

HBT RADII OF CHARGED PIONS WITH SHEAR VISCOUS TRANSPORT DYNAMICS SIMULATIONS IN THE AMPT MODEL

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We utilized the AMPT model to simulate the shear viscous transport dynamics of parton matter with varying specific shear viscosity and phase-transition temperatures and investigate the effect on the HBT radii of charged pions at the $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions. The emission-source size is sensitive to the shear viscosity of parton matter, thus, increasing the shear viscosity to entropy density ratio will reduce the HBT radii. The phase-transition temperature can regulate the evolution duration of the parton and hadron phases. Raising the phase-transition temperature shortens the evolution time of the partonic phase and extends the evolution time of the hadronic phase, resulting in an increase in the total system evolution time. A prolonged hadronic-phase evolution weakens the dependence of the transverse HBT radii on transverse mass and increases the evolution time scale.

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1. Introduction

Exploring the dynamics and properties of the quark–gluon plasma produced at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) is a primary goal of relativistic heavy-ion collision research. The strongly coupled deconfinement QGP produced in nuclear–nuclear collisions exhibits rapid hydrodynamic expansion similar to that of an almost perfect fluid [1–7]. Its temperature-dependent viscosity coefficient can be precisely estimated using the Bayesian parameter estimation method, with the shear viscosity coefficient approaching the KSS bound near the phase-transition temperature [8–10]. Understanding the spatiotemporal evolution of the full system is essential for investigating the characteristics of QGP.

However, due to the complexity of the collision system, experimental observables based on models are frequently employed to study the system properties. A multi-phase transport model (AMPT), which includes partonic- and hadronic-transport processes based on non-equilibration transport dynamics, has been widely applied in heavy-ion collision studies [11–15]. The shear viscous transport dynamics simulation of parton matter enables the quantitative study of shear viscosity and phase-transition temperature, which are closely related to system evolution within the AMPT model [16–18].

The measurements of the space-time distribution at kinetic freeze-out can provide constraints on the system evolution and the HBT interferometry, as an important observable can be used to measure the spatial and temporal scales of systems, and detect the space-time substructure and momentum correlations of the freeze-out configuration in heavy-ion collisions [19–24]. With the shear viscous transport dynamics simulation by the Chapman–Enskog (CE) methods, we simulate Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in the AMPT model, and study the influences of different specific shear viscosity and phase-transition temperatures on the charged pion HBT radii and evolution time scale. In Section 2, we implement the shear viscous transport dynamics simulations of parton matter in the AMPT model. In Section 3, the charged pion HBT radii and evolution time scale are discussed. We conclude in Section 4.

2. Shear viscous simulation of partonic-transport dynamics

The shear viscosity in the transport process of parton matter can be numerically expressed as functions of finite temperature and scattering cross section, despite the differences in numerical estimates obtained from different analytical methods based on kinetic theory [25]. The AMPT model is a hybrid model incorporating partonic- and hadronic-transport processes to generate heavy-ion collision events, with parton interactions based on the two-body forward-angle scattering process of the ZPC cascade model [26]. In the CE method applicable to anisotropic scattering, the shear viscosity can be expressed as

$$\eta = \frac{4T}{5\sigma_p g(w)}, \quad (1)$$

where σ_p is the partonic-scattering cross section, $g(w)$ is the thermal average of $h(a) = 4a(1+a)[(1+2a)\ln(1+\frac{1}{a})-2]$ and can be approximated as [25, 27]

$$g(w) \approx h\left(\frac{w^2}{v^2}\right), \quad (2)$$

with $w = \mu/T$, μ being the screening mass and $v = 11.31-4.847 \exp(-0.1378 w^{0.7338})$. At temperature T , the parton matter of massless three-flavor

quarks is assumed, the shear viscosity-to-entropy density ratio is [18]

$$\eta/s \approx \frac{2\pi w^2}{45g_B\alpha_s^2 g(w)}, \quad (3)$$

where $g_B = 52$ when using Boltzmann distribution and α_s is the QCD coupling constant.

The critical aspect of simulating the shear viscous transport dynamics simulation of parton matter in the AMPT model is the adjustment of μ/T during the system evolution for a fixed α_s , enabling quantitative control of the parameterized $\eta/s(T)$ in parton interactions. Compared to the easily adjustable input parameter μ , which is directly related to the partonic-scattering cross section, $\sigma_p \approx 9\pi\alpha_s^2/(2\mu^2)$, it is more important to obtain the spatiotemporal distribution of the parton phase temperature. The temperature distribution can be estimated using partonic-energy density under the Boltzmann distribution. In a single event, due to the small number of partons in a cell, the statistical nature of the energy density estimated by averaging the parton energy in the volume element is insufficient for extracting the temperature distribution. Hence, we use the Gaussian smearing approximation to obtain the energy density distribution by smearing each pointlike parton with a three-dimensional Gaussian distribution of its total energy. The distribution function of the parton phase temperature at evolution time t is given as

$$T(x, y, z) = \left\{ \frac{\pi^2}{3g_B} \sum N_i \exp \left[-\frac{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}{2\sigma_i^2} \right] \right\}^{\frac{1}{4}}, \quad (4)$$

where $N_i = (\frac{1}{2\pi})^{\frac{3}{2}} \frac{1}{\sigma_i^3} E_i$ provides the proper normalisation, (x_i, y_i, z_i) , σ_i , and E_i are the position vector, Gaussian width, and energy of parton i . A continuous and consistent temperature distribution can be obtained by the Gaussian smearing approximation, which contains the spatial dependence and fluctuations of parton matter. By utilizing the temperature distribution during the evolution of parton matter, quantitative control of parameterized η/s in the model can be achieved by adjusting μ which equates to altering σ_p in two-body scattering processes. To prevent excessively long and unreasonable parton evolution times, when the effective temperature of the cell is below the phase-transition temperature T_{tr} , partons are unable to scatter in that cell and prepare for hadronization.

3. HBT radii and evolution time scale of charged pions

The shear viscous transport dynamics simulation can generate specific parameterized shear viscosity η/s and phase-transition temperature T_{tr}

of the heavy-ion collision events in the AMPT model, enabling the study of the effects of η/s and T_{tr} on system evolution, and measured observables within non-equilibrium transport dynamics. The parameters of the Lund string fragmentation $a_L = 0.55$, $b_L = 0.2 \text{ GeV}^{-2}$, and the coupling constant $\alpha_s = 0.33$ are chosen for Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ in the model.

The two-pion HBT correlation function, which is calculated by the **Correlation After Burner** program without the final-state interactions, can be fitted by the Bertsch–Pratt parametrization in the ‘out–side–long’ coordinate system [28, 29]

$$C(\mathbf{q}) = 1 + \lambda \exp \left(-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2 \right), \quad (5)$$

where q_o , q_s , q_l are three components of the relative momentum difference $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ of a pair of pions. By fitting the correlation function, one can obtain size parameters R_o , R_s , and R_l which are so-called HBT radii.

Figures 1 and 2 show the charged pion HBT radii R_o , R_s , R_l , and the R_o/R_s ratio as a function of the average transverse mass for Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ in 0–10% centrality with $\eta/s = 0.08$ and $\eta/s = 0.16$, respectively, compared to PHENIX data. It can be seen from the figures that for constant shear viscosity, the phase-transition temperature has a significant impact on the variation of HBT radius with transverse mass m_T , and the impact varies for HBT radius in different directions and

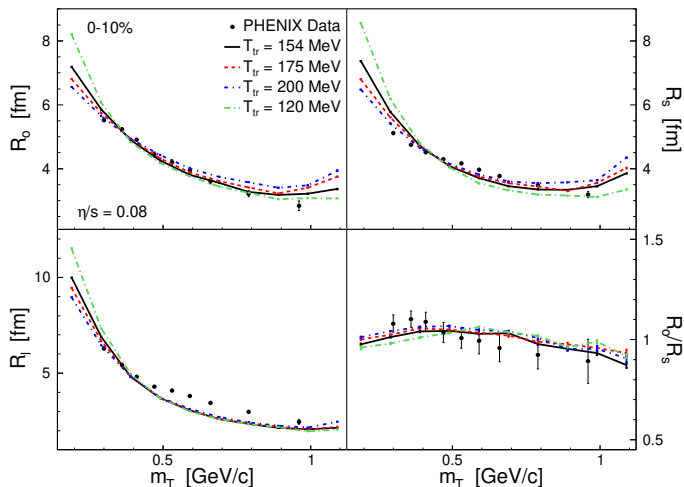


Fig. 1. HBT radii R_o , R_s , R_l , and the R_o/R_s ratio of charged pions varies with average transverse mass for Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ with $\eta/s = 0.08$ in 0–10% centrality compared to experimental data from the PHENIX Collaboration [22].

across different transverse mass ranges. With the same shear viscosity, the transverse mass dependence of the HBT radius weakens as the temperature increases. In the small transverse-mass range, the higher T_{tr} , the smaller the corresponding left initial HBT radius, but higher shear viscosity will reduce the differences between the radii. In the higher transverse mass range, R_l is almost unaffected by changes in T_{tr} , whereas R_o and R_s are increasingly affected by temperature as the transverse mass increases. R_o and R_s in the large- m_T range increases with the rise in T_{tr} , and higher viscosity will enhance the impact on R_o compared to R_s .

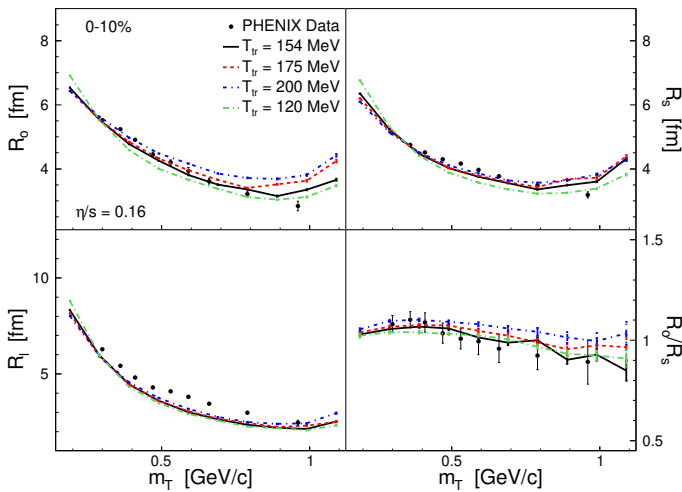


Fig. 2. The same as Fig. 1 but with $\eta/s = 0.16$.

Figure 3 shows the charged pion HBT radii as a function of the average transverse mass for Au+Au collisions with various η/s at the phase-transition temperature $T_{\text{tr}} = 154$ MeV. Similar to the effect of phase-transition temperature on the HBT radius, shear viscosity also reduces the dependence of HBT radius on transverse mass, but the sensitivity of radii in different directions to η/s and T_{tr} varies. R_l , which is less affected by T_{tr} , is sensitive to changes in parton phase η/s , while the impact on R_o is relatively reduced. Compared to the higher T_{tr} resulting in smaller radii only in the small- m_T range, the influence range of η/s on the reduction of radii has significantly expanded and, as shear viscosity increases, its impact on HBT radius gradually diminishes.

Considering that pions from resonance decays can affect the HBT radius, especially in the small transverse-mass range, to distinguish whether the impact of phase-transition temperature and shear viscosity on the HBT radius originates from the source function of pions from resonance decays, we disabled the decay of ω mesons in the hadronic-transport process in the

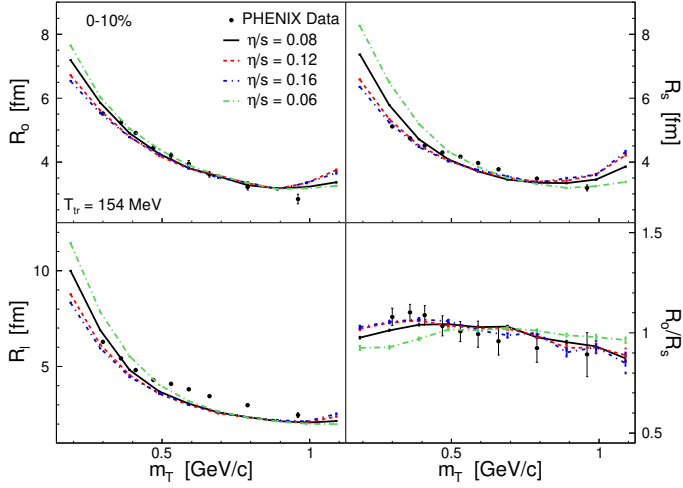


Fig. 3. HBT radii of charged pions with various η/s at $T_{tr} = 154$ MeV.

AMPT model, generating heavy-ion collision events that only include pions directly produced from hadronization. Figures 4 and 5 show the charged pion HBT radii without ω decays as a function of the average transverse mass for Au+Au collisions. It can be observed from the figures that resonance decays only affect the size of the HBT radius in the small- m_T range and do not change the trend of the dependence of the HBT radius on m_T with respect to T_{tr} and η/s , indicating that the changes in T_{tr} and η/s can be reflected in the evolution of the collision system through the HBT radius.

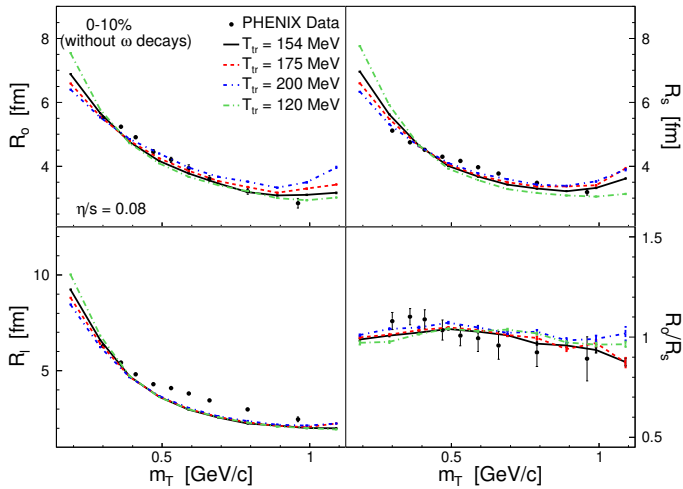


Fig. 4. The same as Fig. 1 but without ω decays.

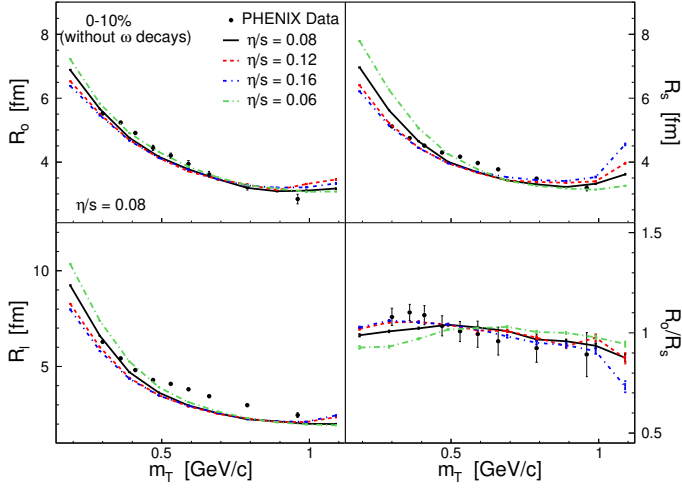


Fig. 5. The same as Fig. 3 but without ω decays.

Figure 6 shows the transverse flow velocity of charged pions as a function of the transverse mass for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in 0–10% centrality, where the left figures present $m_T - m_0 = [0, 0.2]$ GeV/ c^2 , the right figures $m_T - m_0 = [0.2, 1.0]$ GeV/ c^2 . It is noteworthy that the influence trends of T_{tr} and η/s on transverse flow velocity β_T with m_T variation

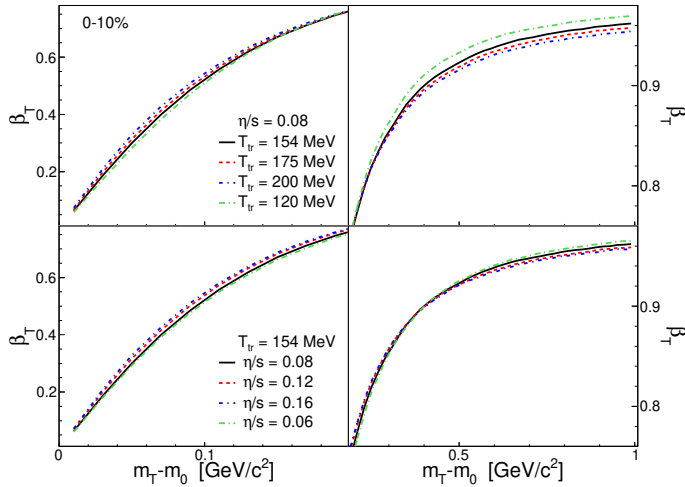


Fig. 6. Transverse flow velocity of charged pions varies with the transverse mass for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in 0–10% centrality. Top: constant η/s . Bottom: constant T_{tr} .

are consistent with the trends of HBT radius dependence on m_T , and the range of m_T affected is also consistent, indicating that T_{tr} and η/s affect the transverse flow velocity in the system evolution process, and the changes in β_T lead to differences in the transverse-mass dependence of the HBT radius.

For constant η/s , the increase in T_{tr} enhances β_T in the small- m_T range and suppresses it in the large- m_T range. This results in particles with increased β_T in the small- m_T range emitting earlier from the source with a reduced transverse radius, while those in the large- m_T range with decreased β_T emit later with an increased transverse radius, and β_T has a smaller effect on longitudinal expansion. The main reason for β_T to be affected by the phase-transition temperature is that a higher T_{tr} shortens the evolution time of the parton phase, thereby extending the evolution duration of the hadron phase. The shear viscosity in the hadronic-transport process is much greater than in the partonic-transport process. The stronger hadron phase viscosity suppresses the outward movement of large transverse-mass particles while promoting the movement of small transverse-mass particles. The longer the hadron phase duration, the greater the impact on particle movement. The transverse HBT radius dependence on transverse mass is mainly influenced by T_{tr} due to changes in the duration of hadron phase evolution.

For constant T_{tr} , shear viscosity similarly impacts transverse flow velocity. An increase in viscosity suppresses the movement of large transverse-mass particles and enhances the movement of small transverse-mass particles. However, compared to the effect of hadron phase viscosity on β_T , the influence of parton phase viscosity is weaker, especially in the large transverse-mass range. This results in R_o , which is most sensitive to transverse flow velocity, being least affected by η/s . Unlike the lower evolution temperature of the hadron phase, the evolution of the parton phase corresponds to the rapid expansion of a high-temperature system. Thus, increasing shear viscosity suppresses system expansion, reducing the system size and leading to the smaller HBT radii with greater η/s , and this is ultimately reflected in R_s and R_l , which are sensitive to system size.

The impact of the combined effects of T_{tr} and η/s on the system evolution is evident not only in the variations in the HBT radius size and its transverse-mass dependence but also directly affects the freeze-out temperature and evolution time scale of the pion source. The transverse-mass spectrum can be parameterized in the following exponential form as [30, 31]:

$$\frac{d^2N}{2\pi m_T dm_T dy} = A \exp\left(-\frac{m_T}{T}\right), \quad (6)$$

where T is the inverse slope parameter and A is a normalization parameter. The freeze-out temperature T_0 is related to the parameter $T = T_0 + m\langle\beta_T\rangle^2$ and $\langle\beta_T\rangle$ is the average transverse flow velocity. By utilizing the dependence

of the longitudinal HBT radius R_l and the corresponding T_0 , the evolution time scale τ of the pion source can be extracted through the Herrmann–Bertsch formula [20]

$$R_l = \tau \sqrt{\frac{T_0}{m_T} \frac{K_2(m_T/T_0)}{K_1(m_T/T_0)}}, \quad (7)$$

which is derived by assuming the thermal spectrum of particles satisfied with the Cooper–Frye formula and the equilibrium thermal distribution as required by the Boltzmann approximation. m_T spectra and R_l of charged pions for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with $\eta/s = 0.08$ in 0–10% centrality are fitted by Eqs. (6) and (7) in Fig. 7.

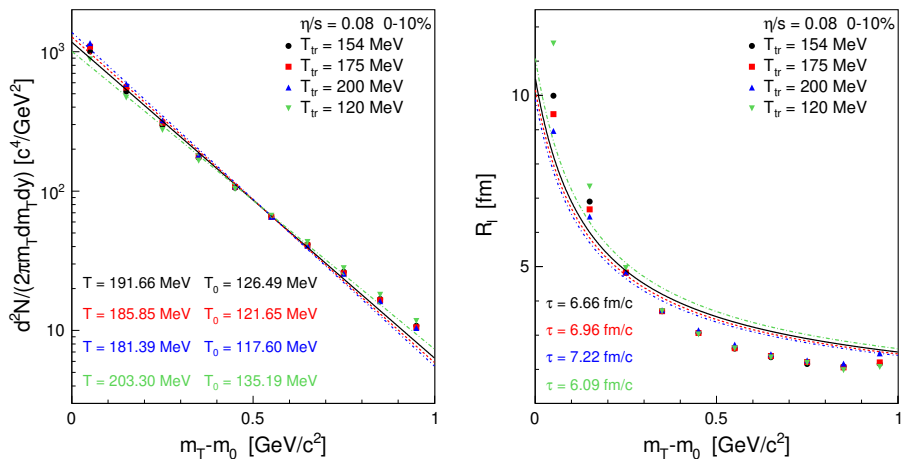


Fig. 7. Transverse-mass spectra and longitudinal HBT radii of charged pions for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with $\eta/s = 0.08$ in 0–10% centrality. Lines represent the fitted results using m_T exponential function (left) and the Herrmann–Bertsch formula (right), respectively.

Figure 8 shows the freeze-out temperature T_0 and evolution time scale τ of the pion source as a function of η/s for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in 0–10% centrality. It is evident that both η/s and T_{tr} significantly affect T_0 and τ . At the same shear viscosity, as phase-transition temperature increases, T_0 decreases and τ increases. This is because a higher T_{tr} causes the system to enter the hadron phase evolution earlier. The stronger shear viscosity in the hadron phase slows down the system expansion and evolution, resulting in a longer evolution time and a lower freeze-out temperature. For the same phase-transition temperature, as η/s increases, likewise, T_0 decreases and τ increases. Similar to the previous reason, the increase

in shear viscosity slows down the system evolution. Here, the parton phase evolution is prolonged due to the increased η/s . Compared to the effect of T_{tr} , the impact of η/s on T_0 and τ tends to saturate as η/s increases. Moreover, the higher T_{tr} , the faster the saturation rate. This is also understandable, as a higher T_{tr} results in a shorter parton phase evolution time, thereby reducing the influence of η/s on parton matter. The results of T_0 and τ , like the previous HBT radii, show that T_{tr} and η/s directly impact the system evolution. η/s acts directly on the parton phase evolution, while T_{tr} regulates the duration of both the parton and hadron phase evolutions.

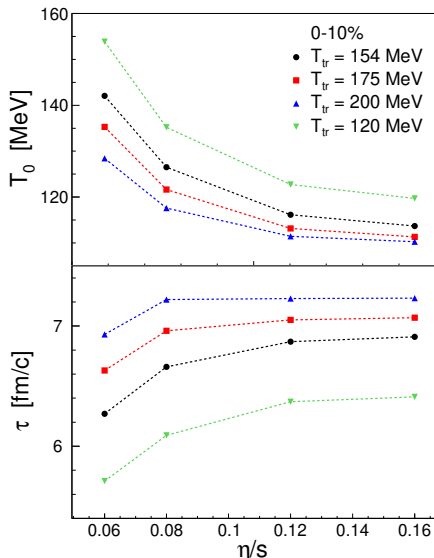


Fig. 8. Freeze-out temperature T_0 and evolution time scale τ of the pion source with various η/s at different T_{tr} .

4. Summary

Based on the AMPT model with the shear viscous transport dynamics simulation of parton matter, we have studied the HBT radii and the evolution time scale of charged pions in the $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions with various η/s and T_{tr} . The phase-transition temperature and shear viscosity have a direct impact on the evolution process, size, and duration of both the parton and hadron phases, and this will be reflected in the final-state observables. Increasing η/s of the parton phase and raising T_{tr} both suppress the outward movement of high- m_T particles and enhance the movement of low- m_T particles. This leads to a reduction in transverse

flow velocity in the high- m_T range and an increase in the low- m_T range, extending the evolution time scale and lowering the freeze-out temperature of the emission source. The shear viscosity η/s of the parton phase acts on the early rapid expansion process of the high-temperature and high-density system, significantly impacting the size more than the evolution time and transverse flow velocity of the system, and gives smaller values of the HBT radii. The phase-transition temperature regulates the evolution duration of the parton and hadron phases and is reflected in the evolution time scale. As T_{tr} increases, the duration of the parton phase decreases while the duration of the hadron phase increases, and the stronger viscosity in the hadronic phase greatly slows down the system evolution speed, thus extending the evolution time. However, after the rapid expansion of the high-temperature and high-density parton phase, the evolution of the hadron phase, with significantly lowered temperature and density, has a much weaker effect on the size compared to its effect on the evolution time and transverse flow velocity of the system. T_{tr} weakens the transverse-mass dependence of the transverse HBT radius, extending the evolution time scale τ . The results help in understanding the evolution of relativistic heavy-ion collisions, allowing for further study of the combined effects of T_{tr} and η/s on measured observables in data from various energy regions.

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