ROLE OF NEUTRON AND PROTON SYSTEMS IN THE ENTROPY OF ^{210,211}Po

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The intrinsic level densities and entropies of ^{210}Po and ^{211}Po nuclei have been extracted within the BCS theory that includes pairing interaction. Then, total level density considering the collective effects is obtained. The results agree well with the recent data obtained from experimental-level densities for the considered nuclei. In addition, the entropy excess of ^{211}Po compared to ^{210}Po has been extracted. Also, the entropy excess ratio, which was introduced in our previous publications, has been calculated as a function of temperature for the neutrons and protons. The neutron system plays a major role in the entropy excess of ^{211}Po at low temperatures.

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1. Introduction

Nuclear level density (NLD) is one of the crucial topics to understand the nuclear structure and nuclear reactions. NLD is a fundamental quantity for calculating thermodynamic quantities and investigating the behavior of nuclei [1, 2]. NLD is a nuclear property that predicts the distribution of nuclear levels and is effective in calculating the cross section of nuclear reactions [3]. Over the past years, there has been a lot of research to find empirical NLD data on theoretical models for NLD prediction [4]. Due to the pairing effects in the NLD, the appropriate model used to calculate the NLD is the Bardeen–Cooper–Schrieffer (BCS) model [5]. Based on the theory of superconductivity (BCS), this model was proposed by Bohr and Mottelson in 1958 for nuclei [6]. On the other hand, due to the dependence of excitation energy on collective effects, consideration of collective effects is essential [7].

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Since the role of neutron and proton systems in thermodynamic quantities is different, the role of each system can be studied by entropy excess. Entropy excess is defined as the entropy difference between two neighboring nuclei [8]. The role of proton and neutron systems in entropy excess by the use of entropy excess ratio was introduced in our previous publication [9]. In this work, the nuclear level densities have been computed for ^{210,211}Po nuclei using the superfluid model with the inclusion of pairing effects. Then, the total level densities considering the collective effects are obtained and compared to their corresponding experimental data [10]. Also, the entropy for ^{210}Po and ^{211}Po nuclei has been calculated and compared with each other. Then, the ratio of proton and neutron entropy excess has been determined and discussed.

2. Summary of the theory

In this section, the total nuclear level densities and thermodynamic properties of neutron and proton systems are presented by using the microscopic model. In the framework of statistical mechanics, the state density is defined as [11]

$$\omega(N, E) = \frac{\exp(S)}{2\pi|D|^{\frac{1}{2}}}\,, \tag{1}$$

where D is the determinant of the second derivatives of the grand partition function taken at the saddle point. Here, the entropy S can be written as

$$S = 2\sum_k \ln\left[(1 + \exp\left(-\beta E_k\right))\right] + 2\beta \sum_k \frac{E_k}{1 + \exp\left(\beta E_k\right)}\,, \tag{2}$$

where $\beta = \frac{1}{T}$ is the inverse of statistical nuclear temperature T, $E_k = [(\epsilon_k - \lambda)^2 + \varDelta^2]^{\frac{1}{2}}$ is the quasi-particle energy, ϵ_k is the single particle energy, $\lambda = \frac{\alpha}{\beta}$ is the chemical potential, and $\varDelta$ is the gap parameter which is a measure of the pairing correlation. The gap parameter is determined by solving the gap equation [12, 13]

$$\frac{2}{G} = \sum_k \frac{1}{E_k} \tanh \frac{\beta E_k}{2}\,, \tag{3}$$

where G is pairing strength.

For a system of N nucleons

$$N = \sum_k n_k\,, \tag{4}$$

with the occupational probability of the level k [14]

$$n_k = 1 - \frac{\epsilon_k - \lambda}{E_k} \tanh \frac{\beta E_k}{2} \,. \tag{5}$$

The energy is

$$E = \sum_k n_k E_k - \frac{\Delta^2}{G} \,, \tag{6}$$

and the excitation energy is

$$U = E - E_0 \,, \tag{7}$$

where E_0 is the energy at $T = 0$.

For a system of N neutrons and Z protons, the total energy is given by

$$E = E_n + E_p \,, \tag{8}$$

and the entropy is

$$S = S_n + S_p \,. \tag{9}$$

The intrinsic level density at an excitation energy $U = U_n + U_p$ is

$$\rho_{\text{int}}(N, Z, U) = \frac{\omega(N, Z, U)}{(2\pi\sigma^2)^{\frac{1}{2}}} \,, \tag{10}$$

where σ^2 is the spin cutoff factor defined as

$$\sigma^2 = \sigma_n^2 + \sigma_p^2 \,, \tag{11}$$

with

$$\sigma_n^2 = \frac{1}{2} \sum_k m_k^2 \sinh^2 \left(\frac{1}{2} \beta E_k \right) \,, \tag{12}$$

where m_k is the magnetic momentum spin quantum number of the state k, and a similar relation can be written for σ_p^2.

The calculation procedure for the total level density considering the collective effects is outlined in our previous publication [15]. In the adiabatic approximation, the total nuclear level density $\rho_{\text{tot}}(U)$ is generally described by [16]

$$\rho_{\text{tot}}(U) = \rho_{\text{int}}(U) K_{\text{coll}} \,, \tag{13}$$

where K_{coll} is the collective enhancement

$$K_{\text{coll}} = K_{\text{rot}} K_{\text{vib}} \,, \tag{14}$$

and K_{rot} and K_{vib} are the rotational and vibrational coefficients [17].

3. Result and discussion

In performing calculations for the number of neutrons and protons of ^{210,211}Po, the single particle energies and their spins were first determined using the modified harmonic oscillator potential according to the Nilsson potential [18].

The total nuclear level density with the inclusion of collective effects as a function of excitation energy is determined for ^{210,211}Po nuclei. The total level densities are shown in Fig. 1 together with the corresponding experimental values [10]. As seen from Fig. 1, the calculations are in good agreement with the experimental level densities at the range of 1.6–5.4 MeV. We have also extracted the entropy for ^{210,211}Po nuclei in the microscopic theory.

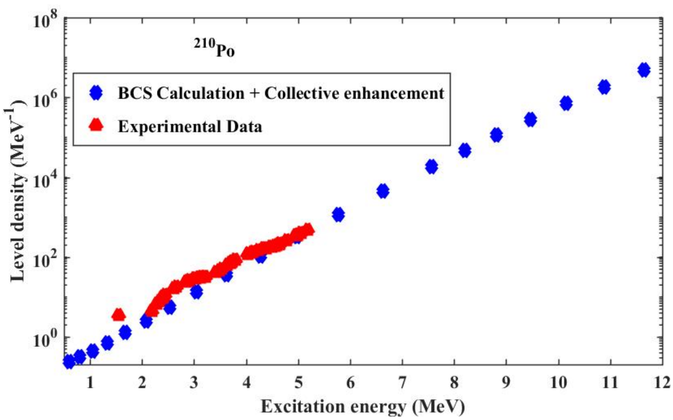

Fig. 1. The experimental [10] and the calculated excitation-energy-dependent level densities for ^{210}Po nucleus.

Since our goal has been the difference between neutrons and proton systems and the role of these systems in these differences, the collective effects in the calculation of the entropy of ^{210,211}Po nuclei are not considered. In the BCS model, there is a critical temperature (T_{c}) at which nucleon pairs break down, then a phase transition occurs. The critical temperature can be obtained by averaging the critical temperatures of proton and neutron systems [12, 19]

$$T_{\mathrm{c}} = \frac{T_{\mathrm{cn}} + T_{\mathrm{cp}}}{2} \,. \tag{15}$$

The temperature-dependent entropies calculated for ^{210,211}Po isotopes up to 1.0 MeV are plotted in Fig. 2. From the figure one can see that the critical temperature is $T_{\mathrm{c}} = 0.6$ MeV, which is corresponds with an excitation energy of $U = 7.56$ MeV.

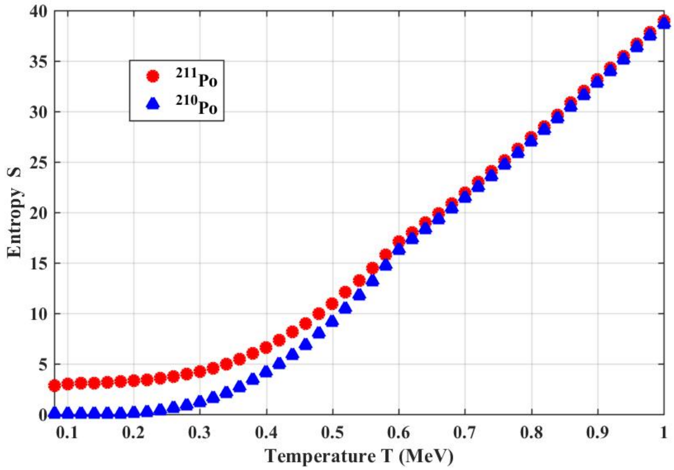

Fig. 2. Temperature-dependent entropies calculated for ^{210,211}Po nuclei.

The entropy excess is given by

$$\Delta S(\text{particle}) = S(\text{odd } A) - S(A-1)\,, \tag{16}$$

the role of the proton and neutron systems can be determined by using the neutron and proton entropy excess ratios $(i = p, n)$ [14]

$$R_i = \frac{\Delta S_i}{\Delta S}\,, \tag{17}$$

where

$$\Delta S_i = S_i(A) - S_i(A \pm 1)\,, \tag{18}$$

and ΔS is the total entropy excess between nuclei.

The evaluated entropy excess ratios for proton and neutron systems in ^{211}Po compared to ^{210}Po are shown in Fig. 3. An examination of this figure reveals that the neutron system plays a major role in the entropy excess at low temperatures, while in our previous work, we showed that the closed-shell neutron system in the yttrium isotopes [9] and the closed-shell proton system in the tin isotopes [8, 14] have a minor role in the entropy excess.

The proton structures of ^{210}Po and ^{211}Po nuclei are similar to each other. The separation energy of a proton (S_p) is $S_p = 4929.9$ keV and $S_p = 4983.5$ keV for the ^{210}Po and ^{211}Po, respectively.

The proton structures of ^{210,211}Po nuclei are very close to the proton structure of the lead nucleus with a magical proton number $Z = 82$. The neutron structure of ^{210}Po with 126 neutrons and with $S_n = 7658.4$ keV indicates a stable structure. The spin parity of ^{210}Po nucleus is $I = 0^+$.

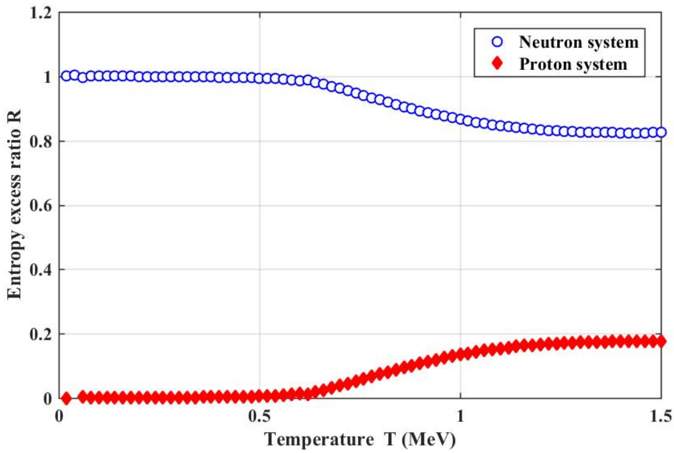

Fig. 3. Entropy excess ratio for proton and neutron as a function of nuclear temperature in ^{211}Po compared to ^{210}Po.

With the addition of a neutron in ^{211}Po, the separation energy of a neutron becomes $S_n = 4550.7$ keV and its spin parity becomes $I = \frac{9}{2}^+$. Therefore, the single extra neutron in ^{211}Po compared to ^{210}Po plays a major role in the entropy excess at low temperatures.

4. Conclusion

In the present study, the nuclear level densities have been extracted using the superfluid model with the inclusion of pairing effects. Then the total level density considering the collective effects is obtained for ^{210,211}Po nuclei. The present investigation shows good agreement between our results and corresponding experimental data. We have also obtained the entropy excess of ^{211}Po compared to ^{210}Po as a function of temperature. The role of the neutron and proton systems has been investigated by entropy excess ratios. Due to the stability and similarity of the proton structure in ^{210,211}Po nuclei and the difference in the neutron separation energies of these nuclei, the proton system plays a minor role and the neutron system plays a major role in the entropy excess.

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