


## DYONS, INSTANTONS, BARYONS, AND AdS/QCD\*

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Dmitri Diakonov has played a significant role in identifying the degrees of freedom underlying hadron spectroscopy. His contributions are discussed with a view on recent developments.

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## 1. Introduction

In the summer 2001, Mitya visited our institute in Bonn and gave a fascinating talk [1] about instantons and baryon dynamics and explained how instantons break the chiral symmetry of strong interactions [2], the ideas Mitya had developed further with Maxim [3]. I was startled. At that time I just had found a mass formula for baryon resonances based on instanton-induced interactions [4], which reproduced the spectrum much better than quark models and with fewer parameters [5]. I had the opportunity to explain to Mitya my new formula and some wild ideas I had about different time scales for color and flavor exchange in baryons and the consequences for confinement, the  $^3P_0$  model, the proton spin, for glueballs and hybrids [6]. Mitya listened with friendly attention, and if he felt bored, he surely hid his feelings.

Later, Mitya developed — jointly with Vitya — ideas to understand confinement by gluon field configurations with asymptotic Coulomb-like chromo-electric and -magnetic fields, called dyons [7, 8]. Dyons are supposed to reveal themselves as Abelian monopoles or as center vortices. The interactions of dyons can be identified with instantons. In essence, in his view, mass generation, confinement, and chiral symmetry breaking are related to a common origin.

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Figure 1 supports this interpretation. It shows a common Regge trajectory for  $N^*$  and  $\Delta^*$  resonances, the leading  $N^*$  resonances with  $J = L + 1/2$ , and  $\Delta^*$  resonances with  $J = L + 3/2$ . The masses, the Regge slope, and the hyperfine structure splitting ( $\Delta(1232)$  versus  $N$ ,  $\Delta(1950)$  versus  $N(1680)$ ,  $\Delta(2420)$  versus  $N(2220)$ ,  $\Delta(2950)$  versus  $N(2700)$ ) are consistently described, in agreement with Mitya's conjecture. But clearly, this is not yet the full story: the fit is not perfect and, in particular, the masses of nucleon and  $N(1680)$  are not well reproduced. Only the  $\Delta^*$  values lie precisely on a straight line, coinciding with zero at  $J = 0$ .

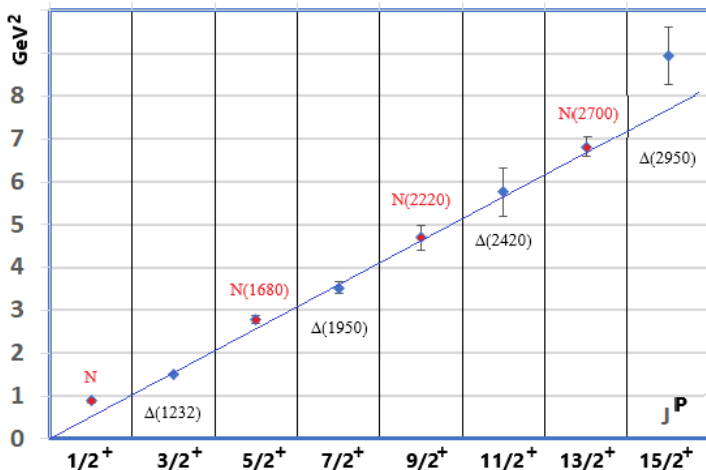


Fig. 1. Squared masses (pole positions) of leading baryon resonances *versus* total angular momentum  $J$ .

## 2. Baryons in AdS/QCD

In an AdS/QCD, the approach developed by Brodsky, Teramond, Dosch, and Ehrlich [9], the spectrum of baryon resonances is reproduced by one constant and two quantum numbers. The nucleon mass, the Regge slope, and the mass splitting of resonances can all be described by just one parameter. This fact again points at a common origin of these three phenomena as Mitya, Vitya, and Maxim have advocated for.

The mass spectrum in AdS/QCD is given by a radial quantum number  $n$  and a quantum number  $\nu$

$$M^2 = 4\lambda(n + \nu + 1). \quad (1)$$

The calculated mass spectrum using Eq. (1) or, alternatively, the relations of Table 4 is compared to the data in Table 1.  $\nu$  needs to be related to the quantum numbers characterizing the different  $N^*$  and  $\Delta^*$  resonances [9], see Table 2.

Table 1. Baryon resonances in the first and second excitation shells and their masses (all masses are given in MeV). Three- and four-star resonances are shown in bold font. The assignments to the 20-plet are questionable.

N	$L^P$	SU(6)	$S$	Baryon	$M_{\text{cog}}$	Instanton induced		AdS/QCD	
0	$0^+$	<b>56</b>	$\frac{1}{2}$	$N(940)\frac{1}{2}^+$	939	$\sqrt{M_\Delta^2 - \frac{\delta}{2}}$	939	$\sqrt{4\lambda}$	988
			$\frac{3}{2}$	$\Delta(1232)\frac{3}{2}^+$	$1210 \pm 1$	$M_\Delta$	1210	$\sqrt{6\lambda}$	1210
1	$1^-$	<b>70</b>	$\frac{1}{2}$	$N(1535)\frac{1}{2}^-$ $N(1520)\frac{3}{2}^-$	$1503 \pm 4$	$\sqrt{M_\Delta^2 + \omega - \frac{\delta}{4}}$	1572	$\sqrt{10\lambda}$	1562
			$\frac{3}{2}$	$N(1650)\frac{1}{2}^-$ $N(1700)\frac{3}{2}^-$ $N(1675)\frac{5}{2}^-$	$1670 \pm 24$	$\sqrt{M_\Delta^2 + \omega}$	1662	$\sqrt{12\lambda}$	1711
			$\frac{1}{2}$	$\Delta(1620)\frac{1}{2}^-$ $\Delta(1700)\frac{3}{2}^-$	$1656 \pm 14$	$\sqrt{M_\Delta^2 + \omega}$	1662	$\sqrt{12\lambda}$	1711
2	$0^+$	<b>56</b>	$\frac{1}{2}$	$N(1440)\frac{1}{2}^+$	$1366 \pm 3$	$\sqrt{M_\Delta^2 + 2\omega - A - \frac{\delta}{2}}$	1372	$\sqrt{8\lambda}$	1386
			$\frac{3}{2}$	$\Delta(1600)\frac{3}{2}^+$	$1550 \pm 15$	$\sqrt{M_\Delta^2 + 2\omega - A}$	1570	$\sqrt{10\lambda}$	1574
	$0^+$	<b>70</b>	$\frac{1}{2}$	$N(1710)\frac{1}{2}^+$	$1696 \pm 10$	$\sqrt{M_\Delta^2 + 2\omega - \frac{A}{2} - \frac{\delta}{4}}$	1724	$\sqrt{12\lambda}$	1711
			$\frac{3}{2}$	$N'(1720)\frac{3}{2}^+$	$1725 \pm 30$	$\sqrt{M_\Delta^2 + 2\omega - \frac{A}{2}}$	1801	$\sqrt{14\lambda}$	1848
			$\frac{1}{2}$	$\Delta(1750)\frac{1}{2}^+$	$1770 \pm 30$	$\sqrt{M_\Delta^2 + 2\omega - \frac{A}{2}}$	1801	$\sqrt{14\lambda}$	1848
	$2^+$	<b>56</b>	$\frac{1}{2}$	$N(1720)\frac{3}{2}^+$ $N(1680)\frac{5}{2}^+$	$1689 \pm 13$	$\sqrt{M_\Delta^2 + 2\omega - \frac{2A}{5} - \frac{\delta}{2}}$	1696	$\sqrt{12\lambda}$	1711
			$\frac{3}{2}$	$\Delta(1910)\frac{1}{2}^+$ $\Delta(1920)\frac{3}{2}^+$ $\Delta(1905)\frac{5}{2}^+$ $\Delta(1950)\frac{7}{2}^+$	$1859 \pm 14$	$\sqrt{M_\Delta^2 + 2\omega - \frac{2A}{5}}$	1850	$\sqrt{14\lambda}$	1848
	$2^+$	<b>70</b> Q.M.	$\frac{1}{2}$	$N(1900)\frac{3}{2}^+$ $N(1860)\frac{5}{2}^+$	$1891 \pm 25$	$\sqrt{M_\Delta^2 + 2\omega - \frac{A}{5} - \frac{\delta}{4}}$	1883	$\sqrt{14\lambda}$	1848
			$\frac{3}{2}$	$N(1880)\frac{1}{2}^+$ $N(1965)\frac{3}{2}^+$ $N(2000)\frac{5}{2}^+$ $N(1990)\frac{7}{2}^+$	$1978 \pm 39$	$\sqrt{M_\Delta^2 + 2\omega - \frac{A}{5}}$	1935	$\sqrt{16\lambda}$	1976
			$\frac{1}{2}$	$\Delta(2000)\frac{5}{2}^+$	$2040 \pm 80$	$\sqrt{M_\Delta^2 + 2\omega - \frac{A}{5}}$	1935	$\sqrt{16\lambda}$	1976
$1^+$	$1^+$	<b>20</b>	$\frac{1}{2}$	$N(2100)\frac{1}{2}^+$ $N(2040)\frac{3}{2}^+$	$2044 \pm 25$	$\sqrt{M_\Delta^2 + 2\omega}$	2016		

Table 2. AdS/QCD values for  $\nu$ .

	${}^2N$	${}^4N$	$\Delta$
$P = +$	$\nu = L$	$\nu = L + \frac{1}{2}$	$\nu = L + \frac{1}{2}$
$P = -$	$\nu = L + \frac{1}{2}$	$\nu = L + 1$	$\nu = L + 1$

The center-of-gravity masses  $M_{\text{cog}}$  in Table 1 are calculated as

$$M_{\text{cog}} = \frac{\sum_J J M_J}{\sum_J J}, \quad (2)$$

where pole masses  $M_J$  are used from Refs. [10] or [11]. The uncertainties are calculated by replacing  $M_J$  by  $(\delta M_J)^2$ .

In this contribution, I give an interpretation of the ADS/QCD quantum numbers directly related to physical effects: The nucleon mass is given by the zero-point oscillation of the excited string, the hyperfine structure splitting by instanton-induced interactions, the quark masses are vanishingly small at the hadronic scale.

First, we notice that  $N^*$  resonances with an intrinsic quark spin  $S = \frac{3}{2}$  belonging to a spin quartet ( ${}^4N$ ) and  $\Delta^*$  resonances have (about) the same mass, independent of the quark-spin of  $\Delta^*$  resonances. Expressed in spin and isospin:  $S = \frac{1}{2}$ ,  $I = \frac{1}{2}$  have a lower mass than all other configurations. If the spatial wave function is symmetric, the spin-isospin wave function is symmetric too, and contains one component which is symmetric, and one which is antisymmetric in spin and isospin. The latter part is influenced by instanton-induced interactions.  ${}^2N$  nucleons have a lower mass than  ${}^4N$ ,  ${}^2\Delta$ , and  ${}^4\Delta$  baryons due to instanton-induced interactions.

The classical rotation of a massless quark and a diquark can be described as a rotating string of length  $D$  with a constant mass distribution. With a mass in an interval  $dr$  given by  $\sigma dr$ , the total mass can be calculated as

$$M = \int_{-D/2}^{+D/2} \frac{\sigma dr}{\sqrt{1-v^2}} = \frac{\pi \sigma D}{2}, \quad (3)$$

while the angular momentum is given by

$$L = \int_{-D/2}^{+D/2} \frac{\sigma v(r) r dr}{\sqrt{1-v^2}} = \frac{\pi \sigma D^2}{8}. \quad (4)$$

Combining these equations, we obtain

$$M^2 = 2\pi\sigma L. \quad (5)$$

In quantum mechanics, both oscillators have a zero-point energy  $\pi\sigma$ . Hence, we write

$$M^2 = 2\pi\sigma \left( L + \frac{1}{2} + \frac{1}{2} \right) = M_0^2 + aL. \quad (6)$$

For the  $L = 0$  ground state, we have  $M_0^2 = 2\pi\sigma$ , just the same as in AdS/QCD. The mass of the proton can be understood as the zero-point energy of the two oscillators.

Equation (1) quantifies the size of instanton-induced interactions as  $2\lambda$ , and thus relates it to the Regge slope  $4\lambda$  and to the nucleon mass of  $4\lambda$ . Mass generation, confinement, and chiral symmetry breaking are all described by a single parameter.

The AdS/QCD approach has great achievements, not only in the calculation of mass spectra of mesons and baryons but also in the derivation of space-like and time-like form factors. However, in some cases, the classification of confirmed baryons in Table 5.4 in Ref. [9] is questionable. For instance, the  $3^* \Delta(1930)5/2^-$  assigned to a 70-plet cannot have intrinsic  $L = 1$ ,  $S = \frac{3}{2}$  (but could exist with  $L = 2$ ,  $S = 1/2$  or  $L = 3$ ,  $S = 1/2$ ). However, it is more likely that this state and the  $3^* \Delta(1900)\frac{1}{2}^-$  and  $2^* \Delta(1940)\frac{3}{2}^-$  belong to a 56-plet. The three- $\Delta$  state must be accompanied by a nucleon doublet. Indeed, there is a spin doublet  $N(1895)\frac{1}{2}^-$  and  $N(1875)\frac{3}{2}^-$ , while no  $N\frac{5}{2}^-$  has been reported. Probably, this is a required doublet. High-mass baryon resonances often have  $I = \frac{1}{2}$ ,  $S = \frac{1}{2}$  or  $I = \frac{3}{2}$ ,  $S = \frac{3}{2}$ , and their spin and orbital angular momenta are aligned. Likely,  $N(2700)\frac{11}{2}^-$  has intrinsic spin  $S = \frac{1}{2}$ .

An interpretation of the baryon spectrum is given in Fig. 2. The Regge trajectory is shown as a function of the intrinsic orbital angular momentum  $L$ , which can be defined since spin-orbit interactions are small. Figure 2 shows the squared masses of  $N^*$  and  $\Delta^*$  as in Fig. 1, but in addition of the squared masses of negative-parity  $\Delta^*$  resonances and of radial excitations.

The only obstacle for AdS/QCD is  $N(1710)\frac{1}{2}^+$ . In Ref. [9], it is the second radial excitation of the nucleon and belongs to a 56-plet. In quark models, it is the companion of the Roper resonance in the  $(70, 0_2^+)$  multiplet with  $S = 1/2$ . As a member of a 70-plet,  $N(1710)\frac{1}{2}^+$  has a wave function given by

$$\mathcal{M}_A = \phi_{00,11}, \quad \mathcal{M}_S = \frac{1}{\sqrt{2}}(\phi_{10,00} - \phi_{01,00}).$$

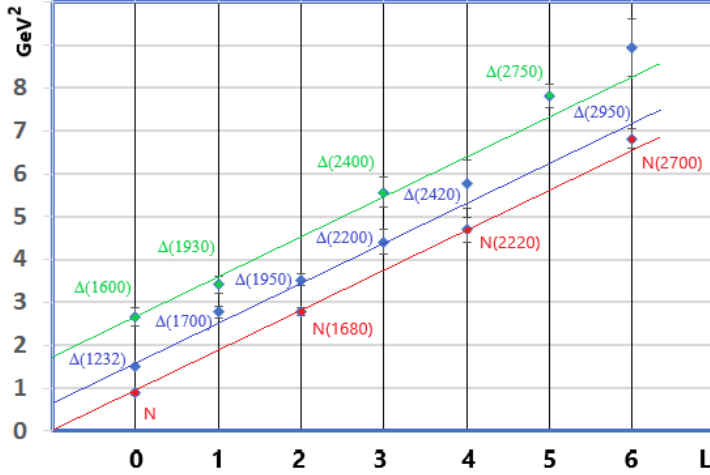


Fig. 2. Squared masses (pole positions) of  $N^*$  (red) and  $\Delta^*$  (blue) resonances *versus* total angular momentum  $L$ .  $\Delta^*$  resonances with radial excitation are shown in green.

In  $M_A$  both,  $\rho$  and  $\lambda$ , are excited to  $l_\rho = l_\lambda = 1$ . In this case, we expect that a sizable fraction of all decays proceed via an intermediate orbital excitation, that is, via  $N(1535)\frac{1}{2}^-\pi$ ,  $N(1520)\frac{3}{2}^-\pi$ , or  $N\sigma$ . In the *Review of Particle Physics*, the branching ratio to  $N(1535)\frac{1}{2}^-\pi$  is listed with (9–21)%, for  $N\sigma$  — an upper limit of 16% is quoted. These decay modes favor the quark-model interpretation of  $N(1710)\frac{1}{2}^+$  as a member of a 70-plet. This interpretation is supported in quark-model calculations.

Table 2 quotes values of  $\nu$  for positive-parity baryons in an SU(6) 56-plet and negative-parity baryons in an SU(6) 70-plet. There are, however, baryons that do not fall into these two categories,  $N(1710)\frac{1}{2}^+$  is one example.  $\Delta(1930)\frac{5}{2}^-$  with  $L = 1$ ,  $S = \frac{3}{2}$  belonging to a 56-plet is a second example. However, with a small change, one can calculate masses of these additional states as well, see Table 3. In the harmonic oscillator approximation,  $N(1710)\frac{1}{2}^+$  contains with 50% probability  $n = 1$ ,  $L = 0$ , but with the same probability ( $n_\rho + n_\lambda = 0$ ,  $l_\rho + l_\lambda = 2$ ). The former yields an excitation of  $10\lambda$ , the latter of  $14\lambda$ , with  $12\lambda$  as mean value.  $\Delta(1750)\frac{1}{2}^+$  is then expected at  $14\lambda$ . In other cases, the wave functions are more complicated, and even fractional values for  $\nu$  can result.

Figure 3 shows the squared masses of (nearly) all resonances as a function of  $\lambda$ . Most resonances are at least compatible with the straight line.

Table 3. AdS/QCD values for  $\nu$ .

$D^P$	${}^2N$	${}^4N$	$\Delta$
$56^+$	$\nu = l$	$\nu = l + \frac{1}{2}$	$\nu = l + \frac{1}{2}$
else	$\nu = l + \frac{1}{2}$	$\nu = l + 1$	$\nu = l + 1$

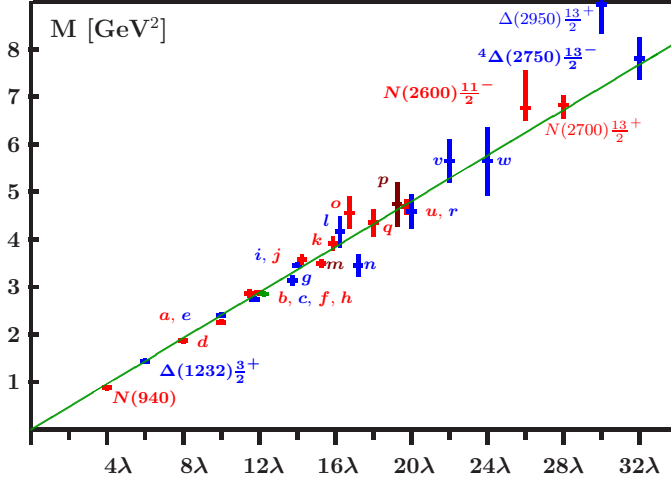


Fig. 3. The squared mass of  $N^*$  and  $\Delta^*$  as a function of the masses predicted in AdS/QCD. For multiplets, the center-of-gravity mass is shown.

a:  $N(1535)_{\frac{1}{2}}^-$ ,  $N(1520)_{\frac{3}{2}}^-$ ; b:  $N(1650)_{\frac{1}{2}}^-$ ,  $N(1700)_{\frac{3}{2}}^-$ ,  $N(1675)_{\frac{5}{2}}^-$ ; c:  $\Delta(1620)_{\frac{1}{2}}^-$ ,  $\Delta(1700)_{\frac{3}{2}}^-$ ;  
 d:  $N(1440)_{\frac{1}{2}}^+$ ; e:  $\Delta(1600)_{\frac{3}{2}}^+$ ; f:  $N(1710)_{\frac{1}{2}}^+$ ; g:  $\Delta(1750)_{\frac{1}{2}}^+$ ; h:  $N(1720)_{\frac{3}{2}}^+$ ,  $N(1680)_{\frac{5}{2}}^+$ ;  
 i:  $\Delta(1910)_{\frac{1}{2}}^+$ ,  $\Delta(1930)_{\frac{3}{2}}^+$ ,  $\Delta(1905)_{\frac{5}{2}}^+$ ,  $\Delta(1950)_{\frac{7}{2}}^+$ ; j:  $N(1900)_{\frac{3}{2}}^+$ ,  $N(1860)_{\frac{5}{2}}^-$ ;  
 k:  $N(1880)_{\frac{1}{2}}^+$ ,  $N(1965)_{\frac{3}{2}}^+$ ,  $N(2000)_{\frac{5}{2}}^+$ ,  $N(1990)_{\frac{7}{2}}^+$ ; l:  $\Delta(2000)_{\frac{5}{2}}^+$ ; m:  $N(1895)_{\frac{1}{2}}^-$ ,  $N(1875)_{\frac{3}{2}}^-$ ;  
 n:  $\Delta(1900)_{\frac{1}{2}}^-$ ,  $\Delta(1940)_{\frac{3}{2}}^-$ ,  $\Delta(1930)_{\frac{5}{2}}^-$ ; o:  $N(2150)_{\frac{1}{2}}^-$ ,  $N(2120)_{\frac{3}{2}}^-$ ; p:  $\Delta(2150)_{\frac{1}{2}}^-$ ,  $\Delta(2190)_{\frac{3}{2}}^-$ ;  
 q:  $N(2060)_{\frac{5}{2}}^-$ ,  $N(2190)_{\frac{7}{2}}^-$ ; r:  $\Delta(2210)_{\frac{5}{2}}^-$ ,  $\Delta(2200)_{\frac{7}{2}}^-$ ; s:  $N(2250)_{\frac{9}{2}}^-$ ; u:  $N(2220)_{\frac{9}{2}}^+$ ;  
 v:  $\Delta(2390)_{\frac{7}{2}}^+$ ,  $\Delta(2300)_{\frac{9}{2}}^+$ ,  $\Delta(2420)_{\frac{11}{2}}^+$ ; w:  $\Delta(2350)_{\frac{5}{2}}^-$ ,  $\Delta(2400)_{\frac{9}{2}}^-$ .

### 3. Quark model approach

Quark models describe the spectrum of baryon resonances as excitations in the dynamics of three constituent quarks. Since the early model of Isgur and Karl [12–14], relativistic corrections have been implemented [15] or fully relativistic calculations have been carried out [16–18]. All models agree on linear confining potential, but a dispute arose on the residual quark–quark interaction. Can these be treated as an effective one-gluon exchange [12–15] or by exchanges of pseudoscalar mesons between quarks [19, 20]? Are instanton-induced interactions at work [16–18]?

In quark models, the three-body system is reduced to two oscillations and a trivial center-of-mass motion. The two oscillators in the variables  $\rho$  and  $\lambda$  represent the oscillation of a diquark relative to the third quark ( $\lambda$ ) and the oscillation within the diquark. Symmetrization guarantees that all quarks are treated equally. Both oscillators can be excited radially ( $n_\rho, n_\lambda$ ) or carry angular momenta ( $l_\rho, l_\lambda$ ). We call  $n = n_\rho + n_\lambda$  radial and  $l = l_\rho + l_\lambda$  orbital excitation quantum number. The orbital angular momentum  $\vec{L} = \vec{l}_\rho + \vec{l}_\lambda$  combines with the total quark spin  $\vec{S}$  to the particle spin  $\vec{J} = \vec{L} + \vec{S}$ . For  $S = 1/2$ , two resonances with  $J = \pm \frac{1}{2}$  are expected, for  $S = 3/2$ , a quartet develops with  $J = -\frac{3}{2}, \dots, J = +\frac{3}{2}$ . In the absence of spin-orbit interactions, the spin-doublet and the spin-quartet would be degenerate in mass.

The quark-quark and quark-diquark potential is linearly increasing, but quark models start with the harmonic-oscillator approximation. Its eigenvalues increase linearly as  $M_0 + (l + 2n)\omega$ . The linear potential is taken into account perturbatively yielding corrections to the eigenvalues for the lowest excitation levels listed in Table 4. As mentioned above, instanton-induced interaction lowers the mass of resonances due to the fraction in their wave function that is antisymmetric in spin and isospin (or spin and flavor). This fraction is shown in Table 5.

Table 4. Mass pattern of baryon resonances in the non-relativistic quark model with a linear confinement potential.

$(D, J^P)_N$	lin. conf.
$(56, 0^+)_0$	$M_0$
$(70, 1^-)_1$	$M_0 + \omega$
$(56, 0^+)_2$	$M_0 + 2\omega - A$
$(70, 0^+)_2$	$M_0 + \omega - \frac{1}{2}A$
$(56, 2^+)_2$	$M_0 + 2\omega - \frac{2}{5}A$
$(70, 2^+)_2$	$M_0 + 2\omega - \frac{1}{5}A$
$(20, 1^+)_2$	$M_0 + 2\omega$

In Table 1, low-mass baryon resonances are listed with their SU(6) classification. The labels  $L$ ,  $S$ , and  $n$  refer to the internal orbital angular momentum, internal spin, and radial excitation quantum number. The mass values suggest that spin-orbit interaction must be small. In the third row, we therefore give the mean (pole) mass of a multiplet, the error covers the range of masses within the multiplet. It is calculated from the maximum spread of results divided by  $\sqrt{12}$ .



Table 5. Fraction of “good diquarks” in the wave function of  $N^*$  resonances. For the nucleon, the fraction is  $\frac{1}{2}$ . It is responsible for the  $\Delta$ - $N$  mass splitting.

$I_{\text{sym}} = \frac{1}{2}$	for $S = \frac{1}{2}N^*$ in a 56-plet
$I_{\text{sym}} = \frac{1}{4}$	for $S = \frac{1}{2}N^*$ in a 70-plet
$I_{\text{sym}} = 0$	for $S = \frac{3}{2}N^*$
$I_{\text{sym}} = 0$	for $N^*$ in a 20-plet
$I_{\text{sym}} = 0$	for $\Delta^*$

In the fourth row, we give formulae and results for the masses of baryons in a multiplet. The potential parameter  $A$ , the string constant  $a$ , and the variable defining instanton-induced interactions  $b$  are chosen as  $A = 1.6$ ,  $a = 1.3$ ,  $b = 1.16$ , all in  $\text{GeV}^2$  units. The similarity of  $a$  and  $b$  reminds of the common origin of the Regge slope and hyperfine structure. In consideration of the simplicity of the formulas, the agreement between data and fit is surprising.

## 4. Conclusions

We have seen that the gross features of the spectrum of light-baryon resonances find simple explanations. Baryon with angular momentum excitation can be understood in a very simple picture of a rotating string. AdS/QCD provides a more detailed and preciser view of the spectrum. Its success underlines the deep connection between the mass of baryons, the string tension, and the hyperfine interaction. Surprisingly, the non-relativistic quark model provides a detailed view of the full richness of the three-body dynamics.

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