ON HARD COHERENT PROCESSES

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A factorization theorem for the hard coherent processes involving high-energy scattering of photons is derived. We elaborate the treatment for the case of the longitudinally polarized photons and extend the treatment to the case of the processes initiated by transversely polarized photons, as well as to the coherent photo-production of heavy quarkonia. A wave function of a hadron being the solution of the Schrödinger equation is presented in the space-time evolution of initially zero-size and zero-color charge wave packet of bare quarks and gluons. The increase of the running coupling constant in QCD with the distance leads to the coexistence of two phases within the hadron wave function. One phase is the pQCD phase, the second phase is dominated by the phenomenon of spontaneously broken chiral symmetry. The critical line at $r = r_{\rm c}$ follows from the gauge symmetry, asymptotic freedom, causality, quantum diffusion, and from the cancellation of soft color fields outside the wave packet. The distinctive properties of QCD are the significant probability of a high-momentum tail of the minimal Fock component (MFC) of the hadron wave function and the strong suppression of the soft part of MFC of hadron WF in the strong coupling regime of QCD. The factorization theorems predict hard coherent processes, color transparency, color fluctuations phenomena, *etc.* We argue that the value of the radius of the region occupied by the pQCD phase, $r_{\rm c}$, is restricted by the radius of the onset of spontaneously broken chiral symmetry $r_{\rm c} \gg r_{\rm CS}$. The success of the constituent quark model for a baryon suggests that the radius of the confinement of color is significantly larger than $r_{\rm c}$. The presence of a color core in hadrons matches well with preQCD field theories at distances larger than the critical line for the onset of the regime of spontaneously broken chiral symmetry. It substitutes Landau nullification of the interaction within preQCD field theories, the preQCD Feynman model at the distances smaller than $r_{\rm c}$.

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1. Introduction

This paper is dedicated to the memory of our friends: Profs. D. Diakonov, V. Petrov, and M. Polyakov with whom we worked for many years in the theory department of PNPI (Gatchina) and regularly discussed the challenging problems of QCD and nuclear theory.

To visualize the role of color and minimal Fock component (MFC) of the WF of a hadron, we consider hard coherent processes (HCP) with the external trigger for the MFC. We argue that these processes are rare within the strong coupling regime of QCD, the preQCD field theories, and the preQCD Feynman parton model. We explain why such processes are not suppressed within the weak coupling regime of QCD. Such processes were observed with high-energy pion beams at FNAL: in the $\pi + T \to 2\,\mathrm{jet} + T$ process and at colliders: $\gamma^* + T \to V + T$ and $\gamma + T \to J/\psi + T$ *etc.* The theoretical analyses of the leading twist hard processes [1] of the static properties of a hadron [2–4] allow us to establish upper boundary on the momentum scale η characterizing the non-perturbative QCD phenomena in the WF of a hadron. We argue that the combination of causality, of quantum diffusion [6], and the increase of running coupling with distance resulting from asymptotic freedom imply the coexistence of the two QCD phases within WF of hadrons. An important role in reaching this conclusion is played by the QCD phenomenon of color screening: a small-size wave packet having zero-color charge has no soft gluon fields outside of the wave packet as the consequence of gauge symmetry and zero-color charge.

The aim of this paper is to demonstrate that color gauge symmetry plays a direct role in the wave function of a photon, hadrons, and in the short-range inter-nucleon forces. Perturbative QCD (pQCD) core of the virtual photon WF of the radius of $r_{\mathrm{c}}^2 \propto 1/Q^2$ was identified as the target rest frame, equivalent of the Bjorken scaling for the structure functions of DIS in [5, 6]. The zero-color charge of a hadron, gauge symmetry of QCD together with the causality and asymptotic freedom imply the finite size of the pQCD core r_{c} of a hadron and a photon [7], as the result of the cancellation of the soft gluon modes around a small size of wave packet leading to the quantum diffusion phenomenon [6], and predict variety of phenomena improbable within the preQCD field models and the preQCD Feynman parton model in the strong coupling regime of QCD.

In Section 2, we consider WF of a bound state as the space-time evolution of the initial condition and impose the additional constraints on the WF at large distances between constituents of a bound state. We suggest the initial condition for the space-time evolution in the form of zero-size color neutral wave packet of quarks and gluons [7, 8]. (In Ref. [8] the role of spontaneously broken chiral symmetry in the space-time evolution was not considered.) The potential advantage of the Feynman path framework is the feasibility to conserve color gauge symmetry during the space-time evolution of the wave packet and to account for the cancellation of color fields outside of the zero charge, small-size wave packet. The smooth matching between strong coupling QCD regime and small coupling pQCD regime is due to the increase in distance of the running coupling constant leading to the bifurcation between phases. Naively, the pQCD regime disappears at the distances where the phenomenon of spontaneously broken chiral symmetry becomes important at the distances $\approx r_{\rm c}$. At this scale, non-perturbative QCD interactions begin to produce the almost massless pion. The observed mass of pion is the product of the bare masses of u, d, s quarks and their interaction with the chiral vacuum condensates [2–4]. This bifurcation, together with the causality, requires the two-phase structure of a hadron and supplies an upper boundary on the radius of the pQCD core [7]. The evident success of the constituent quark models in the description of the hadron spectrum indicates an even larger size of the radius of screening of color significantly larger than the radius of onset of spontaneously broken chiral symmetry and suggests a presence of the third QCD phase within a hadron. To visualize the origin of the pQCD core within a hadron, we consider a non-relativistic model of a heavy quarkonium where the single-gluon exchange dominates in the potential at small relative distances between constituents. The Abelian gauge theory — QED and non-Abelian gauge theory — QCD produce rather similar WF of a ground state at small relative distances between constituents. The major difference between QED and QCD is at large distances between constituents due to the phenomenon of spontaneously broken chiral symmetry.

Within a preQCD quantum field theory, a bare hadron has no inner structure. These theories predicted the nullification of hadron–hadron interactions due to the ultraviolet divergencies [9]. Discussed above the space-time evolution in QCD leads to the finite size of the pQCD core of a hadron which substitutes the nullification of interaction by the multilayer phase structure of a WF of a hadron. One phase is the pQCD core of a hadron *i.e.* weak coupling regime. The second phase is the strong coupling regime consistent with the effective Hamiltonian of spontaneously broken chiral symmetry. The pQCD core of a hadron combined with the asymptotic freedom of QCD and spontaneously broken chiral symmetry prevents the formation of strong meson fields in the center of heavy nuclei predicted within the preQCD field theories in [11]. A related challenge for the nuclear models is the modeling of the high momentum tail of inter-nucleon forces in terms of strongly off mass-shell meson exchanges. Such modeling contradicts the data on the distribution of antiquarks in a nucleon, compare [13]. The pQCD core of a hadron removes this contradiction because the high-momentum tail from the meson exchanges disappears in QCD for $r \leq r_{\rm c}$.

In Section 3, we elaborate on factorization theorem for the coherent electro-production of a longitudinally polarized vector meson off a hadron (nucleus) target derived originally by neglecting specific properties of a bound state in QCD in [18, 19]. We account for the two additional QCD phenomena: cancellation of soft gluon field outside of a small-size wave packet. The second important phenomenon is the decrease in the probability of MFC with the increase of the size of the wave packet, suppression of MFC of a hadron in the strong coupling regime. The resulting scale of a gluon GPD is controlled by the distribution of color within the produced hadron *i.e.* by the interplay between the high-momentum tail of MFC of WF of the produced hadron and the suppression of the interaction of small-size wave packet with a target derived from QCD in [12, 14]. We extend the factorization theorem to the processes initiated by transversely polarized virtual photon in small-x processes and explain how two-dimensional rotational invariance selects special form of MFC of the produced vector meson. We also explain that the pQCD core of a real photon combined with the suppression of the soft gluon field around the produced wave packet and of the small value of the scale of non-perturbative QCD phenomena η justify the applicability of pQCD to the evaluation of the coherent photo-production of heavy quarkonia. These results imply the important role of the MFC within the WF of a produced hadron in HCP.

In Section 3, we also consider coherent photo-production of a vector mesons off a nucleon target $\gamma + N \to J/\psi + N$ which has been investigated within the model ignoring color distribution within MFC of heavy quarkonium. In these approaches, the color distribution within the MFC of the heavy quarkonium WF is accounted for by assuming the effective scale of gluon generalized parton distribution (GPD) $Q_{\mathrm{eff}} \approx m_{\mathrm{c}}$. This approximation does not respect the Ward identities and the energy-momentum conservation. On the contrary, the factorization theorem accounts for the gauge symmetry and the momentum conservation, the cancellation of soft gluon color neutral packet outside of MFC of a hadron and strong suppression of soft part of MFC in the wave functions of a photon and a hadron in the strong coupling regime. We derive an equation for the effective scale of gluon GPD which accounts for the color dynamics. The derivation of factorization theorem for the coherent photo-production of a heavy transversely polarized quarkonium heavily uses color dynamics. As the result of two-dimensional rotational invariance, the form of the dominant high momentum MFC of charmonium WF is unusual. The dominance of non-sense gluon polarizations in the pQCD ladder in the small-x processes, compare [16–18, 22] makes it impossible to neglect the distribution of color within the charmonium WF since it does not allow to use Ward identities to derive the factor b^2 present in Eq. (29). The factor b^2 in Eq. (29), derived in [12, 14], predicts the color transparency (CT) and color fluctuations (CF) phenomena for the processes off a nuclear target within the kinematical window allowed by the coherent length.

Also, we evaluate the high-momentum tail of WFs of bare photon, of W^*, of MFC of WF of a rapid hadron. An important role in the calculations is played by the evaluation of the kinematical window where the quark–gluon wave packet within a hadron WF cannot expand while traversing the target as a result of the Einstein slowing time. In the resulting final-state interaction of a small-size wave packet with a hadron, a nuclear target is suppressed by the factor which is proportional to a power of the transverse radius of the packet occupied by color [12, 14]. Thus, the discovery of hard coherent processes dominated by the high-momentum tail of MFC reveals the onset of the regime of high-resolution nuclear physics.

Conclusions are presented in Section 4.

2. Color dynamics, spontaneously broken chiral symmetry, and two(three) phase structure of a bound state in QCD

To conserve gauge invariance in the description of a bound state and to avoid additional ultraviolet divergencies, we consider WF as the result of space-time evolution of the color-neutral zero-size wave package governed by the QCD Hamiltonian. Besides, additional constraints should be imposed at large distances between constituents on the solution of the Schrödinger equation.

The perturbative QCD regime in the space-time evolution of wave packet from quark, gluon to hadrons is controlled by the scale η characterizing the non-perturbative QCD phenomena. The analysis of the DIS data gives

$$1\ \mathrm{GeV} \geq \eta \gg \Lambda_{\mathrm{QCD}}\,. \tag{1}$$

A smaller value of η was found in the analysis of the data for the quasi-exclusive $e\bar{e} \to X$ processes in the high-energy domain [1]. In the large-p_{t} phenomena, $\eta \approx 0.45$ GeV is the inner transverse momentum of a quark in a nucleon so the gluon scale is expected to be substantially larger.

To separate WF of a bound state from the background of vacuum fluctuations, we analyze LC WF of a bound state where production of the pairs from the vacuum is absent. The soft gluon radiation of a wave packet is additionally suppressed for $k_g \gg k$ by the power of the small coupling constant α_{s}. Here, k is the quark momentum within the wave package and k_g is the gluon momentum. In the kinematics $k_g \ll k$, the radiation is negligible because of zero-color charge of the wave packet and gauge invariance leading to the complete cancellation of the soft radiation. Thus, a color-neutral wave package evolves without radiation of soft gluons up to some distance, according to quantum diffusion phenomenon [6]. This result explains the finite value of $r_{\rm c}$. We want to draw attention that the pQCD fluctuations of the radius $r_{\rm c}$ are a correction which drops with increase of Q^2. It is observed in a number of hard processes.

The small-size wave packet evolves up to the distances $\approx r_{\rm CS}$, where $\alpha_{\rm s}(r_{\rm CS})$ becomes sufficiently large for the formation of the Goldstone boson — pion. Another valuable upper boundary on $r_{\rm c}$ is the radius of confinement of color $r_{\rm conf}$. The restriction on the radius of confinement $r_{\rm conf}$ follows from the success of a constituent quark model where the color of a constituent quark was introduced to guarantee that all quarks in a ground state of a baryon are within the S waves. Thus,

$$r_{\rm CS} \leq r_{\rm conf}\,. \tag{2}$$

The QCD evolution of the initial state leads to the multi-phase structure of a hadron. The first phase is the pQCD core of a hadron for $r \leq r_{\rm c}$. The second phase is developed when running coupling $\alpha_{\rm s}(r)$ achieves the critical value for the formation of Goldstone meson — pion. According to the model, constituent quarks in the second phase have color. So the third phase arises at the distances where color is screened $r \geq r_{\rm conf}$. This phase is described by the effective QCD Hamiltonian.

2.1. QCD inspired constituent quark model and the origin of repulsive core in the inter-nucleon forces

The pion field of a nucleon, $\phi_\pi(r)$ disappears at the inter-quark distances: $r \leq r_{\rm c}$. This property of QCD helps to remove the evident contradiction between the data on antiquark distribution in a nucleon at large $x \sim 0.1\div 0.2$ and the conventional form of the tail of pion–nucleon form factor [13]. The coupling constant in the second phase is significant.

The QCD core of a nucleon prevents the formation of strong meson fields within a nucleon, a nucleus, and the core of a heavy neutron star. The pQCD core of a hadron substitutes the nullification of the interaction predicted for any preQCD field theory [9] which neglects the inner structure of a hadron. At distances larger than r_cs, the interaction of a valence quark with the chiral condensate dominates. As the result, the pQCD evolution of a quark mass is substituted at these distances by the running quark mass evaluated within the mean field approximation to QCD, compare [10].

A QCD-inspired constituent quark model is capable to describe non-perturbative QCD phenomena. The account for the critical point in the space-time evolution due to spontaneously broken chiral symmetry allows us to model description of the interior of a hadron for the region of $r \geq r_{\rm c}$. This model can be derived from QCD if a strong interaction regime is dominated by the interaction of bare constituents with the chiral and gluon condensates. If so, the major effect would be the appearance of the effective mass of a quark together with the pion field. In this model, massive quarks are colored within this kinematical region. Within this model, confinement of color is substituted by the binding as for hydrogen atom in QED.

The similarity between QED and QCD visualizes the origin of short-range repulsive core in the NN forces. Indeed, the repulsive core observed in the interaction of neutral atoms is due to the Pauli principle between electrons of atoms, compare [26]. In the constituent quark model, quarks within a nucleon are in the S states. So the Pauli principle between quarks of different nucleons leads to the strong short-range repulsion between nucleons *i.e.* to the short-range repulsive core in the NN forces. Such an explanation was suggested by Ya.B. Zeldovich within the non-relativistic constituent quark model long before color was introduced.

2.2. QCD or a quark model predictions for the high-momentum tail of MFC of a bound state

The pQCD core of a wave function of γ^*, W^* was observed in the inclusive processes as the bare component of a photon, in the high-Q^2 behavior of cross section of the processes: $e^+e^- \to$ hadrons, *etc.*

To visualize the physical meaning of the pQCD core r_c of a hadron wave function, we consider a non-relativistic quark model of a heavy quarkonium. Within this model, the interaction between heavy quarks at the distance r is described by the potential $V(r) + U(r)$. The phenomenological potential $V(r)$ imitates the non-perturbative QCD phenomena such as confinement of color, SBCS phenomenon, and the fluctuations of the mean positions of constituents within a bound state. A distinctive feature of QCD is the dominance of a single-gluon exchange at the small inter-quark distances

$$U_{\mathrm{Coulomb}}(r) = -(4/3)\frac{\alpha_{\mathrm{s}}(r)}{r}\,. \tag{3}$$

The high-momentum tail of WF can be calculated by analyzing the Lippmann–Schwinger equation for $k \geq \eta$ where potential $V(r)$ gives insignificant contribution

$$\psi(k) = (1/(T(k) - \epsilon)) \int \psi_0\left(k'\right) \mathrm{d}^3k' U\left(k - k'\right) = \psi_0(r = 0)U(k)/T(k)\,. \tag{4}$$

Here, $T \approx k^2/m$ is the kinetic energy of the quark–antiquark system, ϵ is the binding energy. $\phi_0(r)$ is the solution of Schrödinger equation with the potential $V(r)$. Thus, the pQCD core of a hadron is the generalization to the dominance of the QCD Coulomb potential between heavy quarks at small inter-quark distances.

Since QCD is the gauge quantum field theory, the effective number of constituents depends on the configuration. As a result, the effective number of constituents of zero-color charge of the wave packet is decreasing with the decrease of the size of a configuration. In QCD, in the non-relativistic approximation for a heavy flavors MFC of quarkonium is described by equation

$$\begin{aligned} \psi(k) &= (1/(T(k)-\epsilon)) \int F(k-l)'^2\, \psi_0(l)\, \mathrm{d}^3 l\, U(k-l) \\ &= F\left(k^2\right) \psi_0(r=0) U(k)/T(k)\,. \end{aligned} \tag{5}$$

Here, k is the center-of-mass momentum of a heavy quark, $E(k)$ is the energy of the intermediate state built of heavy quarks, perturbative QCD. The factor $\phi(r=0)$ is the S wave of the ground state. The factor $F(k^2)$ accounts for the normalization of MFC and depends on the number of effective degrees of freedom in WF. In a preQCD field theory, the Feynman preQCD parton model, and the strong coupling regime of QCD, the contribution of MFC is strongly suppressed[1]. To visualize the origin of the factor $F(k^2)$, we model it as the Poisson distribution over the number of constituents n

$$F(n;\kappa) = \frac{\kappa^n e^{-\kappa}}{n!}\,. \tag{6}$$

In the case of $n=0$ $F(n=0;\kappa) = e^{-\kappa}$ *i.e.* in the strong coupling regime,

$$F_h \ll 1\,, \tag{7}$$

due to of the dominance of many-body states in the WF in the strong coupling regime.

On the contrary, in QCD in the weak coupling regime where color charge is screened, MFC may dominate. In this limit, $F_h \approx 1$. This is the explanation of why purely lepton decays of mesons occur with a significant probability, as well as characteristic of pion diffraction into exclusive dijet production, in the hard coherent processes. Thus, the major difference between QCD, the preQCD Feynman parton model, and the preQCD quantum field theory is that the high-momentum tail of MFC of a hadron WF is not suppressed in QCD. The practical difference arises from the cancellation of the soft gluon fields outside of expanding zero-color charge, a small-size wave package. The observation in the Lab of the hard coherent processes such as $\gamma_{\mathrm{L}} + T \to V + T$, coherent photo-production of J/ψ, hard diffractive

[1] According to S. Brodsky this argument was put forward by R. Feynman in their discussion of large x regime of structure functions.

$\pi + T \to 2\,\mathrm{jet} + T$ process, significant probability of purely leptonic decays of a pion, ρ, J/ψ *etc.* visualizes that preQCD field theories fail to describe fundamental properties of a hadron.

In the case of a meson consisting of light quarks, ultra-relativistic (universal) regime would dominate at $k \gg m_q$. Here, m_q is the bare mass of a quark, and momentum dependence of the WF of such a meson is close to that for WF of a photon except for an additional power of α_{s}.

2.3. On the coherence of high-energy processes

The Fourier transform allows us to map the standard perturbation series into the distribution over M_n where M_n is the mass of a state n in a Fock column for WF:

$$\mathrm{WF(bound\ state)} = \sum_n \exp i\left[\left(M_n^2 - M_h^2\right)t/2P - iP(z-t)\right]\psi_n\,. \tag{8}$$

Here, $P \to \infty$ is the momentum of a bound state. Thus, in the essential region $t-z \propto 1/P$ but $z = 2P/(M_n^2 - M_h^2)$. Distribution over z characterizes distribution over longitudinal distances of configuration with the mass M_n. The coherence length L_{c} describes the longitudinal distances where the wave package of the small transverse size propagates through a target T without significant distortions

$$L_{\mathrm{c}}(\mathrm{DIS}) = 1/(2m_N x) \geq R_{\mathrm{T}}\,. \tag{9}$$

Here, R_{T} is the radius of a target. Initially, this formula was derived by performing a numerical analysis of the parton model expression for a structure function of a nucleon in the infinite momentum coordinate space [25]. Later, it was reinterpreted as the uncertainty principle in coordinate space applied to the structure function of a nucleon at rest [6]. The concept of coherence length depends on the process.

2.3.1. The coherent electroproduction processes

To simplify our analysis of the space-time evolution, we restrict ourselves to the consideration of MFC of WF of a meson although our consideration is applicable to the case of baryon as well.

The light-cone description of MFC of WF of a hadron justifies the use for the analysis of MFC the vacuum matrix element of the commutators of local currents and causality. Asymptotic freedom of QCD helps introduce ranking between the allowed operators J. The light-cone (LC) description justifies causality of space-time evolution leading to the semiclassical description of a small-size wave packet at small distances between quarks to the point where the coupling constant becomes sufficiently large to produce the Goldstone boson and to the spontaneously broken chiral symmetry, through the interaction of valence quarks with the chiral condensate. The bulk of gluons and antiquarks in the hadron WF arises after the interaction of valence quarks with the condensates. Thus, semiclassical description and causality imply that pQCD dominates in a hadron WF in a finite-size region and that pQCD core corresponds to the high-momentum tail $\geq$ of MFC of the WF of a hadron.

The value of $r_{\rm c}$ for a pion can be extracted from the data of [21] in the $\pi + A \to 2\,\mathrm{jet} + \mathrm{rapidity\ gap} + X$ process where A is the ground state of a nucleus and X includes the sum over nuclear excitations. The selection of two forward jets in the final state enforces the MFC of the pion wave function. The minimal transverse momentum of the identified jet in the data was $\sqrt{(k_{\rm t}^2)} \geq 1.2$ GeV. The application of the uncertainty principle leads to the estimate: $r_{\rm c,t} \geq (\pi/2)(1/k_{\rm t}) \geq 0.26$ fm. The application of the angular momentum constraint predicts $r_{\rm c} \approx 0.32$ fm. The dependence of cross section on the transverse momentum of jets on atomic number, CT phenomenon, were predicted in [23] as the consequence of the pQCD factorization theorem and the dominance of one-gluon exchange in the high-momentum tail of MFC of the pion WF. The predicted form of the WF of MFC is consistent with the data [21].

The value of $r_{\rm c}$ for a nucleon can be extracted from the analysis of the space-time evolution of the process: from the analysis of the space-time evolution of the $V = \langle 0|[J(x), J(0)|0\rangle$ where local current J has quantum numbers of a nucleon. The selection of the contribution of a single-nucleon intermediate state within the V using a method of [3, 4] allows us to evaluate the distance from the production point of color-neutral wave packet to the point where a valence quark hits a condensate. Numerical calculation found that $r_{\rm c} = 0.4$ fm [24]. (The relationship between such a calculation and MFC of the nucleon WF was not considered in the cited paper.)

The high-momentum tail of MFC of WF of transversely polarized virtual photon in the zero order in $\alpha_{\rm s}$ has the form

$$\left(Q^2 + M^2\right) \psi_{\gamma_\mu}\left(\alpha, k_{\rm t}\right) = J_\mu F_\gamma \left(k_{\rm t}^2/{\Lambda_{\rm QCD}}^2\right) . \tag{10}$$

Here, $M^2 = (m_Q^2 + l_{\rm t}^2)/\alpha(1-\alpha)$, m_Q is the mass of a bare quark, and α is the fraction of photon momentum carried by a quark. J_μ is the matrix element of the operator of e.m. current in the zero-order over $\alpha_{\rm s}$. The function F describes the statistical weight of MFC in the regime of strong coupling where for $(k^2/\eta^2) \leq 1$ $F \ll 1$. In the opposite limit: $(k^2/\eta^2) \gg 1$: $F_\gamma \approx 1$.

In the case of the longitudinally polarized virtual photon (virtual W, Z) and large-Q^2 dominance of MFC in a diffractively produced hadron follows from the structure of the photon WF:

$$\left(M^2+Q^2\right)\psi^{\mathrm{L}}(\gamma)=ee^{\mathrm{L}}J_{c\bar{c}}=e\left(\left(\sqrt{Q^2}\right)\Big/q_0\right)\langle c|\bar{c}|j_0|0\rangle=e\left(\sqrt{Q^2}\right). \tag{11}$$

In the derivation, we account for the conservation of e.m. current.

The target rest frame equivalent of the parton model for a structure functions of a target $\sigma(\gamma^*+T)$ is the convolution of MFC of the virtual photon and cross section of the spatially small-size dipole of a target T at small x where the pQCD gluon ladder dominates

$$\sigma\left(\gamma^*+T\right)=\int\psi^2\left(\alpha,b^2\right)\left(\frac{\mathrm{d}\alpha}{\alpha(1-\alpha)}\right)\mathrm{d}^2b\sigma\left(x,b^2\right), \tag{12}$$

$\sigma(x,b^2)$ has been calculated in [12, 14]. In the case of longitudinally polarized photon, this formula is another form of the conventional factorization theorem. In the case of transversely polarized photon, this formula ignores the end-point contribution found in [5, 6]. In the hard coherent processes, the end-point contribution is a small correction to the hard one because of smallness of MFC of a WF of a hadron in the non-perturbative domain. Similar to structure functions, the end-point contribution to the DVCS process is not suppressed in small-x physics. Hard QCD contribution has the form

$$\begin{aligned}&A\left(\gamma^*+T\to\gamma+T\right)\\&=\nu\int\psi_{\gamma^*}\left(\alpha,b^2\right)\psi_\gamma\left(\alpha,b^2\right)\frac{\alpha}{\alpha(1-\alpha)}\mathrm{d}^2b\sigma\left(x_1,x_2,b^2\right)\theta\left(b_0-b\right).\end{aligned} \tag{13}$$

and

$$\sigma\left(x,b^2\right)=\left(\frac{\pi^2}{3}\right)\alpha_{\mathrm{s}}\left(Q^2_{\mathrm{eff}}\right)b^2xG_{\mathrm{T}}\left(x_1,x_2,Q^2_{\mathrm{eff}}\right) \tag{14}$$

is the cross section of the interaction of a small dipole of transverse distance b between heavy quarks within the WF of a photon. The factor b^2 follows from the dominance of non-sense polarizations of exchanged pair of gluons in the ladder and Ward identities. Here, the dominant contribution is given by the high-momentum components of MFC of virtual photon and real photon as explained above. $G_{\mathrm{T}}(x_1,x_2,Q^2)$ is the gluon GPD for the target T. The major effects of skewness were calculated for $G_{\mathrm{T}}(x_1,x_2,Q^2)$ within the leading $\alpha_{\mathrm{s}}\log(Q^2)$ approximation for GPD [20]. Effectively, this physics can be accounted for by including factor $\Theta(k_{\mathrm{t}}^2-\eta^2)$ under the integral of the upgraded factorization theorem. Factor b_0 in the above formulae is the impact parameter space equivalent to η.

The relationship between b_0 and η follows from the Fourier transform of the momentum space amplitude into impact parameter space.

The high-momentum component of WF of a bound state build of heavy quarks is dominated in QCD by a single-gluon exchange between heavy quarks. The form of the component of quarkonium WF in the overlapping integral between WF of a real photon and quarkonium WFs is dictated by the two-dimensional rotational invariance in the momentum plane transverse to the direction of photon momentum

$$\mathrm{WF} = l_{\mu,\mathrm{t}} f\left(l^2\right) . \tag{15}$$

The lack of the anomalous dimension of the MFC of WF of a photon and of transversely polarized Charmonium follows from the two-dimensional invariance of the overlapping integral. To derive this result, we start from the non-relativistic approximation for the non-perturbative quarkonium WF and from the requirement for the high-momentum component to be $\propto l_{\mu,\mathrm{t}}$ to conserve two-dimensional invariance in the overlapping integral for the $\gamma \to J/\psi$ transition in the gluon field of a target.

2.4. Universality of momentum dependence of hard coherent processes

For the quark momenta $l \gg \eta$, the MFC of the wave function of a pion is dominated by one-gluon exchange. In this limit, MFC has the form

$$\psi_\pi(x, l_\mathrm{t}) = c\alpha_\mathrm{s}\left(\sigma l_{\mu,\mathrm{t}}\right)\left(\frac{x(1-x)}{l_\mathrm{t}^2}\right) , \tag{16}$$

where c is the constant calculable in terms of F_π. Power of α_s differs the WF of a bound state from the WF of a bare photon. Ultra-relativistic component of heavy quarkonium WF for $l^2 \gg m_\mathrm{c}^2$ QCD has the same universal form as the above formula for pion. At smaller momenta, one should include into the formula the factor $m_\mathrm{c}^2/(m_\mathrm{c}^2 + k_\mathrm{t}^2)$.

The account of three QCD phenomena discussed above requires to modify previous form of the factorization theorem: (a) Cancellation of color field outside of zero-size wave packet; (b) Suppression of MFC in the strong coupling regime of QCD; (c) The amplitude for the production of MFC is more rapidly increasing with the decrease of x than the soft contribution. A quick way to account for these phenomena is to add under integrand the factor: $\theta(l - \eta)$.

In the analysis of the formula, we account for the leading power of $1/Q$ and pull out the integral of the WF of the longitudinally polarized photon and derive the new form of factorization theorem. Using the discussed above

MFC of WF of vector meson, we derive the formula for the coherent electro-production of a longitudinally polarized vector meson

$$\begin{aligned} & A\left(\gamma_{\mathrm{L}}^{*}+T \rightarrow V+T\right) \\ & \propto\left(\frac{\nu}{Q^{3}}\right) F_{V} \int \frac{\mathrm{d} \alpha}{\alpha(1-\alpha)} \theta(l-\eta) \mathrm{d}^{2} l_{\mathrm{t}} \Delta_{l}\, x G_{\mathrm{T}}\left(x, x_{2}, l_{\mathrm{t}}^{2}\right) . \end{aligned} \tag{17}$$

Here, Δ is the Laplace operator in the two-dimensional momentum space. The practical implication of the specific form of factorization theorem is that the effective scale of GPD Q^2_{eff} differs from that characteristic for LT processes. The effective scale of the GPD is the interplay of the high-momentum tail of MFC of a vector meson, discussed above absence of radiation from MFC of WF of vector meson dictated by zero-color of wave packet, and gauge invariance.

It is easy to show that derived formulae are also applicable to the hard coherent production of transversely polarized vector meson. Another specific feature of this process is that two-dimensional rotational invariance in the momentum space selects special component of WF of transversely polarized vector meson, compare the discussion at the end of the section.

Thus, the factorization theorem for hard coherent production of a vector meson predicts universal momentum dependence in the ultra-relativistic regime where bare masses of quarks can be neglected. The hard coherent processes being strongly suppressed in the preQCD field theories are observed now in a number of collider experiments.

The investigation of exclusive electro-production of longitudinally polarized vector mesons V off a nucleus accompanied by the removal of a nucleons of a nucleus where $V = \rho, J/\psi, \Upsilon$ is a promising tool for the probing of the nuclear structure.

The Ultra-peripheral Collision (UPC) data off a nucleus target found the stronger dependence of the amplitude of coherent forward photo-production of J/ψ on an atomic number, stronger than in the process of the elastic photo-production of ρ meson off a nucleus target indicating onset of pQCD regime of upgraded factorization theorem. b_0 in the above formulae is equivalent of η but in the impact parameter space.

The high-momentum tail of heavy quark MFC of an LC WF of a bare photon is dominated in the lowest order by α_{s} and can be written as

$$\psi(\gamma) = F_{\gamma} e^{\mathrm{T}} \langle c\bar{c} \,|\, j_{\mu,\mathrm{t}} \,|\, 0 \rangle / M^2 \,. \tag{18}$$

The high-momentum component of a bound state built of heavy quarks is dominated in QCD by a single-gluon exchange between heavy quarks. The form of the component of quarkonium within the overlapping integral between WF of a real photon and quarkonium WFs is dictated by the two-dimensional rotational invariance in the momentum plane transverse to the direction of the photon momentum

$$\begin{aligned} \mathrm{WF} &= \int \mathrm{d}^3 l' F \theta\left(l^2 - l'^2\right) \frac{\alpha_{\mathrm{s}}}{\left(l-l'\right)^4} \psi_{\text{non-pert}}\left(l'\right) \\ &\propto cF\alpha_{\mathrm{s}} \left(\frac{l_{\mathrm{t},\mu}}{\left(l'^2\right)^3}\right) \psi_{\text{non-pert}}(r=0)\theta\left(l-l_0\right) . \end{aligned} \tag{19}$$

Thus, the high-momentum tail of the WF of transversely polarized quarkonium has the form

$$WF \propto F(\text{quarkonium})\, l_{\mathrm{t},\mu} \left(1/l^2\right)^3 \psi_{\text{non-pert}}(r=0) . \tag{20}$$

3. The forward hard coherent photo-production of a heavy quarkonium off a hadron target in QCD

Large mass of c, b, t quarks and relatively small value of the scale η of non-perturbative QCD phenomena together with the suppression of MFC of a hadron WF in the strong coupling regime of QCD allow us to expand the region of applicability of pQCD to the description of photo-production of a heavy quarkonium such as $\gamma + T \to J/\psi(\Upsilon) + T$. Besides, the suppression of MFC of a hadron and a photon in the non-perturbative regime and the dominance at sufficiently small x of the exchange by gluon ladder, the end-point contribution is suppressed. So, factorization theorem can be extended to the coherent photo-production of heavy quarkonium. Such hard coherent scale of gluon GPD reads

$$Q^2_{\mathrm{eff}} \approx \left\langle l^2_{\mathrm{t}} \right\rangle \gg \eta^2 . \tag{21}$$

The interpretation of Q^2_{eff} is defined within the leading $\alpha_{\mathrm{s}} \log(Q^2_{\mathrm{eff}}/\Lambda^2_{\mathrm{QCD}})$ approximation where the gluon transverse momenta in the gluon ladder k_{t} should be smaller as compared to the heavy quark momentum l_{t}.

In the pQCD regime, the dominant power of energy ν in the amplitude of a hard diffractive process arises from the Feynman diagrams where the loop of heavy quarks is connected with the target through the two-gluon exchange ladder. In the ladder, the non-sense polarizations of exchanged gluons dominate. The proper technology of calculations of the amplitudes of high-energy processes where masses in the initial and final states are the same was developed initially for the high-energy processes in QED [16] and extended later to the hard QCD processes, for the history and detailed derivation, see [17]. We generalize this technology also used in [18] to the calculation of high-energy processes where masses of the projectile and diffractively produced particles are different. We decompose momenta of exchanged gluons k_1 and k_2 over directions given by the vectors q, p', and k_t. Here, q is the photon momentum, p is four momentum of the target

$$k_1 = x_1 p' + \beta_1 q + k_\mathrm{t}\,, \tag{22}$$

and

$$k_2 = x_2 p' + \beta_2 q - k_\mathrm{t}\,. \tag{23}$$

Here, $p = p' + (p^2 q)$, $p'^2 = 0$, and $\nu' = 2(p'q) \approx \nu = 2(qp)$. Thus, $k_1^2 \approx M^2\beta_1 - k_\mathrm{t}^2 \approx -k_\mathrm{t}^2$ and $k_2^2 \approx (M^2 - M_V^2)\beta_2 - k_\mathrm{t}^2 \approx 4l^2\beta_2 - k_\mathrm{t}^2 \approx -k_\mathrm{t}^2$. We systematically neglect k_t^2 as compared to the l_t^2 — this is the property of the leading $\log(Q^2/\lambda^2_{\mathrm{QCD}})$ approximation. In the leading $\log(x/x_0)$ approximation $k_i^2 \approx -k_\mathrm{t}^2$, x_i are evaluated from the energy-momentum conservation. Here, p_V is the momentum of vector meson. The formulae for the coefficients x_i are given below for the diffractive photo-production of heavy quarkonium

$$x_1\nu = M^2 - k_\mathrm{t}^2 \approx M^2\,. \tag{24}$$

The fraction of target momentum carried by the second exchanged gluon in the gluon ladder is

$$x_2\nu = M^2 - (M_V)^2 - k_\mathrm{t}^2 = 4\left(m_\mathrm{c}^2 - \frac{M_V^2}{4}\right) + 4l^2 - k_\mathrm{t}^2 \approx 4m_\mathrm{c}^2 - M_V^2 + 4l^2\,. \tag{25}$$

Here,

$$M^2(\alpha(1-\alpha)) = \left(m_Q^2 + l_\mathrm{t}^2\right) \tag{26}$$

and M is the mass of the intermediate state, m_Q is the mass of a heavy quark, l is the momentum of a heavy quark within the heavy-quark loop, and α is the fraction of photon momentum carried by a heavy quark. The dominant contribution arises from the contribution of the non-sense polarizations in the a projection operator $P_{\mu,\lambda}$ of the propagator of an exchanged gluon

$$P_{\mu,\lambda} = \frac{2p'_\mu q_\lambda}{(2p'q)}\,. \tag{27}$$

For detailed explanation of this method to QCD, see [17]. We use Ward identities which have the same form in QCD as in QED within the approximation used in the paper. So leading power of ν follows from the equation:

$$M_{\mu,\lambda} p'_\mu\, p'_\lambda = \frac{1}{x_1 x_2}\, M_{mu,\lambda}\, k_{\mathrm{t},\mu}\, k_{\mathrm{t},\lambda}\,. \tag{28}$$

Here, $M_{\mu,\lambda}$ is the loop of heavy quarks. We use the conservation of color current which has for the loop of heavy quarks without gluons the same form in QCD as in QED: $M_{\mu,\lambda}k_{1,\mu} = M_{\mu,\lambda}k_{2,\lambda} = 0$.

The next step is to rewrite Eq. (28) in the more familiar form

$$\frac{1}{\nu}A(\gamma+T\to J/\psi+T)=e_c\int \mathrm{d}\tau N x G_{\mathrm{T}}\left(x_1,x_2,l_{\mathrm{t}}^2\right) . \tag{29}$$

Here,

$$\begin{aligned}N &= F_V l_{\mu,\mathrm{t}}\left(1/\left(m_Q^2+l^2\right)\left(\mathrm{d}^2/\mathrm{d}l_{\mathrm{t}}\right)^2 l_{\mu,\mathrm{t}}/l^2\right)^3\theta(l-l_0)\\ &= \left(8/\left(m_Q^2+l^2\right)\right)l_{\mathrm{t}}^2\left(1/l^2\right)^4\theta(l-l_0)\\ &\approx (16/3)\left(1/\left(m_Q^2+l^2\right)\right)l^2\left(1/l^2\right)^4 .\end{aligned} \tag{30}$$

We took the high-momentum component of charmonium WF as discussed above. The parameters $c, l_0 \gg L$ should be evaluated from the calculation of form factor $F_{J/\psi}$. Thus, N is $\approx 1/(m_Q^2(l^2)^3)$. The final formula reads

$$\begin{aligned}&(1/\nu)A(\gamma+T\to J/\psi+T)\\ &= e_c\, 8/\left(m_Q^2\right)\int \mathrm{d}\tau l_{\mathrm{t}}^2\left(1/l^2\right)^4 x G_{\mathrm{T}}\left(x_1,x_2,l_{\mathrm{t}}^2\right)\theta\left(l-l_0\right) .\end{aligned} \tag{31}$$

This is the upgraded form of the factorization theorem. We pull out the energy denominator from the photon WF outside of integral within the non-relativistic approximation $l^2/m_Q^2 \ll 1$ for WF of heavy quarkonium.

Another new result is the scale for the gluon distribution within a target $Q_{\mathrm{eff}}^2 \approx l_{\mathrm{t}}^2$. The value of Q_{eff}^2 follows from the integral:

$$\int \mathrm{d}^3 l T\theta\left(l-l_0\right)\left(l_{\mathrm{t}}^2\right)/\left(l^2\right)^4 G_{\mathrm{T}}\left(x, Q_{\mathrm{eff}}^2=l_{\mathrm{t}}^2\right) . \tag{32}$$

Here,

$$x\approx\left(x_1+x_2\right)/2\approx 4\left(m_c^2\right)/\nu\,. \tag{33}$$

This equation can be simplified by using kinematical relations Eq. (33) and performing Fourier transform into the coordinate two-dimensional space b. We obtain the formulae for the amplitude of the photo-production of J/ψ in impact parameter place which includes CT factor b^2:

$$\begin{aligned}&(1/\nu)A(\gamma+T\to J/\psi+T)\\ &= \int \mathrm{d}\tau\psi_\gamma(\alpha,b)\sigma\left(b,x_1,x_2,1/b^2\right)\psi_{J/\psi}(\alpha,b)\theta(b_0-b)\,.\end{aligned} \tag{34}$$

The powers of $\log(k_{\mathrm{t}}^2)$ in σ which follow from the k_{t}^2 evolution are converted under Fourier transformation into powers of $\log(1/b^2)$ in the leading log approximation. The distinctive properties of QCD are recorded in the cross section of the interaction of spatially small wave packet of quarks and gluons off a target T convoluted with MFC of the wave functions of colliding particles. In the case when $x_1 = x_2 = x$, the cross section of the scattering of spatially small dipole of a target T has been derived in [12, 14]: The virtualities of GPD are given by the condition:$M_Q \gg Q_{\rm eff} \gg l_0$.

4. Conclusions

QCD predicts the pQCD core of finite size locked in the center of a hadron. The pQCD core of a hadron together with asymptotic freedom substitutes Landau zero-charge puzzle characteristic for a preQCD field theory. We derive the new form of QCD factorization theorem for the higher twist color coherent processes and demonstrate that it is an effective tool for the theoretical description of a high-resolution regime of coherent phenomena. An important role in this theoretical description is played by the high-momentum tail of MFC of wave function of a hadron calculated within pQCD. The investigation of hard diffractive processes allows us to probe MFC of the wave function of a hadron and to predict the variety of phenomena which are beyond the standard framework of low-resolution nuclear theory. The key role in the theoretical description is played by the theoretical object which is absent in the preQCD field theories: zero-size color-neutral wave packet having no external soft modes. Its space-time evolution allows us to account for the color gauge invariance of QCD in the theoretical description of a bound state. Fundamentally new phenomena predicted in the paper include suppression of initial- and final-state interaction for the hard diffractive processes in a special kinematical range controlled by the coherence length- and color-fluctuations phenomena in the hadron–nucleus interactions *etc.* Another important result is the suppression of strong meson fields in the center of a nucleon of a heavy neutron star. Some of the predicted phenomena were observed at FNAL, LHC and discussed in the paper.

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