UNIVERSAL MASS EQUATION FOR EQUAL-QUANTUM EXCITED STATES

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This article proposes a universal mass equation (UME) for the baryon and meson equal-quantum excited states sets that have three or more known states in the set and have no more than one missing state. The conjecture is made that accurately measured masses using Breit–Wigner PDG2024 data at fixed J^P for all equal-quantum baryon excited-states sets (including LHCb exotic $P^+_{c\bar{c}s}$) and at fixed J^{PC} for all equal-quantum meson excited-states sets (including $s\bar{s}$, $s\bar{c}$, $c\bar{c}$, $c\bar{b}$, and $b\bar{b}$) are related by the logarithm function used here; at least for the mass range of currently known excited states. Our study examines the relationship between the ground-state masses and the logarithmic slopes, and finds an approximately inverse relationship. We discuss the measurability of overlapping states. We make an estimate of fundamental properties of the QCD potential.

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Dedication

This paper is dedicated to the memory of our excellent colleague and friend Richard Allen Arndt.

1. Introduction

This research started as a study of the excited states of the proton/neutron. The interesting result of that study led to studies of other equal-quantum combinations of particle-physics resonances/excited states.

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Data for baryons at fixed J^P and mesons at fixed J^{PC} excited states are reported in the Particle Data Listings from the Particle Data Group (PDG) [1]. Resonance masses and widths, with their uncertainties, are reported as Breit–Wigner (BW) values.

A universal mass equation (UME) is presented for equal-quantum excited-states sets. Our Part I [2] (Part II [3]) presented results with three or more (two - "duo" set) BW PDG2024 states.

2. Universal mass equation

In plotting the mass data for the $N1/2^+$ data set, we found an excellent fit using a χ^2 minimization procedure for the logarithm function,

$$M_n = \alpha \ \ln(n) + \beta \,, \tag{1}$$

and used the fit to predict the masses of one missing excited state and four higher excited states of the proton/neutron [2]. Here, n is the radial excitation level and α (logarithmic slope) and β are free parameters. (The parameter $\beta \approx M_1$, the ground-state mass, since $\ln(1) = 0$, especially if the lowest mass is the mass measured most accurately of the set, as the neutron is in the data set $N1/2^+$.)

The success of fitting the $N1/2^+$ data set to the UME emboldened us to attempt similar fits to many other equal-quantum excited states sets that have three or more members, with similar successes. An excited state is a quantum state of a set that has a larger mass than the ground state (M_1).

Our results in Part I [2] consider a set with at least three known states, making the UME over-determined. In Part II [3], we extended our work to cover "duo" sets, sets with only two known states, with masses M_1 and M_2, which have reasonably low measurement uncertainties. For duo sets, the parameter $\beta_1 = M_1$. This work was introduced in a pair of papers [2, 3] analyzing the latest BW resonance parameters from PDG2024 [1].

3. UME samples for baryons

Samples for the baryon case are given in Figs. 1–3. See Part I [2] and Part II [3] for more examples.

3.1. Systematics of baryon excited-states sets

A plot of fit parameter α *versus* M_1 for all baryon sets in Part I [2] and Part II [3] roughly fits a power equation (Fig. 4). The fit is not accurate enough to calculate α for a single ground state. However, it can be used to indicate missing states when the calculated α for a set is far from the power curve; this was used three times in Part II for duo sets.

This illustrates an approximate inverse relationship of the fit parameter α with the ground-state mass M_1.

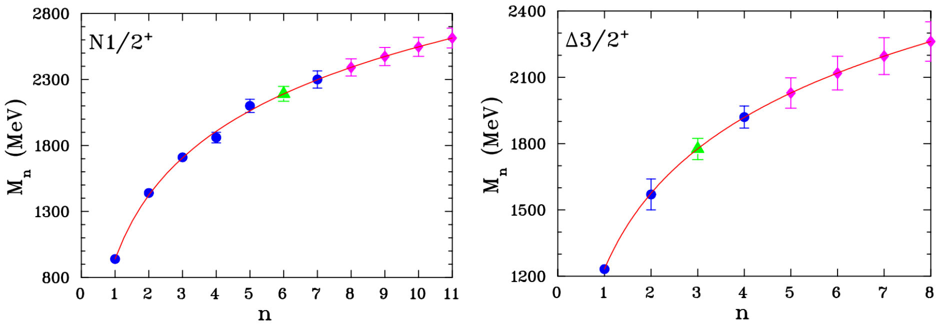

Fig. 1. $N1/2^+$ set (left) and $\Delta3/2^+$ set (right). PDG2024 data (blue circles) [1]. A green triangle is the calculated mass of a missing state. Predicted states are magenta diamonds. The solid red curve presents the best-fit. Part I [2].

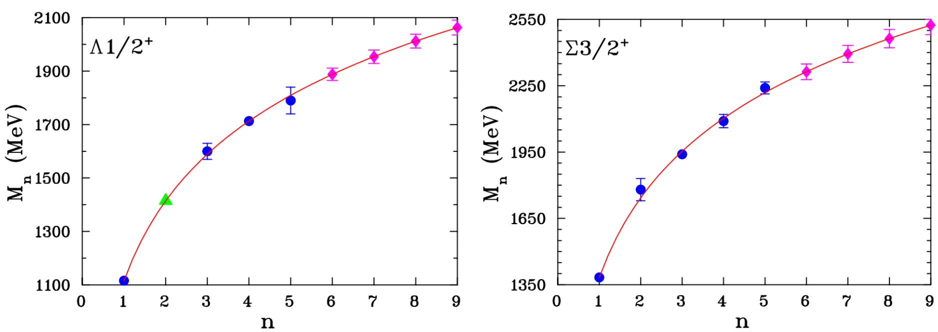

Fig. 2. $\Lambda1/2^+$ set (left) and $\Sigma3/2^+$ set (right). The notation is the same as in Fig. 1. Part I [2].

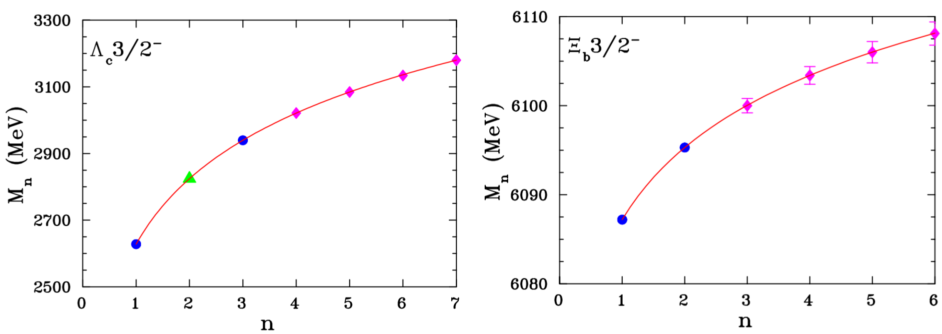

Fig. 3. $\Lambda_c3/2^-$ duo set (left) and $\Xi_b3/2^-$ duo set (right). The notation is the same as in Fig. 1. Part II [3].

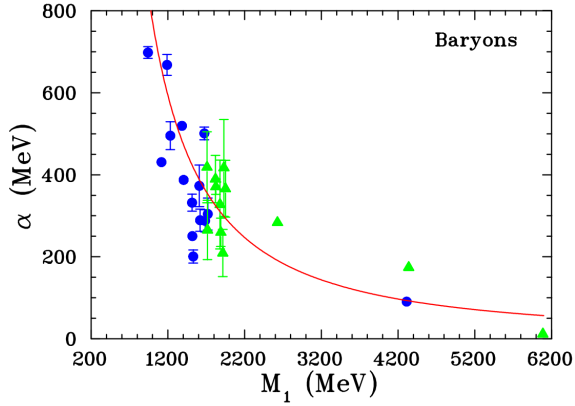

Fig. 4. Baryon α *versus* M_1 power-equation fit [$\alpha = (1.688 \times 10^7) M_1^{-1.446}$ MeV]. Blue filled circles are from Part I baryon sets [2], green filled triangles are from Part II baryon sets [3], and the red curve is Parts I and II fit to the power equation.

4. UME samples for mesons

Samples for the Meson case are given in Figs. 5–8. See Part I [2] and Part II [3] for more examples.

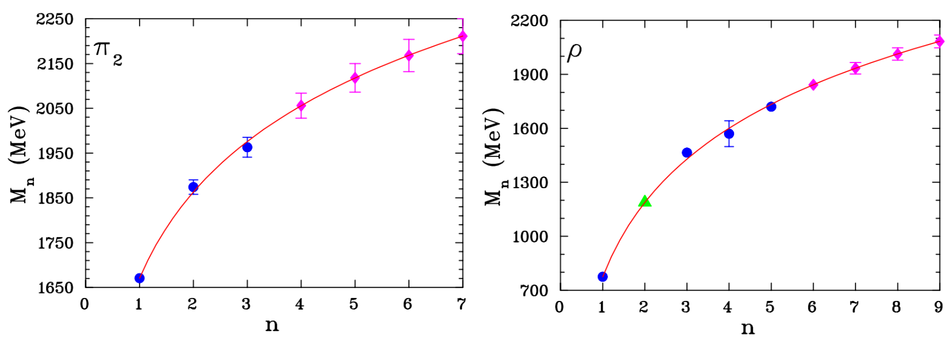

Fig. 5. π_2 set (left) and ρ set (right). The notation is the same as in Fig. 1. Part I [2].

4.1. Systematics of meson excited-states sets

The linear trend of $c\bar{c}$ and $b\bar{b}$ sets' α *versus* mass is shown in Fig. 9. Due to its strong linearity, the two lines can be used to accurately estimate the parameter α for $c\bar{c}$ and $b\bar{b}$ sets for which only the ground-state mass is known [1], to be used to calculate the higher masses of the set. This illustrates an inverse relationship of the fit parameter α with the ground-state mass M_1.

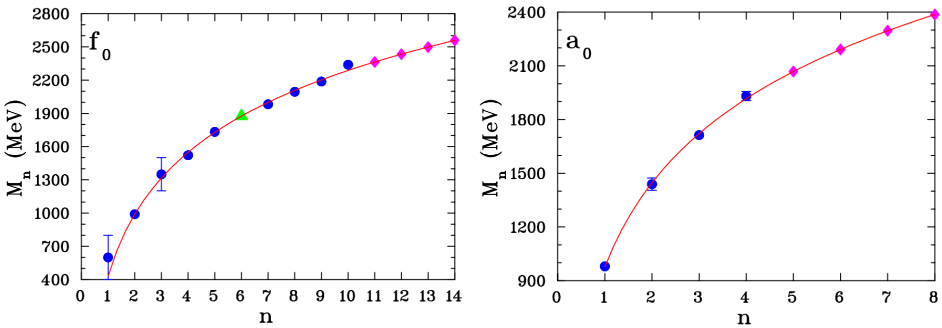

Fig. 6. f_0 set (left) and a_0 set (right). The notation is the same as in Fig. 1. Part I [2].

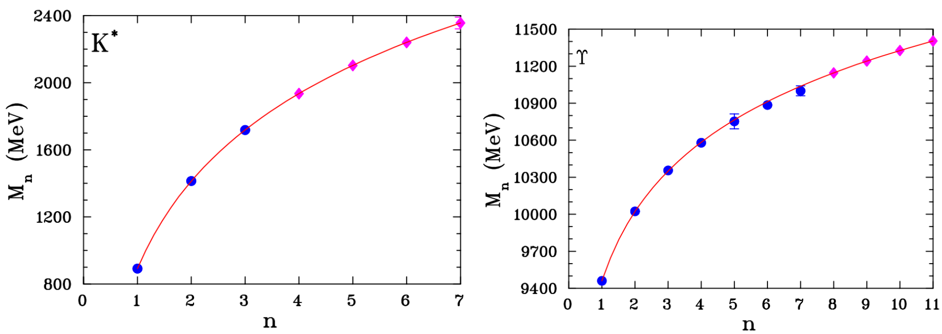

Fig. 7. K^* set (left) and Υ set (right). The notation is the same as in Fig. 1. Part I [2].

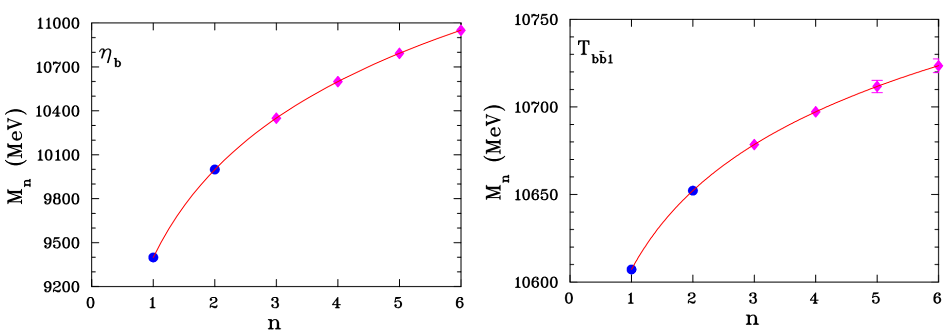

Fig. 8. η_b duo set (left) and $T_{b\bar{b}1}$ duo set (right). The notation is the same as in Fig. 1. Part I [2].

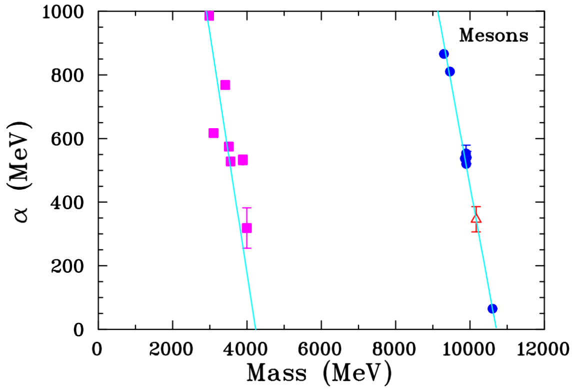

Fig. 9. $c\bar{c}$: $\alpha = -(0.7539\pm0.0045)M_1+(3190.7\pm14.1)$ MeV (magenta filled squares). $b\bar{b}$: $\alpha = -(0.6359\pm0.0028)M_1+(6809.3\pm27.4)$ MeV (blue filled circles). Line fits for $q\bar{q}$ and $s\bar{s}$ are not good. The red open triangle shows Υ_2 (Fig. 10). Part II [3].

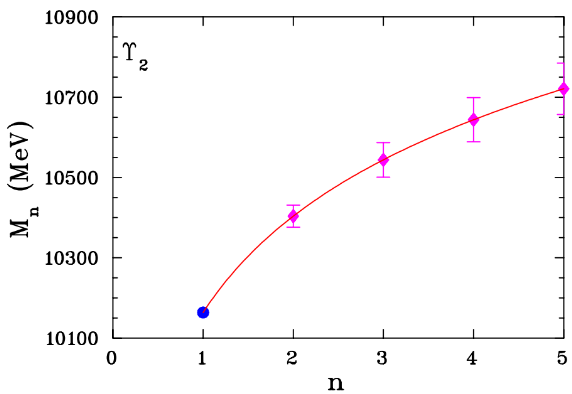

Fig. 10. Υ_2, a $b\bar{b}$ meson, has only one excited state known ($M_1 = 10163.7$ MeV) [1]. The $b\bar{b}$-line fit (Fig. 9) can be used to calculate its $\alpha = 346.2\pm39.5$ MeV. Part II [3].

5. Measurability of predicted excited states

Using the accurate form of the BW resonance equation [4]

$$P(s,M,\Gamma) = \frac{\Gamma}{2\pi}\frac{4M^2+\Gamma^2}{(s^2-M^2+\Gamma^2/4)^2+M^2\Gamma^2}\,, \tag{2}$$

with M and Γ values from [1], one can calculate the BW curves for adjacent predicted excited states to determine their overlap as an indication of their measurability. It illustrates that the "bump hunting" technology is problematic (Fig. 11 (top)) to detect predicted states due to overlapping resonances.

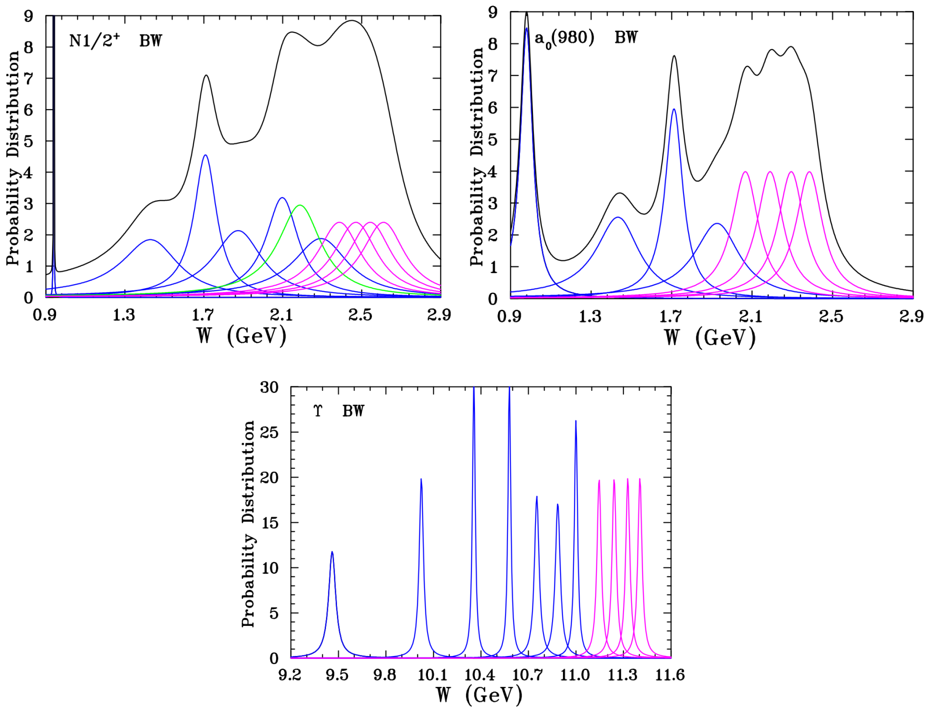

Fig. 11. The curves are for the known (blue), missed (green), and predicted (magenta) states. The unknown widths of the predicted states are assumed to have the average width of the preceding known states. $N1/2^+$ baryon BW curves (top left) — possibly not measurable. a_0 meson BW curves (top right) — possibly measurable. Υ meson BW curves (bottom) — measurable. The notation is the same as in Fig. 1. Part II [3].

6. Potential-energy curve

The de Broglie wavelength for a mass reads

$$\lambda_n = \frac{hc}{M_n} = 2\pi \frac{\hbar c}{M_n} = 2\pi \frac{197.327 \text{ MeV}}{M_n} \text{ fm}\,. \tag{3}$$

Assuming the wavefunction radius takes the form r_n of the excited state n

$$r_n = n\frac{\lambda_n}{4} = \frac{\pi}{2}\frac{\hbar c}{M_n}\,, \tag{4}$$

one can plot M_n *versus* r_n, the potential-energy curve $V(r_n)$. Surprisingly, the data can be fit using the Cornell potential *versus* radius [5]

$$V(r_n) = -\frac{4}{3}A/r_n + Br_n + C\,, \tag{5}$$

which is shown in Fig. 12 for the $N1/2^+$ baryon set and the a_0 meson set. Here, r_n is the effective radius of the $n^{\rm th}$ resonance state, A is the QCD running coupling, B is the QCD string tension, which controls intercepts and slopes of linear Regge trajectories, and C is a constant. The first term of Eq. (5) represents a short distance, $r < 0.1$ fm, and the second term corresponds to a long distance and provides quark confinement.

The fit of the Cornell potential to our rough potential-energy calculation is an interesting result; apparently due to the fact that hadrons are constructed of quarks.

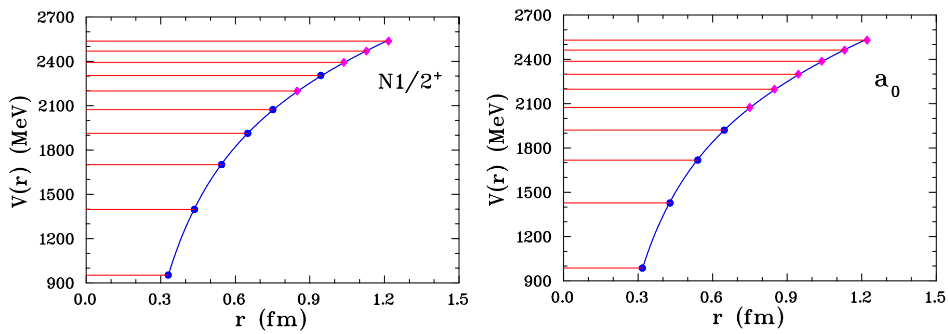

Fig. 12. $N1/2^+$ baryon set (left) and a_0 meson set (right). The blue curves are the Cornell potential-energy function (Eq. (5)) fits to the excited-states set's potential energy. Part I [2].

7. Conclusion

We have identified a universal mass equation (UME) that fits the masses of known equal-quantum baryon and meson excited-states sets and used it to predict missing and higher-mass states. We discuss which of the predicted states are possibly measurable. We roughly define the wavefunction size and plot the excited states of a set *versus* the radius of the wavefunction to get an estimate of a potential energy function and achieve a good fit of it to the Cornell potential.

Our UME should be of interest to experimentalists who are planning future accelerator experiments and to nuclear/particle theorists.

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