MY FRIENDS AND HADRON PHYSICS

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Dmitry Diakonov, Victor Petrov, and Maxim Polyakov were my colleagues and collaborators. Mitya and Vitya were also, and maybe first of all, my close personal friends for many many years. What follows is a mixture of some recollections and a review of our joint works.

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Don't say with anguish: they are gone,
But with a gratitude: they were.
Vassily Zhukovskii (1827)[1]

1. How it all started: Mitya and U(1) problem

I knew Mitya from our student days. He entered the Physical Department of St. Petersburg University a year later than me, and became friends with his peers, who were graduates of the 239$^{\text{th}}$ Physics–Mathematical high school. I also was a graduate of that school, and remained a member of the school's physics, math, and tourism clubs even after graduation. So Mitya's new friends were also my friends. That is how we met, but we did not become close friends in our student years. Our close acquaintance and friendship began later, a few years after graduation from the university, shortly after each of us had defended his Ph.D. dissertation in theoretical physics. The theses were on topics so diverse that it would seem as if we had virtually no common ground. However, it should be taken into account that we were participants in the LNPI theoretical seminar, and practically

[1] From the old Russian poetry, translated by A. Pokidov.
In the original Russian:
Не говори с тоской: их нет,
Но с благодарностию: были.
Василий Жуковский (1827)

Fig. 1. Vitya Petrov, Mitya Diakonov, and Misha Eides (2003).

all of theoretical physics was discussed at the seminar. For a young person, it was just impossible to admit that you do not get something. Of course, initially, we did not know a lot of things, but that just meant that after the seminar, we discussed what we missed, read the original works, and did not stop until figured it out. It was through such discussions that we began our joint work.

These were the years when the modern theory of strong interactions — Quantum Chromodynamics (QCD) was actively developing. Mitya at that moment (together with Yuri Dokshitzer and Serezha Troyan) had already contributed to this theory, as summarized in the famous DDT paper [1]. A comprehensive analysis of hard processes was carried out in that work, and predictions for the cross sections were obtained in the framework of the perturbative QCD. I have not practiced chromodynamics up to that point, but have been working on the first incarnation of the string theory, which was initially considered as a possible theory of strong interactions alternative to chromodynamics. The November Revolution of 1974, which consisted of the discovery of the J/ψ resonance and thus the c-quark, practically put an end to the string theory as a theory of strong interactions. It so happened that at the beginning of the 80s, both Mitya and I were looking for a new area to apply our energies, and we both became very interested in nonperturbative problems of strong interactions. We do not observe any quarks or gluons at low energies. Instead, there is a rich and diverse world of hadrons: protons, neutrons, pions, kaons, *etc.* A description of the actually existing hadrons and their interactions is impossible within the framework of the QCD perturbation theory. At the same time, the field theory is more than just perturbation theory, and most challenging problems are nonperturbative. Mitya and I decided to apply the exact QCD results to the so-called U(1) problem [2] — the long-existing puzzle of why no ninth light pseudoscalar meson exists in nature.

Strong interactions are approximately chiral invariant, which means that light quarks are massless in the leading QCD approximation. There are nine vector and nine axial flavor currents in QCD, which, as well as their corresponding charges, are conserved in the chiral limit. In the simplest scenario, this symmetry should be realized in the spectrum of the observed hadrons. Then all observed hadrons should be massive parity doublets, but parity doubling is not observed experimentally. Thus, we are forced to conclude that chiral symmetry is realized in a spontaneously broken form, *i.e.* condensates that violate chiral symmetry develop in the physical vacuum. It has been known for a long time that in QCD such a condensate is a condensate of light quark–antiquark pairs. The price for the formation of the condensate is the existence of massless Goldstone bosons. There arise as many Goldstone bosons as there are violated symmetries. Experimentally, the octet of pseudoscalar mesons, which are pseudo-Goldstone bosons, has been well-known for a long time. But the quark–antiquark condensate violates also conservation of the flavor singlet axial current, and therefore, there must also be a ninth light pseudoscalar meson. It really exists, but it turns out to be heavy, with a mass of about the proton mass. The theory should explain why it is heavy.

The first clue to the problem of the ninth light boson came about when it was realized that there is an axial anomaly in QCD, and that the ninth axial current is not conserved. The presence of an anomaly alone does not solve the U(1) problem, because the anomaly is the density of the topological charge and, at the same time, can be represented as a divergence of a gauge-noninvariant gluon current. Therefore, in perturbation theory, anomaly does not contribute and does not solve the U(1) problem. Later, 't Hooft [3] discovered that instantons generate a non-zero contribution to the anomaly. But even this did not solve the problem, because instantons predict an incorrect dependence on the number of colors and other parameters [4]. Finally, in the limit of a large number of colors, the problem was solved by Witten [5] and Veneziano [6]. Veneziano's solution required existence of a massless ghost pole in the correlator of topological currents. The reason for the existence of this pole remained obscure. That is the question we decided to address.

Fig. 2. Misha Eides and Mitya Diakonov.

Our work began with discussions after the regular weekly seminars of the LNPI Theory Department at the Ioffe Institute (PhysTech, as everybody called it). The seminars began at ten o'clock in the morning and continued without a time limit, ending usually around two o'clock. After the seminar, a small group of us would go to the dining room of the House of Scientists in Lesnoye (little we knew that years later, Mitya's funeral repast will be held in this very room). At the meal, there were heated discussions on everything in the world, starting with the topics of the just-ended seminar and concluding with politics. Our main interests concentrated on physics, we did not come to physics looking for career success or a high salary, but in search of an answer to the question of how the world around us works. We were confident that many of the answers would be found in our lifetime, and that we could contribute to solving some of nature's most important mysteries. Tired of physics, we switched to freewheeling and noncomplimentary discussion of the "current policies of the Soviet government", which we deeply despised, but then again returned to physics. After dinner, Mitya and I would return to PhysTech and continue to work on the problems of strong interactions. The first thing we had to do was to learn new to us technical methods and approaches, such as current algebra, anomalous currents, anomalous commutators, *etc.* I remember that I was very impressed by the speed at which Mitya was learning new methods and transformed them into a part of his theoretical arsenal. He very quickly went through the learning stage, and pretty soon he was able not only to apply the new technology, but also to develop it further. The creative spirit in him was unusually strong.

Soon we had some interesting ideas, and weekly discussions became insufficient. We began to work together intensively practically every day. At the time, I was employed at the Institute of Metrology, where attendance was mandatory, so our work was organized as follows. I arrived to the large Diakonovs' apartment on the Suvorovsky avenue in the afternoon. Mitya and I would sit across from each other in old-fashioned chairs at a big desk and exchange the ideas that had arisen since we parted the night before. Then came the discussions, calculations, arguments, and all this went on till 1–2 am at night, when I went home to have some sleep.

In our regular discussions, there was usually one break at about 5 pm for an evening tea. The tea was attended by all residents of the Diakonovs' apartment and usually a few more people. Here is a good place to say a few words about the Diakonovs' large family. Mitya's father, Igor Mikhailovich, was a world-renowned scholar of the ancient East (Sumer, Akkad, Assyria, *etc.*) and an unusually bright man. He was elected a fellow of the leading Western academies (but not the Academy of Sciences of the USSR). Mitya's mother, Nina Yakovlevna, was a renowned philologist, who specialized in English literature. Her sister, Elena Yakovlevna, was a physicist and worked at PhysTech. All these members of the older generation of the Diakonovs' family were surrounded by young people, graduate students and researchers, so the five o'clock tea was usually well attended. The conversations over this tea were captivating, it was impossible to name a topic that was not discussed, a wide range of opinions were expressed, with a diverse composition of those present. Igor Mikhailovich used to lead the conversation, and his observations were deep, highly original, and often quite unexpected. I must admit that ideas and opinions of Igor Mikhailovich were perhaps no less interesting than our with Mitya physics, and it was hard to get away from them.

After the tea, Mitya and I would take up physics with a renewed vigor, and we would work until late at night. We lived like that for two or three years, practically every day. At some point, we realized that the reason for the existence of the massless ghost was the structure of the QCD ground state, which is similar to the structure experienced by an electron in a crystal. As in crystal, the ground-state energy in QCD is periodic along a generalized coordinate X (see Fig. 3)

$$X = \int \mathrm{d}^3 x K_0(\boldsymbol{x})\,, \qquad K_0 = \frac{\alpha_\mathrm{s}}{4\pi}\epsilon_{ijk}A_i^a\left(\partial_j A_k^a + \frac{g_\mathrm{s}}{3}f^{abc}A_j^b A_k^c\right)\,. \quad (1)$$

The canonically conjugated variable is an analogue of the generalized conserved quasi-momentum. If the barriers are penetrable, the electron can move practically freely through the crystal, its state is characterized by the values of the effective mass and quasi-momentum. An analogous effect holds in QCD, the system can move freely along the generalized coordinate X and is characterized by a definite value of the quasi-momentum θ. This ability to move freely is reflected in the existence of a ghost pole in the correlator of the topological currents

$$\langle K_\mu K_\nu \rangle_{q\to 0} = \frac{q_\mu q_\nu}{q^2}\,\mathrm{const}\,. \tag{2}$$

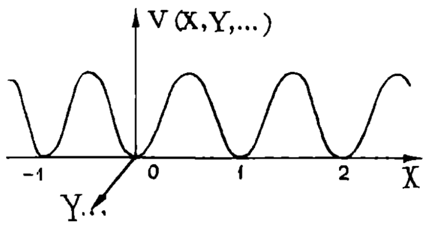

Fig. 3. Periodic structure of QCD vacuum.

In order to relate these abstract considerations to phenomenology, we have written out a complete system of anomalous Ward identities for vector and axial currents, and saturated them with the states from the pseudoscalar nonet. We have found that the existence of a massless ghost makes the ninth pseudoscalar meson significantly heavier than the other mesons. From the Ward identities, we obtained a set of coupling constants and masses of the pseudoscalar nonet, which solve the U(1) problem. From a theoretical point of view, the most interesting conclusion was that the presence of a heavy ninth meson proves the penetrability of the barriers in the ground state of QCD regardless of the instanton approximation [7]. This was important, since the instanton approximation at that time was not self-consistent (later Mitya and Vitya developed a self-consistent instanton approach).

Mitya and I enjoyed working together, even though for an outside observer it would seem that we are in a permanent conflict. While we were working, there were always new ideas, most of them wrong, and it was only after prolonged heated discussions that it became clear what was right and what should go "to the dustbin of history". The most "serious" problems arose when it was time to write our first paper [7] (see also [8]). The point is that all earlier papers with the other co-authors have been written by each of us by himself, and each wanted to do the same this time. So long negotiations followed before we agreed on who would be writing the first draft. At the time, it seemed very important, but, funny to say, now I cannot even recall who wrote it at the end of the day. We were going to send the paper to *JETP*, which was published only in Russian at the time, but first, we were going to prepare an LNPI preprint in English [9]. We did not consider English as an obstacle, each of us was taught English as a kid. But the problem still came up. We should have mentioned "Wick rotation" and there was the difficulty that pursues all native Russian speakers, the problem of articles (I still struggle with articles now). Whether to write "the" before "Wick rotation" or not to write it? It was almost a Hamletian dilemma. We failed to resolve the disagreement on this issue on our own, but there were experts at the Diakonovs' home, whom we could ask. For the peace of mind, we asked Nina Yakovlevna and Igor Mikhailovich separately, they gave us identical answers, and the problem was solved.

We were encouraged by the success with the U(1) problem, and turned to an effective chiral Lagrangian. By construction, an effective chiral Lagrangian has the same symmetries as the fundamental QCD Lagrangian, but unlike the latter, it is formulated in terms of the observable light degrees of freedom, not quarks and gluons. The exact relationship between the fundamental QCD Lagrangian and the effective chiral Lagrangian remained at the time elusive. We assumed that after spontaneous symmetry breaking, the phases of the quark fields turn into the pseudoscalar fields of the meson nonet. Under the chiral transformation of the quark fields, the QCD Lagrangian in the functional integral also transforms due to the presence of the axial anomaly. Using this transformation, we have been able to integrate the anomaly equation and formally calculate the effective chiral Lagrangian starting with the QCD functional integral. As a result, we have unambiguously derived an effective chiral Lagrangian, which included the then recently discovered Wess–Zumino–Witten term [10]. Later, this chiral Lagrangian was derived in the instanton liquid model by Mitya and Vitya.

Afterward, our scientific paths diverged. Mitya continued to work on the development of a nonperturbative approach to low-energy interactions based on instantons and, together with Vitya, achieved great success in this direction. I was not fascinated with instantons and my interests turned to the theory of bound states in quantum electrodynamics. Our everyday discussion on physics gradually ceased, and I became only an occasional visitor to the apartment on the Suvorovsky avenue.

Mitya and I remained close friends for the rest of our lives, and there is a lot to remember besides physics. For many years, we celebrated almost every New Year's Eve in a large company either in the apartment on Suvorovsky or at Mitya's dacha in Ushkovo. On New Year's Eve, we used to have a lot of fun, played charade games, and often self-produced performances of short plays, which were written collectively. Some of the scenarios, which were not complimentary to the Soviet system, can now be found in the online Moshkov Library. Mitya was usually one of the leading authors and performers. He had many talents and his creative nature showed itself not just in physics. A list of questions–predictions was usually prepared for New Year's Eve, and they were answered by all the people present. For each subsequent New Year's Eve, the old predictions were analyzed, and it was revealed who had the most realistic perspective on the unfolding of events. It would be very interesting to find these old predictions from the 80s, and see what we were expecting.

But even without that, I remember the euphoria that gripped us during the era of Perestroika. It seemed as if a bit more, and the country would become normal (there were no disagreements on what the "normal" means). The world opened up. I remember how Mitya, returning from his first trip to America, talked about the universities he visited, seminars he gave, and conversations he had with the American colleagues. Almost everything over the ocean turned out to be familiar, and people's interests were mostly similar to ours, the scientific level was quite comparable to ours, and no significant cultural barriers showed up. The major difference was, of course, in the level and structure of everyday life, American highways, cars, and so on. It was amazing to hear the story of how he rented a car to move from one university town to another, missed a necessary exit from the highway, and had to make an extra hundred miles. These everyday differences seemed easily surmountable at the time, all that was needed was to get rid of the unnatural social order that was decaying before our eyes. The challenge of the transition to a normal (read democratic) society was seen as easily solvable. Now, more than thirty years later, we know that our hopes were not realized.

In the years after our work on the U(1) problem Mitya's scientific work flourished. He and Vitya Petrov developed a successful QCD-based instanton liquid model, which allowed them to address numerous problems of low-energy strong interaction processes. With only two parameters, they were able to calculate almost all experimental constants with an accuracy of about 10%. They constructed a highly successful model of the nucleons and calculated some nucleon characteristics (multiquark components of wave functions, structure functions, *etc.*), which could not be even addressed in other frameworks. As a byproduct of their research, Mitya, Vitya, and Maxim Polyakov predicted the existence of a fundamentally new long-lived exotic hadron Θ^+ — a light pentaquark. Experimental searches for this hadron at the leading scientific centers first seemed to confirm its existence, but later turned negative. I think that the final verdict on Θ^+ is still pending.

Mitya has worked successfully in many different fields, supervised undergraduate and graduate students. He taught, tutored, and actively interacted with colleagues from all over the world. Mitya has always been concerned about the future of the country and Russian science. At the beginning of the 2010s, he has begun to make active attempts to change the situation, at least in Russian science. He became a notable social activist, one of the principal founders of the Society of Scientific Workers, and actively fought for the survival of Russian science. Unfortunately, his achievements in this field were not great, but not for the lack of effort.

It so happened that starting the second part of the 90s, I mainly resided in the US, but remained a member of the PNPI Theory Department, and usually returned to St. Petersburg a few times a year, both for scientific and personal reasons. In December 2012, I, as usual, came to SPB to spend the holidays with my friends and celebrate the New Year. It was a Friday evening, and I gave Vitya a call to tell him about my arrival and to discuss holiday plans. He stunned me with the news that Mitya was in the hospital with a massive heart attack, which happened that day. Next day Mitya's wife, Vitya, and I spent in the hospital, talking to the physicians and trying to find the best outside doctors, who could provide a second opinion on the way to save Mitya. Nothing worked and in a few days Mitya passed away. There was no New Year celebration for us that year, instead, in a few days there was a funeral and we paid the last respects to our friend of many years. Mitya's sudden death caught him at the peak of scientific and public activity. It was a colossal and irreparable loss to his family, friends and colleagues, and to physics at large.

2. Vitya and heavy pentaquarks

Vitya, for me, was first of all a close friend, and then a wonderful physicist and co-author. I remember very well the moment we first met. It was at one of the LNPI winter schools in the second part of the 70s, I think in Komarovo. At that moment, it was already a few years since I graduated from the university, and Vitya was writing his diploma. Life at a winter school is usually hectic and not conducive to theoretical calculations. So I was greatly surprised, when, walking down the corridor, I saw Vitya sitting in one of the small halls with sheets of paper, completely switched off the surroundings and concentrated on writing something. I asked him what he was doing and he told me that he was working on the master thesis under the guidance of Igor Tikhonovich Dyatlov. At this moment, he was calculating what would happen if the Reggeon had a nonzero vacuum expectation value (maybe I am confusing something). This episode made a strong impression on me. Later, while working on the U(1) problem with Mitya, we organized a small private seminar on related topics. Among the participants were Vitya, Alyosha Yung, and some other people. This is how I started to know Vitya better and was quite impressed by his physics proficiency and enthusiasm.

It turned out that we have a lot of common interests besides physics, our worldviews were finely tuned, and after some time we became friends. The end of the 80s and the first part of the 90s were transitional times in Russia, which, to a large extent, determined its future. A lot happened in those turbulent years, a time of great hopes and expectations. Our lives went on, punctured by some memorable events. At the end of the 80s,

Fig. 4. Victor Petrov.

Yusuf Musakhanov organized a series of conferences in the Central Asia Fann Mountains, not far from Tashkent, Uzbekistan. Vitya and I decided to use this occasion to visit ancient cities of Bukhara and Samarkand. So we boarded a plane from SPB to Bukhara. The funny thing was that Bukhara is to the west of Samarkand, but our plane first landed in Samarkand, where part of the passengers disembarked, and then turned around and flying to the west landed in Bukhara. We spent a day in fantastic Bukhara, full of eastern exotic. A few mosques and madrasas built by Tamerlane were just renovated and were incredibly magnificent. It was about 40 degrees centigrade, but we spent a whole day in the sun, going from one eastern fairy tale to another. The next day we took a bus to Samarkand and again had a feeling that we were in an enchanted land. The architecture was splendid and mosques and madrasas, turned into museums, looked as if they were built just yesterday. From Samarkand we went to the Fann Mountains, where the conference took place. After another conference in these mountains, Mitya, Vitya, and I went hiking. It was great until we needed to return to civilization. There was an obstacle in the form of a mountain river after a rain. We needed to cross it, which was not simple or safe, but after a few attempts, we succeeded. The road to Tashkent was then open and we were descending through an incredibly impressive sea of poppy. We took a bunch of poppy flowers with the idea to bring this beauty to Tashkent, but the flowers almost immediately withered and we had no other option but to throw them away.

At the end of December 1991, I returned to SPB from my first trip to the US, and almost immediately, on December 26$^{\text{th}}$, Gorbachev resigned as the President of the USSR, and the Soviet Union was no more. Great political events were accompanied by some personal disasters. Here, I need to mention that Vitya, Mitya, and I were living in the center of SPB, not too far from

Fig. 5. Left: Vitya on the lap of Einstein; Right: Misha Eides and Vitya Petrov.

each other. Vitya's apartment was closer to mine, maybe about three blocks apart. I was peacefully sleeping in my apartment, when in the middle of the night, about one or two o'clock on December 30$^{\text{th}}$, my doorbell rang. I opened the door and saw Vitya's wife Natasha with his two sons. Soon came Vitya himself with his mother-in-law. It turned out that their apartment building was on fire, and they had to flee. We arranged places to sleep for the kids and mother-in-law, and the three of us, Vitya, Natasha, and I went back to their apartment building to watch how firefighters fought the flames. The fire was extinguished in the morning, but the building was declared structurally unstable and the whole Vitya's family became homeless.

Here is one characteristic feature of the time. In the morning, we needed to feed the kids, and it turned out that there was no bread at home. I went outside to buy some bread. There were a few bakeries near my house. At the entrance to the first, there was a long queue for at least an hour, at the second, the line was even longer, the third bakery was closed, there was no bread at all in the fourth, but people were waiting for delivery. I joined the crowd and relatively soon was able to buy two loaves. There was an expectation of coming famine in the city and the most realistic hope was for humanitarian food supplies from the West. Anyway, I returned home with my catch and the kids were fed. The next immediate problem was to arrange lodging for the homeless Vitya's family. After some considerations, the kids were sent to their grandparents in Gatchina and the mother-in-law moved to her friend's apartment. Natasha and Vitya moved to Mitya's apartment, which was larger than mine, and resided there for about a month until they got some temporary accommodation of their own. There are many other episodes that come to my mind, but I will stop here and turn to physics.

Vitya was an unusually gifted, versatile, and creative theoretical physicist. Besides particle theory, he was actively involved in many problems of nuclear physics. Well-known are his works on μ-catalysis, which he carried out together with S.S. Gerhstein, his father Yu.V. Petrov, L.I. Ponomarev, and others. Vitya also made significant contributions to the study of constraints on the time dependence of fundamental constants on the basis of analysis of the characteristics of the natural reactor at Oklo. His interests extended far beyond the physics of fundamental interactions. Thus, in one of his last series of works with V.V. Afonin, he solved the problem of conductivity restoration in a system of one-dimensional strongly interacting electrons with a single impurity and connected it to the long-range order in the Luttinger model.

Vitya loved to teach and knew how to teach. He gave lectures on the field theory at the St. Petersburg University and the Academic University, endeavored to instill in students an interest in physics and research work, and then to attract them to work at the Department of Theoretical Physics at PNPI. In the years after the collapse of the Soviet Union, many members of the Department moved abroad, the flux of new students almost came to a halt. The department badly needed new blood. The situation deteriorated further in 2010–20s, when the Institute was expelled from the umbrella of the Academy of Sciences and fell in the hands of a highly unqualified administration in Moscow. For many years Vitya was the head of the Department of Theoretical Physics at PNPI. He made Herculean efforts to preserve the scientific level and traditions of the department, but, as it is abundantly clear now, his quixotic efforts could not succeed since they were going against the general tendencies developing in Russia at the time.

It so happened that while we were friends for a long time, we never collaborated before the mid of 2010s. At that time, I was a bit tired and bored with working on the bound state QED, and Vitya was also looking for a new direction. To describe our disposition at that moment, it is appropriate to recall one episode which happened about that time. We celebrated my birthday, and Vitya made a speech where he said that since I (Misha Eides) am not so young anymore, I can spend my future years working on topics just for the love of the subject and pleasure, not caring too much about practical implications, such as grants, *etc.* So we decided to collaborate and address some of the interesting problems which each of us had in mind for many years but could not find time for them as we were busy with our routine topics. Then I had resided already for many years in the US and Vitya visited me for a summer in Kentucky, where we were going to start working on one of these topics. But our plans were ruined in a few days after Vitya's arrival. Looking at the list of new preprints at the `arXiv` Vitya discovered a report of the LHCb Collaboration on the discovery of heavy pentaquarks with hidden charm. It was immediately clear that these new states have nothing to do (except the word "pentaquark") with the pentaquarks discussed many years ago by Mitya, Vitya, and Maxim. The history of Θ was always in the minds of all three authors and they always kept hope that their predicted pentaquarks would be at some moment discovered experimentally, rehabilitating their prediction. But now, there was an experimentally discovered pentaquark of an apparently different nature and Vitya considered it a matter of honor to explain it. Maxim felt the same way, they recruited me, and this is how our work on the LHCb pentaquarks started. Vitya and I worked for two months in a hot Kentucky summer, and talked and exchanged ideas with Maxim via Skype.

Our idea was to consider LHCb pentaquarks with hidden charm as hadrocharmonia — bound states of a charmonium state and nucleon. Such an approach is theoretically justified in the limit of heavy quarks and large N_c. A heavy quark–antiquark bound state is a small (compared to the size of a nucleon) heavy color-neutral object. Its interaction with a nucleon is relatively weak even when the distance between quarkonium and nucleon is small. Quarkonium can easily penetrate the nucleon and form a true pentaquark state. In this state, the distances between the three quarks of the nucleon and the compact heavy meson are all of the same order. Interaction of a small-size heavy quarkonium with other hadrons can be considered in the framework of the QCD multipole expansion, the role of the small parameter plays the ratio of the quarkonium size and the gluon wavelength.

Binding in this approach arises as a result of an attractive chromoelectric dipole interaction between the small heavy quarkonium and a large nucleon, and was earlier considered in the problem of a heavy quarkonium interaction with nuclei [11–13]. This interaction is dominated by the virtual emission of two chromoelectric dipole gluons in a color singlet state. Then the effective interaction potential between the heavy quarkonium and the nucleon is proportional to the product of the quarkonium chromoelectric polarizability and the local gluon energy-momentum density inside the nucleon [13]. We calculated quarkonium chromoelectric polarizability in the heavy-quark mass limit at large N_c following [14]. To estimate the gluon energy-momentum density inside a nucleon, we used the chiral theory of nucleon [15, 16], developed by Mitya, Vitya, Pasha Pobylitsa, and Michał Praszałowicz much earlier. We calculated the gluon energy-momentum density inside a nucleon in this model in the mean-field framework. Here, Maxim's experience with the energy-momentum tensor was invaluable. In our first paper on the LHCb pentaquarks [17], we interpreted $P_c(4450)$ as a bound state of $\psi(2S)$ charmonium state and the nucleon, and made a bold prediction that there should be two almost degenerate states $J^P = (1/2)^-$ and $J^P = (3/2)^-$ at the position of the $P_c(4450)$ pentaquark. We also considered unitary multiplet partners of $P_c(4450)$.

Paper [17] was the first in a series of our works on the LHCb pentaquarks. We were working on the development of the hadrocharmonium scenario for the LHCb pentaquarks for a few years. Vitya used to come to Kentucky every summer and we usually spent the daytime for work, saving the evenings for freewheeling discussions on everything, from current events to science and history. In a few follow-up papers [18–20], we further developed the hadrocharmonium scenario of the LHCb pentaquarks following its internal theoretical logic and under the influence of the new experimental LHCb data. In [18], we calculated partial decay width $\Gamma(P_c(4450) \to J/\psi + N)$, predicted existence of two almost degenerate octets of hidden-charm pentaquarks, calculated their masses and decay widths. We also considered pentaquarks properties in an alternative scenario, where they are loosely bound $\Sigma_c\bar{D}^*$ and $\Sigma_c^*\bar{D}^*$ deuteron-like states. We presented a general quantitative comparison of molecular and hadrocharmonium scenarios for the LHCb pentaquarks.

In [19], decays of the hidden charm LHCb pentaquarks in the hadrocharmonium and molecular scenarios were considered. We developed a semirelativistic framework for calculation of the partial decay widths that allows the final particles to be relativistic. Using this approach, we calculated the decay widths in the hadrocharmonium and molecular pictures. Molecular hidden charm pentaquarks are constructed as loosely bound states of charmed and anticharmed hadrons. Calculations have shown that molecular pentaquarks decay predominantly into states with open charm. Strong suppression of the molecular pentaquark decays into states with hidden charm is qualitatively explained by a relatively large size of the molecular pentaquark. The decay pattern of hadrocharmonium pentaquarks that are interpreted as loosely bound states of excited charmonium $\psi(2S)$ and nucleons is quite different. Decays into states with hidden charm dominate in this scenario, but suppression of the decays with charm exchange is weaker than in the respective molecular case. The weaker suppression is explained by a larger binding energy and, respectively, smaller size of the hadrocharmonium pentaquarks. These results combined with the experimental data on partial decay widths could allow to figure out which of the two theoretical scenarios for pentaquarks (if either) is chosen by nature.

In [20], we discussed new LHCb Collaboration results on pentaquarks with hidden charm [21]. These results fit nicely in the hadrocharmonium pentaquark scenario [17, 18]. In the new data, the old LHCb pentaquark $P_c(4450)$ splits into two states $P_c(4440)$ and $P_c(4457)$. We interpreted these two almost degenerate states as a result of hyperfine splitting between two color singlet hadrocharmonium states with $J^P = 1/2^-$ and $J^P = 3/2^-$ that was predicted in [17]. We also improved the theoretical estimate of hyperfine splitting [17, 18] that is compatible with the experimental data. The new

$P_c(4312)$ state was interpreted as a bound state of χ_{c0} and the nucleon, with $I = 1/2$, $J^P = 1/2^+$, and binding energy of 42 MeV. As a bound state of a spin zero meson and a nucleon, the hadrocharmonium pentaquark $P_c(4312)$ does not experience hyperfine splitting in the hadrocharmonium scenario. We found a series of new hadrocharmonium states in the vicinity of the wide $P_c(4380)$ pentaquark what could explain its apparently large decay width. The hadrocharmonium and molecular pentaquark scenarios were once again compared in [20], and their relative advantages and drawbacks were discussed.

Our work was abruptly terminated when in the spring of 2019, it was discovered that Vitya had a cancer. The disease developed fast and Vitya suffered a lot. In 2020, he had undergone a major surgery in a Tel-Aviv hospital, which gave us hope that the disease would be vanquished. I saw Vitya last time in the beginning of August of 2021, when I visited SPB. He looked exhausted but kept a good disposition and was optimistic. In the middle of August his health fast deteriorated. On August 25$^{\rm th}$, I was struck by an untimely and sudden death of Maxim. It was a heavy loss. I did not want to tell Vitya about it but somebody informed him anyway. Vitya's condition worsened from day to day, and on September 22$^{\rm th}$, I got a call from Vitya's wife that he passed away. I immediately returned to SPB from the USA, and it was my sad duty to write an obituary and give an eulogy at the funeral. Needless to say, I was overwhelmed with grief after losing Vitya and Maxim in fast succession.

3. Bound state mass decomposition and quantum anomalous energy

Maxim was one of the researchers who revived interest in the energy-momentum tensor (EMT) and its properties [22]. A few years after he passed away, I became interested in this topic, and especially in the problem of decomposition of the hadron mass, which was actively discussed in the literature, see, *e.g.*, [23–27] and references therein. Especially interesting from my point of view is the idea that the Quantum Anomalous Energy (QAE) term $H_{\rm a}$ should be included as a separate term in the decomposition of the hadron mass, see [24] and references therein. The QAE term $H_{\rm a}$ is one-fourth of the anomalous part of the EMT trace $T_{\rm a} = \int {\rm d}^3r[\gamma_m m_0\bar{\psi}_0\psi_0 + (\beta/2e_0)F_0^2]$. In numerous works, $H_{\rm a} = T_{\rm a}/4$ was included as a separate term in the QCD quantum field Hamiltonians. I felt unease about this idea because QCD and QED are structurally similar and if $H_{\rm a}$ is a separate term in the QCD Hamiltonian, a similar term should exist in the QED Hamiltonian as well. Meanwhile, from my experience with the QED bound states, I got no indications of such a separate term.

In [28], I considered the decomposition problem in the bound state QED. QED admits perturbation theory calculations and I calculated matrix element of $H_{\rm a}$ for hydrogen in the one-loop external field approximation. The contribution of the fermion term in $H_{\rm a}$ turned out to be

$$\left\langle H_{\rm a}^{\rm ferm}\right\rangle = \frac{\gamma_m}{4}E_{nj}\,, \tag{3}$$

where E_{nj} is the exact eigenvalue of the Dirac Hamiltonian with the Coulomb external field

$$E_{nj} = m\left[1+\left(\frac{Z\alpha}{n-\left(j+\frac{1}{2}\right)+\sqrt{\left(j+\frac{1}{2}\right)^2-(Z\alpha)^2}}\right)^2\right]^{-\frac{1}{2}}. \tag{4}$$

The contribution of the gauge boson term as calculated in [28] is

$$\begin{aligned}\langle H_{\rm a}^{\rm gauge}\rangle &= \frac{\alpha(Z\alpha)^2m}{6\pi}\frac{n-\left(j+\frac{1}{2}\right)+\frac{\left(j+\frac{1}{2}\right)^2}{\sqrt{\left(j+\frac{1}{2}\right)^2-(Z\alpha)^2}}}{\left(\left[\sqrt{\left(j+\frac{1}{2}\right)^2-(Z\alpha)^2}+n-\left(j+\frac{1}{2}\right)\right]^2+(Z\alpha)^2\right)^{3/2}}\\ &\approx \frac{\alpha(Z\alpha)^2m}{6\pi n^2}-\frac{\alpha(Z\alpha)^4m}{4\pi n^4}\left[1-\frac{4n}{3\left(j+\frac{1}{2}\right)}\right]+\dots\end{aligned} \tag{5}$$

It is well known that in the external field approximation, one-loop calculation generates the leading contribution to the Lamb shift of order $\alpha(Z\alpha)^4m/n^3$, which depends on the orbital momentum ℓ and not on the total angular momentum j. As we see from Eqs. (4) and (5), the contribution of the QAE term $H_{\rm a}$ to the energy level has wrong dependence on the principal quantum number n and the total electron angular momentum j and, therefore, completely cancels with other contributions to the energy level. The wrong parametric dependence and complete cancellation of the QAE term $H_{\rm a}$ contribution to the hydrogen energy levels in the one-loop approximation indicate that decomposition of the QED field Hamiltonian, which contains $H_{\rm a}$ as a separate term is unwarranted. This conclusion most probably survives in QCD and proves that the QAE term $H_{\rm a}$ should not be included as a separate term in the proton mass decomposition.

4. Conclusions

Some results obtained together with my friends and colleagues Dmitri Diakonov, Victor Petrov, and Maxim Polyakov are reviewed above. My personal recollections are also included.

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