ASYMPTOTIC NORMALIZATION COEFFICIENT OF ^{9}Be IN THE $p + {}^8$Li CHANNEL FROM THE ^{9}Be$(d, {}^3$He$)^8$Li REACTION

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The analysis of the differential cross section of the ^{9}Be$(d, {}^3$He$)^8$Li reaction measured at an incident energy of 52 MeV was performed within the framework of the modified distorted wave Born approximation. The peripheral character of the reaction was tested. The results of the calculations show that the considered reaction is peripheral at the main maximum of the angular distribution of the reaction under consideration. The squared asymptotic normalization coefficient (ANC) values of ^{9}Be in the $p + {}^8$Li channel were extracted from the experimental data.

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1. Introduction

ANCs determine the asymptotics of the wave functions of bound nuclear states a in binary channels $b + c$. The first thorough review of the nuclear vertex constant (NVC) and its relationship with the ANC was presented in Ref. [1]. In this work, the theoretical foundation of the ANC was also introduced and its role in the nuclear reaction theory was discussed. The role of the ANC in the theory of nuclear reactions with the charged particles was given in Ref. [2]. The residue in the poles of the elastic-scattering S-matrix corresponding to bound states [3] and in resonances [4] can be expressed in terms of the ANC in the scattering theory. The comprehensive review [5] presents the main properties and definitions of the ANC, its relation to the

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reduced width, subthreshold resonances and resonances, and the effective-range expansion of the elastic-scattering amplitude. It also discusses the role of the ANC in nuclear reactions, the ANC method in nuclear astrophysics, and the ANC within the R-matrix approach.

The ANC is a fundamental nuclear characteristic of bound states [1] and the partial resonance width is expressed in terms of the ANC [6]. The ANC plays an important role in low-energy elastic scattering, nuclear transfer, and radiative capture reactions [5]. The experimentally determined ANCs can be used to calculate the astrophysical S-factors of the direct radiative capture [7, 8] and resonance [9, 10] reactions.

In contrast to the spectroscopic factors, the ANCs are the on-shell observables [11]. They cannot be directly measured in the experiment and special methods are used to extract them from the experimental data. There are several approaches to determining the ANCs from the experimental data such as the modified distorted wave Born approximation (MDWBA) [12], analytic continuation of the differential cross section to the pole [13], the modified two-body potential approach [14], and analytic continuation of elastic-scattering data to the negative-energy region [15]. MDWBA is one of the most widely used methods for determining ANC from transfer reactions.

The radiative capture $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ reaction is important in the inhomogeneous Big Bang nucleosynthesis. The $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ reaction breaks the bridge of gap elements and heavier nuclei are formed after the breaking of $A = 8$ [16]. The $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ reaction is also of considerable astrophysical importance since it is one of the elements of the primordial nucleosynthetic chain [16].

The astrophysical S-factor for the $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ reaction has been studied via several theoretical methods (or models) such as the modified potential cluster model with forbidden states with the classification of the orbital cluster states according to Young diagrams [17], the potential model [18, 19], and direct capture model using spectroscopic factor [20]. In Ref. [20], the spectroscopic factor of $^{9}\mathrm{Be}$ was obtained to be $Z_{1\,3/2} = 0.64$ from the analysis of the angular distribution of the $^{8}\mathrm{Li}(d,n)^{9}\mathrm{Be}$ reaction and astrophysical S-factor for the $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ process was calculated via direct capture model. The authors obtained the S-factor at zero energy to be $S(0) = 0.16 \pm 0.08$ keV b. In Ref. [19], the spectroscopic factors $Z_{1\,3/2} = 1.50 \pm 0.28$ and $Z_{1\,1/2} = 0.17 \pm 0.03$ were obtained from the analysis of the $^{9}\mathrm{Be}(^{8}\mathrm{Li},^{9}\mathrm{Be})^{8}\mathrm{Li}$ reaction. The S-factor obtained in [19] for the $^{8}\mathrm{Li}(p,\gamma)^{9}\mathrm{Be}$ reaction is higher by a factor of 2.6 than the value obtained from Ref. [20]. This discrepancy comes from the spectroscopic factors used in the calculations in [19] and $Z_{1\,3/2} = 0.64$ in Ref. [20]. In addition, only the proton transfer to the $1p_{3/2}$

orbit in ^{9}Be was taken into account, while the contribution from the $1p_{1/2}$ orbit was neglected in Ref. [20]. On the other hand, experimental studies in the astrophysical energy range are insufficient. The measurement of this reaction was performed in this energy range more than 60 years ago [21]. To date, no modern measurements of this reaction exist in the astrophysical energy range. In this regard, up to now, the problem of reliable extrapolation of measured astrophysical S-factors to experimentally inaccessible energy regions (≤ 25 keV) and of correct theoretical predictions has not been entirely solved, and it may considerably influence the predictions of the solar models. In turn, the solution of this problem is impossible without obtaining the very low energy S-factors for this reaction.

One of the possible solutions of this problem is based on the idea that at stellar energies, the amplitude of the direct capture reaction of astrophysical interest proceeds through the tail of the overlap integral [22], and, hence, is completely determined by the respective ANC. The ANC for the $^8\mathrm{Li}+p \to {}^9$Be system specifies the amplitude of the tail of the radial overlap function for the system. It follows from the results that the direct astrophysical S-factors for the $^8\mathrm{Li}(p,\gamma)^9$Be reaction can be correctly determined through the ANC for the $^8\mathrm{Li}+p \to {}^9$Be system which is extracted from the experimental data of the peripheral transfer reactions within the framework of MDWBA. However, until now, the ANC of ^{9}Be nucleus in the $^8\mathrm{Li}+p$ channel has not been extracted from the experimental data. In this regard, it is of great interest to extract the ANC value of ^{9}Be nucleus in the $^8\mathrm{Li}+p$ channel from the experimental data of the $^9\mathrm{Be}(d,{}^3\mathrm{He})^8$Li reaction.

In this study, we extracted the squared ANC value of ^{9}Be nucleus in the $^8\mathrm{Li}+p$ channel from the analysis of the proton transfer reaction $^9\mathrm{Be}(d,{}^3\mathrm{He})^8$Li, which was measured in [23], within the framework of MDWBA [12].

The paper is organized as follows. Section 2 is devoted to the analysis of the differential cross section of the $^9\mathrm{Be}(d,{}^3\mathrm{He})^8$Li reaction and the determination of the ANC of ^{9}Be nucleus in the $^8\mathrm{Li}+p$ channel. In Section 3, conclusions are given.

2. Analysis of the ^{9}Be(d,^{3}He)^{8}Li reaction

For the considered reaction, the orbital angular momentum of the transferred proton in the $^9\mathrm{Be} \to {}^8\mathrm{Li}+p$ configuration is equal to $l_{p^8\mathrm{Li}}=1$. The corresponding total angular momentum values are equal to $j_{p^8\mathrm{Li}}=1/2$ and $j_{p^8\mathrm{Li}}=3/2$. For the $^3\mathrm{He} \to d+p$ configuration, the orbital and total angular momenta are equal to $l_{pd}=0$ and $j_{pd}=1/2$, respectively.

2.1. Basic formulas of the modified DWBA

In the framework of MDWBA, the differential cross section (DCS) in the region of the main peak of the angular distribution for the peripheral transfer reaction $^9\mathrm{Be}(d,{}^3\mathrm{He})^8\mathrm{Li}$ ($^3\mathrm{He} \to d+p$ and $^9\mathrm{Be} \to {}^8\mathrm{Li}+p$) is parametrized in terms of the product of the squares of ANCs, and the DCS has the form [12] (see Refs. [24, 25] and references therein)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = C^2_{1\,1/2} C^2_{dp;\,0\,1/2} \left(R_{1\,1/2}\left(E,\theta; b_{1\,1/2}, b_{dp;\,0\,1/2}\right) + \lambda R_{1\,3/2}\left(E,\theta; b_{1\,3/2}, b_{dp;\,0\,1/2}\right)\right). \tag{1}$$

Here,

$$R_{1\,1/2}\left(E,\theta; b_{1\,1/2}, b_{dp;\,0\,1/2}\right) = \frac{\sigma^{\mathrm{DW}}_{1\,1/2}\left(E,\theta; b_{1\,1/2},, b_{dp;\,0\,1/2}\right)}{b^2_{dp;\,0\,1/2}\; b^2_{1\,1/2}} \tag{2}$$

and

$$R_{1\,3/2}\left(E,\theta; b_{1\,3/2}, b_{dp;\,0\,1/2}\right) = \frac{\sigma^{\mathrm{DW}}_{1\,3/2}\left(E,\theta; b_{1\,3/2}, b_{dp;\,0\,1/2}\right)}{b^2_{dp;\,0\,1/2}\; b^2_{1\,3/2}}, \tag{3}$$

where $\sigma^{\mathrm{DW}}_{1\,1/2}(E,\theta; b_{1\,1/2}, b_{dp;\,0\,1/2})$ and $\sigma^{\mathrm{DW}}_{1\,3/2}(E,\theta; b_{1\,3/2}, b_{dp;\,0\,1/2})$ are the single-particle cross sections corresponding to the total angular momenta $j_{p^8\mathrm{Li}} = 1/2$ and $j_{p^8\mathrm{Li}} = 3/2$, respectively. $b_{dp;0\,1/2}$ is a single-particle ANC which determines the amplitude of the tail of the shell model wave function corresponding to the bound state $^3\mathrm{He} = d + p$. $b_{1\,1/2}$ and $b_{1\,3/2}$ are the single-particle ANCs which determine the amplitudes of the tails of the shell model wave functions of the $p_{1/2}$ and $p_{3/2}$ components in the bound state $^8\mathrm{Li} + p = {}^9\mathrm{Be}$, respectively. $C_{1\,1/2}$ and $C_{1\,3/2}$ are the ANCs of the $p_{1/2}$ and $p_{3/2}$ components in the ^{9}Be ground state, respectively. $C_{dp;0\,1/2}$ is the ANC for the $d + p \to {}^3\mathrm{He}$ system. $\lambda = C^2_{13/2}/C^2_{1\,1/2}$ is the ratio of ANCs.

According to Ref. [12], at fixed values of $C^2_{dp;\,0\,1/2}$, λ and the optical-model parameters for the initial and final states, the peripheral character of the reaction under consideration in the region of the main peak of the angular distribution is ensured by the condition

$$R_{l_{p^8\mathrm{Li}}\, j_{p^8\mathrm{Li}}}\left(E,\theta; b_{l_{p^8\mathrm{Li}}\, j_{p^8\mathrm{Li}}}, b_{dp;\,0\,1/2}\right) = f(E,\theta) = \mathrm{const}. \tag{4}$$

This means that for all scattering angles and a fixed value of the energy E, the left-hand side of Eq. (4) should not depend on $b_{1\,1/2} = b_{1\,1/2}(r_0, a)$, $b_{1\,3/2} = b_{1\,3/2}(r_0, a)$, and $b_{0\,1/2} = b_{dp;\,0\,1/2}(r_0, a)$. These quantities, in turn, are functions of the geometrical parameters of the Woods–Saxon (W–S)

potential, namely the radius r_0 and the diffuseness a. Then, from Eqs. (1) and (4), the following condition

$$C^2_{1\,1/2} = \frac{\left(\frac{\mathrm{d}\sigma^{\mathrm{exp}}}{\mathrm{d}\Omega}\right)_{\theta=\theta_{\mathrm{peak}}}}{C^2_{dp;\,0\,1/2}\left(R_{1\,1/2}\left(E,\theta;b_{1\,1/2},b_{dp;\,0\,1/2}\right)+\lambda R_{1\,3/2}\left(E,\theta;b_{1\,3/2},b_{dp;\,0\,1/2}\right)\right)} = \text{const.}\,, \tag{5}$$

must be fulfilled for each fixed energy E, the scattering angle θ, and the function of $R_{l_{p^8\mathrm{Li}}\,j_{p^8\mathrm{Li}}}(E,\theta;b_{l_{p^8\mathrm{Li}}\,j_{p^8\mathrm{Li}}},b_{dp;\,0\,1/2})$ from Eq. (4). Fulfillment (or weak violation within the errors of the experimental differential cross section) of conditions (4) and (5) makes it possible to obtain the experimental value of squared ANC of ^{9}Be nucleus in the $^8\mathrm{Li} + p$ channel.

We note that the spectroscopic factor $Z_{1\,1/2}$, which is the norm of the radial overlap function of the bound state ^{9}Be wave function in the $^8\mathrm{Li} + p$ channel, is related to the ANC $C_{1\,1/2}$ by the equation [1]

$$C_{1\,1/2} = Z^{1/2}_{1\,1/2}\, b_{1\,1/2}\,. \tag{6}$$

The same relation holds for the ANC $C_{1\,3/2}$ and the spectroscopic factor $Z_{1\,3/2}$.

2.2. Analysis of the $^9Be(d,{}^3He)^8Li$ reaction within MDWBA

The DCS of the proton transfer reaction $^9\mathrm{Be}(d,{}^3\mathrm{He})^8\mathrm{Li}$ was measured at a deutron incident energy of 52 MeV in [23], and the spectroscopic factor was extracted from the experimental data using the DWBA calculation. In this section, we performed analysis of the DCS of this reaction.

The calculations were performed using the DWUCK5 code within the Nuclear Reaction Video (NRV) program [32]. The ANC value for the $d + p \to {}^3$He channel was taken to be 4.28 ± 0.5 fm^{-1} from Ref. [26]. The optical potential (OP) sets for the entrance and exit channels are given in Table 1. The parameters of OPs for the entrance $^9\mathrm{Be} + d$ channel, labelled A1 and A2, were taken from Refs. [27, 28]. The parameters of OPs for the exit $^8\mathrm{Li} + {}^3$He channel were taken from Refs. [29, 30], and they are labelled as B1 and B2. No dedicated optical potential parameters are available in the literature for the $^3\mathrm{He} + {}^8$Li system. For the $^3\mathrm{He} + {}^8$Li exit channel, the B1 optical potential parameters, originally derived for the $^3\mathrm{He} + {}^{16}$O system [24], are adopted. This choice is supported by their successful use in Ref. [23] to reproduce the angular distribution of the $^9\mathrm{Be}(d,{}^3\mathrm{He})^8\mathrm{Li}$ reaction within the zero range DWBA framework. In Table 1, we have used standard notations: V and W are the depths of the real and imaginary volume potentials, r_V (r_W) and a_V (a_W) are the radius and diffuseness parameters of the

real (imaginary) potential. $W_{\rm D}$ and $V_{\rm SO}$ are the depths of the surface and spin-orbit potentials, $r_{\rm D}$ ($r_{\rm SO}$) and $a_{\rm D}$ ($a_{\rm SO}$) are the radius and diffuseness parameters of the surface (spin-orbit) potential. $r_{\rm C}$ is the Coulomb radius parameter. To determine the ratio of ANCs λ in Eq. (1), according to the results of Ref. [31], the squared ANC values of ^{9}Be in the $p + {}^8$Li channel for the total angular momenta $j_{p^8{\rm Li}} = 1/2$ and $j_{p^8{\rm Li}} = 3/2$ were taken equal to 8.07 fm^{-1} and 53 fm^{-1}, respectively. Hence, for the ratio of ANCs, $\lambda = C^2_{13/2}/C^2_{1\,1/2} = 6.58$ was used in the calculations.

Table 1. The parameters of OPs for the initial and exit channels for the ^{9}Be$(d, {}^3$He$)^8$Li reaction. The depth of the potentials is given in MeV and the geometry parameters are in fm.

Parameters	A1	A2	B1	B2
V_0	63.68	80.45	190.0	152.39
r_0	1.250	1.136	1.140	0.980
a_0	0.760	0.745	0.675	0.792
W		4.338	11.20	1.721
r_W		1.353	2.170	1.350
a_W		0.647	0.426	0.801
$W_{\rm D}$	13.00	9.239		14.11
$r_{\rm D}$	1.250	1.407		1.350
$a_{\rm D}$	0.668	0.660		0.801
$V_{\rm SO}$	6.000	3.557		
$r_{\rm SO}$	1.250	0.972		
$a_{\rm SO}$	0.750	1.011		
$r_{\rm C}$	1.300	1.303	1.400	1.300
Refs.	[27]	[28]	[29]	[30]

We tested the peripheral character of the reaction under consideration. To test this, we systematically varied the parameters r_0 and a of the adopted W–S potential within the ranges $1.13 \le r_0 \le 1.37$ fm and $0.59 \le a \le 0.71$ fm, relative to the standard values $r_0 = 1.25$ fm and $a = 0.65$ fm. For each (r_0, a) pair, the depth of the W–S potential was adjusted to reproduce the proton binding energy.

Variations of the parameters r_0 and a result in changes of the single-particle ANCs within the interval $5.76 \le b_{1\,1/2} \le 8.76$ fm$^{-1/2}$ for $j_{p^8{\rm Li}} = 1/2$, $6.31 \le b_{1\,3/2} \le 9.45$ fm$^{-1/2}$ for $j_{p^8{\rm Li}} = 3/2$, and $1.73 \le b_{dp;0\,1/2} \le$

1.93 fm$^{-1/2}$. Figure 1 illustrates the dependence of the $R_{1\,1/2}(\theta, b_{1\,1/2})$ and $R_{1\,3/2}(\theta, b_{1\,3/2})$ functions on the single-particle ANCs of the $p_{1/2}$ and $p_{3/2}$ components in the ^{9}Be ground state, respectively. The width of the band reflects the minor residual dependence of the R functions on the parameters r_0 and a. Variations of the parameters r_0 and a cause R functions to change in the range $8.13 \times 10^{-2} \leq R_{1\,1/2} \leq 9.03 \times 10^{-2}$ mb/sr fm and $7.04 \times 10^{-2} \leq R_{1\,3/2} \leq 7.54 \times 10^{-2}$ mb/sr fm for the $p_{1/2}$ and $p_{3/2}$ components in the ^{9}Be ground state, respectively.

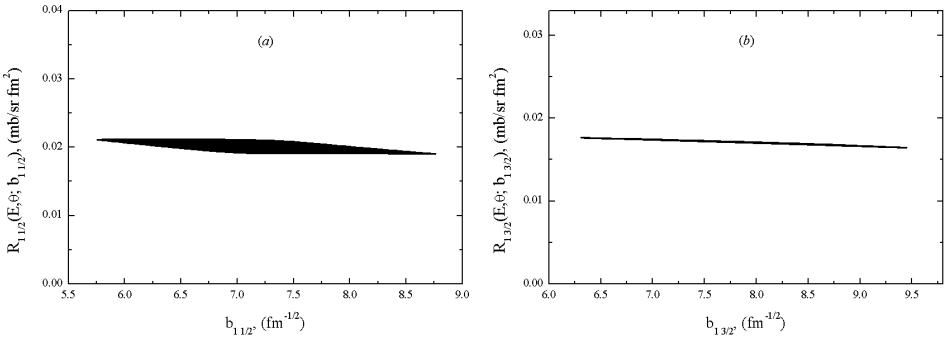


Fig. 1. The dependence of the R functions on the single-particle ANCs of the $p_{1/2}$ and $p_{3/2}$ components in the ^{9}Be ground state at an energy of 52 MeV using optical potential set (A1+B1) at the angle $\theta_{\rm peak} = 13.09°$ at $b_{0\,1/2} = 1.84$ fm$^{-1/2}$. In (a) and (b), the calculated R function is plotted for $j_{p^8{\rm Li}} = 1/2$ and $j_{p^8{\rm Li}} = 3/2$, respectively. The band widths in the graphs correspond to the variation of the geometric parameters r_0 and a within the ranges $1.13 \leq r_0 \leq 1.37$ fm and $0.59 \leq a \leq 0.71$ fm.

For each value of the single-particle ANC, we calculated the spectroscopic factors and ANCs of ^{9}Be in the p+^{8}Li channel. The dependence of the ANCs and the spectroscopic factors on the single-particle ANCs at $\theta_{\rm peak} = 13.09°$ are presented in Fig. 2. The squared ANCs varies slightly within the narrow ranges of $9.95 \leq C^2_{1\,1/2} \leq 10.72$ fm^{-1} and $65.49 \leq C^2_{1\,3/2} \leq 70.57$ fm^{-1} when changing the W–S potential parameters r_0 and a within the above-mentioned interval. In Table 2, the dependence of the single-particle ANCs, the squared ANCs, and R functions at $\theta_{\rm peak} = 13.09°$ on the parameters (r_0, a) in the region $1.13 \leq r_0 \leq 1.37$ fm and $0.59 \leq a \leq 0.71$ fm are presented. The uncertainty in the ANC $C^2_{1\,1/2}$ corresponding to the relative difference between the central value of $C^2_{1\,1/2}$ (for $r_0 = 1.25$ fm and $a = 0.65$ fm) and the lowest and highest values of the geometrical parameters varying in the above-mentioned region is ±4.7%. The same uncertainty in the ANC $C^2_{1\,3/2}$ is ±4.5%. This implies that ANCs are relatively insensitive to the single-particle ANC vari-

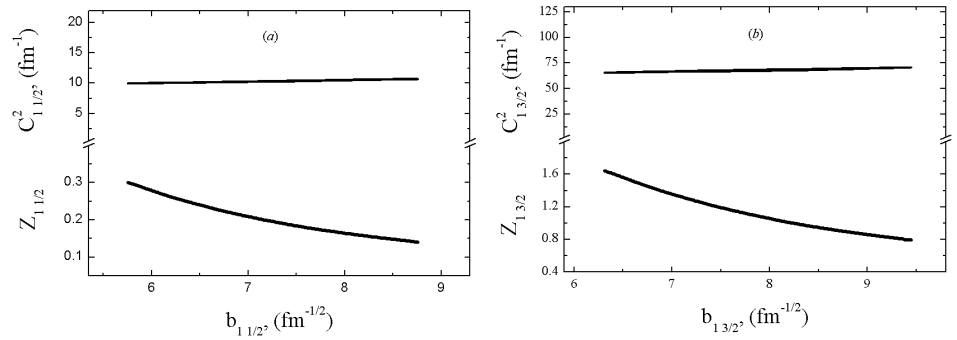


Fig. 2. The behavior of the squared ANCs $C^2_{1\,3/2}$ and $C^2_{1\,1/2}$ [upper bands in panels (a) and (b)] on the single-particle ANCs $b_{1\,3/2}$ and $b_{1\,1/2}$, respectively, at an energy of 52 MeV using optical potential set (A1+B1) at the angle $\theta_{\rm peak} = 13.09°$. The dependence of the spectroscopic factors $Z_{1\,3/2}$ and $Z_{1\,1/2}$ [lower bands in panels (a) and (b)] on the single-particle ANCs $b_{1\,3/2}$ and $b_{1\,1/2}$, respectively, at an energy of 52 MeV using optical potential set (A1+B1) at the angle $\theta_{\rm peak} = 13.09°$. The width of the bands corresponds to the variation of the geometric parameters r_0 and a within the ranges $1.13 \le r_0 \le 1.37$ fm and $0.59 \le a \le 0.71$ fm.

Table 2. The dependence of the R functions on the single-particle ANCs of the $p_{1/2}$ and $p_{3/2}$ components in the ^{9}Be ground state for the ^{9}Be$(d,{}^3$He$)^8$Li reaction. The calculation have been carried out with the optical potential set (A1+B1) at the angle $\theta_{\rm peak} = 13.09°$.

r_0 [fm]	a [fm]	$b_{1\,1/2}$ [fm$^{-1/2}$]	$b_{1\,3/2}$ [fm$^{-1/2}$]	$C^2_{1\,1/2}$ [fm^{-1}]	$C^2_{1\,3/2}$ [fm^{-1}]	$R_{1\,1/2}$ [mb/sr fm^2]	$R_{1\,3/2}$ [mb/sr fm^2]
1.13	0.59	5.76	6.31	9.96	65.53	2.11×10^{-2}	1.76×10^{-2}
1.13	0.71	7.05	7.41	10.29	67.67	1.91×10^{-2}	1.73×10^{-2}
1.25	0.65	7.12	7.74	10.26	67.51	2.01×10^{-2}	1.71×10^{-2}
1.37	0.59	7.37	8.20	10.25	67.45	2.10×10^{-2}	1.70×10^{-2}
1.37	0.71	8.76	9.45	10.73	70.57	1.90×10^{-2}	1.64×10^{-2}

ations. In contrast, the spectroscopic factors change significantly in the ranges $0.140 \le Z_{1\,1/2} \le 0.301$ and $0.790 \le Z_{1\,3/2} \le 1.647$ under the same variations of the parameters r_0 and a. This indicates a strong dependence of the spectoroscopic factors on the single-particle ANCs. Similar trends, namely weak ANC dependence and strong sensitivity of the spectroscopic factor, are observed at other experimental angles. In summary, the results demonstrate the reliability of the ANC determination of ^{9}Be in the $p + {}^8$Li channel from the transfer reaction ^{9}Be$(d,{}^3$He$)^8$Li, while highlighting the

strong model dependence that affects the extraction of the spectroscopic factor. For all optical potential sets listed in Table 1, condition (5) for the R functions is fulfilled at angles within the main maximum of the angular distribution when the W–S potential parameters r_0 and a are varied across the above-mentioned ranges. The results indicate that the reaction under study is peripheral at the main maximum of the angular distribution. Therefore, the experimental DCS of the ^{9}Be$(d, {}^3$He$)^8$Li reaction can be employed to extract the values of ANCs in a manner that is independent of the geometric parameters (r_0 and a) of the W–S potential.

2.3. Determination of the ^9Be ANC in the $p + {}^8Li$ channel

By normalizing the calculated DCS within the framework of MDWBA to the experimental one at the forward angles $\theta < 20°$, we extracted the ANC values of ^{9}Be in the $p + {}^8$Li channel. For each of the five experimental points of angles θ, the ANCs values of ^{9}Be in the $p + {}^8$Li channel have been determined by using relation (6) and the central values of the $R_{1\,1/2}$ and $R_{1\,3/2}$ functions, corresponding to the standard values of the parameters r_0 and a of the adopted W–S potential ($r_0 = 1.25$ fm and $a = 0.65$ fm).

The calculated values of the squared ANCs of ^{9}Be in the $p + {}^8$Li channel are presented in Fig. 3 for each experimental data point in the main maximum of the angular distribution, considering all sets of OPs. Additionally, the weighted ANCs of ^{9}Be in the $p + {}^8$Li channel associated with each OP set are summarized in Table 3. The weighted mean uncertainty in the ANC values represents the root-mean-square error derived from the combined uncertainties of the calculated R functions, which depends on the free parameters

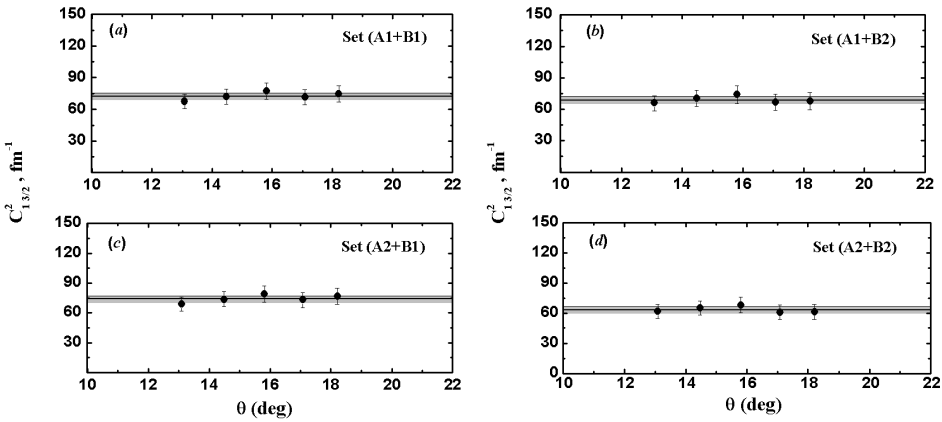


Fig. 3. The values of ANC $C^2_{1\,3/2}$ of ^{9}Be in the $p + {}^8$Li channel for each experimental point of angles in the main maximum of the angular distribution. The band widths are the corresponding weighted uncertainties.

$b_{1\,1/2}$ and $b_{1\,3/2}$, alongside the experimental DCS error and the uncertainty of the ANC for $d+p \to {}^3\mathrm{He}$. In addition to the weighted mean uncertainties given in Table 3, the systematic uncertainty associated with the choice of OPs was evaluated from the spread of the ANC values obtained using different OP sets. The corresponding OP uncertainties were found to be 0.66 fm^{-1} and 4.63 fm^{-1} for $C^2_{1\,1/2}$ and $C^2_{1\,3/2}$, respectively. Our recommended ANC values, obtained in the present work, are $10.54 \pm 0.36\,(\mathrm{wm}) \pm 0.66\,(\mathrm{OP})$ and $69.22 \pm 2.38\,(\mathrm{wm}) \pm 4.63\,(\mathrm{OP})$ for $j_{p^8\mathrm{Li}} = 1/2$ and $j_{p^8\mathrm{Li}} = 3/2$, respectively.

Table 3. The weighted values of ANCs of ${}^9\mathrm{Be}$ in the $p + {}^8\mathrm{Li}$ channel, obtained from the analysis of the peripheral reaction ${}^9\mathrm{Be}(d, {}^3\mathrm{He}){}^8\mathrm{Li}$ for different sets of OPs. The last row presents the weighted mean values obtained from all considered OPs. The uncertainties include both the weighted mean uncertainties (wm) and the systematic uncertainties associated with the choice of OPs. ANCs are given in fm^{-1}.

Sets	$C^2_{1\,1/2}$	$C^2_{1\,3/2}$
Set (A1+B1)	10.99 ± 0.76	72.30 ± 5.02
Set (A1+B2)	10.44 ± 0.75	68.71 ± 5.01
Set (A2+B1)	11.22 ± 0.78	73.81 ± 5.11
Set (A2+B2)	9.61 ± 0.70	63.25 ± 4.64
Weighted mean	$10.54 \pm 0.36\,(\mathrm{wm}) \pm 0.66\,(\mathrm{OP})$	$69.22 \pm 2.38\,(\mathrm{wm}) \pm 4.63\,(\mathrm{OP})$

To examine the dependence of the extracted ANCs on the adopted ratio $\lambda = C^2_{13/2}/C^2_{1\,1/2}$, a sensitivity test was performed by varying the ratio λ by $\pm 10\%$ around its adopted value $\lambda = 6.58$. It was found that $C^2_{1\,1/2}$ changes by about 8.4%, while $C^2_{1\,1/2}$ varies only by about 1.3%. At the same time, their sum changes by only 0.16%. This indicates that, although the ANCs $C^2_{1\,1/2}$ and $C^2_{1\,3/2}$ show some dependence on λ, the total ANC, constrained by the experimental data, is essentially insensitive to its variation.

The weighted ANC values listed in Table 3 for each set of optical potentials were employed to calculate the DCSs using Eq. (1). The calculated DCS results for the ${}^9\mathrm{Be}(d, {}^3\mathrm{He}){}^8\mathrm{Li}$ reaction, populating the ground state of the ${}^9\mathrm{Be}$ nucleus, are presented in Fig. 4 alongside the corresponding experimental data from Ref. [23]. The curves, shown in Fig. 4, represent calculations performed within the framework of MDWBA, utilizing the weighted ANC values from Table 3, across different OP sets listed in Table 1. As demonstrated in the figure, the calculated DCSs within the framework of MDWBA

exhibit good agreement with the experimental data, particularly within the main maximum region of the angular distribution. As discussed in Ref. [12], this angular region is the most reliable for extracting the ANC since the pole mechanism (see Fig. 1 of Ref. [12]) dominates. However, at larger angles ($\theta > 30°$), the calculated cross sections deviate significantly from the experimental values for all OP sets. Obviously, the main reason for this is that the contribution of other reaction mechanisms increase at backward angles.

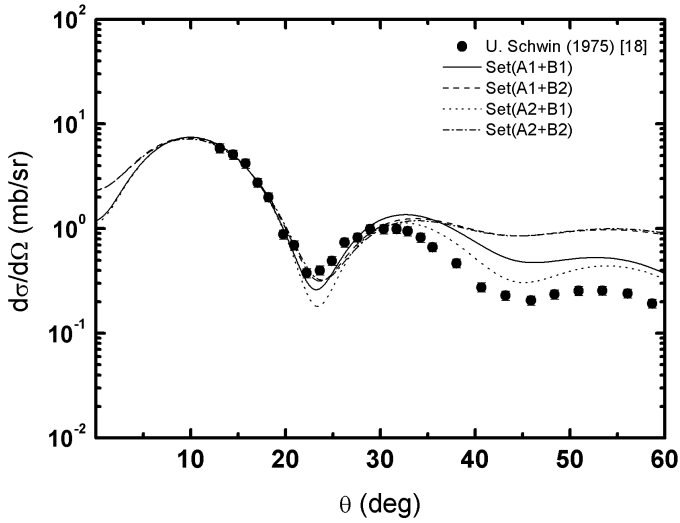


Fig. 4. The DCSs for the ^{9}Be$(d, {}^3$He$)^8$Li reaction at an energy of 52 MeV. The experimental points are taken from [23]. The solid, dashed, dotted, and dash-dotted lines are the calculated DCSs corresponding to set (A1+B1), set (A1+B2), set (A2+B1), and set (A2+B2), respectively.

3. Conclusion

The analysis of the differential cross section of the ^{9}Be$(d, {}^3$He$)^8$Li reaction measured at an incident energy of 52 MeV was performed within the framework of MDWBA. The peripheral character of the reaction was tested. The results of the calculations show that the considered reaction is peripheral at main maximum of the angular distribution of the reaction under consideration. It was established that the spectroscopic factor demonstrates a significant sensitivity to the parameters of the W–S potential. The squared ANC of ^{9}Be in the $p + {}^8$Li channel was found from the experimental data for the first time.

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