

RELAXATION OF PARTICLE–HOLE-TYPE EXCITATION IN A FERMI SYSTEM WITHIN THE DIFFUSION APPROXIMATION OF KINETIC THEORY FOR THE CASE OF CONSTANT DIFFUSION AND DRIFT COEFFICIENTS

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The time evolution of the distribution function for a particle–hole excitation in a Fermi system was calculated using the direct numerical solution of a nonlinear diffusion equation in momentum space. A phenomenological expression for calculating the relaxation time of such an excitation to its equilibrium value has been proposed. It is shown that the relaxation time is dependent on both the excitation energy and the mass number.

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1. Introduction

The study of physical phenomena occurring in finite-size Fermi systems remains an active area of research [1]. Among these, the relaxation processes of collective and single-particle excitations in atomic nuclei are particularly important and intriguing. The relaxation processes in many-particle Fermi systems are conveniently analyzed using quantum kinetic theory methods. Within this framework, the system is described by the kinetic equation for the Wigner distribution function in the phase space of coordinates and momenta. The advantage of this approach is that it allows for a relatively straightforward description of average quantities such as nucleon density, density flux, pressure, and others in a quantum Fermi system. However, complications arise due to the presence of the collision integral on the right-hand side of the kinetic equation. To address these challenges, various simplification methods are employed [2–4].

One of these methods is the so-called diffusion approximation [5]. For instance, in [6], a schematic model describing the equilibrium state in a finite-size Fermi system was considered. The master equation for single-particle states was transformed into a nonlinear diffusion equation, taking the Pauli

exclusion principle into account. Within this model, an analytical solution was obtained in the simplified case of constant transport coefficients. This made it possible to both describe the time evolution of the initial-distribution function for the Fermi system and derive an expression for the equilibrium relaxation time of this excitation: $\tau = 4D/v^2$, where D is the diffusion coefficient and v is the drift coefficient. In subsequent studies [7], the authors presented another, alternative method for solving the nonlinear diffusion equation in the approximation of constant kinetic coefficients.

In works [8, 9], it was demonstrated that the Landau-Vlasov kinetic equation with the nine-dimensional collision integral in phase space can be reduced to a diffusion equation for the Wigner distribution function in momentum space. Explicit expressions for the diffusion and drift coefficients were obtained. At low temperatures in a Fermi system, the small transferred momentum approximation is used to simplify calculations involving particle scattering near the Fermi surface [2]. However, this assumption alone was insufficient to obtain a physically accurate result. It was necessary to make an additional assumption regarding the nature of the interaction between the scattering particles. Previously, the approximation of isotropic nucleon scattering probability was considered adequate [2, 4]. However, in this case, the calculation of kinetic coefficients led to divergent integral expressions. This divergence was avoided by imposing a short-range condition on the internucleon potential. Specifically, for a Gaussian potential, it was possible to obtain convergent expressions for the kinetic coefficients, calculate their numerical values, and determine their temperature dependencies based solely on phenomenological parameters of the internucleon interaction, such as the potential depth and its effective range [9].

The diffusion equation can be used to calculate the time evolution of the distribution function in phase space starting from an initial distribution. In the approximation of constant kinetic coefficients, as demonstrated by the authors in [6, 7], this can be done analytically, which is one of the rare cases where a nonlinear equation can be solved exactly. In works [8, 9], the nonlinear diffusion equation was solved through a direct numerical calculation. It is evident that the solution in this case exhibits properties similar to the analytical approach, namely, the gradual spreading of the initial-distribution function over time until it reaches the equilibrium Fermi distribution, with a temperature determined by the ratio of the kinetic coefficients. However, the advantage of this method is that the direct numerical calculation allows for future solutions in the more general case of momentum-dependent kinetic coefficients.

In this paper, Section 2 presents the diffusion approximation for the kinetic equation with the collision integral. In Section 3, a phenomenological method is proposed for calculating the equilibrium relaxation time τ_{eq} for the

initial-distribution function in a spherical Fermi system, which models an atomic nucleus in its ground state. Section 4 examines the case of nucleon–hole-type excitation in an atomic nucleus and uses the proposed formula to calculate the dependence of τ_{eq} on the excitation energy E_{ex} of the nucleon–hole pair and the mass number A . Conclusions are presented in Section 5.

2. Diffusion approximation

Let us consider the kinetic equation with the collision integral on the right-hand side

$$\frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} + \hat{L}f(\mathbf{r}, \mathbf{p}, t) = \text{St}\{f\}, \quad (1)$$

where $f(\mathbf{r}, \mathbf{p}, t) \equiv f(\mathbf{p})$ is the Wigner distribution function in phase space, $\text{St}\{f\}$ is the collision integral, and the operator $\hat{L}$ is given by the expression

$$\hat{L} = \frac{1}{m} \mathbf{p} \cdot \nabla_{\mathbf{r}} - (\nabla_{\mathbf{r}} U) \cdot \nabla_{\mathbf{p}}, \quad (2)$$

where m is the particle mass. In the general case, the single-particle potential U includes both the self-consistent and external fields. For the collision integral, we use the expression obtained in the diffusion approximation [8, 9]

$$\begin{aligned} \text{St}\{f\} = & -\nabla_{p_\nu} \left[K_p(\mathbf{p}) f(\mathbf{p}) (1 - f(\mathbf{p})) \frac{p_\nu}{m} + f(\mathbf{p})^2 \nabla_{p_\nu} D_p(\mathbf{p}) \right] \\ & + \nabla_{p_\nu}^2 [f(\mathbf{p}) D_p(\mathbf{p})], \end{aligned} \quad (3)$$

where the quantities $D_p(\mathbf{p})$ and $K_p(\mathbf{p})$ define the diffusion and drift coefficients in momentum space.

We study a Fermi system that models a spherical atomic nucleus in its ground state. To do this, we consider the following sequence of approximations. Let us assume infinite, uniformly distributed nuclear matter in coordinate space. Then, in the kinetic equation (1), we set $f(\mathbf{r}, \mathbf{p}, t) = f(\mathbf{p}, t)$, and thus, $\hat{L}f(\mathbf{r}, \mathbf{p}, t) = 0$. In this nuclear matter, we isolate a spherical region with radius R . The distribution function of such a system will exhibit spherical symmetry in momentum space, *i.e.*, $f(\mathbf{p}) = f(p)$. For the expressions $D_p(\mathbf{p})$ and $K_p(\mathbf{p})$ in the collision integral (3), we apply the approximation of constant kinetic coefficients [6–10], namely: $D_p(\mathbf{p}) = D_{p,0}$ and $K_p(\mathbf{p}) = K_{p,0}$. Taking into account the aforementioned approximations, the kinetic equation (1) transforms into a nonlinear diffusion equation in momentum space

$$\begin{aligned} \frac{\partial f(p)}{\partial t} = & -\frac{K_{p,0}}{m} \left[p(1 - 2f(p)) \frac{\partial f(p)}{\partial p} + 3f(p)(1 - f(p)) \right] \\ & + D_{p,0} \left[\frac{\partial^2 f(p)}{\partial p^2} + \frac{2}{p} \frac{\partial f(p)}{\partial p} \right]. \end{aligned} \quad (4)$$

The diffusion equation (4) must be supplemented with an initial condition. As such, we choose a step-distribution function

$$f(p, t = 0) = f_{\text{in},0}(p) = \Theta(p_F - p), \quad (5)$$

where p_F is the Fermi momentum, which is determined from the condition of particle number conservation

$$\frac{4\pi g V}{(2\pi\hbar)^3} \int_0^{p_F} dp p^2 = A. \quad (6)$$

Here, $g = 4$ is the degeneracy factor for nucleons, and $V = 4\pi R^3/3$ is the volume of the nucleus, where its radius R is related to the mass number A by the relation $R = r_0 A^{1/3}$. In the subsequent calculations, we use the value for the coefficient $r_0 = 1.2$ fm. After integrating over momentum and substituting the expression for the nucleus volume, we obtain the expression for the Fermi momentum

$$p_F = \left(\frac{9\pi}{8} \right)^{1/3} \frac{\hbar}{r_0}, \quad (7)$$

where $\hbar$ is the Planck constant. Substituting the numerical values of $\hbar$ and the coefficient r_0 into equation (7), we obtain the numerical value of the Fermi momentum: $p_F \approx 8.4 \times 10^{-22}$ MeV·fm⁻¹·s. This value of the Fermi momentum p_F corresponds to the Fermi energy: $E_F = p_F^2/2m \approx 33.7$ MeV, where m is the nucleon mass.

When performing numerical calculations, we use the values of the kinetic coefficients obtained in our work [10]. In the approximation of constant coefficients, their values are calculated at momenta equal to the Fermi momentum, such that $D_{p,0} = D_p(p_F)$ and $K_{p,0} = 3K_p(p_F)$. The value of the diffusion coefficient $D_{p,0} \approx 3.4 \times 10^{-22}$ MeV²·fm⁻²·s describes the experimental data well and is consistent with the estimates of other authors [6, 7, 11]. For the numerical value of the drift coefficient, we use $K_{p,0} \approx -8.3 \times 10^{-23}$ MeV·fm⁻²·s, which satisfies the relation between the equilibrium temperature T_{eq} and the kinetic coefficients (see relation (10)).

In Fig. 1, the time dependence of the distribution function $f(p, t)$ calculated using the diffusion equation (4) is shown. The momentum dependence is presented in units of the Fermi momentum p/p_F , so the Fermi surface corresponds to a unit value of the relative momentum. As seen from the figure, the obtained distribution function $f(p, t)$ gradually broadens over time and evolves toward the equilibrium Fermi distribution

$$f_{\text{eq}}(p) = \left(1 + \exp \frac{p^2/2m - E_{F,\text{eq}}}{T_{\text{eq}}} \right)^{-1}, \quad (8)$$

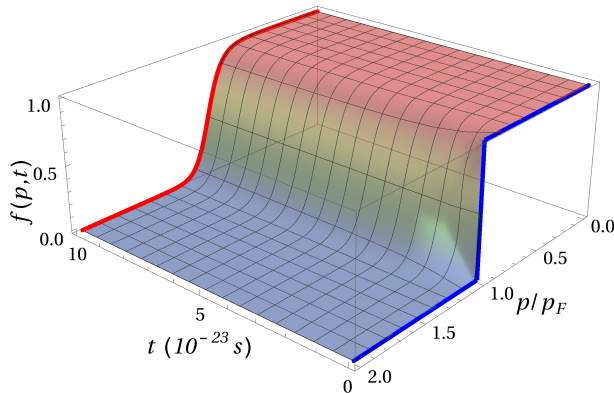


Fig. 1. Time evolution of the distribution function $f(p, t)$ obtained using the diffusion equation (4). The blue line represents the initial-distribution function (5), while the red curve represents the equilibrium Fermi-distribution function (8).

where $E_{F,eq}$ is the Fermi energy, which differs from the Fermi energy E_F for the step distribution and is determined from the condition

$$\frac{4\pi gV}{(2\pi\hbar)^3} \int_0^\infty dp \, p^2 f_{eq}(p) = A. \quad (9)$$

Here, T_{eq} is the equilibrium temperature, which, according to [6, 7, 12], is determined by the ratio of kinetic coefficients

$$T_{eq} = -\frac{D_{p,0}}{K_{p,0}}. \quad (10)$$

At the given values of the kinetic coefficients, the equilibrium temperature is $T_{eq} \approx 4$ MeV. For this equilibrium temperature, from equation (9), we numerically find the Fermi energy for the equilibrium distribution (8): $E_{F,eq} \approx 33.3$ MeV. As we can see, the obtained value $E_{F,eq} < E_F$, which is due to the diffuseness of the equilibrium distribution function.

3. Relaxation time

Let us consider the deviation of the distribution function from its equilibrium value

$$\delta f(p, t) = f(p, t) - f_{eq}(p). \quad (11)$$

It is clear that at the initial moment, $t = 0$, the distribution function is in its initial state

$$f(p, t = 0) = f_{in}(p). \quad (12)$$

We then denote the initial deviation from the equilibrium distribution as

$$\delta f(p, t = 0) \equiv \delta f_{\text{in}}(p) = f_{\text{in}}(p) - f_{\text{eq}}(p). \quad (13)$$

Let us introduce the root-mean-square deviation

$$\Delta(t) = \sqrt{\int d\mathbf{p} [\delta f(p, t)]^2}. \quad (14)$$

Its initial value will be

$$\Delta(t = 0) \equiv \Delta_0 = \sqrt{\int d\mathbf{p} [\delta f_{\text{in}}(p)]^2}. \quad (15)$$

In the future, it will be more convenient to consider not the function $\Delta(t)$ itself, but the quantity normalized to its initial value, $\Delta(t)/\Delta_0$. Clearly, at time $t = 0$, the ratio $\Delta(t)/\Delta_0$ equals one, indicating that the deviation of the distribution function from its equilibrium value is at its maximum.

In Fig. 2, the calculated dependence of the ratio $\Delta(t)/\Delta_0$ on time (red curve), according to expressions (11)–(15) is shown for the previously obtained time evolution of the distribution function, depicted in Fig. 1. As can be seen from the figure, over time, $\Delta(t)/\Delta_0$ decreases and asymptotically approaches zero. This dependence is nonexponential in nature.

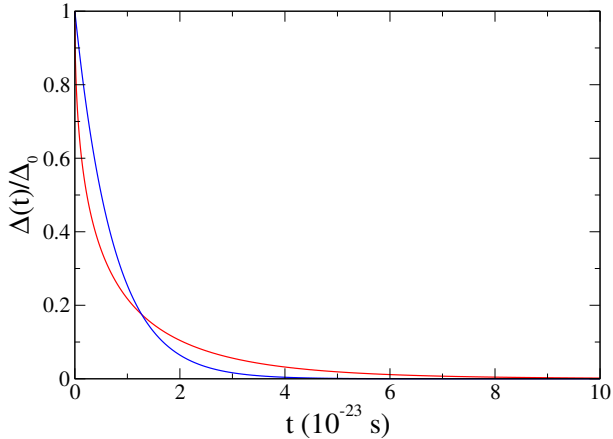


Fig. 2. The dependence of the ratio $\Delta(t)/\Delta_0$ on time t . The red curve represents the numerical calculation, while the blue curve corresponds to the exponential dependence (17) with a relaxation time $\tau_{\text{eq}} \approx 7.3 \times 10^{-24}$ s.

Let us assume that the relaxation of the deviation of the distribution function from its equilibrium value is described by an exponential dependence

$$\delta f(p, t) = \delta f_{\text{in}}(p) \exp(-t/\tau_{\text{eq}}), \quad (16)$$

where τ_{eq} is the relaxation time. Substituting (16) into expression (14), we obtain

$$\Delta(t) = \Delta_0 \exp(-t/\tau_{\text{eq}}). \quad (17)$$

Integrating both sides of expression (17) with respect to time yields

$$\int_0^\infty \Delta(t) dt = \Delta_0 \int_0^\infty \exp(-t/\tau_{\text{eq}}) dt. \quad (18)$$

By performing the integration on the right-hand side of (18), we obtain

$$\tau_{\text{eq}} = \int_0^\infty \frac{\Delta(t)}{\Delta_0} dt. \quad (19)$$

Substituting the exponential dependence (17) into this expression gives a trivial identity. However, in the case of a nonexponential time dependence of $\Delta(t)$, we obtain a value of τ_{eq} that characterizes the relaxation time in this scenario.

Thus, by substituting expressions (14) and (15) into (19), one can obtain the relaxation time for an arbitrary time dependence of the root-mean-square deviation of the distribution function from its equilibrium value. In this case, the area under the curve $\Delta(t)/\Delta_0$ will be equal to the area under the exponential dependence $\exp(-t/\tau_{\text{eq}})$ and will correspond to the relaxation time τ_{eq} calculated using formula (19). In Fig. 2, the exponential dependence $\exp(-t/\tau_{\text{eq}})$ with a relaxation time $\tau_{\text{eq}} \approx 7.3 \times 10^{-24}$ s, obtained using (19) and expressions (14) and (15) for the evolution of the distribution function depicted in Fig. 1, is shown in blue. As can be seen from the figure, the areas enclosed by the blue and red curves are equal.

4. Particle–hole-type excitation

Let us consider the case of a particle–hole-type excitation, which is described by an initial-distribution function of the form

$$f_{\text{in}}(p) = [1 - \Theta(p - p'_1) + \Theta(p - p'_2)] [1 - \Theta(p_F - p)] + [1 - \Theta(p - p_2)] \Theta(p - p_1) \Theta(p_F - p). \quad (20)$$

The distribution (20) indicates that the particle is localized beyond the Fermi surface with momentum $p_1 > p_F$, while the hole is accordingly located below the surface with $p'_1 < p_F$. The width of the interval in momentum space,

describing the particle, $\Delta p = p_2 - p_1$, is determined from the following condition

$$\frac{4\pi gV}{(2\pi\hbar)^3} \int_{p_F}^{\infty} dp p^2 f_{\text{in}}(p) = 1, \quad (21)$$

and the width of the interval describing the hole, $\Delta p' = p'_2 - p'_1$, is determined accordingly from the condition

$$\frac{4\pi gV}{(2\pi\hbar)^3} \int_0^{p_F} dp p^2 f_{\text{in}}(p) = A - 1. \quad (22)$$

Note that these two conditions automatically satisfy the conservation of the total particle number. It is also worth mentioning that the Fermi momentum in this case is not equal to the Fermi momentum for the step distribution (5). However, their difference is very small, so we will neglect it.

The time evolution of the distribution function for the initial particle-hole-type distribution (20) is illustrated in Fig. 3. As can be seen from the figure, similar to the previous case, the initial-distribution function becomes blurred and eventually approaches the equilibrium distribution (8). According to the calculations in Appendix A, the widths of the intervals in momentum space, which describe the particle and the hole, are inversely proportional to the mass number and the square of the momentum: $\Delta p \approx p_F^3/(3Ap_1^2)$ and $\Delta p' \approx p_F'^3/(3Ap_1'^2)$. Therefore, as shown in the figure, the relative width of the dip associated with the hole, $\Delta p'/p_F$, is greater than

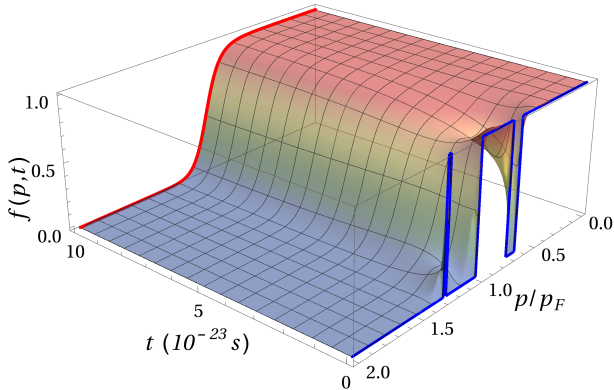


Fig. 3. The illustration of the time evolution of the distribution function $f(p, t)$ for the initial distribution describing the particle-hole-type excitation (20) is shown by the blue line. The red line represents the equilibrium Fermi-distribution function (8) at temperature of $T_{\text{eq}} = 4$ MeV.

the relative width of the peak associated with the particle, $\Delta p/p_F$, since the widths in both cases are inversely proportional to the squares of the momenta of the hole and the particle. It should be noted that, due to the smallness of the relative widths $\Delta p/p_F$ and $\Delta p'/p_F$ for heavy and medium nuclei, $A = 10$ was chosen for clarity in calculating the evolution of the distribution function. Additionally, to separate the peak and dip from the Fermi surface background, arbitrary energy values for the particle and hole, equidistant from the Fermi surface, were chosen.

The excitation energy of the particle–hole pair, E_{ex} , is the difference between the energies of the nucleon, E_1 , and the hole, E'_1 . In this study, the energy of the hole was chosen arbitrarily, but with consideration of its small deviation from the Fermi energy. Specifically, we chose $E'_1 = E_F - 2$ MeV arbitrarily.

The average binding energy per nucleon in atomic nuclei is approximately $B/A \approx 8$ MeV. Therefore, it makes sense to consider excitation energies, E_{ex} , where the excited nucleon remains bound within the nucleus, *i.e.*, $E_{\text{ex}} - 2 \leq B/A$. Given this constraint, we will focus on the excitation of the nucleon–hole pair within the range of $2 \leq E_{\text{ex}} \leq 10$ MeV.

Figure 4 shows the dependence of the relaxation time, τ_{eq} , calculated according to expression (19), for the particle–hole-type distribution as a function of the excitation energy, E_{ex} , for nuclei with different mass numbers. As seen in the figure, for all nuclei, the relaxation time, τ_{eq} , decreases nonlinearly with increasing excitation energy, E_{ex} .

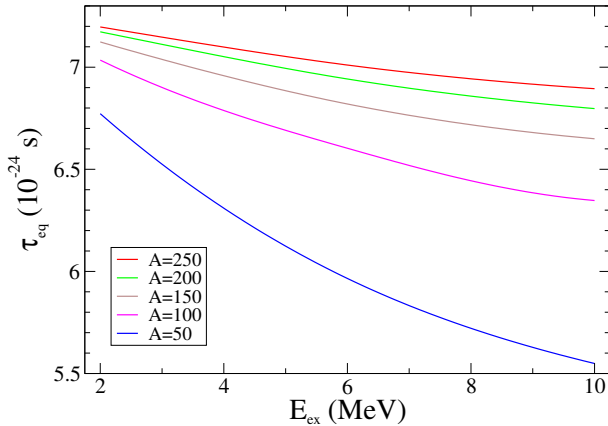


Fig. 4. The relaxation time τ_{eq} as a function of the nucleon excitation energy, E_{ex} , for nuclei with different mass numbers. The nucleon is excited from the $E_F - 2$ MeV level.

It should also be noted that at all excitation energies, the value of τ_{eq} increases with the mass number, A . This dependence is illustrated in Fig. 5, where the relaxation time, τ_{eq} , is plotted as a function of the mass number for different values of the particle–hole excitation energy, E_{ex} . As seen in the figure, the relaxation time increases with the mass number at all excitation energies.

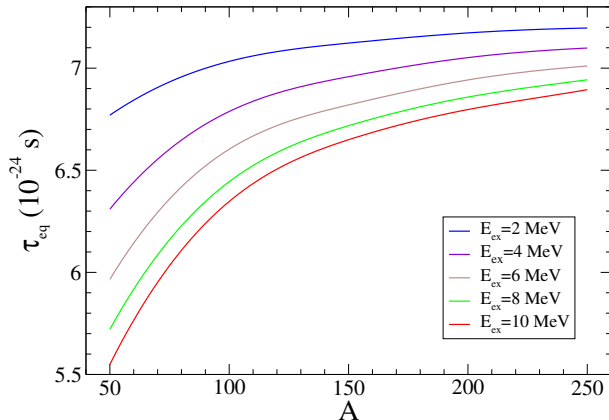


Fig. 5. The relaxation time τ_{eq} as a function of the mass number A at different values of the particle–hole excitation energy, E_{ex} . A nucleon is excited from the $E_{\text{F}} - 2 \text{ MeV}$ level.

Finally, it is worth noting that the numerical calculations performed are technically resource-intensive, requiring substantial computational power to achieve satisfactory accuracy in the obtained results.

5. Conclusions

In this work, within the framework of the diffusion approximation in kinetic theory, a nonlinear diffusion equation for the distribution function was derived for the case of constant diffusion and drift coefficients. Through direct numerical calculations, the time evolution of the distribution function was obtained for both an initial step-like distribution and a case of particle–hole excitation in a Fermi system, modeling the ground state of spherically symmetric atomic nuclei. It was demonstrated that in both cases, the initial distribution gradually blurs over time, evolving towards the equilibrium Fermi distribution, whose temperature, as is known, is determined by the ratio of the diffusion and drift coefficients.

We proposed a phenomenological method for calculating the relaxation time of an arbitrary initial excitation to its equilibrium value. The relaxation time is determined by the area under the equivalent exponential time

dependence, normalized to the initial value of the root-mean-square deviation of the distribution function. Using this method, the relaxation time for the step-like initial distribution was calculated as $\tau_{\text{eq}} \approx 7.3 \times 10^{-24}$ s.

We also calculated the dependence of the relaxation time on the particle–hole excitation energy in a spherically symmetric Fermi system, where the arbitrarily chosen hole energy is close to the Fermi energy (in our case, the difference is 2 MeV). The calculations showed that as the excitation energy increases, the relaxation time decreases in a nonlinear manner. This trend holds for all values of the mass number. Additionally, it was demonstrated that as the mass number increases, the relaxation time also increases across different values of the particle–hole excitation energy.

Theoretical studies revealed several issues that require further investigation. In particular, the relaxation time, τ_{eq} , obtained using this method was approximately an order of magnitude smaller than expected. It can be hypothesized that the reason for this discrepancy may lie in the method used to determine the relaxation time. Another possible explanation is that the momentum-integrated quantity $\Delta(t)/\Delta_0$ likely reaches equilibrium much faster than the distribution function at the Fermi momentum itself does. This occurs because, in a large fraction of momentum space, the initial distribution is already equal to the equilibrium distribution due to the Pauli principle. Additionally, further clarification is needed regarding the energy imbalance for the initial particle–hole distribution, as it leads to the equilibrium temperature, T_{eq} , being independent of the excitation energy, E_{ex} .

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Appendix A

Conditions for widths in momentum space for a particle and a hole

Let us examine conditions (21) and (22) in more detail. We denote the corresponding integrals as I_1 and I_2 . Then, expressions (21) and (22) will take the form of

$$\frac{4\pi gV}{(2\pi\hbar)^3} I_1 = A - 1, \quad (\text{A.1})$$

$$\frac{4\pi gV}{(2\pi\hbar)^3} I_2 = 1. \quad (\text{A.2})$$

For the distribution function (20), the integral I_1 can be expressed as follows:

$$I_1 = \int_0^{p_F} dp \, p^2 f_{\text{in}}(p) = \int_0^{p'_1} dp \, p^2 + \int_{p'_2}^{p_F} dp \, p^2 = \frac{1}{3} (p_1^3 + p_F^3 - p_2'^3) . \quad (\text{A.3})$$

Let us express the momentum p'_2 through the initial momentum p'_1 and the width of the interval of the hole in momentum space $\Delta p'$, then $p'_2 = p'_1 + \Delta p'$. Thus,

$$p_2'^3 = (p'_1 + \Delta p')^3 = p_1'^3 + 3p_1'^2 \Delta p' + 3p'_1 \Delta p'^2 + \Delta p'^3 . \quad (\text{A.4})$$

After substituting this expression into (A.3), we will have

$$I_1 = \frac{p_F^3}{3} (1 - 3x_1'^2 \Delta x' - 3x_1' \Delta x'^2 - \Delta x'^3) , \quad (\text{A.5})$$

where the notation $x'_1 = p'_1/p_F$, $\Delta x' = \Delta p'/p_F$ is introduced.

Similarly, we will write the integral I_2

$$I_2 = \int_{p_F}^{\infty} dp \, p^2 f_{\text{in}}(p) = \int_{p_1}^{p_2} dp \, p^2 = \frac{1}{3} (p_2^3 - p_1^3) . \quad (\text{A.6})$$

Let us express the momentum p_2 through the initial momentum p_1 and the width of the particle interval in momentum space Δp , then $p_2 = p_1 + \Delta p$. Thus,

$$p_2^3 = (p_1 + \Delta p)^3 = p_1^3 + 3p_1^2 \Delta p + 3p_1 \Delta p^2 + \Delta p^3 . \quad (\text{A.7})$$

After substituting this expression into (A.6), we will have

$$I_2 = \frac{p_F^3}{3} (3x_1^2 \Delta x + 3x_1 \Delta x^2 + \Delta x^3) , \quad (\text{A.8})$$

where the notation $x_1 = p_1/p_F$, $\Delta x = \Delta p/p_F$ is introduced.

After substituting the obtained expressions for integrals (A.5) and (A.8) into expressions (A.1) and (A.2), we obtain the following conditions:

$$\frac{4\pi gV}{(2\pi\hbar)^3} \frac{p_F^3}{3} (1 - 3x_1'^2 \Delta x' - 3x_1' \Delta x'^2 - \Delta x'^3) = A - 1 , \quad (\text{A.9})$$

$$\frac{4\pi gV}{(2\pi\hbar)^3} \frac{p_F^3}{3} (3x_1^2 \Delta x + 3x_1 \Delta x^2 + \Delta x^3) = 1 . \quad (\text{A.10})$$

Since $\Delta x' \ll 1$ and $\Delta x \ll 1$, it makes sense in expressions (A.9) and (A.10) to consider only the contribution from the linear terms

$$\frac{4\pi gV}{(2\pi\hbar)^3} \frac{p_F^3}{3} (1 - 3x_1'^2 \Delta x') \approx A - 1, \quad (\text{A.11})$$

$$\frac{4\pi gV}{(2\pi\hbar)^3} \frac{p_F^3}{3} 3x_1^2 \Delta x \approx 1. \quad (\text{A.12})$$

We will consider that with sufficient accuracy, the Fermi momentum p_F can be determined using expression (6) for the step distribution

$$\frac{4\pi gV}{(2\pi\hbar)^3} \frac{p_F}{3} \approx A. \quad (\text{A.13})$$

Therefore, taking into account the law of conservation of particle number (A.13), conditions (A.11) and (A.12) will be written in their final form as

$$\Delta x' \approx 1 / (3Ax_1'^2), \quad (\text{A.14})$$

$$\Delta x \approx 1 / (3Ax_1^2). \quad (\text{A.15})$$

REFERENCES

- [1] V.M. Kolomietz, S. Shlomo, «Mean Field Theory», *World Scientific*, Singapore 2020.
- [2] A.A. Abrikosov, I.M. Khalatnikov, «The theory of a Fermi liquid (the properties of liquid ^3He at low temperatures)», *Rep. Prog. Phys.* **22**, 329 (1959).
- [3] G. Bertsch, «The collision integral in nuclear matter at zero temperature», *Z. Phys. A* **289**, 103 (1978).
- [4] V.M. Kolomietz, S.V. Lukyanov, V.A. Plujko, S. Shlomo, «Collisional relaxation of collective motion in a finite Fermi liquid», *Phys. Rev. C* **58**, 198 (1998).
- [5] E.M. Lifshitz, L.P. Pitaevsky, «Physical Kinetics», *Pergamon Press*, Oxford–New York–Seoul–Tokyo 1993.
- [6] G. Wolschin, «Equilibration in Finite Fermion Systems», *Phys. Rev. Lett.* **48**, 1004 (1982).
- [7] T. Bartsch, G. Wolschin, «Equilibration in fermionic systems», *Ann. Phys.* **400**, 21 (2019).
- [8] V.M. Kolomietz, S.V. Lukyanov, «Diffusion on the distorted Fermi surface», *Ukr. J. Phys.* **59**, 764 (2014).
- [9] V.M. Kolomietz, S.V. Lukyanov, «Diffuse approximation to the kinetic theory in a Fermi system», *Int. J. Mod. Phys. E* **24**, 1550023 (2015).

- [10] S.V. Lukyanov, «Properties of the diffusion and drift kinetic coefficients in momentum space for a cold Fermi system», *Nucl. Phys. At. Energy* **24**, 5 (2023).
- [11] S.V. Lukyanov, «Properties of kinetic diffusion and drift coefficients in momentum space for a finite Fermi system», in: «Proceedings of the XXX annual scientific conference of Institute for Nuclear Research», *National Academy of Sciences of Ukraine*, Kyiv, Ukraine, 25–29 September, 2023, pp. 27–28.
- [12] S.V. Lukyanov, «Diffuse relaxation approximation in a heated Fermi system», *Int. J. Mod. Phys. E* **30**, 2150060 (2021).