SECONDARY REACTIONS IN RELATIVISTIC FRAGMENTATION OF NUCLEI

Agata Pawełkiewicz,

Wrocław University of Science and Technology, Wrocław, Poland

Volha Charviakova, Zygmunt Patyk

National Center for Nuclear Research, Warsaw, Poland

Timo Dickel, Emma Hättner, Wolfgang Plass
Christoph Scheidenberger

II. Physikalisches Institut, Justus-Liebig-Universität Gießen, Gießen
and
GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

Received 18 November 2024, accepted 10 January 2025,
published online 23 January 2025

Advanced facilities like GSI-FAIR in Germany or RIKEN in Japan are dedicated to the research of nuclei far from the stability line. In this paper, we study the fragmentation of relativistic projectiles as a production method for these nuclei, with particular emphasis on the role of secondary reactions.

DOI:10.5506/APhysPolB.56.1-A3

1. Introduction

The knowledge of cross sections in the fragmentation of relativistic nuclei is crucial for the production and study of exotic nuclei in modern laboratories (*e.g.* GSI-FAIR, RIKEN, MSU). In this paper, we discuss two-step reactions. In the first step, a nucleus is produced as a result of relativistic fragmentation. Subsequently, this nucleus, still possessing relativistic energy, can undergo further fragmentation, leading to the formation of an exotic nucleus. The estimation of the cross section in the secondary reaction of a specific exotic nucleus is the primary objective of this study.

2. Cross section in a secondary projectile fragmentation reaction

Cross section is one of the most important properties of a nuclear reaction. It is a measure of the effective area that in a specific reaction is seen by a projectile nucleus when collides with a target nucleus. Therefore, here are presented key points for calculating the cross section for a single and secondary projectile fragmentation reaction.

One of the main variables is the target thickness, denoted by ρ and expressed in g/cm^2. Let us consider a fixed hypothetical surface S positioned at the front of the target, in the direction of the incoming beam. The number of atoms, N, located behind this surface, divided by the surface area, is denoted by R

$$R \equiv \frac{N}{S} = \frac{\rho N_{\mathrm{A}}}{M}\,, \tag{1}$$

where N_{A} is Avogadro's number, and M is the molar mass of the target, expressed in grams. Although the number of atoms per unit area, R, is typically a large value, it is usually multiplied by a cross section of the order of millibarns. The cross-section $\sigma(P,\phi)$ for a specific reaction ($P + \text{target} \to \phi$) is calculated directly using the EPAX code [1–3].

Let us assume that the first reaction occurs at a relative distance $0 < x_1 < 1$ from the front of a target. The term "relative distance" refers to the distance divided by the width of the target. Then the probability that the fragment X_1 with mass number A_{X_1} and atomic number Z_{X_1} will be produced in a layer $\mathrm{d}x_1$ located around x_1 is expressed as

$$\mathrm{d}P_1 = R \times S \frac{\sigma\left(P, X_1\right) \mathrm{d}x_1}{S} = R \times \sigma\left(P, X_1\right) \mathrm{d}x_1\,. \tag{2}$$

Here, P refers to the projectile and $\sigma(P, X_1)$ is the cross section for the reaction $(A_{\mathrm{P}}, Z_{\mathrm{P}}) + (A_{\mathrm{t}}, Z_{\mathrm{t}}) \to (A_{X_1}, Z_{X_1})$, where t refers to the target. The probability is a ratio of two surfaces: cross section $\sigma(P, X_1)$ multiplied by the number of atoms behind the surface S in the relative distance $\mathrm{d}x_1$, and the surface S. The probability $\mathrm{d}P_1$ does not depend on the surface S.

If the created fragment X_1 reacts again, we get the second reaction $(A_{X_1}, Z_{X_1}) + (A_{\mathrm{t}}, Z_{\mathrm{t}}) \to (A_{X_2}, Z_{X_2})$ with the cross section $\sigma(X_1, X_2)$, thus producing fragment X_2. Let us assume, that the second reaction takes place at a relative distance x_2 fulfilling the simple relation $0 < x_1 < x_2 < 1$ in a thin layer $\mathrm{d}x_2$. The probability of such reaction occurring and fragment X_2 being produced is a sum of products (or integral) of two probabilities $\mathrm{d}P_1$ and $\mathrm{d}P_2$

$$
\begin{aligned}
P(X_1, X_2) &= \int\limits_0^1 \left(\int\limits_{x_1}^1 R^2 \sigma(P, X_1) \sigma(X_1, X_2) \mathrm{d}x_2 \right) \mathrm{d}x_1 \\
&= R^2 \sigma\left(P, X_1\right) \sigma(X_1, X_2)/2\,, \qquad (3)
\end{aligned}
$$

where $\int_0^1 (\int_{x_1}^1 \mathrm{d}x_2) \mathrm{d}x_1 = 1/2$ and the expression under integral does not depend on x_1 or x_2. Note that $P(X_1, X_2) \neq P(X_2, X_1)$.

In a similar way, we calculate three-step reactions where a fragment denoted as X_1 is produced near x_1 in a layer $\mathrm{d}x_1$, a fragment X_2 near x_2 in a layer $\mathrm{d}x_2$, and a final fragment X_3 near x_3 in a layer $\mathrm{d}x_3$. Of course, relative distances fulfill $0 < x_1 < x_2 < x_3 < 1$. The probability is a product of $\mathrm{d}P_1$, $\mathrm{d}P_2$, and $\mathrm{d}P_3$, and as independent events could be summed or integrated

$$
\begin{aligned}
&P\left(X_1, X_2, X_3\right) \\
&= \int\limits_0^1 \left(\int\limits_{x_1}^1 \left(\int\limits_{x_2}^1 R^3 \sigma\left(P, X_1\right) \sigma\left(X_1, X_2\right) \sigma\left(X_2, X_3\right) \mathrm{d}x_3 \right) \mathrm{d}x_2 \right) \mathrm{d}x_1 \\
&= R^3 \sigma(P, X_1) \sigma(X_1, X_2) \sigma(X_2, X_3)/6\,, \qquad (4)
\end{aligned}
$$

where $\int_0^1 (\int_{x_1}^1 (\int_{x_2}^1 \mathrm{d}x_3) \mathrm{d}x_2) \mathrm{d}x_1 = 1/6$ and expression under integral does not depend on x_1, x_2 or x_3.

The final expression, for the probabilities $P(\phi)_{\mathrm{P}}$ for primary and $P(\phi)_{\mathrm{S}}$ for secondary reactions to produce the desired fragment ϕ are given by formulas

$$
P(\phi)_{\mathrm{P}} = R \times \sigma(P, \phi) - R^2 \sigma(P, \phi) \sum_i \sigma(\phi, X_i)/2\,, \qquad (5)
$$

$$
\begin{aligned}
P(\phi)_{\mathrm{S}} &= P(\phi)_{\mathrm{P}} + R^2 \sum_i \sigma(P, X_i) \sigma(X_i, \phi)/2 \\
&\quad - R^3 \sum_{i,j} \sigma(P, X_i) \sigma(X_i, \phi) \sigma(\phi, X_j)/6\,, \qquad (6)
\end{aligned}
$$

where sums are over fragments. The first term in (5) describes the probability of creating the desired fragment directly from the projectile.

The second term expresses the probability that the fragment will be lost by reacting again with the target before exiting it. The sum goes over all nuclei with atomic masses smaller than that of the fragment ϕ.

The term proportional to R^2 in (6) calculates the probability that the fragment X_i as a projectile will produce the next fragment ϕ as a result of secondary reaction. Here, we sum over the fragments X_i with masses larger than the fragment ϕ, but still smaller than the starting projectile.

The final term describes the probability of absorption of a fragment ϕ produced through secondary reactions.

Thus, the effective cross section that a fragment ϕ will be produced from fragmentation of the projectile P for secondary nuclear reactions equals

$$\sigma'(P,\phi) = \frac{P(\phi)_\mathrm{S}}{R}\,, \tag{7}$$

and depends on a density of a target through R as defined in Eq. (1). Similarly, we calculate the cross section for primary reaction.

Let us estimate the order of probability $P(\phi)$ that a fragment, denoted as ϕ, will be produced from the projectile P in a fragmentation reaction for reasonable target thickness and discussed reactions. By integrating equation (2), we obtain

$$P(\phi) = \sigma(P,\phi) \times \rho \times N_A/M\,, \tag{8}$$

where ρ is the target thickness expressed in g/cm^2 and M the molar mass of target in g. The number of atoms in one mole known as the Avogadro number N_A is approximately equal to 6.022×10^{23}.

In the present paper, the cross section is expressed in millibarns mb, a unit of area where 1 mb $= 10^{-27}$ cm^2 $= 0.1$ fm^2. We estimate the order of probability $\sum_\phi P(\phi)$, where $P(\phi)$ is given by equation (8). Assuming a ^{9}Be target thickness of $\rho = 1\mathrm{g/cm}^2$, a molar mass for ^{9}Be of $M = 9$ g, and a maximal value for the sum of cross sections $\sum_\phi \sigma(P,\phi) = 100$ mb, the probability is no greater than

$$\sum_\phi P(\phi) = \frac{1\ \mathrm{g/cm}^2}{9\ \mathrm{g}} \times 6.022 \times 10^{23} \times 100 \times 10^{-27}\ \mathrm{cm}^2 = 6.691 \times 10^{-3}\,. \tag{9}$$

For a typical sum of cross sections for all fragments of the order of 100 mb, we find that the probability is a few per thousand and remains significantly smaller than unity. This conclusion is particularly valid for the target thicknesses considered in this paper, ranging from 1 to 17 g/cm^2.

3. Results

All calculations were performed for five different projectiles: ^{238}U, ^{209}Bi, ^{180}W, ^{152}Sm, and ^{124}Xe, colliding with a ^{9}Be target, representing typical fragmentation reactions used at the GSI facility. The basic properties of primary and secondary reactions were studied for a sample target thickness of 12 g/cm^2. In the first step, we studied the cross-section distribution for fragments obtained in primary reactions. To achieve this, a fragment with a chosen proton number Z was selected, and cross sections for its isotopes (*i.e.*, different neutron numbers N) were computed. We observed that the cross sections followed a Gaussian distribution with well-defined mean values and widths. The neutron numbers corresponding to the mean values for a fixed proton number Z formed a path on the nuclear chart (Z, N). Five such paths for different projectiles are shown in figure 1.

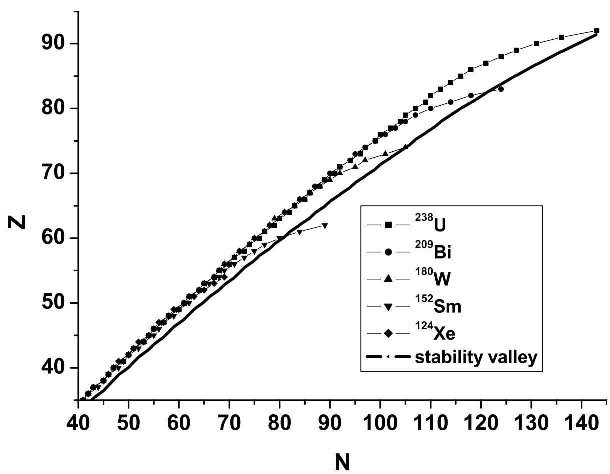

Fig. 1. Fragment's paths for five different projectiles.

Each path starts from the number of protons and neutrons in the projectile. The fragment's path can be divided into two parts: a common path for all projectiles, referred to as the 'asymptotic', and a second, much shorter path that converges toward the asymptote. Each path, close to the projectile, runs almost parallel to the neutron axis — the loss of one proton corresponds to the loss of several neutrons. As shown in figure 1, it is evident that lighter projectiles reach the asymptotic path much faster, with the number of lost neutrons ranging from 2 for ^{124}Xe to 8 for ^{238}U.

The cross sections (in mb) along the fragment's trajectory are shown in figure 2. These indicate that the reaction exhibits the highest cross sections for fragments located closer to the projectile. If only primary reactions are considered, this value decreases rapidly, suggesting that fragments farther from the projectile are produced at a lower rate.

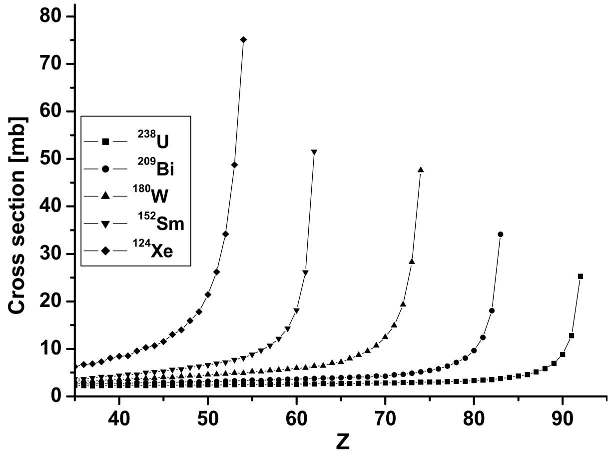

Fig. 2. Cross sections (in mb) on the fragment's path.

However, this changes when secondary reactions are included in the calculations, particularly for heavier projectiles. In figure 3, the ratio of cross sections along the fragment's path is plotted. For heavier projectiles, it can be observed that after an initial decrease, a significant increase in the ratios of secondary-to-primary cross-section values occurs, indicating that smaller fragments may also be produced. This result highlights the importance of secondary reactions, as their inclusion can increase the cross-section value by up to threefold.

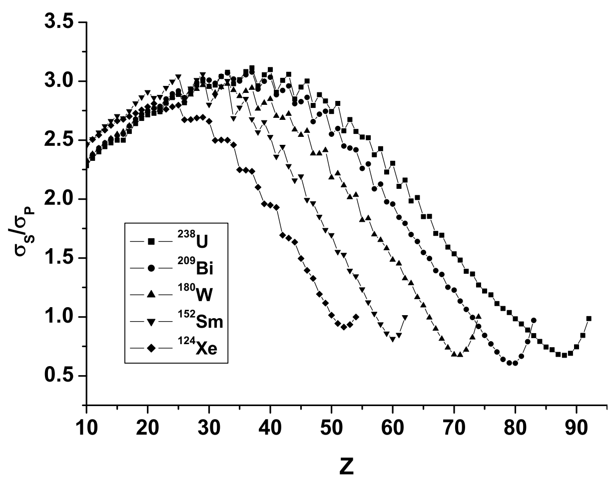

Fig. 3. Ratio of cross sections for the secondary and primary relativistic fragmentation.

The cross section for fragments with fixed proton number Z has almost Gaussian distribution as a function of neutron numbers N_Z. We define moments for cross-section distributions as

$$m_k(Z) = \sum_{N_Z} \sigma(N_Z, Z) N_Z^k \,. \tag{10}$$

We can estimate the width ΔN of the cross-section distribution as a function of the neutron number N_Z for a fixed proton number Z using the formula

$$\Delta N = \sqrt{\frac{m_2(Z)}{m_0(Z)} - \frac{m_1(Z)^2}{m_0(Z)^2}} \,. \tag{11}$$

The deviations ΔN are plotted in figure 4 and are nearly the same for both primary and secondary reactions. For both types of reactions, ΔN is very small, of the order of 2–5 neutrons, illustrating how rapidly the cross section decreases from the fragment's path. This decrease is more rapid for lighter fragments, where the deviations are smaller. The production of exotic nuclei

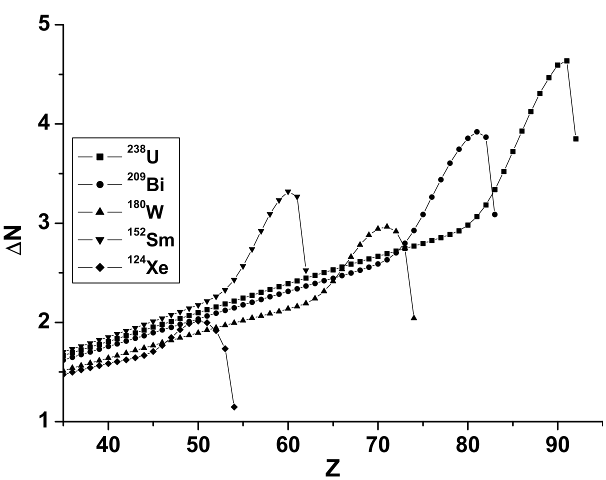

Fig. 4. Width ΔN of cross section distributions.

depends on the competition between fragmentation and absorption in the target, as well as on the target thickness. In figure 5, the production probability of the ^{98}Cd ion in the ^{124}Xe + ^{9}Be $\rightarrow$ ^{98}Cd reaction is shown as a function of the target thickness. It can be observed that the production probability for fragments in secondary reactions is optimal around 14 mg/cm^2, and is nearly seven times higher, on average, than for primary reactions.

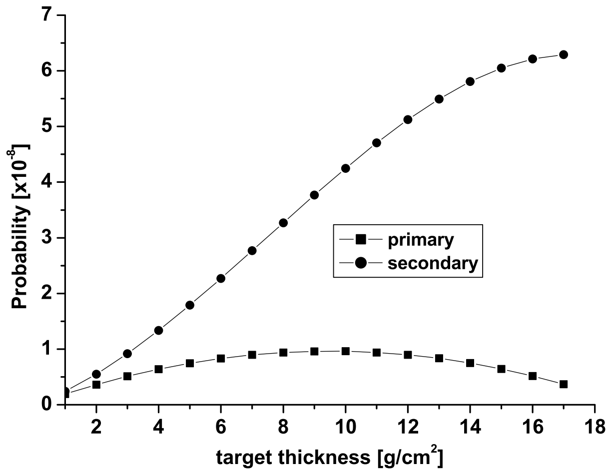

Fig. 5. Probability for primary and secondary reactions as a function of the target thickness.

4. Conclusion

In this paper, the relativistic fragmentation of heavy projectiles on a light target was extensively studied. Five different projectiles colliding with a beryllium target were considered. The results indicate that proton-rich nuclei are much easier to produce, while neutron-rich nuclei are significantly more difficult to generate due to the rapid decrease in cross sections as one moves away from the asymptotic path.

Subsequently, the maximal cross-section values, in terms of proton number, were compared between primary and secondary reactions for all five projectiles. These findings already highlight the importance of secondary reactions. The calculated cross sections show that deviations from the fragment path correspond to a few neutrons.

The maximal cross sections for all five projectiles were compared for different target thicknesses. The calculations clearly show that the path remains unchanged regardless of the target thickness; however, the maximal values do exhibit slight variations, particularly for fragments close to the projectile.

Finally, the maximal cross sections in terms of proton number were compared between the five projectiles for a chosen target thickness. These comparisons indicate that the optimal choice of projectile for producing the desired fragment is likely the lightest one.

We gratefully acknowledge the late Prof. Dr. Hans Geissel for his many inspiring discussions, as well as the anonymous referee for constructive criticism, which helped to improve the manuscript. The paper was partly financed by the international project PMW of the Polish Minister of Science and Higher Education; active in the period 2022–2024, grant Nr 5237/GSI-FAIR/2022/0.

REFERENCES

[1] K. Sümmerer *et al.*, «Target fragmentation of Au and Th by 2.6 GeV protons», *Phys. Rev. C* **42**, 2546 (1990).

[2] K. Sümmerer, B. Blank, «Modified empirical parametrization of fragmentation cross sections», *Phys. Rev. C* **61**, 034607 (2000).

[3] K. Sümmerer, «Improved empirical parametrization of fragmentation cross sections», *Phys. Rev. C* **86**, 014601 (2012).