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FROM INSTANTONS TO PENTAQUARKS

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In memory of Dmitry Diakonov

March 30, 2014 would have marked the 65$^{\mathrm{th}}$ anniversary of the birth of D.I. Diakonov, an outstanding theoretical physicist who worked in the Theoretical Department of PNPI for more than 40 years. We collaborated with him for over 30 years and were co-authors of over 70 papers. This lecture is dedicated to his memory.

Fig. 1. D.I. Diakonov, 30.03.1949–26.12.2012.

† PNPI — Petersburg Nuclear Physics Institute. Translated from Russian by M. Eides (Ed.)

Mitya Diakonov turned up in the Theoretical Department of the Leningrad Nuclear Physics Institute in 1972, having entered graduate school after graduating from the University. At that time, the theoretical department, headed by V.N. Gribov, was the world center for the Regge theory approach to high-energy scattering. Quantum field theory was not particularly popular: it was believed that a self-consistent field theory (due to the zero charge) could not exist.

Mitya's advisor was Alexey Andreevich Anselm, one of the few people in the department who retained a love for quantum field theory and continued to actively study it. As one of his first tasks, he assigned Mitya to explore spontaneous chiral symmetry breaking in a model 2-dimensional theory[1] [1]. This was a model with a four-fermion interaction, which is now called the Gross–Neveu model. This model was asymptotically free, but these events precede the formal discovery of asymptotic freedom[2]. The mechanism of formation of chiral condensate and spontaneous breaking of chiral invariance was quite similar to the theory of superconductivity. However, even then, Mitya clearly realized [2] that this approach is not parametrically justified and cannot be applied in QCD (although such works appeared later in large number).

Fig. 2. Mitya Diakonov, 1977.

[1] This text mainly contains references only to the works of D. Diakonov, which is natural for a lecture of this kind. Attempts to restore the full list of literature would lead to its expansion almost to infinity.

[2] Asymptotic freedom in the Yang–Mills theory had already been discovered by I.B. Khriplovich several years earlier. There were also works by some other authors. A.A. Anselm himself had observed asymptotic freedom in some 2-dimensional theories. I have always been amazed why these facts had attracted so little attention before the famous works of Gross, Wilczek, and Politzer.

A.A. Anselm was always interested in the theory of electroweak interactions and grand unified theories. Mitya's next two works are focused on the calculation of the Coleman–Weinberg potential and radiative corrections to the Weinberg angle in the Standard Model. At that time, these were quite complicated problems in quantum field theory, the description of the Higgs effect in different gauges presented certain difficulties. The Weinberg angle in different grand unification models was calculated in [3], it was Mitya's solo work, without co-authors. CP-violation based on Weinberg's mechanism and the limitations on this violation that could be obtained at that time were discussed in [4]. This work played a certain role in establishing the fact that this mechanism is not realized in nature.

Mitya's youth as a theorist coincided with the "revolution" of 1973–74, as a result of which QCD arose in the form in which we now know it. Mitya (unlike many) accepted it with enthusiasm. In 1975, he (together with Mark Strikman) translates into Russian the R. Feynman book «Photon–Hadron Interaction», which influenced him very much, and he begins to try his hand at the physics of hard processes in QCD. His first paper on this topic [5] arose on his own initiative (in the second year of graduate school), and in his next work [6] he managed to involve his advisor as well. Both of these papers were influenced by Feynman's ideas and were the result of careful reading of his book.

In passing, by that time the parton model and the questions associated with it were in the center of attention of the entire Theoretical Department of PNPI. The problem of combining the parton model and quantum field theory has always been of concern to Mitya. After it was successfully resolved with the discovery of the asymptotic freedom, for the first time in the theory of strong interactions it became possible to calculate something that is within the scope of the theory. This possibility was very attractive, and Mitya joined Yu.L. Dokshitzer and S.I. Troyan (DDT), who already were engaged in hard processes in QCD. The traditions of such calculations in our department were laid in the famous works of V.N. Gribov and L.N. Lipatov [7, 8].

In a couple of years, DDT completely understood the physics of the main hard processes and published several works on this topic [9–13]. Particular attention was paid to the *Drell–Yan process* (see Fig. 3). They obtained the famous *DDT formula* for the distribution of lepton pairs in transverse momentum

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2\mathrm{d}q_\mathrm{t}^2\mathrm{d}y} = \frac{4\pi\alpha^2}{9}\frac{1}{sq^2q_\mathrm{t}^2}\frac{\partial}{\partial\log q_\mathrm{t}^2}\left[\sum_{F=q,\bar{q}} e_F^2 D_a^F\left(x_1,q_\mathrm{t}^2\right)D_b^F\left(x_2,q_\mathrm{t}^2\right)T_F^2\left(q_\mathrm{t}^2,q^2\right)\right], \tag{1}$$

where D_a^F are the parton distributions (which can be measured in deep inelastic scattering), and T_F is the double logarithmic Sudakov form factor

$$T_F = \exp\left[-\frac{\alpha_s}{3\pi}\log^2\frac{q^2}{q_t^2}\right].$$

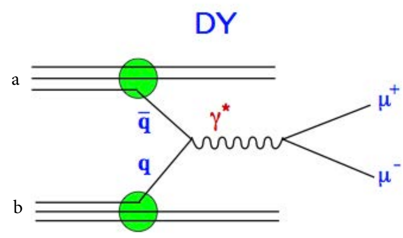

Fig. 3. Drell–Yan process.

This was one of the first quantitative calculations in perturbative QCD, and the authors managed to predict not just one number, but an entire function of the transverse momentum. In 1978, they wrote a *Physics Reports* review [14], which made them widely known and which became a classics of QCD (it was published only in 1980). This review has collected about 800 references in INSPIRE, with the latest dated by 2013[3].

It would seem that now an obvious path of a QCD theorist, an expert in perturbation theory and hard processes, was open to Mitya. However, he believed that the answers to the most interesting questions in QCD cannot be obtained within the framework of perturbation theory[4]. Therefore, at this time he abruptly changes the field of his activity (as he did repeatedly later), switching to non-perturbative QCD.

At that time, the QCD sum rules came into fashion. Mitya finds here an untested topic: calculating the mass of a "hermaphrodite" — an exotic particle consisting of gluon, quark, and antiquark, and published a paper on this topic together with A. Yung and I. Balitsky [15]. Interestingly, this is one of the few works on the sum rules, which is a prediction, not a description of already known quantities. Since the mass was unknown in advance, the authors calculated the contributions of the terms with unusually high dimensions. In addition, it turned out that this problem is not so simple and contains some subtleties. As a result, this work did not engage the scientific community, and they had to return to this topic later [16].

[3] 928 citations as of December 2024 (ME).

[4] He liked to repeat that, in essence, the entire theory of hard processes follows from the Lienard–Wiechert potential for an ultrarelativistic particle, and therefore cannot answer important and interesting questions. This is, of course, an exaggeration, but there is some truth in it.

By that time, Mitya was deeply interested in non-perturbative QCD. A small circle for discussions on all non-perturbative topics, which he headed, formed naturally. One of the central issues was the mechanism of spontaneous chiral symmetry breaking (SχSB) in QCD. Sum rules do not address such questions, they simply take the existence of condensates in QCD as a given. Meanwhile, SχSB determines the properties of hadrons almost to a greater extent than the confinement of quarks. It is thanks to SχSB that there is a nearly massless octet consisting of π- and K-mesons, as well as the η-meson[5]. What may be surprising, however, is that there are 8 Goldstone's bosons, not 9, as it should be when $\mathrm{U}(3)_\mathrm{R} \otimes \mathrm{U}(3)_\mathrm{L}$ is broken. For some reason, the mass of the ninth meson η' is not small (and does not vanish, like the masses of the octet mesons, in the limit of massless quarks). This observation is known as the "U(1)-problem". The solution to the U(1)-problem was found by E. Witten and, as it turned out, is related to the most subtle properties of QCD.

First of all, due to the Adler anomaly, conservation of the singlet axial current (with which the η'-meson is associated) is violated. The non-conservation of this current in QCD is associated with the so-called *topological charge*

$$Q_\mathrm{t} \;=\; \frac{1}{32\pi^2}\int \mathrm{d}^4x\; \varepsilon_{\mu\nu\rho\lambda}\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\lambda}\,. \tag{2}$$

Therefore, the Goldstone theorem for the singlet current can be violated. However, for this violation to happen, gluon field fluctuations with non-zero topological charge should exist in vacuum. This is not trivial, because expression (2) is a total derivative. In fact, as was pointed out in the work of Mitya and M. Eides [17], existence of a special "ghost" pole in QCD correlators containing topological charge is necessary for violation of the Goldstone theorem to occur. The mixing of this pole with the Goldstone mode explains the appearance of the η'-meson non-zero mass. In the limit of a large number of colors, this mass is given by the Witten–Veneziano formula

$$m^2_{\eta'} \;=\; 4N_f\,\frac{\langle Q_\mathrm{t} Q_\mathrm{t}\rangle}{F_\pi^2}\,, \tag{3}$$

where $\langle Q_\mathrm{t} Q_\mathrm{t}\rangle$ is the correlator of topological charges in the QCD vacuum in the absence of quarks (pure gluodynamics), N_f is the number of flavors, and F_π^2 is the axial decay constant of the π-meson.

[5] Also notice that in a world with an unbroken chiral symmetry, the nucleon must be massless or, at least, degenerate with $N(\frac{1}{2}^-)$. Neither is observed in nature.

The problem can be looked at from another angle. Potential energy in the Yang–Mills theory, as is well known, is periodic with respect to a certain coordinate K,

$$K = \varepsilon_{ijk} \int \mathrm{d}^3 x \left[A_i \partial_j A_k + \frac{2}{3} A_i A_j A_k \right] , \tag{4}$$

which is the QCD Chern–Simons term (see Fig. 4). There are many minima, and configurations with non-zero topological charge describe, as was explained by V. N. Gribov, the *under-barrier* transitions between different minima: $Q_\mathrm{t} = K_\mathrm{final} - K_\mathrm{initial}$. The topological charge in such processes can be only an integer.

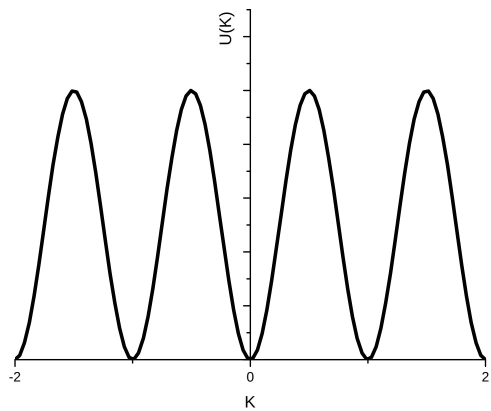

Fig. 4. Potential energy in the Yang–Mills theory.

The existence of a non-zero mass for the η'-meson demonstrates that the barriers are penetrable and tunnel transitions do occur. However, the solution of the U(1)-problem in QCD leads to a new, so-called Θ*-problem*. The point is that, as is well known in solid state physics, in a system with a periodic potential and penetrable barriers states are characterized by a certain conserved quasi-momentum Θ [17]. Thus, instead of a unique vacuum in QCD, there must exist a set of states corresponding to different Θ, and there is no reason to believe that we live in the world with $\Theta = 0$. The problem is that the CP symmetry of strong interactions is broken in the states with $\Theta \neq 0$, what leads, in particular, to a non-zero EDM of the neutron. Despite all efforts, the Θ-problem has not been resolved yet.

Having figured out the physics of the U(1)- and Θ-problems in [17], D. Diakonov and M. Eides gave an excellent lecture at our school [18], which I recommend to anyone interested in the details.

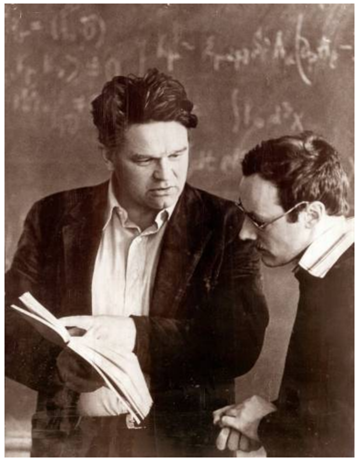

Fig. 5. Mitya and L. Faddeev, 1981.

Working on the U(1)-problem made a very strong impression on Mitya — he started to realize that fluctuations with non-zero topological charge, implementing under-barrier transitions could play a principal role in the QCD vacuum. Such fluctuations — *instantons* — have been already known for a long time, starting with the works of Belavin–Polyakov–Schwartz–Tyupkin. Therefore, we decided to try the hypothesis that non-perturbative vacuum of QCD consists of instantons. By this time, there appeared also a series of works by E. Shuryak, who showed that the hypothesis of dominance in vacuum of instantons with the size $\rho \approx (600\,\mathrm{MeV})^{-1}$ and the average distance between them approximately three times larger, allows one to explain a number of phenomenological facts.

However, before starting, it was necessary to come up with a method for taking into account arbitrary fluctuations in the functional integral that determines the partition function of QCD. This method turned out to be *Feynman's variational principle* — it can be considered as a generalization of the usual variational principle for the Schrödinger equation. We tested Feynman's principle in a quantum mechanical framework [20], developed an approximate method for estimating functional determinants (which is a prerequisite) [21] and returned to QCD.

We adopted the simplest ansatz for non-perturbative fluctuations in vacuum, which is a sum of instantons and anti-instantons

$$A_\mu(x) = \sum_{I,\bar{I}} A^I_\mu(x,\rho,U,z) + \bar{A}^I_\mu(x,\rho,O,z)\ , \qquad A^I_\mu = O^{ab}\frac{\eta^b_{\mu\nu}(xz)_\nu}{(xz)^2+\rho^2}\ , \quad (5)$$

where z_μ is the position, ρ is the size, O^{ab} is the color orientation matrix of an instanton. The gluodynamics functional integral on this configuration reduces to a very non-trivial problem of statistical physics

$$\mathcal{Z} = \int \prod_{I,\bar{I}} \frac{\mathrm{d}^4 z_I \mathrm{d}\rho_I}{\rho_I^5} (\rho_I \Lambda)^b \exp\left(-\beta U_{\mathrm{int}}(\rho_I, z_I, O_I)\right) . \tag{6}$$

This partition function describes a gas of pseudoparticles interacting with the potential U_{int} and characterized by the sizes and color orientations. Gas temperature $\beta = 8\pi^2/g^2$ (g is the QCD coupling constant) should be determined as a result of a self-consistent solution to the problem.

The results of the solution we obtained in [22] turned out to be surprisingly favorable. First of all, the *instanton liquid*, due to the interaction, stabilized. The interaction turned out to be not small, $\beta U_{\mathrm{int}} \sim 1$, but the ratio of the average instanton size to the distance between them remained small

$$\frac{\bar{R}}{\bar{\rho}} \approx 3.1 \tag{7}$$

and it turned out to be exactly the one, which was required to agree with the phenomenology[6].

The coupling constant "froze" also at a relatively small value

$$\beta(\bar{\rho}) = \begin{cases} 12\,, & \mathrm{SU}(2) \\ 15\,, & \mathrm{SU}(3) \end{cases} , \tag{8}$$

which ensured self-consistency of the entire approach. The calculations were carried out in 2 loops (one was not enough) without any fitting parameters. The distribution of instantons by sizes turned out to be

$$\mu(\rho) = \rho^{b-5} \exp\left(-\nu \frac{\rho^2}{\bar{\rho}^2}\right) , \qquad \nu = \frac{b-4}{2} , \qquad b = \frac{11}{3} N_c \,. \tag{9}$$

About ten years after our work it was measured on a lattice (see Fig. 6).

The results of the calculations were various correlators in the vacuum of gluodynamics: gluon condensate, topological charge correlator, $\langle \mathcal{F}^3_{\mu\nu} \rangle$, *etc.* As is expected in the renormalization-invariant theory, they were expressed

[6] A curious confirmation of the large ratio $\bar{R}/\bar{\rho}$ comes from a comparison with Mitya's work on the mass of glukonium [15, 16]. The point is that the mass of this particle is related to the correlator $\langle G^3_{\mu\nu} \rangle$. In the instanton vacuum, this correlator turns out to be much larger than naive expectations: $\langle G^3_{\mu\nu} \rangle \approx \frac{R^2}{\rho^2} \langle G^2_{\mu\nu} \rangle^{3/2}$. If $R^2/\rho^2 \approx 1$, then the mass of this exotic particle would be, according to the sum rules, less than 1 GeV, which is absolutely excluded by the experiment. For the parameters we obtained, the "hermaphrodite" mass is pushed to at least 1.5 GeV, which is consistent with other estimates.

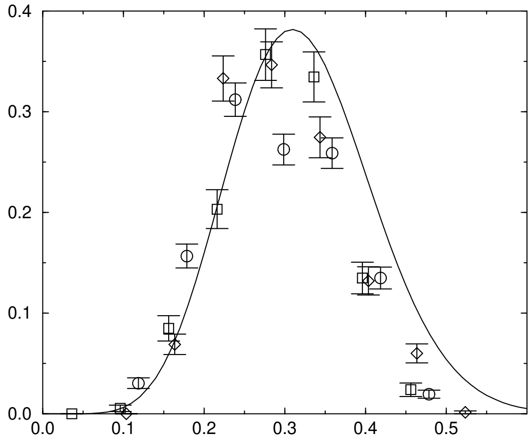

Fig. 6. Distribution of instantons by sizes (density in fm^{-5} *versus* ρ in fm). The curve corresponds to (9), the dots — lattice measurements [23]. (Figure from Ref. [24].)

through the cutoff Λ (we preferred the Pauli–Villars scheme, Λ_{PV}). We obtained approximately

$$\frac{\left\langle \mathcal{F}^2_{\mu\nu} \right\rangle}{32\pi^2}\,(200\ \mathrm{MeV})^4 \;\approx\; \langle Q_{\mathrm{t}} Q_{\mathrm{t}} \rangle\;(190\ \mathrm{MeV})^4 \approx\; (0.7\Lambda_{\mathrm{PV}})^4 \tag{10}$$

(experimental values are given in the brackets). The variational principle guaranteed that this is an estimate from below for the gluon condensate. Unfortunately, Λ_{PV} was then (and is now) very poorly known, so we preferred to formulate our results as a prediction for it. We needed $\Lambda_{\mathrm{PV}} \approx 280\,\mathrm{MeV}$ or slightly less. In those years, it was believed that it was in the region of 100 MeV, and we were told that instanton fluctuations constitute only a small fraction of all vacuum fluctuations of gluodynamics. However, since then, its value has grown significantly, and now the hypothesis that instanton fluctuations make up the lion's share of fluctuations in the vacuum seems plausible. Our estimate of the QCD coupling constant was also confirmed — indeed, $\alpha_{\mathrm{s}} \approx 0.5$, as predicted in (8).

The natural next step was to introduce quarks into the instanton vacuum of pure gluodynamics. For a long time, we could not figure out what is the mechanism of SχSB in the instanton vacuum. Quarks have zero modes (*i.e.*, non-trivial solutions of the Dirac equation $\nabla\!\!\!/(A)\psi = 0$) in the instanton field, it was obvious that they were related to SχSB, but how it is realized was unclear.

This mechanism turned out to be *delocalization* and collectivization of individual zero modes in the instanton medium [25]. This phenomenon was well known in the physics of disordered systems (the Anderson model). It is important that, due to the slow decrease of the overlap integrals of zero modes, it occurs at *any* density of the instanton liquid. In addition, due to the presence of zero mode, the relative magnitude of the chiral condensate becomes large at low instanton density. Therefore, the calculation of the chiral condensate in the instanton liquid is parametrically justified, which distinguishes this case favorably from the formation of condensate in the perturbative approach.

In the thermodynamic limit distribution of eigenvalues of the Dirac equation in the instanton medium becomes a continuous function $\nu(\lambda)$,

$$\nu(\lambda) \;=\; \left\langle\left\langle \sum_i \delta(\lambda-\lambda_i)\right\rangle\right\rangle \tag{11}$$

($\langle\langle\ldots\rangle\rangle$ means averaging over the instanton ensemble), which is smeared due to overlap of the zero modes. According to the Casher–Banks relation the chiral condensate is connected with the value of this distribution function at zero

$$\langle\bar\psi\psi\rangle \;=\; -\pi\frac{N}{V}\nu(0) \tag{12}$$

(N/V — density of the instanton fluid).

In [25] (and some others), we managed to calculate function $\nu(\lambda)$ and compute the chiral condensate

$$\langle\bar\psi\psi\rangle \;\approx\; -(255\text{ MeV})^3\,, \tag{13}$$

which agrees very well with the experimental value of $-(250\,\mathrm{MeV})^3$ (at a low normalization point).

Scattering of quarks off chiral condensate leads to the emergence of an effective quark mass $M(p)$, which depends on its virtuality (see Fig. 7). This dependence is determined by the zero modes in the instanton field. The mass changes on the scale $\sim 1/\rho$. Its value at zero determines the mass of the *constituent quark*

$$M(0) \;\approx\; 350\,\mathrm{MeV}\,. \tag{14}$$

Averaging over the instanton ensemble leads to an effective interaction between constituent quarks. In the limit of a large number of colors N_c, this interaction can be bosonized, and in this way, we can find the spectrum of mesons in our theory. First of all, in agreement with the Goldstone theorem, we observe a *massless* π-meson. The U(1)-problem in the instanton vacuum

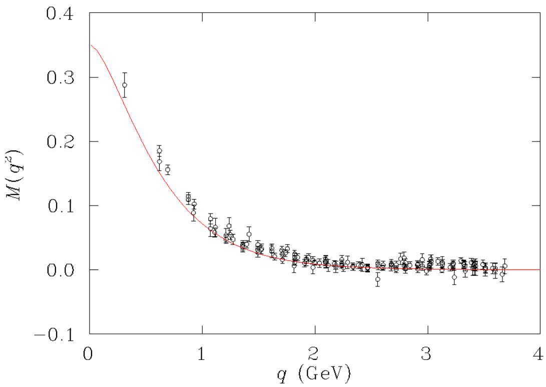

Fig. 7. Effective quark mass as a function of virtuality, dots — lattice measurements. (Figure from Ref. [26] after Ref. [27]).

is solved, and the η'-meson mass follows the Witten formula in the $N_f \to 0$ limit[7]

$$m^2_{\eta'} \;=\; 4N_f \frac{N}{VF_\pi^2} \tag{15}$$

(in the limit of weak interaction of instantons, the topological susceptibility $\langle Q_{\rm t} Q_{\rm t}\rangle = N/V$). The axial constant F_π^2 can also be calculated

$$F_\pi^2 \;=\; 4N_c \int \frac{{\rm d}^4 p}{(2\pi)^4} \frac{M^2(p)}{(p^2+M^2(p))^2} \;\approx\; 98\,{\rm MeV}\,. \tag{16}$$

Experimentally $F_\pi^2 \approx 93.5$ MeV.

The smallness of the packing parameter ρ/R leads to the emergence of *two scales*, existence of which has long been suspected in the theory of strong interactions. The scale $1/\rho$ determines the masses of all other mesons and the size of the constituent quark (and all mesons). The scale $1/R$ determines the gluon condensate and topological susceptibility. The quark mass occupies an intermediate position $M \sim 1/\sqrt{R\rho}$. It is interesting that the same smallness of the packing parameter explains the smallness of the axial constant, its ratio to the gluon condensate is determined by the packing parameter

$$\frac{F_\pi^4}{(G_{\mu\nu}^2)/(32\pi^4)} \;\sim\; \left(\frac{\rho}{R}\right)^4 . \tag{17}$$

[7] The behavior of $m^2_{\eta'}$ in the instanton vacuum is not as usual: it is a constant in N_c, and does not decrease as it should. This has some deep reasons. Therefore, in order to make it small (only then formula (3) can be applied), one has to use the $N_f \to 0$ limit.

The theory of instanton vacuum makes it possible to calculate the correlators of meson currents in all channels. This allows for a detailed verification of the theory. Agreement with the experiment is generally satisfactory, although not in all channels (for an example, see Fig. 8).

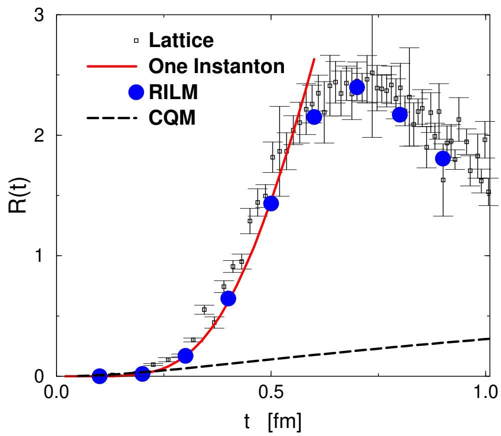

Fig. 8. The ratio of correlators in scalar and pseudoscalar channels $\left(\mathit{\Pi}_{\rm scal}(q^2) - \mathit{\Pi}_{\rm pseudo}(q^2)\right) / \left(\mathit{\Pi}_{\rm scal}(q^2) + \mathit{\Pi}_{\rm pseudo}(q^2)\right)$. (Figure from Ref. [28]).

The existence of two scales in the instanton vacuum makes it meaningful to calculate the *low-energy Lagrangian*, valid for the momenta, smaller than ρ^{-1}. At these scales, only constituent quarks (with constant mass M) and π-mesons remain. The effective Lagrangian, which we calculated in [25], has an extremely simple form

$$\mathcal{L} \;=\; \bar{\psi}\left[i\hat{\partial} + iM\exp\left(i\frac{\hat{\pi}\gamma_5}{F_\pi}\right)\right]\psi\,. \tag{18}$$

Here, $\hat{\pi} = \pi^a\lambda^a$ is the field describing the *octet* of π-mesons. Notice that the $\hat{\pi}$ field enters without the kinetic term. It arises as a result of integration over quarks.

Integrating over quarks, we obtain the effective chiral Lagrangian (EχL), which is valid in the entire momentum region, up to $1/\rho$

$$\mathcal{L}_{\rm E\chi L} \;=\; \mathbf{Det}\left[i\hat{\partial} + iM\exp\left(i\frac{\hat{\pi}\gamma_5}{F_\pi}\right)\right]. \tag{19}$$

It is curious that exactly this EχL was proposed in the work of Mitya and M. Eides [19] (for completely different reasons) several years earlier. We have verified that this Lagrangian is consistent with the known experimental data not only in the leading but also in the next terms of the pion field gradient expansion.

The next stage of our work was a theory of *baryon states* [29]. Several years before this, E. Witten proposed a theory of baryon states in the limit of a large number of colors. Witten had shown that in the $N_c \to \infty$ limit, the nucleon should be a soliton of meson fields, and proposed to implement this idea in the well-known Skyrme model[8]. Having in hand a much more effective chiral Lagrangian derived from the instanton model, it was natural for us to try to build a nucleon.

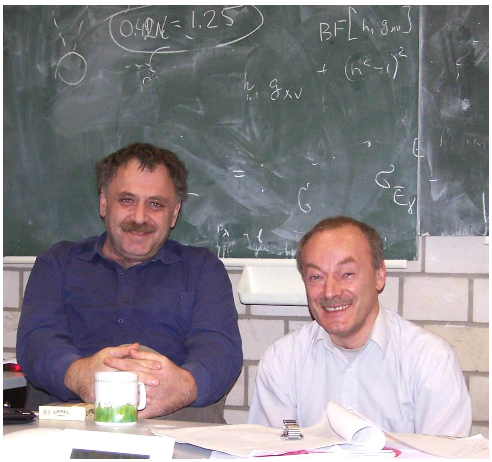

Fig. 9. V. Petrov and D. Diakonov. Chiral Theory of the Nucleon, Bochum, 2005.

In the language of our low-energy Lagrangian, the picture of the nucleon at $N_c \to \infty$ turned out to be very simple: the $N_c \to \infty$ limit meant that the mean-field method can be applied to the problem, and it was necessary to find a discrete level for N_c valence quarks in this field. The nucleon mass was composed of energies of the valence levels in a self-consistent pion field and the energy of the pion field, which is obtained as a sum of the Dirac sea energy levels in this field. The minimum of this mass corresponds to the nucleon and determines the self-consistent field (see Fig. 10).

It took us many years to work on the theory of the nucleon. Our former students — P. Pobylitsa and M. Polyakov took part in this work. In 1988, we were also joined by a group from Bochum (Germany), headed by Professor K. Goeke. All the main characteristics of the nucleon were determined: mass, σ-term, electromagnetic and axial form factors, coupling constants with mesons. Generalizing the theory to the case of the SU(3) flavor group,

[8] Mitya was very interested in this work. With a lot of effort we worked out its details (or rather, re-derived all its statements ourselves) and realized Witten's program in the Skyrme model. Unfortunately for us, the famous paper of Witten, Adkins, and Nappi came out earlier. Then we turned our work into a lecture, which Mitya delivered at the ITEP Winter School in 1984 [30].

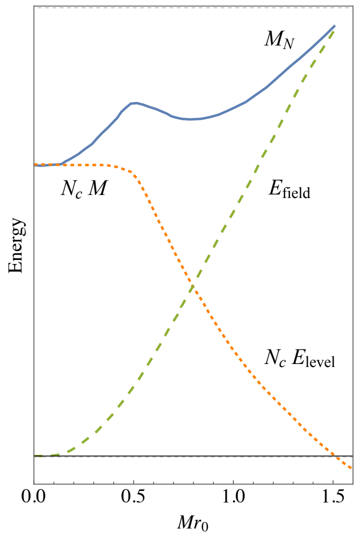

Fig. 10. Discrete level (orang) and pion field energy (green) as a function of field size. The minimum total energy corresponds to the nucleon–soliton. (Figure from M. Praszałowicz, this volume, after [31]).

we calculated the splittings in the two main baryon multiplets: the nucleon octet and the decuplet containing the Δ-resonance (for some examples — see Table 1). All static properties of the nucleon were described pretty well and the accuracy of the theory turned out to be somewhere at the level of 10–15%.

Table 1. Dependence of the characteristics of the nucleon on N_c.

Theor.		Exp.	
σ-term	54 MeV	N_c	45–60
g_a	1.31	N_c	1.25
$g_{\pi N \Delta}$	$3/2 g_{\pi NN}$	$N_c^{3/2}$	
$\mu_p - \mu_n$	5.2	N_c^2	4.71
$\frac{\mu_{\Delta N}}{\mu_p - \mu_N}$	0.70	1	0.71 ± 0.1

More important than the coincidence of the numbers was that we were dealing with a consistent relativistic field theory. Therefore, it was possible to pose such questions, which, in principle, cannot be placed in either the quark or Skyrme model. An example of such a question is calculation of the structure (see work [32]) or wave functions of the nucleon. It was possible to calculate the probability of finding 5 or 7 quarks in a nucleon [33] and so on and so forth. It was also very important for us that both approaches — the quark model and the Skyrme model — were contained in the theory as limiting cases, and our model smoothly interpolated between them. The quark model, which, for large N_c, can also be formulated in the mean-field approximation, is not, however, a relativistic theory, since it does not take into account components of the nucleon wave function with additional quark–antiquark pairs (Fig. 11). These components, however, are not suppressed in the $N_c \to \infty$ limit, moreover, the valence quarks in the nucleon turn out to be fully relativistic particles. It is also significant that the quark model does not take into account the physics of spontaneous chiral symmetry breaking, which is the determining factor for the octet of π-mesons, and, ultimately, for the nucleon. Finally, a wrong mean field symmetry is assumed in the quark model (SU(6)-symmetry instead of the hedgehog symmetry in the chiral theory of the nucleon), what leads to an incorrect set of multiplets. Thus, the quark model at large N_c is a nonrelativistic limit of the quark chiral soliton. The Skyrme model, on the contrary, corresponds to the limit of a large soliton with ultrarelativistic valence quarks.

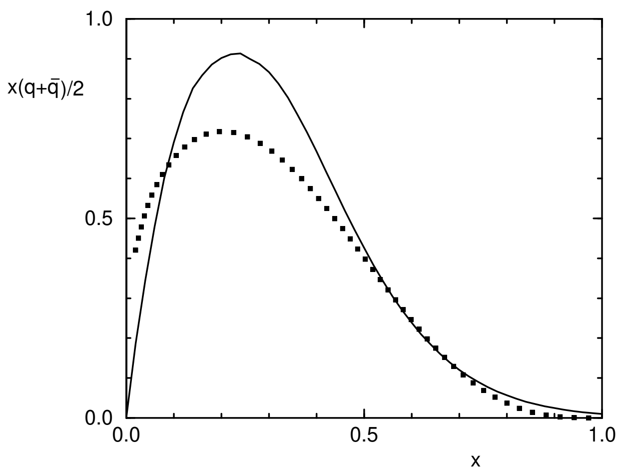

Fig. 11. The structure function of the nucleon: antiquarks $x(\bar{u}(x) + \bar{d}(x))/2$ [32]. The dots are one of the popular parameterizations of experimental data. This structure function in principle cannot be calculated in the quark model.

The quark baryon-soliton model and the $N_c \to \infty$ limit allow to bring order in many questions, in particular in the meson scattering off nucleons, calculation of the nucleon potential, *etc.* Mitya always wanted to explain also nuclear matter with the help of this approach. We have not achieved much success here, although Mitya and A. Mirlin published an article back in 1988 [34], in which they described what is now called a "skyrmion crystal", and what is so intensively discussed in connection with the heavy-ion collision experiments at the LHC. Around the same time, we did some work on calculating the mass of the singlet dilambda — a new type of 6-quark state [35]. Experiments to search for it are currently planned at JPARC (Japan).

Fig. 12. Pentaquark. D. Diakonov, V. Petrov, M. Polyakov, Bochum, 2004.

Having accumulated experience on the properties of ordinary baryons, in 1997 we wrote a paper with a prediction of a new exotic antidecuplet of pentaquark states [36]. This question has had a long history for us. That the next rotational excited state after the octet and decuplet is an antidecuplet, we realized back in 1984 (and not only we, of course) and mentioned this in our lecture [30]. For us, this was a flaw in the theory, and we hoped that this state would turn out to be so wide that it will be unobservable (roughly speaking as in the bag model). We acquired even more reasons for this point of view, when we showed that for highly excited rotational states with $J \sim N_c$, the soliton is deformed, and as a result, linear Regge trajectories for baryons are obtained [37]. We considered the Skyrme model prediction of mass to be very unreliable, but we expected this state to be quite heavy.

The situation changed with the arrival of our chiral soliton model. Encouraged by its successes, we decided to try to calculate the mass and width of the exotic antidecuplet. The first results amazed us: the antidecuplet turned out to be very light, and most importantly, extremely narrow. The width was so small that we even could not estimate it reliably, since it was obtained by cancellation of two large contributions.

In the published article, at Mitya's insistence, we used a slightly different strategy. Since the absolute mass of the antidecuplet was precarious to calculate, we "tied" it to a known particle from the Particle Data Tables, namely the nucleon $N(1710)$. We just noted an acceptable agreement of its mass with the predictions of the chiral soliton model (at the level of the

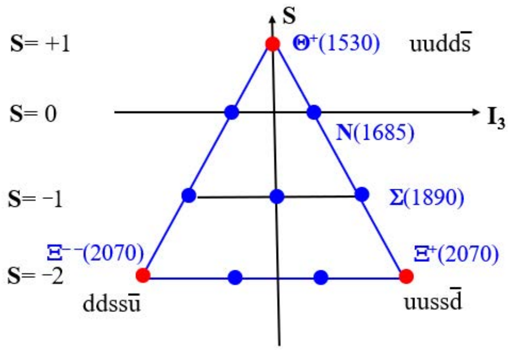

Fig. 13. $\overline{10}$-plet of exotic baryons.

same 10%), and its mass was taken as a reference point. Then the mass of the lightest particle from the antidecuplet (later, according to Mitya's proposal, it was called Θ^+) turned out to be 1530 MeV. The cancellation in calculations of its width worried us for a long time until we proved that in a certain limiting case it should be zero. Nevertheless, we clearly realized that we could not truly (accurately) calculate it, so in the paper, we limited ourselves to the estimate of "less than 15 MeV".

From this moment on, the history of the pentaquark began, which became one of the most dramatic events in baryon spectroscopy. First of all, it was necessary to persuade experimentalists to test existence of the antidecuplet. The main role here belonged to Mitya, who managed to persuade T. Nakano (LEPS, Osaka) to make an experiment, and in 2003, he reported the observation of Θ^+ with a mass very close to the one we predicted, and a small width. Almost simultaneously with him, Θ^+ was observed in the experiments of A. Dolgolenko *et al.* (DIANA experiment, ITEP). The next were two CLAS experiments from JLAB.

Afterwards, Θ^+ was observed in dozens of experiments with varying degrees of reliability. It was even included in the Particle Data Tables. However, starting in 2004, experimental works with high statistics began to appear, reporting "non-observation" Θ^+. It is believed that a follow-up CLAS experiment [40], in which a very low upper limit on the Θ^+ production cross section was established, put an end to the story of Θ^+.

Our prediction was also received skeptically in the theoretical community. Several works have been published criticizing our paper [36], and we had to respond. Thus, it was claimed that the existence of Θ^+ is an artifact of our model. We took the most popular (although very inaccurate) Skyrme model and showed that Θ^+ also arises in it, and with a close mass [41]. However, the width of this state is not small, since the physics responsible for the small width is omitted in this model. Predictions of the Skyrme

Fig. 14. T. Nakano and D. Diakonov.

model for the KN-scattering cross section are in stark contradiction with experiment: in the low-energy region, a giant peak corresponding to the Θ^+ baryon dominates. The only way to make them compatible with the experiment is to reduce the width of the pentaquark.

We were also concerned about our proof of the smallness of the width: we needed a self-evident way to explain its smallness. Such an explanation was obtained by determining the decay constant in the infinite momentum frame, in which the calculation of baryon characteristics is greatly simplified. It turned out that the decay constant is proportional to the overlap integral of the pentaquark wave function and the *5-quark component of nucleons* [42]. The smallness of this constant (which looked like a cancellation of two large contributions in the rest frame) is due to the small probability of finding 5 quarks in a nucleon. In [42], we estimated the pentaquark width at only 2 MeV.

Mitya was also keenly interested in the experimental side of the issue. Together with M. Amaryan and M. Polyakov he proposed a new method for detecting the Θ^+-baryon [43], based on interference. The cross section of the Θ^+ production, which is amplitude squared, seems to be small, so instead of measuring the production cross section one can use interference. The Θ^+ signal can be enhanced in the interference of two processes, where the product of the strong ϕ production amplitude and the Θ^+ production amplitude is measured. Thus, by studying this interference, one can detect the exotic Θ^+. The idea was implemented by a *part* of the CLAS collaboration, which, by analyzing *the same data*, based on which CLAS concluded that Θ^+ was missing [40], observed this exotic resonance at a level better than 5σ [9]. In new experiments and with a new analysis, the LEPS and

[9] The rest of the collaboration was quick to distance themselves from this result.

DIANA collaborations confirmed their old observations (also at the level of about 5σ). Apparently, there is also other data that speaks in favor of Θ^+. Finally, new experiments to detect pentaquarks are planned (they were just discussed at the HADRON-2013 conference in Japan). Lastly, thanks to the efforts of M. Polyakov, V. Kuznetsov and others, another member of the exotic antidecuplet — $N(1685)$ (5-quark cryptoexotic nucleon) (Fig. 13) was probably discovered. However, in general, it can be said that the scientific community has not yet been convinced.

Pentaquarks became a theme of Mitya's constant meditations for many years. In recent years, he has also proposed a new type of pentaquarks, composed of heavy quarks. The idea is to consider a heavy quark to be roughly at rest and weakly interacting with the other, light quarks. The latter are again described by the quark–soliton model. This scheme works very well for description of ordinary non-exotic baryons containing c- and b-quarks. Mitya applied it to exotic particles. The pentaquarks he predicted are structured differently than the exotic antidecuplet (they are not rotational states), but the arguments for their existence seem to be even more compelling. From the point of view of SU(3) they form a $\overline{15}$-plet. It turns out to be very light, some particles may even be stable under strong interaction. The lightest particle, which Mitya called the β_c-baryon, should have a mass of only 2420 MeV (Fig. 15). Under the most conservative assumptions, about $\sim 10^6$ of such baryons should be produced at the LHC per year.

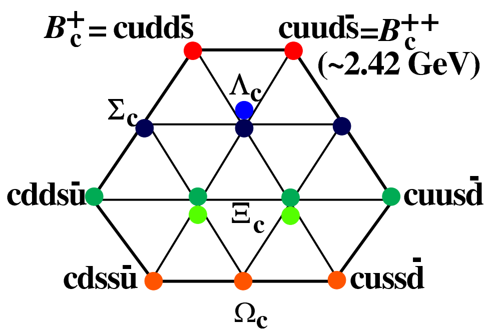

Fig. 15. $\overline{15}$-plet of exotic baryons containing the c-quark.

We have repeatedly returned to the theory of instanton vacuum, trying to clarify some of its elements. An interesting situation arose in the early to mid-90s in connection with the *baryon number violation* (BNV) in the Standard Model. It is known that BNV is possible in the Standard Model due to instanton transitions, but the probability of it is negligibly small ($\exp(-4\pi/\alpha_{\mathrm{EW}}) \approx 10^{-70}$). A. Ringwald drew attention to the fact that in BNV processes at high energies, production of each additional particle contributes a factor of $1/g^2$. Thus, we can hope that in processes with creation of a huge number of particles, $n \sim 1/g^2$, the exponential factor will be absent and the cross section will be of order one. Ringwald's work generated great interest and led to a large number of publications on this topic. It was rather quickly realized that in the leading semiclassical approximation, the cross section of the process with BNV is described by the formula

$$\sigma_{\mathrm{BNV}} = \exp\left(-2 \times \frac{8\pi^2}{g^2} f(\varepsilon)\right), \tag{20}$$

where ε is the ratio of the energy of colliding particles to the mass of the sphaleron $\varepsilon = E/M_{\mathrm{sphl}}$ ($M_{\mathrm{sphl}} \sim 1/g^2$). Function f, universal for a given theory, was named the *Holy Grail function.*

In my opinion, the task of fully computing the Holy Grail function is one of the most interesting in quantum field theory. In the electroweak theory for small ε, it is a powers series in $\varepsilon^{2/3}$

$$f(\varepsilon) = 1 - \frac{3}{4}\varepsilon^{4/3} + \left(\frac{3}{8} - \frac{3}{16} + \frac{1}{16}\right)\varepsilon^2 + c\varepsilon^{8/3}\log\varepsilon + \ldots \tag{21}$$

The first nontrivial term (tree) was calculated in the work of V. Zakharov, the second (1-loop) — in our work [46] (the first term in the parentheses corresponds to the massless pure Yang–Mills theory, the second arises from taking into account the mass of the W boson, and the third — from multiple production of Higgs bosons) and simultaneously by several other authors. The third term (2-loop) was calculated in the work of Mitya and M. Polyakov [47].

Although the computation of (21) is possible in the language of diagrams, it is clear that the Holy Grail function has an entirely *semiclassical origin.* As has been shown in our work [48], function (21) (and with it the cross section of BNV) can be calculated if a complex *singular solution* of the equations of motion in Minkowski space, that satisfies certain boundary conditions, is known. The imaginary part of the action on this singular solution determines function (21), and the real part determines the evolution of the configuration in real space-time and at $t \to \infty$ describes the momentum distribution of the produced particles. We have developed a generic semiclassical approach, allowing to solve the problems of this type. In general, the problem turned out to be extremely meaningful, it combines such questions as the behavior of high orders of perturbation theory in field theory, multiple particle production at the threshold, counting of high-order diagrams, *etc.* It has not been fully solved to this day.

In [48], we showed that the function $f(\varepsilon)$ cannot become less than 1/2 (*i.e.* the BNV cross section always remains small, although at high energies, one can win back a factor of $\sim 10^{35}$!). It is a product of two factors. The first one is the probability of a tunneling sub-barrier (instanton) transition, which increases with energy and at $\varepsilon \sim 1$ reaches values of order one. The second one contains the square of the overlap amplitude of the original wave function (2 particles with high energy) with the final one (many particles $\sim 1/g^2$ with small energies). This factor decreases monotonically with energy. Long ago, V.N. Gribov gave simple quantum-mechanical arguments, which show that the overlap amplitude is equal to the square root of the sub-barrier factor at energy $\varepsilon \sim 1$. Therefore, $f(\varepsilon) = 1/2$ and at higher energies, it begins to increase.

The practical significance of these calculations for the theory of the instanton vacuum is that, as shown in [48], the Holy Grail function is related to the *instanton–anti-instanton* interaction by a Fourier transform

$$\mathrm{e}^{-\beta U_{\mathrm{int}}^{\bar{I}I}(R)} = \int \frac{\mathrm{d}\varepsilon}{2\pi} \mathrm{e}^{i\varepsilon R} \exp\left[-\beta(f(\varepsilon) - 2)\right] , \tag{22}$$

where R is the distance between I and $\bar{I}$. The marker that BNV places on this process allows to separate the purely non-perturbative contribution from the perturbation theory contribution and correctly determine the instanton–anti-instanton contribution, as well as the interaction between them (see Fig. 16). The contribution of instanton and anti-instanton can be reconstructed from the BNV cross section, by squaring the well-defined single-instanton amplitudes and using the unitarity relation. This procedure determines the imaginary part of the corresponding contribution, while the real part is restored using dispersion relations. The situation here is exactly the same as for perturbation theory diagrams: one can reconstruct the entire series of perturbation theory from the Born graph and the unitarity relations. In the same way, single-instanton BNV amplitude is sufficient to obtain the contribution of an arbitrary number of instantons and anti-instantons.

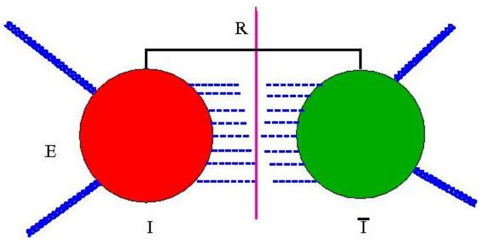

Fig. 16. Contribution of I and $\bar{I}$ determined through the BNV cross section.

With the help of the derived relations, we obtain the interaction of instantons and anti-instantons at large distances in the most attractive orientation

$$U_{\rm int}(R,\rho_1,\rho_2) = -6\frac{\rho_1^2\rho_2^2}{R^4} + 12\frac{\rho_1^2\rho_2^2\left(\rho_1^2+\rho_2^2\right)}{R^6} - 72\frac{\rho_1^4\rho_2^4}{R^8}\log\frac{R^2}{\rho^2} + O\left(\frac{\rho^8}{R^8}\right), \tag{23}$$

where $\rho_{1,2}$ are the sizes of I and $\bar{I}$, R is the distance between them. At small distances, the instanton and anti-instanton *repel* (according to the results [48], logarithmically), what is ultimately connected with the growth of $f(\varepsilon)$ for large ε. This contradicts the usual idea of the attraction between I and $\bar{I}$ at small distances, which arises because $I\bar{I}$ field and zero field are not separated by a barrier. This $I\bar{I}$ configuration corresponds to the already taken into account contribution of perturbation theory, and the attraction itself arises due to the mixing of this configuration with the perturbative sector. Notice that the formulae like (23) are exact, in contrast to our representation of $U_{\rm int}$, obtained from the variational ansatz.

We have always been aware that the main drawback of the QCD instanton vacuum model is the *absence of quark confinement*. The problem, however, seemed too difficult: confinement means the area law for the Wilson loop

$$W(C) = \left\langle P\exp i\int\limits_C {\rm d}x_\mu A_\mu \right\rangle \sim \exp(-\sigma S)\,. \tag{24}$$

However, even the original object, the non-Abelian P-exponent (Wilson loop), cannot be written down in an acceptable and computable form. This was the first problem, with which we started: we managed to find some new representation for the Wilson loop [49]. The formula is a functional integral over directions of the field n^a in color space, where the action is the Chern–Simons term. Our formula turned out to be quite general — it is valid for any gauge theory (for example, for the theory of gravity, where the role of the field A_μ is played by the Christoffel symbols, and the role of the Wilson loop is played by parallel transport along a closed contour) and in an arbitrary representation for the Wilson loop. Using this representation, we also proved a *non-Abelian Stokes' theorem* [50], expressing the Wilson loop through an integral over a surface. We subsequently used these tools more than once in the study of the Wilson loop.

We have calculated the potential between two heavy quarks in the instanton vacuum [51]: as expected, at infinity it goes to a finite limit and generates only a finite (and also small, of the order of 100 MeV) renormalization of the quark mass. We looked at confinement from another perspective as well. The most popular confinement mechanism, proposed by S. Mandelstam, is the *dual Meissner effect* — a condensate of monopoles should form in the QCD vacuum. Unfortunately, even this very notion is not defined, since there are no elementary monopoles in QCD. Therefore, it was necessary to reformulate the Mandelstam criterion (existence of a monopole condensate) in another language. We introduced the probability $\Phi(L)$ of finding a monopole loop of size L in vacuum (this quantity is gauge-invariant and can be measured on a lattice). In work [52], it was shown that if for large loop sizes $\Phi(L) \sim 1/L^3$, then a massless pole is formed in the correlator of monopole currents (this statement is equivalent to condensation), and the Wilson loop decreases exponentially with area. Thus, this statement completely replaces the Mandelstam criterion. It is interesting to notice that there is a monopole loop of size ρ inside an instanton of size ρ. Therefore, if the instanton size distribution behaved like $1/\rho^3$, then there would be confinement in the instanton vacuum [53]. Unfortunately, this is apparently impossible: such a distribution would lead to a long-range order in the instanton medium.

By this point, we had realized the main contradiction of the confinement phenomenon: confinement means very long-range correlations in a certain type of quantities (such as the Wilson or Polyakov loop) and, conversely, a small correlation radius of local operators, such as the correlators of field strengths squared. The latter are well described in the instanton vacuum, while the former are not described at all. It is very difficult to imagine a vacuum that has both properties at the same time. Quantities of the first type are gauge-invariant only globally, not locally, so we came to the idea of formulating the Yang–Mills theory in gauge-invariant terms, in the hope that the resulting theory will have a finite correlation radius.

We solved this problem in 3 and 4 dimensions (completely only for the color group SU(2)) [54, 55], introducing dual variables on the lattice and taking the continuous limit. To our surprise, we got some version of the Regge gravity, but with a certain "ethereal" term, violating general covariance (which the Yang–Mills theory, of course, does not possess). The entire content of the Yang–Mills theory was determined by this ether term (of a simple form), without it the theory was empty and led to some variant of topological theory. At $d = 3$ and for the group SU(2), the theory reduced to Einstein's gravity, which is thus a topological theory. This statement had already been derived before us by E. Witten. We, however, proposed new types of topological theories, including those in 4 dimensions. All of them belonged to the class of the so-called BF (Batalin–Fradkin) theories, which are now being intensively studied as candidates for the role of a consistent theory of gravity. Our theories had a grandiose symmetry, which included as a small subgroup the group of diffeomorphisms. The "ethereal" term softly violated this symmetry, turning the topological theory into the Yang–Mills gauge theory. We have not been able to figure out the dynamics of these theories, although we have returned to this issue several times. I still think it is a very interesting topic to think about.

Fig. 17. Discussing confinement. G. 't Hooft and D. Diakonov.

Gradually, we came to the conclusion that the properties of confinement could be understood, only by considering QCD at non-zero temperature (and density). The point is that at $T \neq 0$, all possible degrees of freedom become excited. Confinement means that a significant part of them have infinite mass. This is especially evident in the limit of a large number of colors, where only N_c-independent number of hadrons survive, when numbers of N_c^2-1 gluons and N_c quarks go to infinity. Therefore, the behavior of thermodynamic quantities cannot be described in a model in which confinement does not take place.

Mitya has already made forays in this direction — so, he calculated the corrections to the well-known Weiss potential for the Polyakov loop in the limit of high temperatures [56]. He also applied the instanton vacuum model to figure out the QCD behavior at very high densities. In this case, he showed that the phenomenon of *color superconductivity* (in the spirit of F. Wilczek) arises, and calculated the magnitude of the color condensate [57]. The work of P. van Baal, who showed that at temperature $T \neq 0$, instantons *melt into dyons* in states with the non-zero Polyakov loop, which appeared at that time, played an important role. This work was all the more important because by that time we had made an excursion into the SUSY version of QCD [58] (with the aim of explaining the well-known paradox with the gluino condensate in supersymmetric theories) and knew well that it is the dyons that are responsible for confinement in compactified, exactly solvable supersymmetric theories. As a result, we came up with the following plausible picture of confinement emergence in QCD at non-zero temperature.

It is well known that the order parameter for the confinement and deconfinement phases is the so-called *Polyakov loop*

$$\mathcal{P} \;=\; \left\langle P\exp\int_0^{1/T} \mathrm{d}tA_4(x) \right\rangle . \tag{25}$$

In the confinement phase $\mathcal{P} = 0$, and in the deconfinement phase, it is different from zero, which is a manifestation of the center group (a subgroup of the color group $\mathrm{SU}(N_c)$) symmetry violation. Matrix $\mathcal{P}$ is not invariant with respect to gauge transformations, but its eigenvalues are gauge-invariant. In the SU(2) group, they can be parameterized as $\{\mathrm{e}^{iv}, \mathrm{e}^{-iv}\}$ (vT is the average field A_4 in a given state).

Let us construct effective potential for v. At high temperatures, this can be done using perturbation theory, the corresponding potential is called the *Weiss* potential. It is a periodic function with minima at $v = 0, 2\pi, \ldots$ corresponding to $\mathcal{P} = 1$ (see Fig. 18). The average value of the field v is close to this minimum, *i.e.* in vacuum $\mathcal{P} = 1$, *i.e.* we are in the deconfinement phase. Perturbative corrections in the coupling constant and non-perturbative contributions arise at lower temperatures.

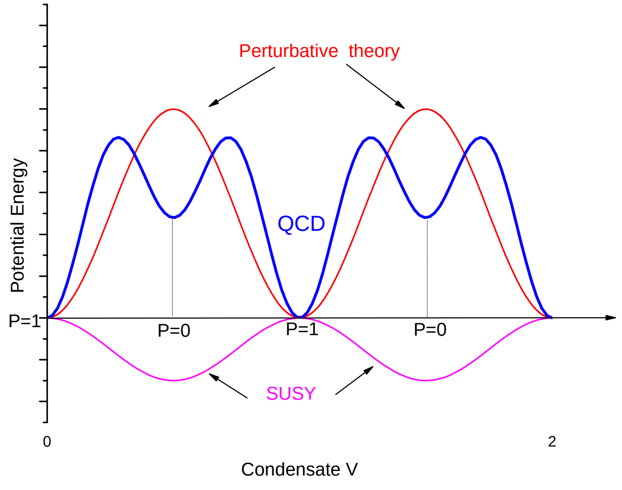

Fig. 18. Effective potential for the Polyakov loop in perturbation theory, dyonic vacuum, and SUSY theories.

At $v \neq 0$ (and temperatures not low), instantons melt, as already mentioned, into dyons, so it is natural to construct a vacuum model based on dyons. However, this theory differs from the theory of instanton vacuum in many ways. By definition, dyons are pseudoparticles that have non-zero electric and magnetic color charges. There are 4 types of dyons in the SU(2) group, which correspond to four possible values ± 1 of both charges and interact via a weakly decreasing (Coulomb) potential. Under no circumstances, this interaction is small at large distances (for instantons, the interaction decreases at least as $1/R^4$). This interaction is quantum, *i.e.* it is contained in the 1-loop correction to the dyonic configuration. The existence of strong long-range interactions greatly complicates the theory. On the other hand, a dyon has fewer collective coordinates, it has neither size nor color orientation. The only collective coordinate is the position of a dyon in 4-dimensional space-time. When two dyons merge into an instanton, these $2 \times 4 = 8$ variables form 8 collective coordinates of instanton (group SU(2)): 4 — for position, one — for size and 3 coordinates for color orientation.

First of all, it was necessary to calculate quantum corrections to the dyonic configuration. Unlike the instanton, this was not done, and we, together with our students N. Gromov and S. Slizovsky, determined the quantum weight of the dyons in [59]. We constructed a theory of dyonic vacuum in [60].

It turned out that the effective potential for the Polyakov loop induced by dyons has minima at $v = \pi, 3\pi, \ldots$ (not at $v = 0, 2\pi, \ldots$), where $\mathcal{P} = 0$. Qualitatively, this effective potential coincides with the potential in SUSY theories, but in those theories, the situation is simpler, since the contribution of perturbation theory is missing (the contributions of gluons and gluinos completely cancel out). Therefore, there is no phase transition in supersymmetric theories. In QCD, however, there is a competition between the perturbation theory and dyon contributions. As the temperature decreases, the dyon contribution begins to dominate, and at a certain temperature T_{c} the minimum abruptly (first-order phase transition in all groups except SU(2)) shifts to the point $v = \pi$, corresponding to $\mathcal{P} = 0$. Hence, the *confinement–deconfinement phase transition* occurs at $T = T_{\mathrm{c}}$.

The theory of the dyon vacuum agrees with the lattice measurements not only qualitatively but also quantitatively. We calculated phase transition temperatures and topological susceptibility for different values of N_c and compared them with the available lattice data. The agreement is at the level of 10%, the trends of change of the values in our data and on the lattice coincide (see Table 2).

Table 2. Comparison of the dyon vacuum theory with lattice measurements for the phase transition temperature $T_{\rm c}$ and topological susceptibility $\langle Q_{\rm t}^2\rangle$ in units of the string tension σ.

N_c	2	3	4	∞
$T_{\rm c}$	0.7425	0.6430	0.6150	0.5830
lattice	0.7091	0.6462	0.6344	0.5970
$\langle Q_{\rm t}^2\rangle^{1/4}$	0.5	0.439	0.4831	434
lattice	0.420	0.399	0.387	0.376

By calculating the correlator of two Polyakov loops, we showed that the potential between heavy quarks actually increases linearly with distance, and at the point of the phase transition, the linear potential abruptly turns into zero. It is interesting that at the same time the spatial Wilson loop almost does not change. This corresponds to the lattice data. On the other hand, we have verified that at low temperatures, the "spatial" tension becomes equal to the tension determined from the linear potential. This shows that at low temperatures the theory indeed goes to the 4-dimensionally invariant limit.

An interesting question is about the tension of the string in different representations. In a theory with the group $\mathrm{SU}(N_c)$, there should exist N_c-1 different string tensions corresponding to N_c-1 antisymmetric irreducible representations (other representations can be reduced to these by screening with some number of perturbative gluons). We have obtained the following general formula:

$$\sigma_k \;=\; \sigma_0 \sin\frac{\pi k}{N_c}\,. \tag{26}$$

This formula agrees very well with the lattice data, even better than the famous "Casimir scaling" which is thought to be derived from the lattice data. Interestingly, the same formula arises in the exactly solvable supersymmetric Seiberg–Witten model.

Since dyons are produced by melting instantons, our theory of the dyon vacuum should smoothly match the instanton model. There is one exception: the topological susceptibility is of order one in the dyon vacuum (as it should be), and not of order N_c, as in the instanton vacuum. The gluon condensate is proportional to N_c and is equal (for $N_c=3$)

$$\left\langle \frac{G_{\mu\nu}^2}{32\pi^2}\right\rangle \;\approx\; (240\ \mathrm{MeV})^4\,. \tag{27}$$

This is quite close to both the value obtained in the instanton vacuum and to the experimental data.

Unfortunately, we did not manage to fully trace the connection between the dyon vacuum and the instanton model, since our methods lose their applicability at low temperatures.

In recent years, Mitya has become interested in the theory of gravity. We wanted to have an example of a consistent gravity theory based on field theory (rather than on strings). However, it seemed obvious that Einstein's gravity is just a low-energy limit of some more general self-consistent theory. Mitya treated gravity in exactly the same way as an effective chiral Lagrangian, considering Einstein's gravity as a first term in a gradient expansion. In this regard, he did work that, oddly enough, no one had done before him: classified all possible actions of a given order in gradients.

A more general, than Einstein's, theory of gravity, of course, contains torsion, so the authors of [61] still had to figure out what it leads to. The conclusion was that torsion only induces a 4-fermion interaction of the "current-by-current" type, with a structure similar to the weak interaction, but with a constant many orders of magnitude smaller. Therefore, it is unlikely to be observed under any conditions.

Fig. 19. Quantum gravity. A. Vladimirov and D. Diakonov, 2012.

Any quantum theory of gravity contains a fundamental problem that worried Mitya a lot and which we discussed with him many times: its action is not positive definite. The functional integral with such an action diverges for large values of the gravitational field. It seems there are only two possible solutions to this problem: either the integration domain in the correct theory of gravity is bounded in some way (this is not easy to do, because the boundary should not violate unitarity and causality), or the theory contains only fermion fields for which the functional integral converges with any action, and the graviton is a composite particle. Mitya (unlike me) adhered to the second point of view. In [62], he and A. Vladimirov formulated a program for constructing theories with a composite graviton and gave some simple (two-dimensional) examples, how such a theory could work. Mitya had high hopes for this program, but he did not have time to move along this path far enough.

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On Thursday, December 20, 2012, Mitya and I, as always, were working at my place in Gatchina — discussed baryon resonances in the chiral theory of the nucleon[10]. I parted with him Friday evening, and the next day, in the morning, he suffered a severe heart attack, and three days later, despite all medical efforts, he died.

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