ESTIMATION OF NUCLEAR TEMPERATURE DURING FUSION INTERACTIONS: A DYNAMIC APPROACH

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This study aimed at investigating the impact of temperature effects on participating nuclei during fusion reactions and emphasized the need for employing dynamic models to improve the accuracy of static approaches in the fusion process. To achieve this, the researchers utilized the improved quantum molecular dynamics model (ImQMD). Previous research on the influence of temperature on nuclear reactions has shown that static approaches can estimate the temperature of compound nuclei at the point of statistical pre-equilibrium, but they are limited when it comes to the fusion reaction process. To conduct a more detailed study, various methods based on statistical ensembles have been introduced. One of the oldest techniques used to determine nuclear temperature is Weisskopf's theory, which has been widely applied in temperature studies of various nuclear reactions using the proximity potential. This highlights the importance of considering temperature in the reaction process. However, due to certain limitations, alternative approaches for studying temperature effects have been explored. The ImQMD model, known for its successful capture of dynamic reaction information, provides an opportunity to investigate the temperature effects on compound nuclei during fusion processes. A comparison between the results obtained using the ImQMD model and other statistical models such as Weisskopf's theory and codes like GEMINI indicates that the dynamic ImQMD model is valuable for examining temperature fluctuations during interactions, unlike static approaches that describe nuclear processes before reaching statistical pre-equilibrium.

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1. Introduction

The introduction of the compound nucleus and highly excited nuclei has led to the development of the concept of nucleus temperature. This concept, initially proposed by Weisskopf [1, 2] to explain the formation and decay of

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the compound nucleus in light projectile reactions, is based on the equilibrium condition. The temperature of the nucleus is an important thermodynamic quantity for analyzing nuclear reactions. Extensive experimental and theoretical studies have been conducted to determine the properties of hot nuclear systems, which are highly excited, in reactions involving heavy-ion projectiles [3]. Furthermore, the role of temperature effects on the compound nucleus in alpha and cluster decays has been investigated in other studies using the proximity model, resulting in improved results that are consistent with experimental data [4–6]. These studies have significantly contributed to our understanding of the temperature effects in nuclei. Consequently, statistical hypotheses have always been employed to analyze the temperature effects in nuclear reactions. Nuclear temperature is closely linked to the concept of equilibrium, specifically the energy distribution among all the degrees of freedom in the system. Therefore, statistical physics and thermodynamics have become vital tools for investigating nuclear reactions and the study of many particle systems in which the excitation energy is distributed as thermal energy among the particles [7]. In general, the study of the nuclear ground state involves a degenerate fermion system with limited compounds. In this system, nucleons move within an average field that arises from the nuclear force. Thermodynamically, zero temperature is associated with this system [8]. However, following a nuclear projectile–target nucleus collision in reactions such as heavy-ion reactions and fusion processes, a system containing hot nuclear matter is formed momentarily due to an increase in temperature and density. During this process, the kinetic energy of the projectile is transferred to the target nucleus, exciting the newly formed compound nucleus. Specifically, the kinetic energy, angular momentum, and mass of the projectile are transferred to the target nucleus, causing the compound system to transition from a ground-state energy level to a higher-energy level. This new nuclear system, known as the compound nucleus (CN), possesses an excitation energy greater than the ground-state energy and has a sufficient lifetime. After the uniform distribution of the excitation energy among all nucleons, the compound nucleus decays through the most favorable channels, such as the emission of light particles (LP), intermediate mass fragments (IMF), or heavy mass fragments (HMF). In macroscopic systems, temperature is determined by the thermal contact of standard external test devices with the target system [9]. However, due to the impracticality of this method for measuring the temperature of microscopic systems like atomic nuclei, alternative methods based on the evaluation of reaction products and emitted particles are necessary to determine the temperature of the nucleus. Such studies are conducted under the assumptions of statistical particle emission and employ various statistical ensembles. Depending on the system being studied, the temperature

of the nuclear system can be measured by considering the kinetic energy of the emitted particles, the relative number of emitted particles, and the population of excited states of these particles [10]. The microscopic model of Weisskopf's theory for hot nuclei is based on fitting the spectroscopy gradient of emitted particles and is one of the first models used for experimentally measuring nucleus temperature [1]. This theory assumes that the hot compound nucleus is in statistical equilibrium. Bethe's approach in analyzing the nuclear level density also uses statistical mechanics to determine the thermal properties of the nucleus [11]. He demonstrated that statistical analysis is valid when the number of states in a system with a limited number of particles is sufficiently large [2]. As the excitation energy increases, the nuclear level density, which is the most important quantity for describing the statistical properties of the nucleus, increases exponentially. For high excitation energies, the nucleus has a continuum of energy states, and the transition between states can be statistically justified [12]. However, there are limitations in temperature measurement based on the statistical model of Weisskopf's theory [12], which is also used to investigate the fusion process and the formation of a compound nucleus using the proximity potential approach [4, 5]. It is challenging to experimentally determine the spectrum of vaporized light particles because it is difficult to distinguish between colliding particles and vaporized particles during hot nucleus de-excitation. Additionally, the Coulomb field of other colliding particles and the existing nuclear field can affect the quality of the vaporized particle spectrum. Cascading evaporation effects should also be taken into account when analyzing the spectrum. However, Weisskopf's theory is only valid for the emission of a single particle and does not consider cascade emissions that occur at high excitation energy. Furthermore, the energy distribution of the emitted particle is not related to the initial temperature of the compound nucleus due to the system reaching equilibrium after the particle emission. However, it does exhibit the residual temperature of evaporation. Ensuring the condition of statistical equilibrium is also a significant concern, especially in heavy-ion collisions with high energy, due to the large fluctuations in energy during the collision of nuclei. On the other hand, the static nature of this theory requires that such changes be disregarded during the formation of the compound nucleus. Another method considered in studying the temperature effects in excited compound nuclei is the simulation of compound nucleus de-excitation using evaporation models. Among the available simulation codes, we can mention GEMINI and GEMINI++ codes. GEMINI, used in this work, is a basic code applied for simulating heavy-ion reactions involving light-to-medium mass nuclei, while GEMINI++ is an advanced version with additional features for simulating nuclear reactions involving heavier nuclei and more complex reaction mechanisms. The GEM-

INI statistical code, which describes the decay of charged particles and the emission of fission fragments in heavy-ion fusion reactions in a wide range of mass and excitation energy, calculates the successive decay of the compound nucleus using the Monte Carlo method [13, 14]. Since this code is also based on equilibrium statistical mechanics for the compound nucleus, it is not as effective as Weisskopf's theory in describing temperature changes during the fusion process. Therefore, to consider the temperature during the creation of the compound nucleus and before reaching the state of statistical equilibrium, dynamic methods must be used. The improved quantum molecular dynamics (ImQMD) model studies the temperature effects of the nuclear system by considering the location and momentum evolutions of nucleons, which are reflected in the kinetic energy of the nuclear system. In our previous research using ImQMD, we introduced new coefficients for IQ3 and SKP* and demonstrated the role of this model in dynamically justifying the diffusion parameter [15]. We also discussed the dynamic description of certain fusion processes using this model. Recently, we evaluated the role of this dynamic model in the interaction of neutron-rich nuclei, showing its ability for heavy-ion interactions [16]. In this article, our objective is to examine the influence of temperature on nuclei using the ImQMD model. Specifically, we will analyze the temperature variations in the nuclear system during fusion reactions. Our approach involves comparing the temperatures of the compound nucleus from both static and dynamic perspectives to construct a comprehensive model for studying the effects of temperature on nuclei.

2. Weisskopf's theory and its application in nucleus temperature measurement

Weisskopf's theory describes a nuclear reaction process using thermodynamic similarities. This theory calculates the hot-nuclei temperature in the Fermi energy region using the energy spectrum obtained from neutron emission or evaporating charged particles using standard statistical mechanics methods. In this theory, the energy distribution of the emitted particles is obtained as follows [1]:

$$\omega_b\left(\varepsilon_b\right) = \text{const.}\ \sigma_{bY}\ \varepsilon_b\ \mathrm{e}^{\frac{\varepsilon_b}{T_Y}}\ \mathrm{e}^{-f(\varepsilon_b)}\,. \tag{1}$$

Weisskopf's studies to describe this energy spectrum showed that independent of the excitation energy, temperature analysis in nuclear interactions is impossible [17], which is similar to Bethe's result for finding a statistical relation for the nuclear level density. In these studies, the nuclear level density is evaluated using the principles of statistical mechanics and based on the Fermi gas model. It is always noted that the level density is evaluated as the function of the excitation energy and using the energy spectrum of

the reaction products. Finally, the following relation was obtained for the nuclear level density:

$$\rho(E) = Ce^{2\sqrt{aE}} . \tag{2}$$

On the other hand, the difference between the nuclear system and other macroscopic systems is significant in evaluating the density of states (or entropy). In standard methods, the entropy of the Fermi gas is evaluated based on the macrocanonical ensemble. At low excitation energies, E^*, the entropy is expressed as [9]

$$S_{\text{grandcanonical}}\,(E^*N) = 2\ \ (a\ E^*)^{\frac{1}{2}}\ , \tag{3}$$

where a is the constant proportional to the number of particles and the single-particle level density in the Fermi energy, E^* is the excitation energy, and S is the system's entropy. In equation (3), if $S_{\text{grandcanonical}}$ is replaced by $S_{\text{microcanonical}}$, the relation between temperature and excitation energy, expressed by the equation $T = \left(\frac{E^*}{a}\right)^{\frac{1}{2}}$, is obtained. For the isolated nuclear system, it is necessary to make the following correction:

$$S_{\text{microcanonical}} = S_{\text{grandcanonical}} + \Delta S\,. \tag{4}$$

Here, by increasing the excitation energy, ΔS compared to $S_{\text{grandcanonical}}$ will become negligible. At low excitation energies, the approximate expression S for the Fermi gas is given as

$$\Delta S\ \approx\ \gamma\ \ln(E^*)\ . \tag{5}$$

Depending on isospin and momentum, γ varies from 1 to 2. By using the appropriate $S_{\text{microcanonical}}$ to calculate the nucleus temperature, we can write

$$\frac{1}{T} = \frac{\partial\ S_{\text{grandcanonical}}}{\partial E} + \ \frac{\partial \Delta S}{\partial E}\,, \tag{6}$$

thus, we have

$$\frac{1}{T}\ \approx\ \left(\frac{a}{E^*}\right)^{\frac{1}{2}} - \left(\frac{\gamma}{E^*}\right)\ . \tag{7}$$

For high excitation energies, the correction term of γ vanishes and can be written as

$$\frac{1}{T} = \sqrt{\frac{a}{E^*}} - \frac{1}{E^*}\,. \tag{8}$$

Therefore, the relation between the excitation energy and the temperature of the compound nucleus is expressed as follows:

$$E^*_{\text{CN}} = E_{\text{cm}} + Q_{\text{in}} = \frac{1}{a}\ A_{\text{P}}\ T^2 - T\,. \tag{9}$$

In this relation, $E^*_{\rm CN}$, $Q_{\rm in}$, and $E_{\rm cm}$ are the excitation energy of the compound nucleus with the mass number A_p, the Q-value parameter of the system input channel, and the center-of-mass energy of the projectile, respectively.

Equation (9) shows the necessity of considering the excitation energy in the compound nucleus system in the analysis of the temperature effects of the interacting system. In a static approach, this equation provides the possibility of examining temperature effects in nuclear interactions. In these studies, it is crucial to choose a suitable potential that can effectively improve the consistency between theoretical and experimental data by adding the temperature effects. The proximity potential is one of the practical models that investigates the thermal effects of the compound nucleus through the intrinsic properties of nuclei. According to this model, the nuclear potential between two colliding nuclei is defined as follows [18]:

$$V_{\rm P} = 4\pi\ \bar{R}(T)\gamma(T)b(T)\Phi\left(S_0(T)\right)\ , \tag{10}$$

where $\bar{R}$, b, and γ are the average curvature radius, surface thickness, and surface energy coefficient of the system, respectively. The correction of the temperature effects can be applied through equation (9) in all these three parameters. The temperature in the static proximity potential model is also related to the interacting system in the statistical equilibrium state.

3. ImQMD model

The behavior of each particle in the molecular dynamics models is analyzed based on Newton's equations of motion and the laws of statistical physics [15]. In statistical mechanics, the state of a particle is determined by three spatial coordinates and three momentum coordinates. Therefore, the state of the system at any point in space represents a microscopic state (or microstate) of the system, which includes information about the location and momentum of all particles in the system. Over time, the microstates of the system evolve as the locations and momenta of the particles change. Thus, depending on the system under study, the use of statistical ensembles allows for the determination of an average of the system's microscopic states. According to Liouville's theorem, when the number of microscopic states of the system remains constant over time, the mean of the corresponding ensemble will remain unchanged. Therefore, by considering a certain number of particles and numerically solving the equations of motion for each particle, while taking into account the initial conditions (particle location and velocity) as well as the forces between the particles, the future state of the system can be predicted based on its current state. By calculating the particle's trajectory at different time steps and obtaining the location and velocity of the

particle at each step, the macroscopic properties of the system, including the temperature, can be determined at each step. In quantum molecular dynamics models such as ImQMD, which are used to study many-body nuclear systems, the time evolution of the interacting system is investigated using a classical picture. It can be said that the temperature of the system is the ensemble average of the kinetic energy of the microstates at each point in the phase space. In the ImQMD model, preparing the initial nuclei is a critical point. In this research, a method for preparing the initial nuclei for quantum molecular dynamics (QMD) calculations is detailed. First, neutron- and proton-density distributions are obtained through RMF calculations. Then, nucleon positions are sampled based on these distributions. The local Fermi momentum is determined using the local-density approximation. For light nuclei, a slight adjustment is made to the momentum. The stability of the prepared nuclei is verified through a 2000 fm/c evolution, and only stable nuclei are selected for further simulations. In this model, each nucleon is represented as a Gaussian wave function as follows [19]:

$$\phi_i\left(\vec{r}\right) = \frac{1}{\left(2\pi\sigma_r^2\right)^{\frac{3}{2}}} \quad \exp\left[-\frac{\left(\vec{r}-\ \vec{r}_i(t)\right)^2}{4\ \sigma_r^2} + \frac{i}{\hbar}\ \vec{r}\cdot\vec{p}_i\right], \tag{11}$$

where $\vec{r}_i$ and $\vec{p}_i$ are the centers of the i^{th} Gaussian wave function in coordinate and momentum space. σ_r shows the spread of the wave function, whose value is directly related to the strength of the force between nucleons and the radius of the nucleus. The whole wave function of the nucleus is obtained by multiplying the wave functions of the nucleons. The density and momentum distribution of the system are expressed as follows:

$$\begin{aligned} \rho(\vec{r}) &= \sum \rho_i(\vec{r}), \\ g(\vec{p}) &= \sum g_i(\vec{p}). \end{aligned} \tag{12}$$

The density and momentum distribution functions of the i^{th} nucleon are also calculated as follows:

$$\begin{aligned} \rho_i\left(\vec{r}\right) &= \frac{1}{\left(2\pi\sigma_r^2\right)^{\frac{3}{2}}} \exp\left[-\frac{\left(\vec{r}-\vec{r_i}\right)^2}{2\sigma_r^2}\right], \\ g_i(\vec{r}) &= \frac{1}{\left(2\pi\sigma_p^2\right)^{\frac{3}{2}}} \exp\left[-\frac{\left(\vec{p}-\vec{p_i}\right)^2}{2\sigma_p^2}\right], \end{aligned} \tag{13}$$

where σ_r and σ_p are the width of the wave functions in the coordinate and momentum space and satisfy the uncertainty relation

$$\sigma_r \sigma_p = \frac{\hbar}{2}\,. \tag{14}$$

The propagation of nucleons takes place under the influence of the self-consistent field and momentum distribution based on the Fermi constraint by Hamilton's equations of motion

$$\dot{r}_i = \frac{\partial H}{\partial p_i}\,, \qquad p_i = \frac{\partial H}{\partial r_i}\,, \tag{15}$$

where the Hamiltonian H is the sum of the kinetic energy T and the effective interaction potential energy U

$$\begin{aligned} H &= T + U\,, \\ T &= \sum_i \frac{p_i^2}{2m}\,, \end{aligned} \tag{16}$$

and the effective interaction potential energy U includes two parts of the nuclear interaction $U_{\rm loc}$ and the Coulomb interaction energy $U_{\rm Coul}$

$$U = U_{\rm loc} + U_{\rm Coul}\,, \tag{17}$$

where we have

$$U_{\rm Coul} = \frac{1}{2}\sum_{i\neq j} \rho_i(r) \frac{e^2}{|\vec{r} - \vec{r}\,'|}\ \rho_j\left(\vec{r}\,'\right) \mathrm{d}^3 r\ \mathrm{d}^3 r'\,, \tag{18}$$

$$\begin{aligned} U_{\rm loc} = {} & \frac{\alpha}{2}\sum_i \frac{\rho_i}{\rho_0} + \frac{\beta}{\gamma+1}\ \sum_i \left(\frac{\rho_i}{\rho_0}\right)^\gamma + \frac{g_0}{2}\sum_i \sum_{i\neq j} f_s \frac{\rho_{ij}}{\rho_0} + g_r \sum_i \left(\frac{\rho_i}{\rho_0}\right)^\eta \\ & + \frac{C_{\rm S}}{2}\ \sum_i \sum_{i\neq j} t_i t_j \frac{\rho_{ij}}{\rho_0}\left(1 - k_{\rm S}\ f_{\rm S}\right)\,. \end{aligned} \tag{19}$$

The energy density coefficients for the sets IQ1, IQ2, and IQ3 are given in Refs. [15, 20].

4. Temperature calculation using the ImQMD model

In the ImQMD simulation, by calculating the location and momentum evolutions of nucleons in the nuclear many-body interacting system, the system's temperature is calculated in each step by the time averaging of the

kinetic energy. Using the energy conservation theorem, the instantaneous temperature $T(t)$ is defined as follows:

$$T(t) = \frac{2}{3Nk_{\mathrm{B}}} K(t)\,. \tag{20}$$

In this relationship, $K(t)$ is the total instantaneous kinetic energy of the system at time t, k_{B} is Boltzmann's constant, and $3N$ is the number of degrees of freedom of the system. Due to the fluctuation of temperature with respect to time, the macroscopic temperature T is obtained by the time averaging of the instantaneous temperature [1]

$$T(t) = \frac{1}{3Nk_{\mathrm{B}}} \sum_{i=1}^{N} m_i \ \left\langle v_i^2(t) \right\rangle . \tag{21}$$

According to this relation, by calculating the location and momentum of nucleons for the projectile and target system at the time t, the average kinetic energy and then the temperature of the system is calculated. Therefore, in this way, the temperature evaluation is possible not only at the equilibrium and the formation of the compound nucleus, but also during the interaction time. In the ImQMD simulation, we investigate the evolution of the system using the laws of statistics and probability. This is done by determining the target and projectile nuclei, as well as selecting the energy of the system (E_{cm}). Since random behavior dominates, it is expected that different processes will occur with varying probabilities. Therefore, before discussing temperature calculation, it is necessary to evaluate the likelihood of the fusion process in the studied reactions. One way to determine the time of compound nucleus formation is by analyzing the change in distance between the colliding nuclei during their interaction and comparing it with the sum of their radii. By considering the distance graph of the colliding nuclei as a measure of time, it is possible to calculate the time of compound nucleus formation and, subsequently, measure the temperature of the equilibrium compound nucleus.

In this study, we investigate the fusion reactions of different nuclei to study the effects of temperature on nuclear processes. We analyzed the effects of temperature in the following fusion reactions: ^{16}O+^{92}Zr, ^{20}Ne+^{59}Co, ^{20}Ne + ^{40}Ca, ^{16}O + ^{94}Zn, ^{28}Si + ^{64}Ni, and ^{40}Ca + ^{92}Zr. Actually, the choice of these reactions in this range of mass has been used as an example, and this method can be used for other reactions and different energies. It is worth mentioning that this energy range is based on the experimental data of the fusion cross section. For example, in ^{16}O + ^{92}Zr, ^{20}Ne + ^{40}Ca, and ^{28}Si + ^{64}Ni reactions, the experimental fusion cross sections are in the range of [37.35–69.76] , [44.1–70.4], and [47.3–63.4], respectively. By considering

the distance graphs between target and projectile nuclei, we find that the probability of creating a fusion process and forming a compound nucleus varies depending on the type of reaction and the energy of the interacting system. Therefore, by increasing the energy, more events will lead to the formation of a compound nucleus. For example, let us consider the $^{16}O + {}^{92}Zr$ reaction at 35 MeV and 45 MeV energies. Figure 1 shows that at 35 MeV energy, the probability of the pseudo-fission process is very high in comparison with the fusion process. However, at 45 MeV energy, the possibility of the fission process and the compound nucleus formation is higher.

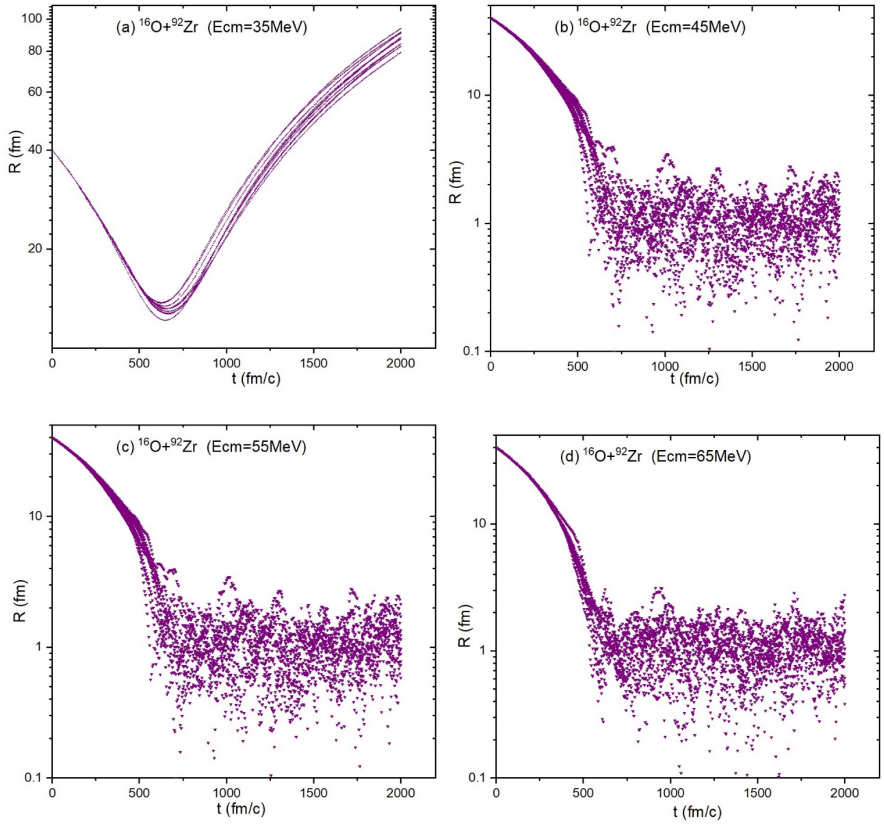

Fig. 1. Time evolution of the r.m.s. radii of the system for the fusion reaction $^{16}O + {}^{92}Zr$ at different incident energies.

The graphs in figures 1–6 illustrating the formation time of compound nuclei provide evidence that an increase in reaction energy leads to a higher likelihood of compound nucleus formation. After the stable projectile and target nuclei at the initial time (as illustrated in the previous section), when

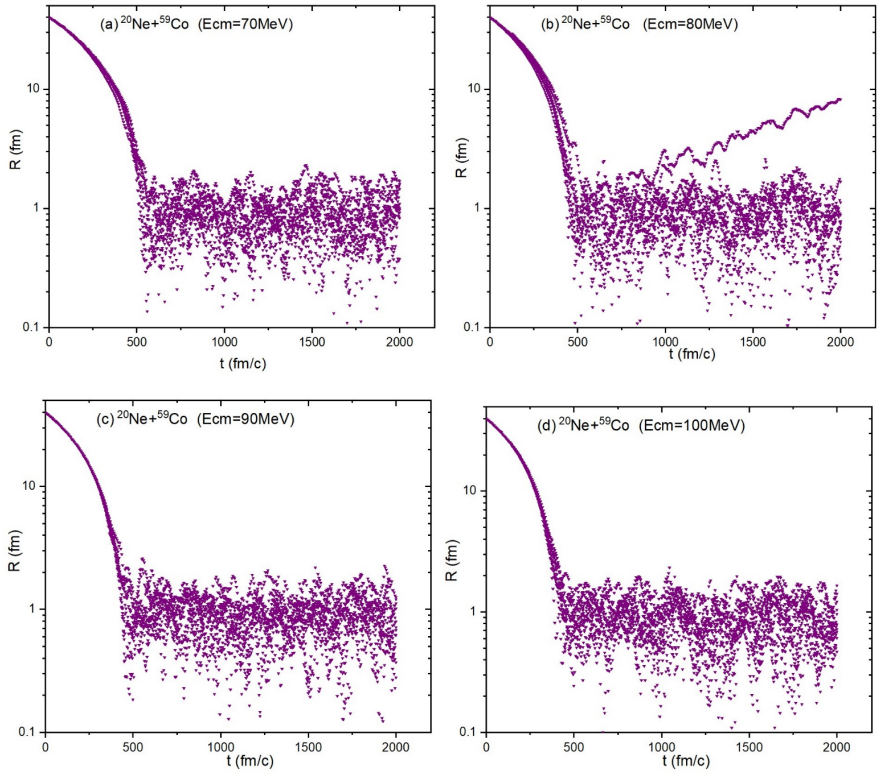

Fig. 2. Time evolution of the r.m.s. radii of the system for the fusion reaction $^{20}\mathrm{Ne} + {}^{59}\mathrm{Co}$ at different incident energies.

the participating nuclei rotate randomly, new ensembles can be obtained. By evaluating each ensemble and averaging over all ensembles, macroscopic properties such as temperature can be calculated. In our calculations, due to the time-consuming nature of dynamic simulation, we chose 500 ensembles for each reaction at each reaction energy. In figures 1–6, we present some of ensembles as an example. This trend is commonly observed as the energy of the center of mass rises and the width of the Coulomb barrier decreases, resulting in a faster attainment of statistical equilibrium and an increased probability of fusion. The variations in compound nucleus formation time suggest that changes in the position and momentum of nucleons affect temperature fluctuations in the interacting system during fusion formation. Therefore, a dynamic approach is necessary to accurately assess temperature fluctuations during the reaction. The continuous alterations in nucleon location and momentum imply dynamic changes within the system during the interaction, rendering the use of static models invalid for considering temperature changes in fusion reactions. The distinction between static

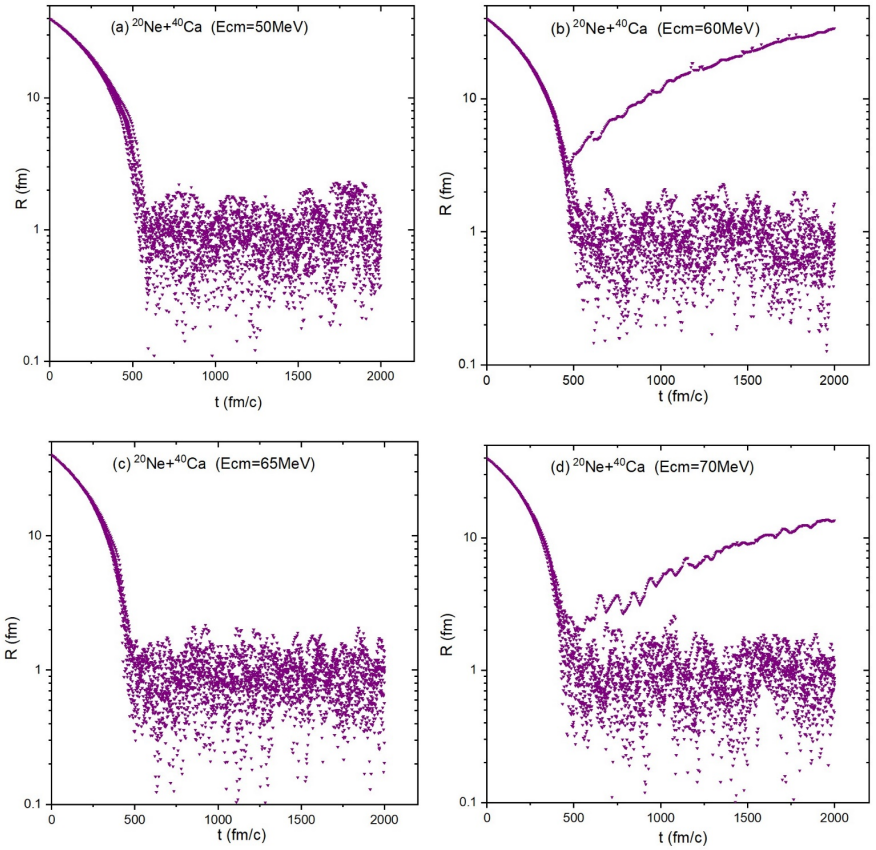

Fig. 3. Time evolution of the r.m.s. radii of the system for the fusion reaction $^{20}\mathrm{Ne} + {}^{40}\mathrm{Ca}$ at different incident energies.

and dynamic approaches becomes apparent when examining the graphs of temperature fluctuations during the reaction. In summary, from Figs. 1–6, we can conclude that:

(a) The initial distance between the nuclei is evident.

(b) These graphs clearly show that the type of fusion process, quasi-fission process, or elastic process are observable.

(c) The type of process depends on the energy of interaction. At some energies, the fusion process is dominant, while at other energies, the elastic or quasi-fission process dominates.

(d) The time to reach pre-equilibrium state at each energy and reaction is observable.

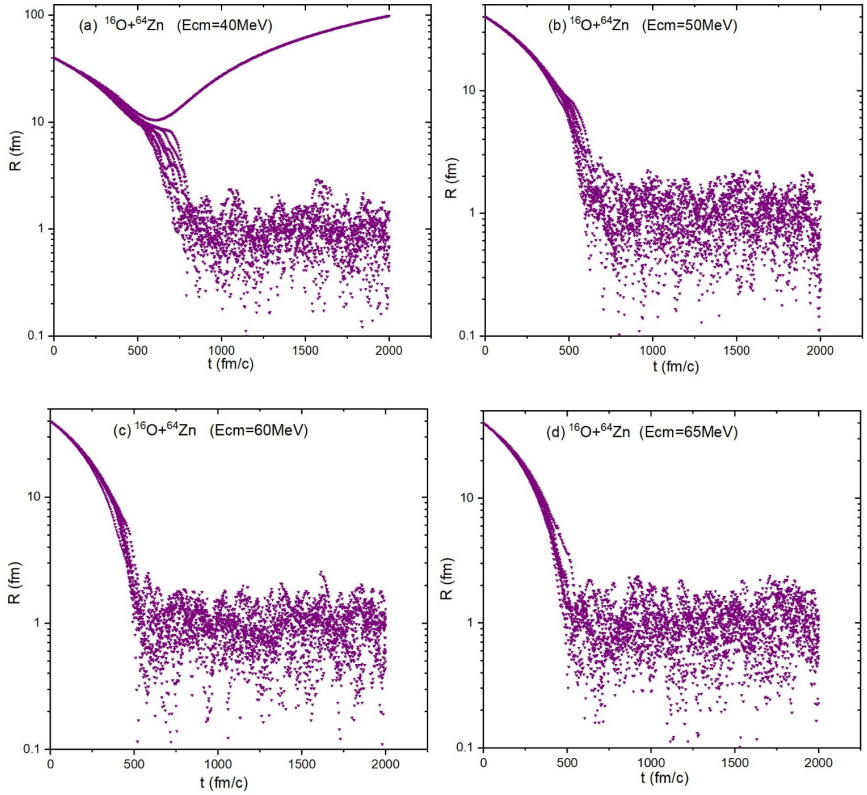

Fig. 4. Time evolution of the r.m.s. radii of the system for the fusion reaction $^{16}\mathrm{O} + {}^{64}\mathrm{Zn}$ at different incident energies.

To better illustrate the relation between formation time and the center-of-mass energy (E_{cm}) of the participating nuclei in fusion reactions, Fig. 7 depicts the formation time relative to E_{cm} energy. It is evident that as the center-of-mass energy increases, the formation time of the compound nucleus shows a decreasing trend. In order to validate this method, we obtained the temperature at the point of statistical pre-equilibrium when the compound nucleus is formed by referring to the results presented in Figs. 8–12. We compared this temperature with the results obtained from the equilibrium statistical mechanics approach (Weisskopf's theory) and the GEMINI statistical code.

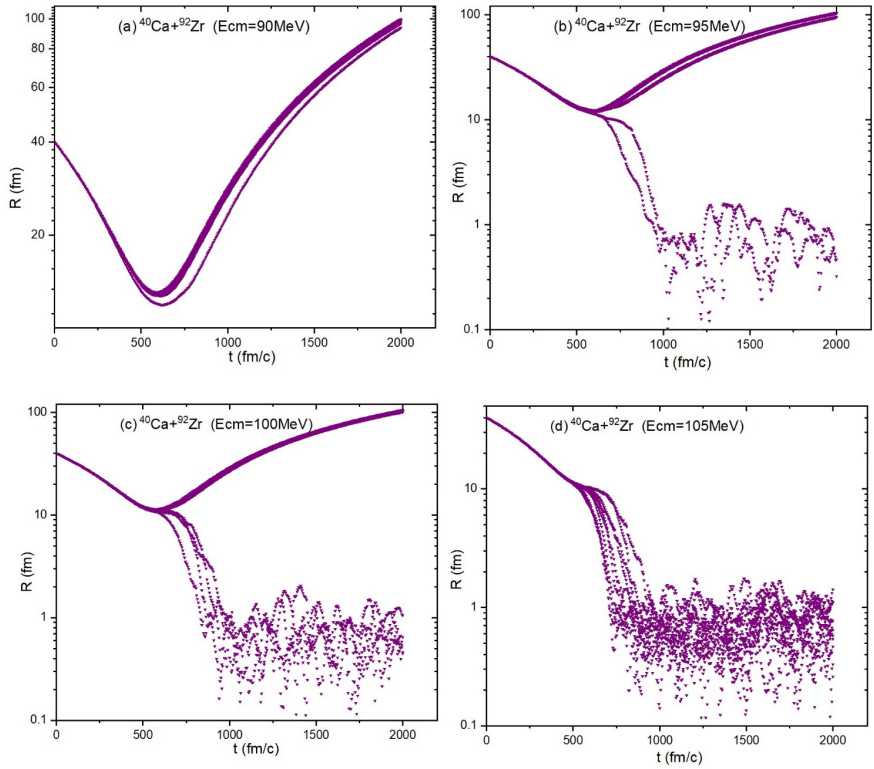

Fig. 5. Time evolution of the r.m.s. radii of the system for the fusion reaction $^{40}\mathrm{Ca} + {}^{92}\mathrm{Zr}$ at different incident energies.

The temperature fluctuations during the reaction, depicted in Figs. 8–12, indicate that the dynamic analysis of nuclear reactions, which takes into account the distribution of nucleons throughout the process, accurately captures the temperature evolution. Therefore, the investigation of temperature effects in hot nuclear systems using the ImQMD model provides a more precise description of the nuclear process, particularly prior to reaching statistical pre-equilibrium. Despite variations in the temperature of the compound nuclei obtained from these models, the temperature range derived from the ImQMD model closely aligns with Weisskopf's theory and the GEMINI statistical code. These differences are minimal for reactions with less mass asymmetry in the inlet channel. However, at lower energies, a significantly greater discrepancy in the obtained temperature values can be observed. It is worth mentioning that the hot compound nucleus is formed in an excited state and enters various channels through the evaporative and fission processes, involving different factors. The model presented in this

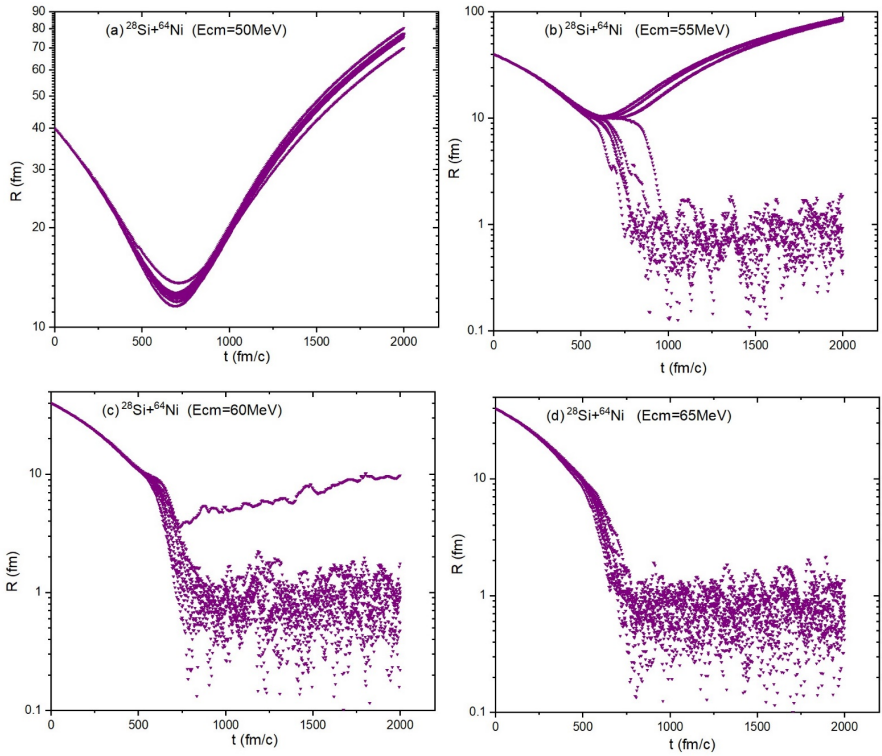

Fig. 6. Time evolution of the r.m.s. radii of the system for the fusion reaction $^{28}\mathrm{Si} + {}^{64}\mathrm{Ni}$ at different incident energies.

section requires various corrections, including corrections to motion equations, different effects on potential, and so on. At the pre-equilibrium point and the formation of the hot compound nucleus, the values obtained from other models have been compared. It is worth noting that these temperatures have been calculated only for ensembles that have led to the fusion process. Therefore, as can be clearly seen in figures 8–12, the temperature values we predict are higher than temperatures obtained from other models. We conducted similar investigations on other reactions and compared the temperature of the compound nuclei obtained from the aforementioned models. These findings demonstrate a strong agreement between the temperatures recorded at the moment of compound nucleus formation and the predictions of statistical models.

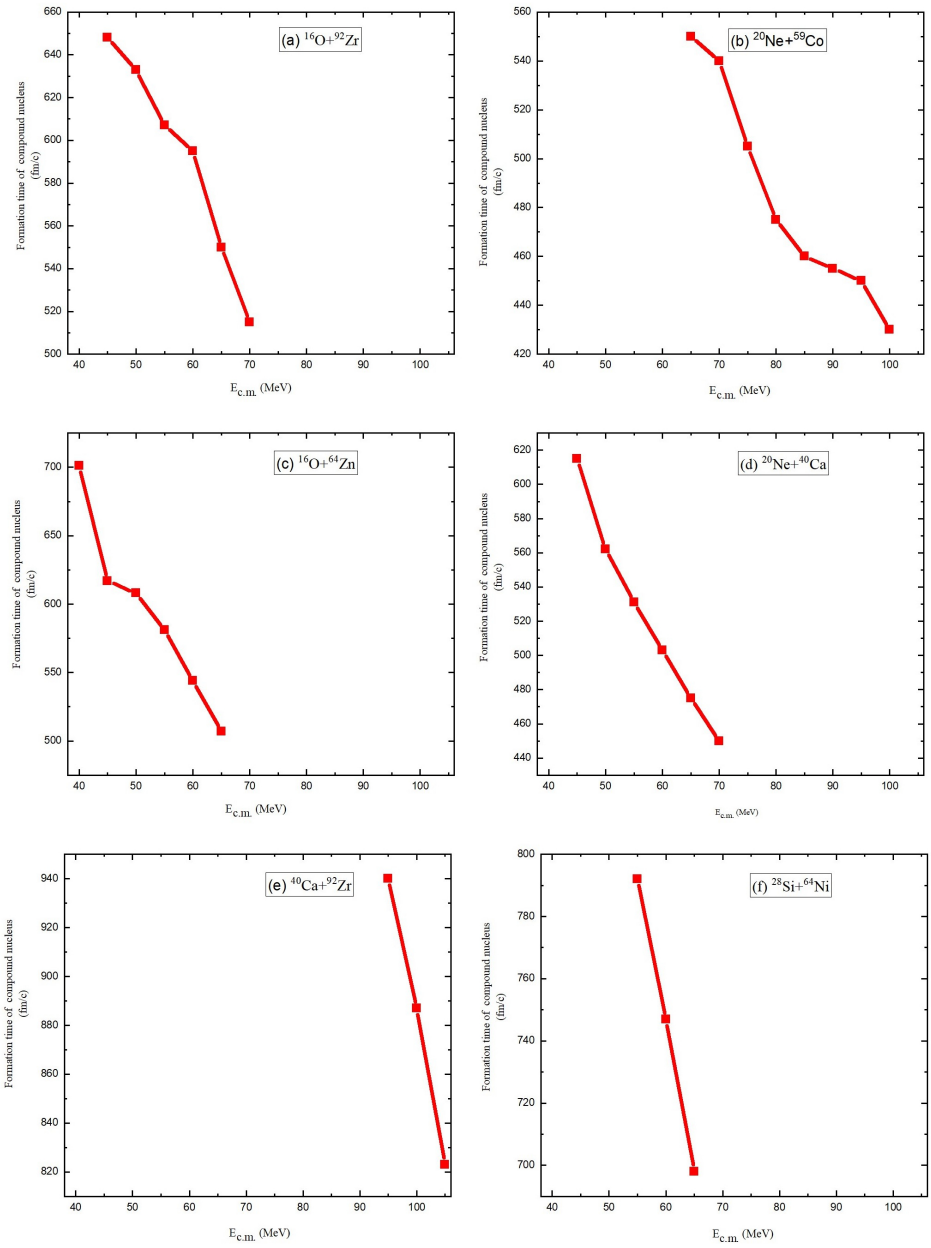

Fig. 7. Formation time of the compound nucleus with respect to $E_{\rm cm}$ in the studied reactions.

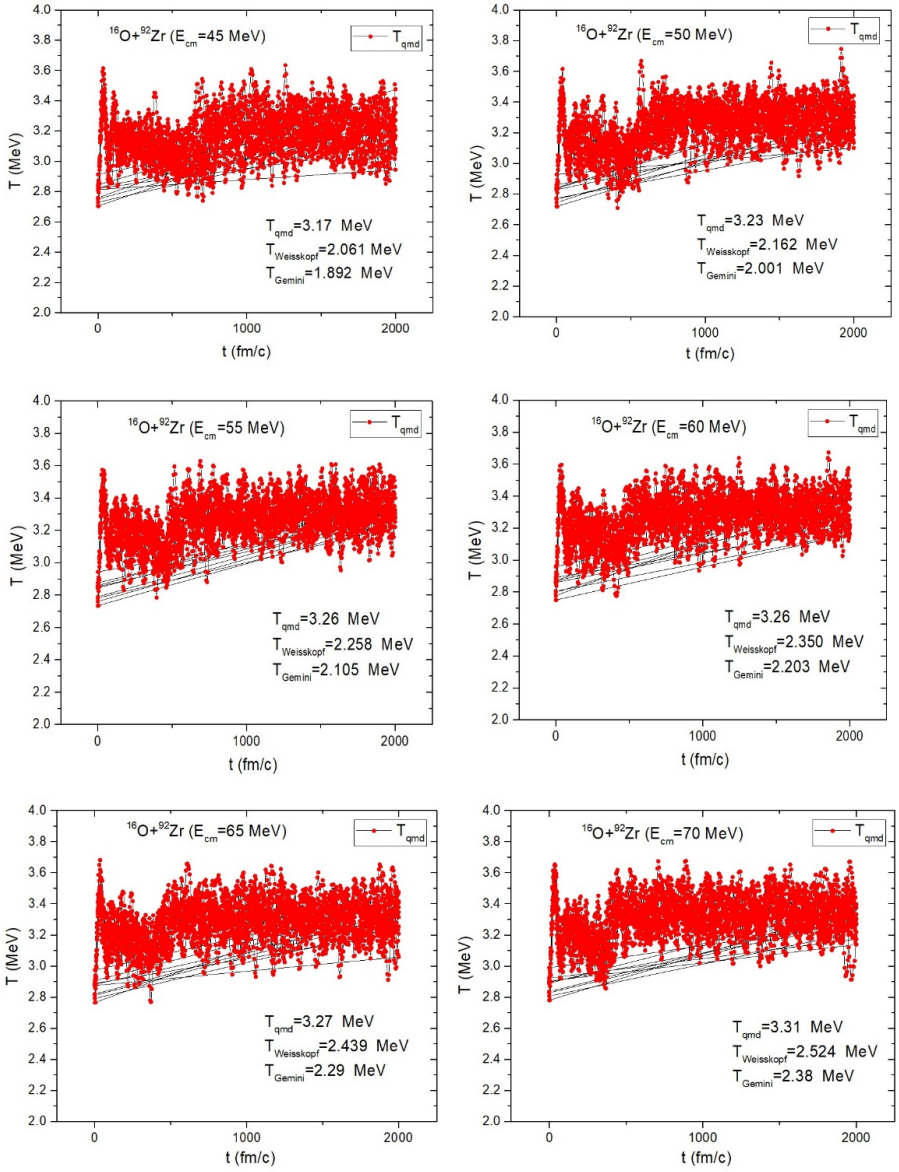

Fig. 8. Temperature fluctuations according to the ImQMD model results for $^{16}O + ^{92}Zr$ reactions.

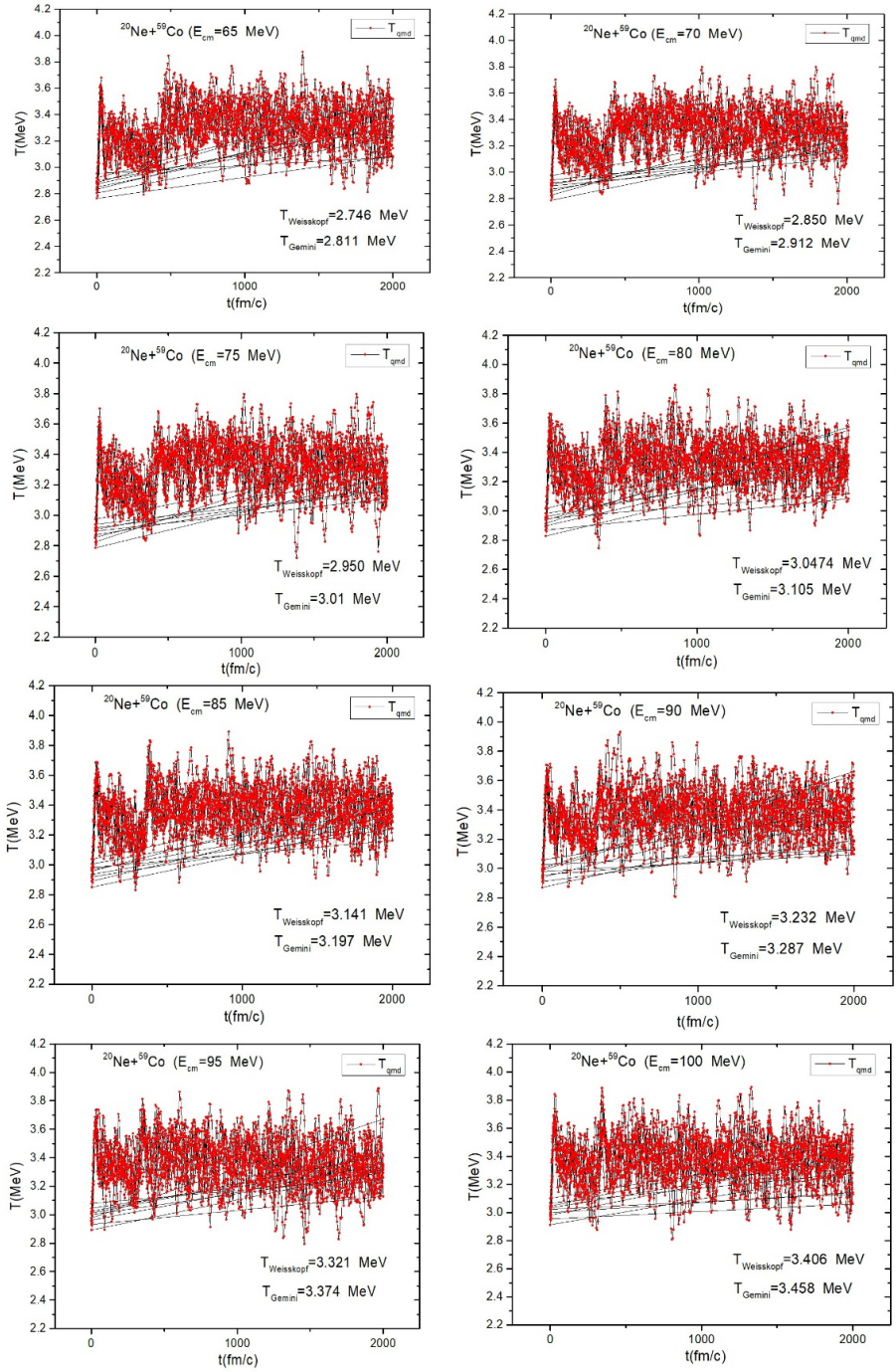

Fig. 9. Temperature fluctuations according to the ImQMD model results for ^{20}Ne+^{59}Co reactions.

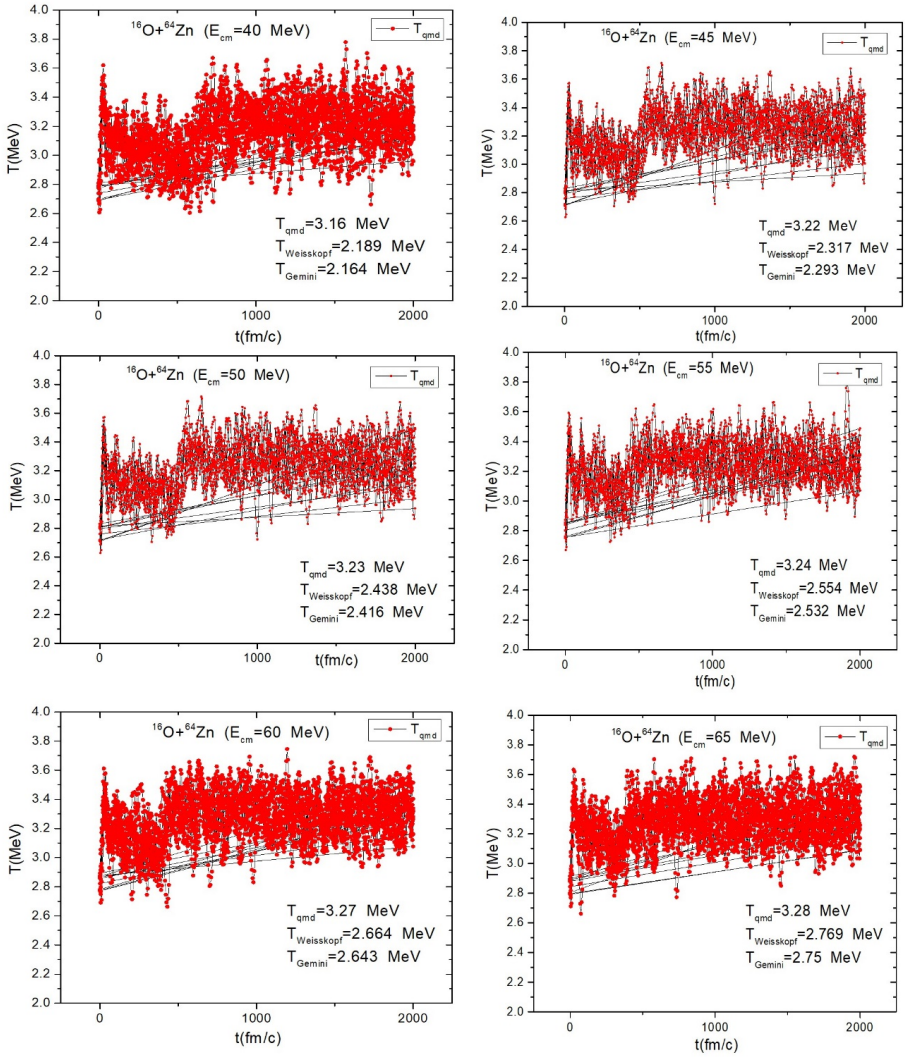

Fig. 10. Temperature fluctuations according to the ImQMD model results for $^{16}O+^{64}Zn$ reactions.

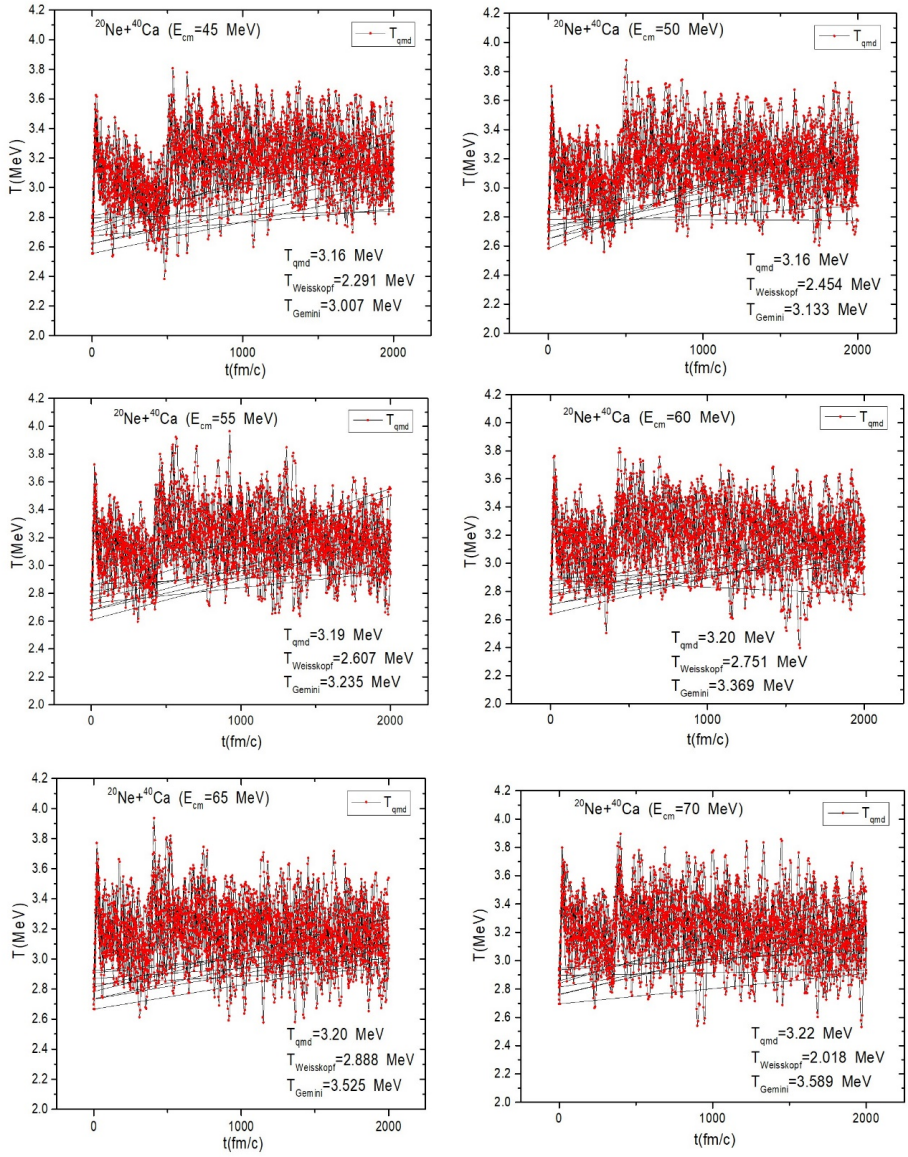

Fig. 11. Temperature fluctuations according to the ImQMD model results for $^{20}\mathrm{Ne} + {}^{40}\mathrm{Ca}$ reactions.

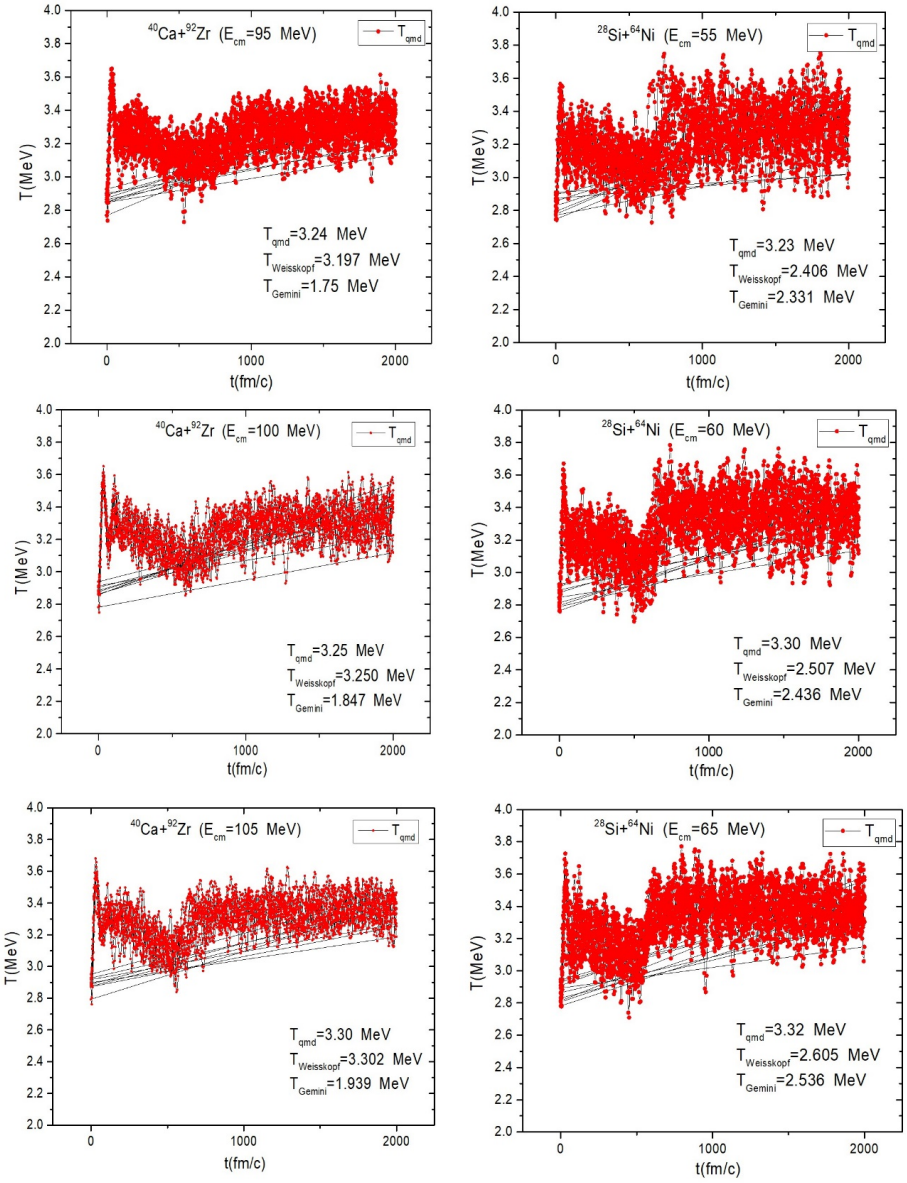

Fig. 12. Temperature fluctuations according to the ImQMD model results for $^{40}\mathrm{Ca} + {}^{92}\mathrm{Zr}$ and $^{28}\mathrm{Si} + {}^{64}\mathrm{Ni}$ reactions.

5. Conclusion

This article investigates the temperature evolution in fusion reactions using the ImQMD model. The study emphasizes the significance of dynamic analysis in understanding temperature effects in heavy nuclear systems. The research methodology involves evaluating the feasibility of fusion processes, determining the equilibration time of the compound nucleus, and calculating the compound nucleus temperature using the ImQMD model. The average kinetic energy of the nuclei is computed to track the system's temperature by considering microstate and ensemble averaging. The results demonstrate that higher incident particle center-of-mass energy leads to a shorter equilibration time. The analysis of temperature fluctuations during the fusion process highlights the limitations of static models that overlook changes in location and momentum distribution of nucleons. The dynamic approach ($T_{\rm qmd}$) yields higher temperature values compared to static approaches, such as Weisskopf's theory and the GEMINI statistical code. However, this discrepancy diminishes as the center-of-mass energy increases, underscoring the importance of considering temperature effects at lower energies. Moreover, temperature fluctuations are more significant at the initial stages of the interaction, gradually decreasing as the interacting nuclei transition into the compound nucleus state. Reactions with less mass asymmetry exhibit smaller discrepancies between dynamic temperature values ($T_{\rm qmd}$) and those obtained from the GEMINI approach. Furthermore, as the energy of the interacting nuclei increases, the difference between the temperature values provided by GEMINI and the dynamic model becomes less pronounced.

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