INTRA-BAND REDUCED E2-TRANSITION PROBABILITIES OF EXCITED COLLECTIVE STATES OF TRIAXIAL EVEN–EVEN NUCLEI

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The intra-band reduced E2-transition probabilities depending on the effective triaxiality parameter $\gamma_{\rm eff}$ are studied within the framework of the Davydov–Chaban model. The Davidson potential for β variable has been used. The intra-band reduced E2-transition probabilities ground-state, β, and γ bands are presented and expressed in terms of three parameters: g, $\gamma_{\rm eff}$, and Q_0. Permitted and forbidden intra-γ-band E2 transitions were discussed. The sensitivity of intra-band reduced E2-transition probabilities to the effective triaxiality parameter $\gamma_{\rm eff}$ was analyzed.

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1. Introduction

Obtaining a general solution of the Schrödinger equation with the Bohr Hamiltonian [1] is an extremely complex task. Therefore, various simplified approximate solutions have been proposed [2]. More recent results in this direction are presented in Ref. [3] and are associated with the discovery of critical point symmetries that led to the development of specific models and improved approaches.

One of such approximate solutions is the Davydov–Chaban model, which effectively accounts for transverse γ vibrations [4]. In Ref. [5], an analytical solution of the Davydov–Chaban Hamiltonian with a sextic oscillator potential was considered for the triaxial parameter value $\gamma = 30^\circ$, where the energy spectra of the ground-state and first β and γ bands, as well as the corresponding B(E2) transitions, were determined with $Z(4)$-sextic model.

In the framework of the Davydov–Chaban model, excited collective states of even–even lanthanide and actinide nuclei were studied [6] over the entire range of the effective triaxial parameter $\gamma_{\rm eff}$, and the obtained results were compared with experimental data [7] for the nuclei: ^{154}Sm ^{158,160,162}Gd, ^{166}Dy, ^{162,166}Er, ^{168}Hf, ^{168}Yb, ^{170}W, ^{228,230,232}Th, ^{232,234,236,238,240}U, and ^{242,244}Pu.

It should be noted that the energy spectrum of excited collective states is a static quantity, because it is a set of discrete energy levels of the nucleus. To study the dynamic properties of excited collective states, it is necessary to examine the probability of multipole transitions and their branching [8]. They are sensitive to collective modes in atomic nuclei, such as rotational and vibrational excitations, as well as give information about the shape and collectivity of the spectrum of the nucleus [9–11].

The energy levels of the collective states of these nuclei are classified by the ground-state, γ, and β bands. Intensive or passive intra-/inter-band E2 transitions occur in these bands. These transitions are described by the reduced E2-transition probabilities [11].

In Ref. [12], the branching ratios of inter-/intra-band reduced E2-transition probabilities in the ground-state, γ, and β bands of heavy even–even nuclei are studied, including high-spin states. The sensitivity of the intra-band reduced E2-transition probabilities to the triaxiality parameter γ_0 is analyzed in the full range of its variations γ ($0\leq \gamma_0 \leq \pi/6$). However, in this paper, we did not investigate in detail the E2 transitions within the ground-state, γ, and β bands, especially the intra-band transitions in the γ band, where permitted and forbidden E2 transitions exist. In the present work, permitted and forbidden E2 transitions are studied in detail.

The structure of the article is organized as follows. Section 2 briefly discusses the Davydov–Chaban model formalism with the Davidson potential, as well as the analytical expressions for the energy levels and wave functions obtained. The dependence of the γ-band energy levels in the asymmetric-rigid-rotator model on the effective triaxiality parameter $\gamma_{\rm eff}$ is described in Section 3. In Section 4, the analytical expressions for intra-band reduced E2-transition probabilities in the ground-state, γ, and β bands are obtained. E2-transition probabilities in the ground-state, γ, and β bands were analyzed in Section 5. The intensive or passive intra-band reduced E2-transition probabilities are discussed in Section 6. The results are presented in Section 7. The conclusions are given in Section 8.

2. Davydov–Chaban model

The Bohr Hamiltonian in the Davydov–Chaban model has the following form [13, 14]:

$$\hat{H}_\beta = \hat{T}_\beta + \frac{\hat{T}_{\rm rot}}{\beta^2} + V(\beta)\,, \tag{1}$$

where

$$\hat{T}_\beta = -\frac{\hbar^2}{2B}\frac{1}{\beta^3}\frac{\partial}{\partial\beta}\left(\beta^3\frac{\partial}{\partial\beta}\right) \tag{2}$$

is the kinetic energy operator of the β-vibrations and B is the mass parameter,

$$\hat{T}_{\mathrm{rot}} = \frac{1}{4B}\sum_{\lambda=1}^{3}\frac{\hat{I}_\lambda^2}{\sin^2\left(\gamma_{\mathrm{eff}} - \frac{2\pi}{3}\lambda\right)} \tag{3}$$

is the rotational energy operator with effective triaxiality γ_{eff} (in degrees), where $\hat{I}_\lambda$ is the component of the total angular-momentum operator ($\lambda = 1;$ $2;\ 3$) and $V(\beta)$ is the potential energy of β vibrations.

We write the Schrödinger equation with the Bohr Hamiltonian (1) as

$$\left[-\frac{\hbar^2}{2B}\frac{1}{\beta^3}\frac{\partial}{\partial\beta}\left(\beta^3\frac{\partial}{\partial\beta}\right) + \frac{1}{4B\beta^2}\sum_{\lambda=1}^{3}\frac{\hat{I}_\lambda^2}{\sin^2\left(\gamma_{\mathrm{eff}} - \frac{2\pi}{3}\lambda\right)} + V(\beta) - E_{I\tau}\right]$$
$$\times\Psi_{nIM\tau}(\beta,\theta_i) = 0\,, \tag{4}$$

where n is the quantum number of β vibrations, I is the total angular momentum, M is the component of the spin onto the axial axis in the laboratory coordinate system, τ is the quantum number that enumerates the energy levels of the rigid asymmetric rotator of the identical IM [15], and θ_i are the Euler angles.

Then, we find the wave functions of the Schrödinger equation (4) in the following form [14]:

$$\Psi_{nIM\tau}(\beta,\theta_i) = F_n(\beta)\phi_{IM\tau}(\theta_i)\,, \tag{5}$$

where

$$\phi_{IM\tau}(\theta_i) = \sqrt{\frac{2I+1}{16\pi^2(1+\delta_{0K})}}\left[D_{IK}^{M}(\theta_i) + (-1)^I D_{M,-K}^{I}(\theta_i)\right]A_{IK}^{\tau}\,, \tag{6}$$

and $D_{IK}^{I}(\theta_i)$ is the Wigner function, K is the component of the spin onto the axial axis in the coordinate system associated with the nucleus, δ_{0K} is the Kronecker symbol, and A_{IK}^{τ} is the normalized wave function of the rigid triaxial rotator [15].

The general solution of the Schrödinger equation (4) is very complicated [16]. Therefore, different simplifications [5, 14, 16–24] are used. In Ref. [25], different solutions of the Schrödinger equation with the Bohr Hamiltonian (1) are considered for different modifications.

We write equation (4) for $F_{I\tau}(\beta)$ [6] as

$$\left[-\frac{\hbar^2}{2B}\frac{\partial^2}{\partial\beta^2}-\frac{3\hbar^2}{2B\beta}\frac{\partial}{\partial\beta}+\frac{\hbar^2\varepsilon_{I\tau}}{4B\beta^2}+V(\beta)-E_{I\tau}\right]F_{I\tau}(\beta)=0\,. \tag{7}$$

Now, we choose $V(\beta)$ as Davidson's potential

$$V(\beta)=V_0\left(\frac{\beta}{\beta_0}-\frac{\beta_0}{\beta}\right)^2,$$

where β_0 is the position of the minimum of the potential and V_0 is the potential energy of the ground state of an even–even nucleus.

Then, we obtain the wave functions $F_n(\beta)$ of the Schrödinger equation (7) in the following form [6]:

$$F_n(x)=Nx^s\exp\left(-\frac{x}{2}\right)F(-n,2s+2,x)\,, \tag{8}$$

where N is the normalization coefficient, $F(-n,2s+2,x)$ is the confluent hypergeometric function of the first kind, and

$$x=\frac{\sqrt{2BV_0}}{\hbar\beta_0}\beta^2\,;\qquad s=-0.5+\sqrt{\frac{2+\varepsilon_{I\tau}+4g}{8}}\,;\qquad g=\frac{BV_0\beta_0^2}{\hbar^2}\,.$$

Here, g is the dimensionless inverse nonadiabaticity parameter and $\varepsilon_{I\tau}$ is the eigenvalue of the triaxial rotator [13].

The energy levels of the excited states have the following form [6]:

$$\Delta E_{nI\tau}=\hbar\omega\left(n+\frac{1}{2}\sqrt{1+\frac{\varepsilon_{I\tau}}{2}+2g}-\frac{1}{2}\sqrt{1+2g}\right), \tag{9}$$

where $\hbar\omega$ is the energy factor in keV.

It is evident that the structure of the energy levels of the excited collective states is determined by the quantum numbers: n, K, and τ. Yes, that is right, K is a bad quantum number in the present model, because it is taken into account through the summation [13]. We consider its first value, *i.e.* $K=2$, in the structure of energy bands. This value plays a very important role in determining the forbidden and permitted intra-band E2 transitions associated with the γ band. Thus, we determine the structure of the bands of excited collective states by the quantum number (n,K,τ). For the ground-state band, $(n,K,\tau)=(0;0;1)$. For the γ band, $(n,K,\tau)=(0;2;2)$ if I is an even number, and $(n,K,\tau)=(0;2;1)$ if I is an odd number. For the β band, $(n,K,\tau)=(1;0;1)$.

The adjustable model parameters for the description of energy levels of the excited collective states are $\hbar\omega$, $\gamma_{\rm eff}$, and g.

Table 1. The values of energy levels of the γ band with spins from 2 to 10 depending on the triaxiality parameter γ_{eff} (in degrees).

γ_{eff}	2	3	4	5	6	7	8	9	10
1	6567.6	6571.6	6576.9	6583.6	6591.6	6600.9	6611.6	6623.6	6636.9
2	1643.4	1647.4	1652.7	1659.4	1667.4	1676.8	1687.5	1699.5	1712.9
3	731.5	735.5	740.9	747.6	755.6	765.1	775.7	787.8	801.2
4	412.3	416.4	421.8	428.5	436.6	446.1	456.8	468.9	482.4
5	264.6	268.7	274.1	280.9	289.1	298.5	309.3	321.5	335.1
6	184.4	188.5	193.9	200.7	208.9	218.4	229.4	241.6	255.4
7	136.1	140.1	145.6	152.5	160.8	170.3	181.5	193.7	207.8
8	104.6	108.8	114.3	121.2	129.7	139.3	150.6	162.8	177.3
9	83.1	87.3	92.9	99.9	108.5	118.1	129.8	141.8	157.1
10	67.7	72.1	77.7	84.7	93.5	103.1	115.2	127.1	143.1
11	56.3	60.6	66.5	73.6	82.6	92.2	104.8	116.4	133.4
12	47.7	52.1	58.1	65.2	74.5	84.1	97.3	108.5	126.8
13	41.1	45.4	51.5	58.7	68.4	77.8	92.1	102.6	122.5
14	35.7	40.2	46.4	53.7	63.9	73.1	88.2	98.2	119.9
15	31.4	36.1	42.4	49.7	60.4	69.4	85.8	94.8	118.7
16	27.9	32.5	39.3	46.6	58.1	66.6	84.3	92.2	118.4
17	25.1	29.8	36.8	44.1	56.2	64.3	83.7	90.2	118.8
18	22.6	27.5	34.8	42.1	55.1	62.6	83.7	88.8	119.8
19	20.6	25.6	33.3	40.4	54.6	61.3	84.2	87.7	121.1
20	18.9	24.1	32.2	39.2	54.5	60.3	85.1	86.8	122.3
21	17.4	22.7	31.5	38.2	54.8	59.7	86.2	86.2	123.4
22	16.2	21.6	31.1	37.4	55.3	59.1	87.3	85.6	124.1
23	15.2	20.6	30.8	36.9	56.1	58.8	88.4	85.2	124.4
24	14.3	19.9	30.9	36.5	57.1	58.5	89.1	84.9	124.5
25	13.6	19.3	31.2	36.2	58.1	58.3	89.6	84.6	124.4
26	13.1	18.8	31.7	36.1	58.9	58.2	89.9	84.4	124.3
27	12.5	18.4	32.3	36.1	59.5	58.1	89.9	84.2	124.2
28	12.2	18.2	33.1	36.1	59.8	58.1	90.1	84.1	124.1
29	12.1	18.1	33.7	36.1	59.9	58.1	90.1	84.1	124.1
30	12	18	34	36	60	58	90	84	124

3. Dependence of the energy levels of the γ band of the asymmetric-rigid-rotator model on the effective triaxiality parameter γ_{eff}

In this section, we study the behavior of the energy levels of the γ band of the asymmetric-rigid-rotator model [15] as a function of the effective triaxiality parameter γ_{eff}. In our opinion, this behavior plays an important role in considering the intra-band reduced E2 transitions.

Table 1 shows the changes in the energy levels of excited states of the γ band for each value of spin as a function of the triaxiality parameter γ_{eff}. It is evident from the table that the energy levels of excited states with odd spin decrease monotonically with the increase of triaxiality parameter γ_{eff} over the entire range of its variation. The energy levels of excited states with even spin first decrease monotonically and then increase with increasing γ_{eff}, with the exception of the 2γ state. It should be noted that, for some large values of spin, the energy levels first decrease, then increase slightly, and finally decrease slightly again as γ_{eff} increases.

Such calculations were previously performed in Ref. [13] (Table 6, p. 79), where the ratios $R_{I\gamma} = E_{I\gamma}/E_{2gr}$ were calculated. Therefore, these changes were not observed there, with the exception of the last values of $R_{4\gamma}$.

In the subsequent sections, we examine how these changes are reflected in the behavior of the reduced E2-transitions probabilities.

4. Reduced E2-transition probabilities

The reduced E2-transition probabilities between the $I_{n\tau}$ and $I'_{n'\tau'}$ [14] states are given by

$$B\left(\mathrm{E2}; I_{n\tau} \to I'_{n'\tau'}\right) = \frac{5}{16\pi(2I+1)} \sum_{MM'\mu} \left|\left\langle nI\tau|Q_{2\mu}|n'I'\tau'\right\rangle\right|^2 , \tag{10}$$

where

$$Q_{2\mu} = \frac{\beta}{\beta_0}\left[Q_0 D^2_{\mu 0} + Q_2\left(D^2_{\mu 2} + D^2_{\mu,-2}\right)\right] , \tag{11}$$

and Q_0 (in units of $\sqrt{\text{W.u.}}$) is the quadrupole moment along the symmetry axis. Q_2 is the measure of the shape asymmetry with respect to the symmetry axis [1]

$$Q_2 = \frac{\tan\gamma_0}{\sqrt{2}} Q_0 .$$

The reduced E2-transition probabilities between $i \equiv \{I_{n\tau}\}$ and $f \equiv \{I'_{n'\tau'}\}$ can be represented as

$$B\left(\mathrm{E2}; i \to f\right) = B_{\mathrm{a}}\left(\mathrm{E2}; I_\tau \to I'_{\tau'}\right) S^2_{if} , \tag{12}$$

where $B_{\mathrm{a}}(\mathrm{E2}; I_\tau \to I_{\tau'})$ denotes the reduced E2-transition probability in an asymmetric-rotator approximation [13]

$$B_{\mathrm{a}}\left(\mathrm{E2}; I_\tau \to I'_{\tau'}\right) = \sum_{KK'} A^{\tau}_{IK} A^{\tau'}_{I'K'} \Bigg\{ Q_0\sqrt{2I+1}\left(I2K0|I'K'\right)$$
$$+Q_2\sqrt{\frac{2I+1}{2\left(1+\delta_{K'0}\right)\left(1+\delta_{K0}\right)}}\left[\left(I2K2|I'K'\right)+\left(I2K-2|I'K'\right)\right]\Bigg\}^2 , \quad (13)$$

where $(I2K0|I'K')$ are the Clebsch–Gordan coefficients.

$$S_{if} = \int\limits_0^\infty F_i(\beta)\frac{\beta}{\beta_0}F_f(\beta)\beta^4 \mathrm{d}\beta\,, \quad (14)$$

which takes into account the β-deformability of an even–even nucleus.

The general expression for matrix elements S_{if} is very complicated. Therefore, we consider specific cases.

The intra-band reduced E2-transition probabilities in the ground-state and γ bands, $n = 0$ and $n' = 0$, are

$$S_{if} = \frac{1}{\sqrt[4]{2g}}\frac{\Gamma\left(s_i+s_f+\frac{5}{2}\right)}{\sqrt{\Gamma(2s_i+2)}\sqrt{\Gamma(2s_f+2)}}\,, \quad (15)$$

where $\Gamma(x)$ is the Gamma function[1].

The intra-band reduced E2-transition probabilities in the β band, $n=1$ and $n'=1$, are

$$S_{if} = \frac{1}{\sqrt[4]{2g}}\sqrt{\frac{2s_i+2}{\Gamma(2s_i+2)}}\sqrt{\frac{2s_f+2}{\Gamma(2s_f+2)}}\Bigg[1-\frac{2s_i+2s_f+4}{(2s_i+2)(2s_f+2)}$$
$$\times\left(s_i+s_f+\frac{5}{2}\right)+\frac{\left(s_i+s_f+\frac{7}{2}\right)\left(s_i+s_f+\frac{5}{2}\right)}{(2s_i+2)(2s_f+2)}\Bigg]\Gamma\left(s_i+s_f+\frac{5}{2}\right). \quad (16)$$

The reduced E2-transition probabilities, normalized to the transition probability from the first excited 2^+ state to the ground state, are given by

$$B\left(\mathrm{E2}; I_{n\tau} \to I'_{n'\tau'}\right) = \frac{B\left(\mathrm{E2}; I_{n\tau}\to I'_{n'\tau'}\right)}{B\left(\mathrm{E2}; 2_{01}\to 0_{01}\right)}\,. \quad (17)$$

[1] Formula (15) can be used to calculate the inter-band transitions between the ground-state and γ bands.

The effective triaxiality parameter γ_{eff} will be used to study the behavior of intra-band reduced E2-transition probabilities. We designate the energy levels as $I_{n\tau}$. The intra-band E2 transitions are denoted as $2I_{n\tau} \to 2(I-2)_{n'\tau'}$ ($I = 0, 1, 2, 3, \ldots$ here and below). We will now study the energy states corresponding to a specific value of angular momentum I. These rules play an important role in calculating the intra-band E2-transition probabilities.

It is known that, for a fixed value of angular momentum I, the number of its possible projections K is equal to $1/2(I+2)$ if I is even and equal to $1/2(I-1)$ if I is odd [13]. Hence, in expression (13), the summation is carried out over the quantum numbers K and K' [13]. Thus, in the sum (13), for states with $I = 2$, only the values $K = 0$, 2 are taken into account; for $I = 3$, only $K = 2$; for $I = 4$, only $K = 0$, 2, 4; for $I = 5$, only $K = 2$, 4; *etc.*

Intra-band E2 transitions between energy levels in the ground-state, γ, and β bands are shown in Fig. 1.

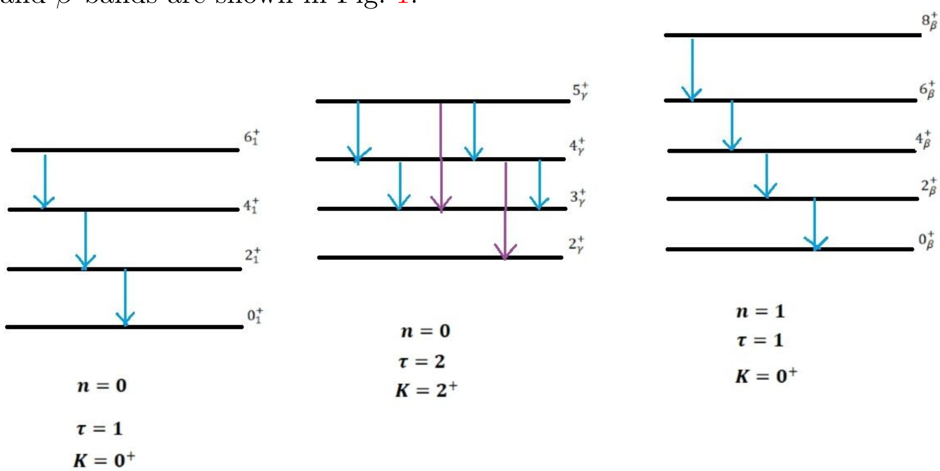

Fig. 1. Intra-band E2 transitions between energy levels in the ground-state, β, and γ bands (here, in the γ band, we can observe the branching of E2 transitions).

5. Intra-band E2-transition probabilities

Note that the intra-band E2 transitions occur with $\Delta I = 2$ or $\Delta K = 2$. There are permitted and forbidden transitions in the $\Delta K = 2$ case.

Intra-ground-state-band E2 transitions have the form $2I_{01} \to 2(I-2)_{01}$ and these E2 transitions are permitted. In this case, all members of the product $A^{\tau}_{IK} \times A^{\tau'}_{I'K'}$ of two normalized wave functions of the rigid triaxial rotator in Eq. (13) are taken into account.

In the same way, intra-β band denoted as $2I_{11} \to 2(I-2)_{11}$ and these E2 transitions are permitted. The product of the normalized wave functions of the rigid triaxial rotator has the same values as in ground-state-band transitions.

The E2 transitions in the γ band are more complex than those in the previous two because in this band, branching of E2 transitions between the odd and even or even and odd energy levels is observed, with the exception of $3_{01} \to 2_{02}$. For example, the E2 transition from the 4_{02} transition branches into two $4_{02} \to 2_{02}$ (as a result, E2 transition with $\Delta I = 2$) and $4_{02} \to 3_{01}$ (as a result, E2 transition with $\Delta K = 2$), and so on.

All E2 transitions between states with even values of the spins in the γ band, $2I_{02} \to 2(I-2)_{02}$, and odd values of the spins in the γ band, $(2I+1)_{01} \to (2I-1)_{01}$, are permitted. In this case, the product $A^{\tau}_{IK} \times A^{\tau'}_{I'K'}$ of the normalized wave functions of the rigid triaxial rotator has the same values as in ground-state band.

Intra-band E2 transitions between even and odd values of spins in the γ bands, $2I_{02} \to (2I-1)_{01}$, and *vice versa*, $(2I+1)_{01} \to 2I_{02}$, occur with $\Delta K = 2$; both permitted and forbidden E2 transitions exist in this case. Therefore, the contributions from forbidden E2 transitions must be excluded. For example, the E2 transition $3_{01} \to 2_{02}$ is forbidden. Instead, an M1+E2 transition is possible because $\Delta K = 0$ in this case [7].

The E2 transition $5_{01} \to 4_{02}$ is forbidden when $I_i = 5$ ($K_f = 2$) and $I_f = 4$ ($K_f = 2$). In contrast, the E2 transition $5_{01} \to 4_{02}$ is permitted when $I_i = 5$ ($K_f = 4$) and $I_f = 4$ ($K_f = 2$). In exactly the same way, the E2 transition $4_{02} \to 3_{01}$ is forbidden, whereas $4_{042} \to 3_{021}$ is permitted. The same reasoning can be applied to other E2 transitions considered here. In general, for equal values of the initial and final states of the components of the total angular momentum $K_i = K_f$, *i.e.* in the product $A^{\tau}_{IK} \times A^{\tau'}_{I'K'}$ of the normalized wave functions of the rigid triaxial rotator in expression (13) are excluded because E2 transitions between these states are forbidden. As a result, for convenience, the ratios of these probabilities in Eq. (17) can be rewritten as $B(\mathrm{E2};\, 2I_{n\tau} \to (2I-2)_{n\tau})$. However, the notation used in expression (17) is retained in Table 2 as well as in Figs. 2–7.

The dependences of intra-band reduced $B(\mathrm{E2};\, 2I_{01} \to (2I-2)_{01})$-transition probabilities on the triaxiality parameter γ_{eff} are presented in Figs. 2–7.

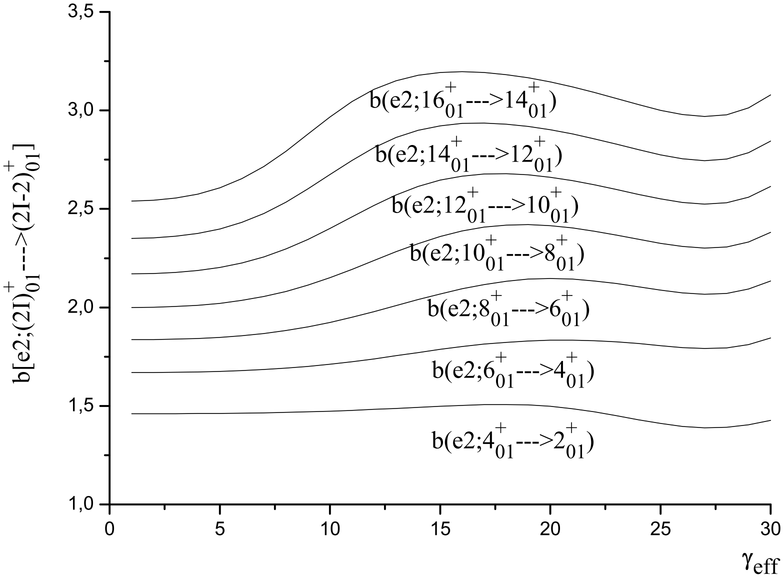

Fig. 2. Dependence of intra-band reduced $B(\mathrm{E}2;\ (2I)_{01} \to (2I-2)_{01})$-transition probabilities on the triaxiality parameter γ_{eff}. Here and in subsequent figures, $I = 1{,}2{,}3\ \ldots$

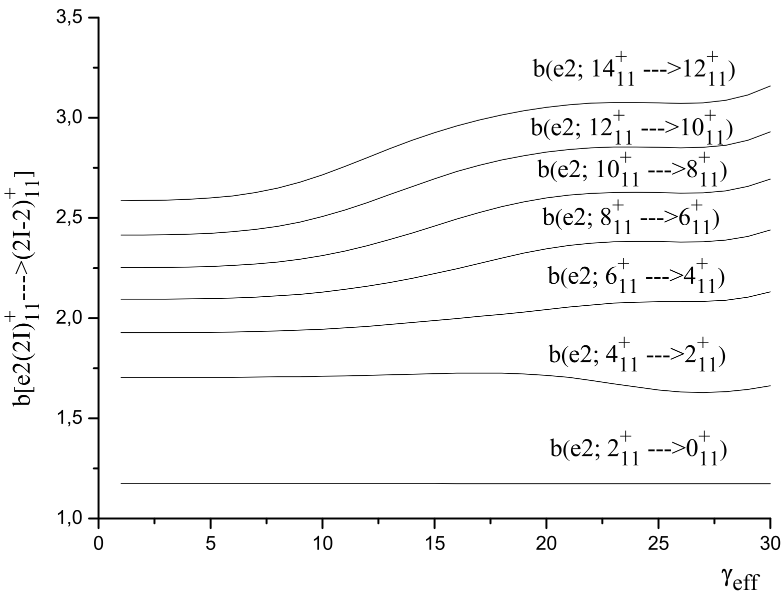

Fig. 3. The same as in Fig. 2, but for $B(\mathrm{E}2;\ (2I)_{11} \to (2I-2)_{11})$.

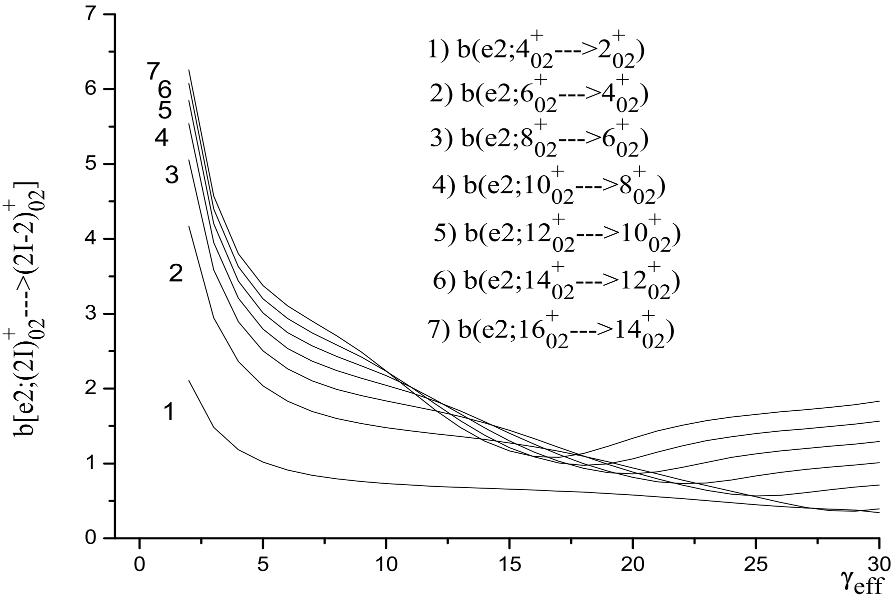

Fig. 4. The same as in Fig. 2, but for $B(\mathrm{E2};\ (2I)_{02} \to (2I-2)_{02})$.

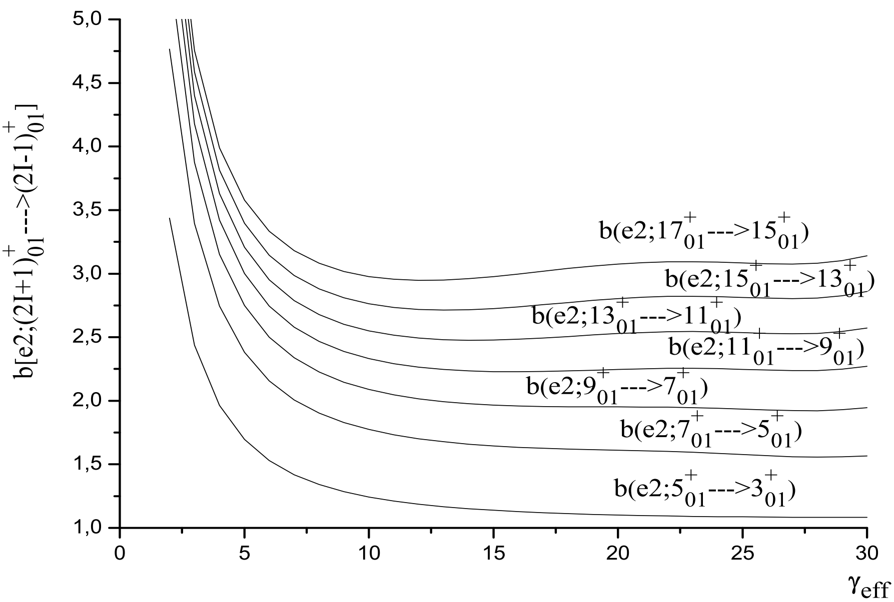

Fig. 5. The same as in Fig. 2, but for $B(\mathrm{E2};\ (2I+1)_{01} \to (2I-1)_{01})$.

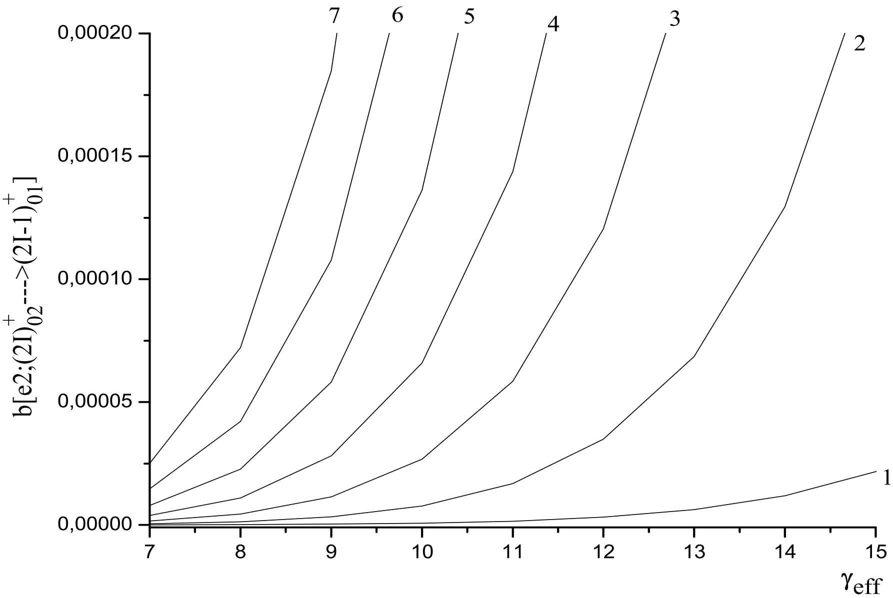

Fig. 6. The same as in Fig. 2, but for $B(\mathrm{E2};\,(2I)_{02} \to (2I-1)_{01})$.

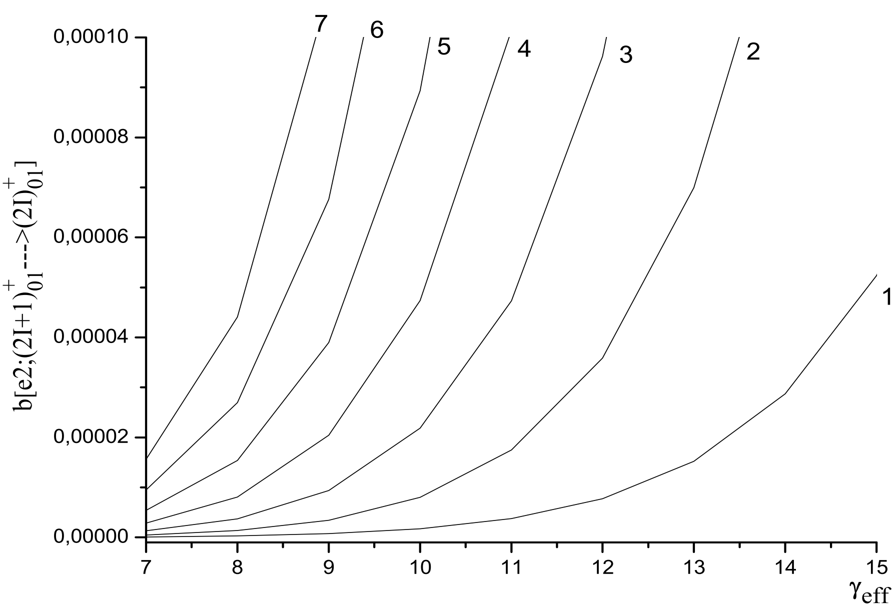

Fig. 7. The same as in Fig. 2, but for $B(\mathrm{E2};\,(2I+1)_{01} \to (2I)_{01})$.

6. Discussion

The dependence of intra-band reduced $B(\mathrm{E2};\, 2I_{01} \to (2I-2)_{01})$-transition probabilities on the parameter γ_{eff} is presented in Fig. 2. The value of $B(\mathrm{E2};\, 2I_{01} \to (2I-2)_{01})$ first increases, then decreases slightly, and finally increases slightly again with increasing triaxiality parameter γ_{eff}. The behavior of $B(\mathrm{E2};\, 2I_{01} \to (2I-2)_{01})$ at small values γ_{eff} approximately corresponds to that of spherical nuclei. $B(\mathrm{E2};\, 2I_{01} \to (2I-2)_{01})$ increases with increasing the values of the spin of the energy levels.

The dependence of intra-band reduced $B(\mathrm{E2};\, 2I_{11} \to (2I-2)_{11})$-transition probabilities on the parameter γ_{eff} is presented in Fig. 3. The value of $B(\mathrm{E2};\, 2I_{11} \to (2I-2)_{11})$ first increases with increasing γ_{eff}. The value of $B(\mathrm{E2};\, 2I_{01} \to (2I-2)_{01})$ slightly increases with increasing γ_{eff}. $B(\mathrm{E2};\, 2I_{11} \to (2I-2)_{11})$ increases with increasing the values of the spin of the energy levels.

The dependence of intra-band reduced $B(\mathrm{E2};\, 2I_{02} \to (2I-2)_{02})$-transition probabilities between even values of the spin on the parameter γ_{eff} is presented in Fig. 4. The $B(\mathrm{E2};\, 2I_{02} \to (2I-2)_{02})$ value first decreases with increasing triaxiality parameter γ_{eff}. However, the decrease of $B(\mathrm{E2};\, 2I_{02} \to (2I-2)_{02})$ transition between even spins is not monotonic with increasing γ_{eff}, and it increases again at last large values of the triaxiality parameter γ_{eff}. In our opinion, this is related to the behavior of the energy levels of the γ band in the asymmetric-rigid-rotator model, as noted in Section 3.

The dependence of intra-band reduced $B(\mathrm{E2};\, (2I+1)_{01} \to (2I-1)_{01})$-transition probabilities between odd values of the spin on the parameter γ_{eff} is presented in Fig. 5. It is evident from this figure that $B(\mathrm{E2};\, (2I+1)_{01} \to (2I-1)_{01})$ transition between odd spins decreases monotonically with increasing triaxiality parameter γ_{eff} over the entire range of its variation. The $B(\mathrm{E2};\, (2I+1)_{01} \to (2I-1)_{01})$ value increases with increasing the values of the spin of the energy levels.

Table 1 shows that, for small values of the triaxiality parameter γ_{eff}, the energy levels of the γ band in the asymmetric-rigid-rotator model take large values. Therefore, we do not show the corresponding values of the intra-band E2-transition probabilities in Fig. 4 and 5.

In Figs. 2–5, the intra-band E2 transitions occur with $\Delta I = 2$.

The dependence of intra-band reduced $B(\mathrm{E2};\, 2I_{02} \to (2I-1)_{01})$-transition probabilities between even and odd values of the spin on the parameter γ_{eff} is presented in Fig. 6. The numbers in the figure denote:

(1) $B(\mathrm{E2};\, 4_{02} \to 3_{01})$;

(2) $B(\mathrm{E2};\, 6_{02} \to 5_{01})$;

(3) $B(\mathrm{E2};\, 8_{02} \to 7_{01})$;

(4) $B(\mathrm{E2};\, 10_{02} \to 9_{01})$;

(5) $B(\mathrm{E2};\, 12_{02} \rightarrow 11_{01})$;
(6) $B(\mathrm{E2};\, 14_{02} \rightarrow 13_{01})$;
(7) $B(\mathrm{E2};\, 16_{02} \rightarrow 15_{01})$.

The dependence of intra-band reduced $B(\mathrm{E2};\, (2I+1)_{01} \rightarrow (2I-2)_{02})$-transition probabilities between odd and even values of the spin on the parameter γ_{eff} is presented in Fig. 7. The numbers in the figure denote:

(1) $B(\mathrm{E2};\, 5_{01} \rightarrow 4_{02})$;
(2) $B(\mathrm{E2};\, 7_{01} \rightarrow 6_{02})$;
(3) $B(\mathrm{E2};\, 9_{01} \rightarrow 8_{02})$;
(4) $B(\mathrm{E2};\, 11_{01} \rightarrow 10_{02})$;
(5) $B(\mathrm{E2};\, 13_{01} \rightarrow 12_{02})$;
(6) $B(\mathrm{E2};\, 15_{01} \rightarrow 14_{02})$;
(7) $B(\mathrm{E2};\, 17_{01} \rightarrow 16_{02})$.

In Figs. 6–7, the intra-band E2 transitions occur with $\Delta K = 2$. These transition probabilities increase steeply with increasing the triaxiality parameter γ_{eff}.

7. Result

A comparison of the theoretical and experimental [7] values of the reduced E2-transition probabilities (in W.u.) within the ground-state, γ, and β bands, as well as the values of the theoretical parameters g, γ_{eff}, and Q_0 is presented in Table 2 for the nuclei: ^{154}Sm, ^{158}Gd, ^{162,166}Er, ^{168}Hf, ^{232}Th, ^{236}U, ^{238}U, including high-spin states. The theoretical and experimental values of the reduced E2-transition probabilities are in good agreement.

This table shows that the theoretical and experimental values of the reduced probabilities of E2 transitions within the fundamental band are in good agreement. The values of such E2 transitions within the β, and γ bands are also listed in this table, although only in small numbers or as ratios. However, they also agree well with the experimental data [7]. It is clear that the results for the ^{166}Er nucleus provide more complex data regarding the energy levels of the γ band, including transitions with $\Delta K = 2$.

In the present work, for the description of the reduced E2-transition probabilities, we used the values of the theoretical parameters g and γ_{eff}, which were chosen as in the case of the description of the energy levels of the collective states in the even–even nuclei under consideration [6].

The values of the triaxiality parameter γ_{eff} deserve special attention. The values of this parameter within present approximation differ little from those obtained in [26, 27]. These values were obtained from microscopic calculations within the Monte Carlo shell model [26] and the proxy SU(3)

Table 2. Comparison of theoretical and experimental values [7] of the reduced E2-transition probabilities (in W.u.) intra ground-state, β, and γ bands. As well as the values of the parameters g, γ_{eff}, Q_0 are presented.

Nuclei	E2 transition	Exp.	Theory
^{154}Sm	$B(\mathrm{E2};\, 2_{01} \to 0_{01})$	176.3	175.65
$g = 20.638$	$B(\mathrm{E2};\, 4_{01} \to 2_{01})$	245.6	259.38
$\gamma_{\mathrm{eff}} = 9.183°$	$B(\mathrm{E2};\, 6_{01} \to 4_{01})$	298.8	301.95
$\mathrm{Q}_0 = 12.4$	$B(\mathrm{E2};\, 8_{01} \to 6_{01})$	319.1	339.62
$\gamma_{\mathrm{hw}} = 4.31°$ [27]	$B(\mathrm{E2};\, 10_{01} \to 8_{01})$	314.1	378.67
	$B(\mathrm{E2};\, 12_{01} \to 10_{01})$	282^{+19}_{-17}	420.51
^{158}Gd	$B(\mathrm{E2};\, 2_{01} \to 0_{01})$	198.5	197.42
$g = 23.467$	$B(\mathrm{E2};\, 4_{01} \to 2_{01})$	290.4	290.89
$\gamma_{\mathrm{eff}} = 10.038°$	$B(\mathrm{E2};\, 6_{01} \to 4_{01})$	—	337.53
$Q_0 = 13.2$	$B(\mathrm{E2};\, 8_{01} \to 6_{01})$	330.3	378.23
$\gamma_{\mathrm{eff}} = 5.9°$ [26]	$B(\mathrm{E2};\, 10_{01} \to 8_{01})$	340.3	420.27
$\gamma_{\mathrm{hw}} = 5.72°$ [27]	$B(\mathrm{E2};\, 12_{01} \to 10_{01})$	310.3	465.30
	$B(\mathrm{E2};\, 4_{11} \to 2_{11})$	456^{+912}_{-67}	415.13
	$B(\mathrm{E2};\, 4_{02} \to 2_{02})$	113^{+166}_{-13}	—
^{162}Er	$B(\mathrm{E2};\, 6_{01} \to 4_{01})/B(\mathrm{E2};\, 4_{01} \to 2_{01})$	1.01	1.35308
$g = 12.7$	$B(\mathrm{E2};\, 8_{01} \to 6_{01})/B(\mathrm{E2};\, 6_{01} \to 4_{01})$	0.91	1.32980
$\gamma_{\mathrm{eff}} = 13.5°$	$B(\mathrm{E2};\, 10_{01} \to 8_{01})/B(\mathrm{E2};\, 8_{01} \to 6_{01})$	0.80	1.29140
$\gamma_{\mathrm{eff}} = 8.5°$ [26]	$B(\mathrm{E2};\, 12_{01} \to 10_{01})/B(\mathrm{E2};\, 10_{01} \to 8_{01})$	0.65	1.24351
$\gamma_{\mathrm{hw}} = 5.92°$ [27]	$B(\mathrm{E2};\, 14_{01} \to 12_{01})/B(\mathrm{E2};\, 12_{01} \to 10_{01})$	0.38	1.20244
	$B(\mathrm{E2};\, 16_{01} \to 14_{01})/B(\mathrm{E2};\, 14_{01} \to 12_{01})$	1.01	1.17252
^{166}Er	$B(\mathrm{E2};\, 2_{01} \to 0_{01})$	217.5	217.73
$g = 34.648$	$B(\mathrm{E2};\, 4_{01} \to 2_{01})$	312.11	319.53
$\gamma_{\mathrm{eff}} = 12.632°$	$B(\mathrm{E2};\, 6_{01} \to 4_{01})$	370.2	368.49
$Q_0 = 14$	$B(\mathrm{E2};\, 8_{01} \to 6_{01})$	373.14	410.09
$\gamma_{\mathrm{eff}} = 8.2°$ [26]	$B(\mathrm{E2};\, 10_{01} \to 8_{01})$	390.17	452.53
$\gamma_{\mathrm{hw}} = 12.12°$ [27]	$B(\mathrm{E2};\, 12_{01} \to 10_{01})$	372.21	497.17
	$B(\mathrm{E2};\, 14_{01} \to 12_{01})$	—	543.25
	$B(\mathrm{E2};\, 16_{01} \to 14_{01})$	330.18	589.89
	$B(\mathrm{E2};\, 4_{02} \to 3_{01})/B(\mathrm{E2};\, 4_{02} \to 2_{02})$	1.5	0.1
	$B(\mathrm{E2};\, 5_{01} \to 4_{02})/B(\mathrm{E2};\, 5_{01} \to 3_{01})$	13	1.6
	$B(\mathrm{E2};\, 6_{02} \to 5_{01})/B(\mathrm{E2};\, 6_{02} \to 4_{01})$	2.5	0.05
	$B(\mathrm{E2};\, 7_{01} \to 6_{02})/B(\mathrm{E2};\, 7_{01} \to 5_{01})$	27.8	1.5

Continuation of Table 2

Nuclei	E2 transition	Exp.	Theory
^{168}Hf	$B(\mathrm{E2};\, 2_{01} \to 0_{01})$	154.71	153.06
$g = 5.9$	$B(\mathrm{E2};\, 4_{01} \to 2_{01})$	244.12	237.37
$\gamma_{\mathrm{eff}} = 14.3^\circ$	$B(\mathrm{E2};\, 6_{01} \to 4_{01})$	285.18	344.45
$Q_0 = 13.4$	$B(\mathrm{E2};\, 8_{01} \to 6_{01})$	350.53	484.66
$\gamma_{\mathrm{eff}} = 13.41^\circ$ [26]	$B(\mathrm{E2};\, 10_{01} \to 8_{01})$	370.61	648.33
$\gamma_{\mathrm{hw}} = 8.2^\circ$ [27]	$B(\mathrm{E2};\, 12_{01} \to 10_{01})$	320.12	823.32
	$B(\mathrm{E2};\, 14_{01} \to 12_{01})$	240.52	1004.37
	$B(\mathrm{E2};\, 16_{01} \to 14_{01})$	260.61	1191.23
^{232}Th	$B(\mathrm{E2};\, 2_{01} \to 0_{01})$	198.1	197.37
$g = 29.979$	$B(\mathrm{E2};\, 4_{01} \to 2_{01})$	286.24	289.26
$\gamma_{\mathrm{eff}} = 9.951^\circ$	$B(E2; 6_{01} \to 4_{01})$	326.22	332.77
$Q_0 = 13.4$	$B(\mathrm{E2};\, 8_{01} \to 4_{01})$	344.15	369.09
	$B(\mathrm{E2};\, 10_{01} \to 8_{01})$	363.2	405.78
	$B(\mathrm{E2};\, 12_{01} \to 10_{01})$	370.3	444.81
	$B(\mathrm{E2};\, 14_{01} \to 12_{01})$	390.3	486.45
	$B(\mathrm{E2};\, 16_{01} \to 14_{01})$	390.4	530.34
	$B(\mathrm{E2};\, 18_{01} \to 16_{01})$	450.7	575.89
	$B(\mathrm{E2};\, 20_{01} \to 18_{01})$	360.6	622.54
	$B(\mathrm{E2};\, 22_{01} \to 20_{01})$	420.1	669.85
	$B(\mathrm{E2};\, 24_{01} \to 22_{01})$	240.7	717.59
	$B(\mathrm{E2};\, 26_{01} \to 24_{01})$	350.12	765.65
	$B(\mathrm{E2};\, 28_{01} \to 26_{01})$	700	813.98
^{236}U	$B(\mathrm{E2};\, 2_{01} \to 0_{01})$	248.8	246.38
$g = 56.705$	$B(\mathrm{E2};\, 4_{01} \to 2_{01})$	340.24	357.12
$\gamma_{\mathrm{eff}} = 8.703^\circ$	$B(\mathrm{E2};\, 6_{01} \to 4_{01})$	385.2	403.42
$Q_0 = 15.1$	$B(\mathrm{E2};\, 8_{01} \to 6_{01})$	390.4	437.24
	$B(\mathrm{E2};\, 10_{01} \to 8_{01})$	360.4	468.64
	$B(\mathrm{E2};\, 12_{01} \to 10_{01})$	410.7	500.73
	$B(\mathrm{E2};\, 14_{01} \to 12_{01})$	450.5	534.56
	$B(\mathrm{E2};\, 16_{01} \to 14_{01})$	380.4	570.38
	$B(\mathrm{E2};\, 18_{01} \to 16_{01})$	490.5	608.07
	$B(\mathrm{E2};\, 20_{01} \to 18_{01})$	510.8	647.34
	$B(\mathrm{E2};\, 22_{01} \to 20_{01})$	520.12	687.84
	$B(\mathrm{E2};\, 24_{01} \to 22_{01})$	670.13	729.26
	$B(\mathrm{E2};\, 26_{01} \to 24_{01})$	670.19	771.33
	$B(\mathrm{E2};\, 28_{01} \to 26_{01})$	1100.5	813.17

Continuation of Table 2

Nuclei	E2 transition	Exp.	Theory
^{238}U	$B(\mathrm{E2};\, 2_{01} \to 0_{01})$	281.4	281.75
$g = 66.513$	$B(\mathrm{E2};\, 4_{01} \to 2_{01})$	—	407.53
$\gamma_{\mathrm{eff}} = 8.1^\circ$	$B(E2; 6_{01} \to 4_{01})$	—	458.70
$Q_0 = 16.2$	$B(\mathrm{E2};\, 8_{01} \to 6_{01})$	410.6	494.80
	$B(\mathrm{E2};\, 10_{01} \to 8_{01})$	480.6	527.42
	$B(\mathrm{E2};\, 12_{01} \to 10_{01})$	500.5	560.21
	$B(\mathrm{E2};\, 14_{01} \to 12_{01})$	491.38	594.53
	$B(\mathrm{E2};\, 16_{01} \to 14_{01})$	490.2	630.81
	$B(\mathrm{E2};\, 18_{01} \to 16_{01})$	480.3	669.07
	$B(\mathrm{E2};\, 20_{01} \to 18_{01})$	460.4	709.12
	$B(\mathrm{E2};\, 22_{01} \to 20_{01})$	490.75	750.67
	$B(\mathrm{E2};\, 24_{01} \to 22_{01})$	530.85	793.41
	$B(\mathrm{E2};\, 26_{01} \to 24_{01})$	585.60	837.07
	$B(\mathrm{E2};\, 28_{01} \to 26_{01})$	540.13	881.43
	$B(\mathrm{E2};\, 30_{01} \to 28_{01})$	185.2	926.33

approximation to the shell model [27]. They are also presented in the first column of Table 2. It should be noted that Ref. [26] yielded different values for γ for the intra-band $(2^+_{01} \to 0^+_{01})$ and inter-band $(2^+_{02} \to 0^+_{01})$ E2 transitions. In our opinion, this issue requires further clarification.

8. Conclusion

In this paper, the dynamics of prolate quadrupole shape excitations in even–even heavy nuclei are considered within the framework of the Davydov–Chaban model. The Davidson potential is used for the β variable. Expressions for the energy levels and wave functions of the excited collective states were obtained. The E2-transition probabilities in the excited collective states of even–even lanthanide and actinide nuclei were studied, and the obtained results were compared with experimental data. The branching of E2 transitions between the energy levels of the γ band was studied in detail. Permitted and forbidden intra-γ-band E2 transitions were discussed. The sensitivity of the intra-band reduced E2-transition probabilities to the effective triaxiality parameter γ_{eff} was analyzed.

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