1950046 (2019).

[19] N. Godani, «Stable Morris Thorne wormholes supported by non-exotic matter», *Int. J. Geom. Methods Mod. Phys.* **18**, 2150170 (2021).

[20] E. Teo, «Rotating traversable wormholes», *Phys. Rev. D* **58**, 024014 (1998).

[21] V.M. Khatsymovsky, «Rotating vacuum wormhole», *Phys. Lett. B* **429**, 254 (1998).

[22] M. Azreg-Aïnou, «Generating rotating regular black hole solutions without complexification», *Phys. Rev. D* **90**, 064041 (2014).

[23] P.K.F. Kuhfittig, «Axially symmetric rotating traversable wormholes», *Phys. Rev. D* **67**, 064015 (2003).

[24] F. Ahmed, «Topologically Charged Rotating Wormhole», *Acta Phys. Pol. B* **54**, 11-A3 (2023).

[25] G. Clément, D. Gal'tsov, «Rotating traversable wormholes in Einstein–Maxwell theory», *Phys. Lett. B* **838**, 137677 (2023).

[26] M. Azreg-Aïnou, «From static to rotating to conformal static solutions: rotating imperfect fluid wormholes with(out) electric or magnetic field», *Eur. Phys. J. C* **74**, 2865 (2014).

[27] D. Hochberg, M. Visser, «Dynamic wormholes, antitrapped surfaces, and energy conditions», *Phys. Rev. D* **58**, 044021 (1998).

[28] S.A. Hayward, «Dynamic Wormholes», *Int. J. Mod. Phys. D* **08**, 373 (1999).

[29] J.P.S. Lemos, F.S.N. Lobo, S.Q. de Oliveira, «Morris–Thorne wormholes with a cosmological constant», *Phys. Rev. D* **68**, 064004 (2003).

[30] F. Rahaman *et al.*, «Wormhole with varying cosmological constant», *Gen. Relativ. Gravit.* **39**, 145 (2007).

[31] T.W.B. Kibble, «Topology of cosmic domains and strings», *J. Phys. A: Math. Gen.* **9**, 1387 (1976).

[32] T.W.B. Kibble, «Some implications of a cosmological phase transition», *Phys. Rep.* **67**, 183 (1980).

[33] T.W.B. Kibble, «Phase Transitions in the Early Universe», *Acta Phys. Pol. B* **13**, 723 (1982).

[34] A. Vilenkin, E.P.S. Shellard, «Strings and Other Topological Defects», *Cambridge University Press*, Cambridge 1994.

[35] R. Brandenberger, «Topological Defects and Structure Formation», *Int. J. Mod. Phys. A* **09**, 2117 (1994).

[36] M. Hindmarsh, T.W.B. Kibble, «Cosmic strings», *Rep. Prog. Phys.* **58**, 477 (1995).

[37] S. Sarangi, S.H.H. Tye, «Cosmic string production towards the end of brane inflation», *Phys. Lett. B* **536**, 185 (2002).

[38] R. Jeannerot, J. Rocher, M. Sakellariadou, «How generic is cosmic string formation in supersymmetric grand unified theories», *Phys. Rev. D* **68**, 103514 (2003).

[39] G. Dvali, A. Vilenkin, «Formation and evolution of cosmic D strings», *J. Cosmol. Astropart. Phys.* **2004**, 010 (2004).

[40] D. Harari, C. Lousto, «Repulsive gravitational effects of global monopoles», *Phys. Rev. D* **42**, 2626 (1990).

[41] A. Vilenkin, «Topological inflation», *Phys. Rev. Lett.* **72**, 3137 (1994).

[42] R. Basu, A.H. Guth, A. Vilenkin, «Quantum creation of topological defects during inflation», *Phys. Rev. D* **44**, 340 (1991).

[43] R. Basu, A. Vilenkin, «Evolution of topological defects during inflation», *Phys. Rev. D* **50**, 7150 (1994).

[44] M. Kalam, P. Das, «Wormhole in the Milky Way galaxy with global monopole charge», *Eur. Phys. J. C* **82**, 342 (2022).

[45] F.R. Klinkhamer, «Defect Wormhole: A Traversable Wormhole Without Exotic Matter», *Acta Phys. Pol. B* **54**, 5-A3 (2023).

[46] F.R. Klinkhamer, «Vacuum-Defect Wormholes and a Mirror World», *Acta Phys. Pol. B* **54**, 7-A3 (2023).

[47] F. Ahmed, «Geodesics motion of test particles around Schwarzschild–Klinkhamer wormhole with topological defects and gravitational lensing», *J. Cosmol. Astropart. Phys.* **2023**, 010 (2023).

[48] F. Ahmed, «A topologically charged four-dimensional wormhole and the energy conditions», *J. Cosmol. Astropart. Phys.* **2023**, 082 (2023).

[49] Z.-L. Wang, «On a Schwarzschild-type defect wormhole», arXiv:2307.01678 [gr-qc].

[50] F.R. Klinkhamer, «Higher-dimensional extension of a vacuum-defect wormhole», arXiv:2307.12876 [gr-qc].

[51] F. Ahmed, «A new five-dimensional vacuum-defect wormhole space-time», arXiv:2308.11938 [gr-qc].

[52] S.W. Hawking, G.F.R. Ellis, «The Large Scale Structure of Space-Time», *Cambridge University Press*, Cambridge 1973.

[53] F.S.N. Lobo, «Wormholes, Warp Drives and Energy Conditions», *Springer International Publishing*, 2017.