TRIAXIALITY IN ^{79}Y AND ^{79}Zr MIRROR NUCLEI: PRM APPROACH

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Triaxiality and the signature splitting in the mirror nuclei ^{79}Y and ^{79}Zr are studied within the particle-plus-rotor model theoretical framework. The Mirror Energy Differences for the rotational band structures calculated using the PRM with triaxial core parameters agree with those reported from the DFT–NCCI calculations.

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1. Introduction

The study of mirror nuclei, defined by the interchange of proton and neutron numbers while preserving the mass number, offers a unique framework for exploring the charge independence of nuclear forces/isospin symmetry and its breaking mechanisms in nuclear systems [1–3]. In an ideal scenario, in the absence of the Coulomb interaction, mirror partners are expected to exhibit identical nuclear structures. However, in realistic nuclei, small energy differences appear between mirror partners as excitation energy and angular momentum increase. These differences arise from Coulomb interaction and

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other symmetry-breaking effects, making them useful probes of nuclear models. From a theoretical perspective, the Mirror Energy Differences (MEDs) [4–6] are very demanding quantities, as they are highly sensitive to details of the changes in configurations as a function of increasing excitation energy and angular momentum.

The Warsaw group has developed a computational code [7–9] that combines the density-functional theory (DFT) with the no-core configuration-interaction (NCCI) approach. Using this framework, reasonable agreement was achieved for the ^{47}Ti / ^{47}Mn mirror pair [10] as well as for the heavy ^{79}Zr / ^{79}Y mirror pair [11]. However, the model fails to reproduce the MEDs in the ^{45}Sc / ^{45}Cr [10], possibly due to differences in nuclear shape parameters. In Ref. [12], triaxiality was incorporated into the calculations, and the results demonstrate that it has a significant impact on the theoretical MEDs.

In deformed nuclei, collective motion gives rise to rotational band structures that follow characteristic energy systematics. In odd-mass nuclei, the coupling of a valence nucleon to a triaxially deformed core introduces signature splitting, a phenomenon that manifests as alternating energy spacings between adjacent spin states. Although signature splitting has been widely studied, its dependence on triaxial deformation in mirror systems remains an area requiring further investigation. The collective behavior of the mirror pairs ^{47}V / ^{47}Cr and ^{49}Cr / ^{49}Mn was discussed within the particle-plus-rotor model (PRM) at low spins in Ref. [13].

Recently, Llewellyn *et al.* [11] studied the MEDs in the $5/2^+[422]$ bands of ^{79}Y and ^{79}Zr mirror nuclei (see Fig. 1) using the DFT-NCCI model [8]. In addition, the $5/2^+[422]$ bands of the neighboring nuclei ^{79,81}Y [14, 15] and ^{81}Zr [16] exhibit rigid-rotor-like behavior. The present study focuses on the theoretical investigation of collective bands in ^{79}Zr / ^{79}Y mirror nuclei using the PRM. Special emphasis is placed on understanding the role of triaxiality in generating signature splitting and its possible differences between mirror partners due to the Coulomb interaction.

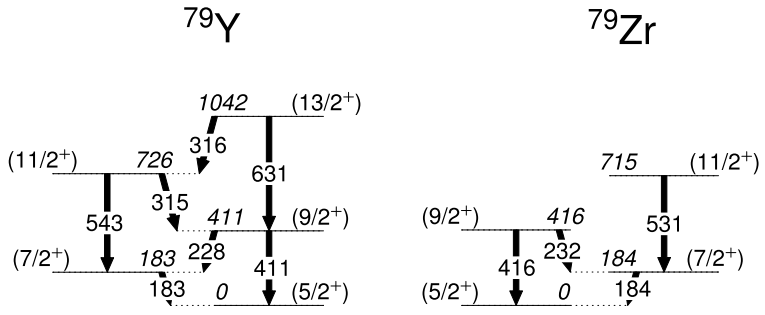


Fig. 1. The $5/2^+[422]$ bands of ^{79}Y from Ref. [11] and ^{79}Zr from Ref. [14].

2. PRM calculations

The PRM Hamiltonian [17–19] is written as

$$H = H_{\rm rot} + H_{\rm sp} + H_{\rm Coriolis}\,,$$

where $H_{\rm rot}$ represents the rotational energy of the core, $H_{\rm sp}$ is the single-particle Hamiltonian describing the motion of the valence nucleon in a deformed field, and $H_{\rm Coriolis}$ denotes the Coriolis coupling between the particle and the rotating core. The Coriolis coupling operator is formally identical for both protons and neutrons, but its effective contribution is different. This difference arises from Coulomb-induced changes in single-particle energies and wavefunctions. As a result, observable differences appear in energy spectra and signature splitting between mirror nuclei.

2.1. Signature splitting, triaxiality parameter, and energy levels

The Nilsson–Strutinsky cranked shell model and total Routhian surface (TRS) approaches [14, 20] predicted a large quadrupole deformation ($\varepsilon_2 \sim 0.38$) and significant triaxiality for the strongly coupled yrast band built on the $g_{9/2}$, $5/2^+[422]$ configuration in the $A \sim 80$ region.

In an axially symmetric nucleus with quadrupole deformation, high-K rotational sequences are expected to exhibit interleaved states with regular spacings and no signature splitting. Signature splitting is expected in nuclei with triaxial deformation. It can also occur as a consequence of mixing with $K = 1/2$ bands. In such cases, the magnitude of the splitting is expected to increase with increasing rotational frequency. The experimental signature splitting between the favored and unfavored members of the $5/2^+[422]$ bands of ^{79,81}Y and ^{79,81}Zr, shown in Fig. 2 (a) and (b), indicates the presence of triaxiality in these nuclei. Here, the signature splitting function $S(I)$ used in the figure is defined as

$$S(I) = \frac{E(I) - E(I-1)}{2I}\,.$$

To investigate the role of triaxiality, calculations for the $5/2^+[422]$ and $5/2^-[303]$ bands of ^{99,101,103}Nb were performed within the framework of the PRM in Ref. [21], using the computer codes GAMPN and ASYRMO [17, 22, 23]. In the present work, similar calculations are carried out for the $5/2^+[422]$ bands of ^{79}Y and ^{79}Zr. The deformation parameter ε_2 was kept at ~ 0.38 [14, 20], while the triaxiality parameter γ was varied in the $5^\circ \leq \gamma \leq 20^\circ$ range. In the PRM framework, the calculated results are insensitive to the sign of the triaxiality parameter γ. All higher-order deformations ε_4 and ε_6 were set to zero. The comparison between the calculated and experimental

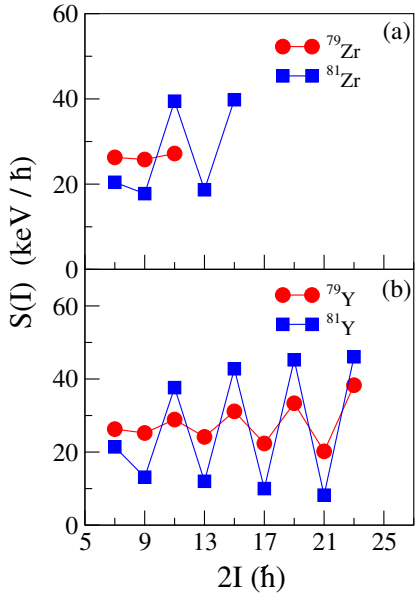


Fig. 2. Experimental signature splitting of the $5/2^+[422]$ bands in ^{79,81}Y (b) and ^{79,81}Zr (a). The lines are drawn as a visual guide for the eye.

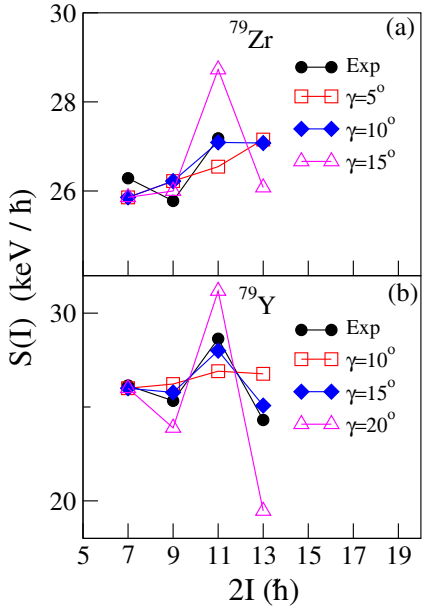


Fig. 3. A comparison of the experimental and calculated signature splitting for triaxiality parameters $\gamma = 5°$, $10°$, $15°$, and $20°$ in the low-spin states of the $5/2^+[422]$ bands in ^{79}Y (b) and ^{79}Zr (a). The lines are drawn as a visual guide for the eye.

signature splitting (see Fig. 3) shows that the theoretical results are in good agreement with the experimental data for ^{79}Y and ^{79}Zr at $\gamma = 15°$ and $\gamma = 10°$, respectively.

2.2. MEDs

The MEDs between ^{79}Y and ^{79}Zr are compared with the experimental data and the theoretical predictions obtained from the DFT-NCCI model reported in Ref. [11]. As shown in Fig. 4, good agreement is observed for the first two excited states, whereas a discrepancy of about 20 keV appears for the $11/2^+$ state.

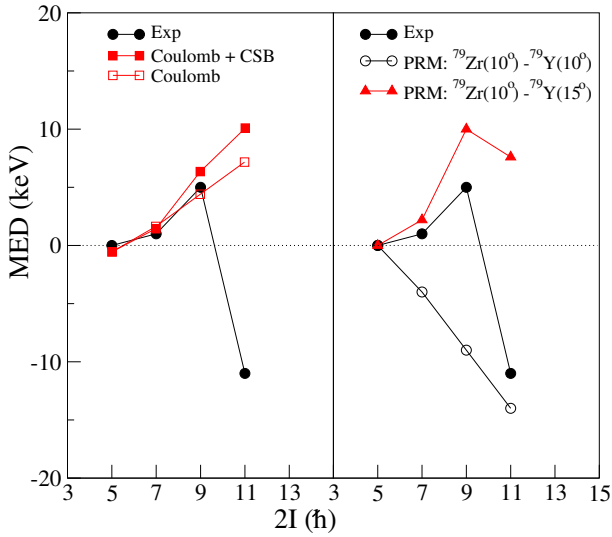


Fig. 4. Level energy differences as a function of spin $2I$ for the excited states of ^{79}Y and ^{79}Zr in Ref. [11]. The calculations based on the DFT+NCCI approach are shown for the Coulomb interaction only, for the Coulomb interaction including charge-symmetry-breaking (CSB) terms from Ref. [11], and for the PRM. The lines are drawn as a visual guide for the eye.

The PRM calculations for mirror nuclei must be structurally identical when performed using the same deformation and core parameters. However, differences arise due to Coulomb-induced modifications of proton single-particle energies, leading to small but observable deviations in energy spectra and signature splitting between mirror partners. The MEDs for both nuclei, calculated at $\gamma = 10°$ for ^{79}Zr; $\gamma = 10°$, $15°$ for ^{79}Y, taking into account signature splitting, are presented in Fig. 4. The MEDs from PRM at $\gamma = 10°$ for ^{79}Zr; $\gamma = 15°$ for ^{79}Y agree well with the DFT and NCCI results reported in Ref. [11].

3. Conclusion

In summary, triaxiality and signature splitting in the mirror nuclei ^{79}Y and ^{79}Zr are investigated within the PRM framework. The PRM provides a simple description of the coupling between a single particle and a triaxial core. The MEDs for the rotational band structures are calculated using the PRM with triaxial core parameters ($\gamma = 10^\circ$ for ^{79}Zr; $\gamma = 15^\circ$ for ^{79}Y). The results show good agreement with those obtained from the DFT–NCCI calculations.

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